Irreversibility and Lagrangian power statistics in Navier-Stokes equations and in reversible Shell Models

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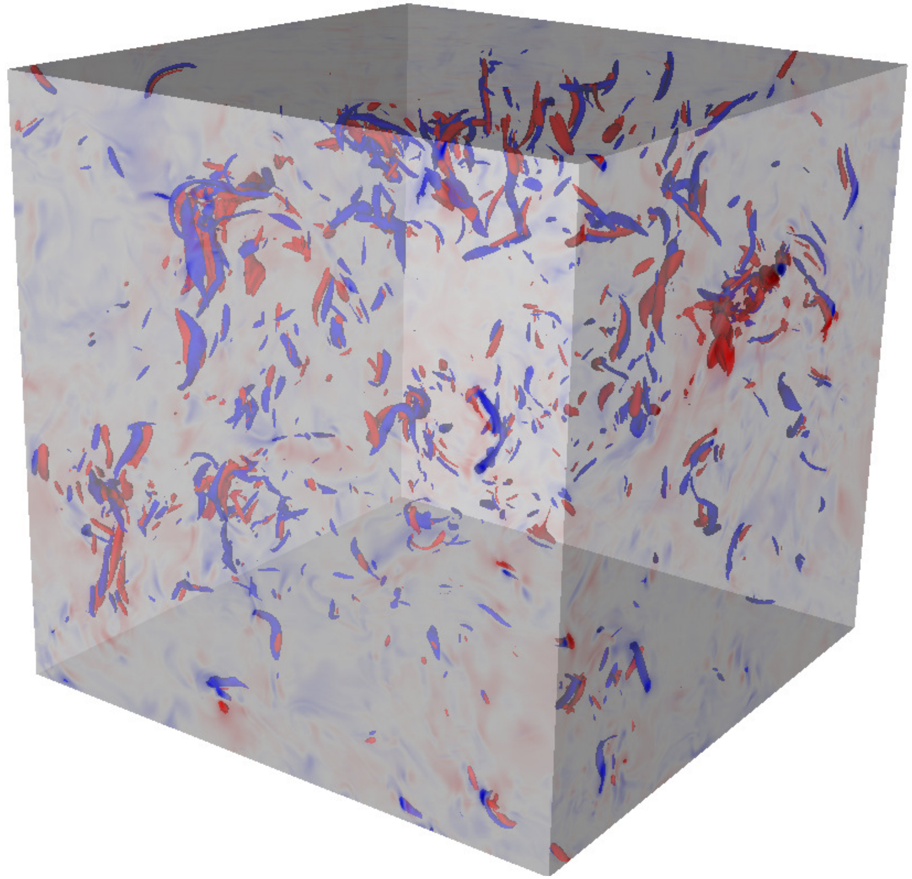

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General issue: turbulence & irreversibility

"*An inviscid-equation symmetry — time reversal invariance — remains broken even as the symmetry-breaking viscosity becomes vanishing small.*
A trained eye viewing a movie of steady turbulence run backwards can tell that something is indeed wrong! " G. Falkovich & K.R. Sreenivasan Phys. Today 2006

Euler

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p$$

Equilibrium physics:
time reversibility t→-t u→-u

Navier-Stokes

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \Delta \boldsymbol{u} + \boldsymbol{F}$$

Non-equilibrium and time-reversibility breaking also in the limit $\nu \to 0$

irreversibility in the Eulerian Frame : asymmetry of two-point statistics

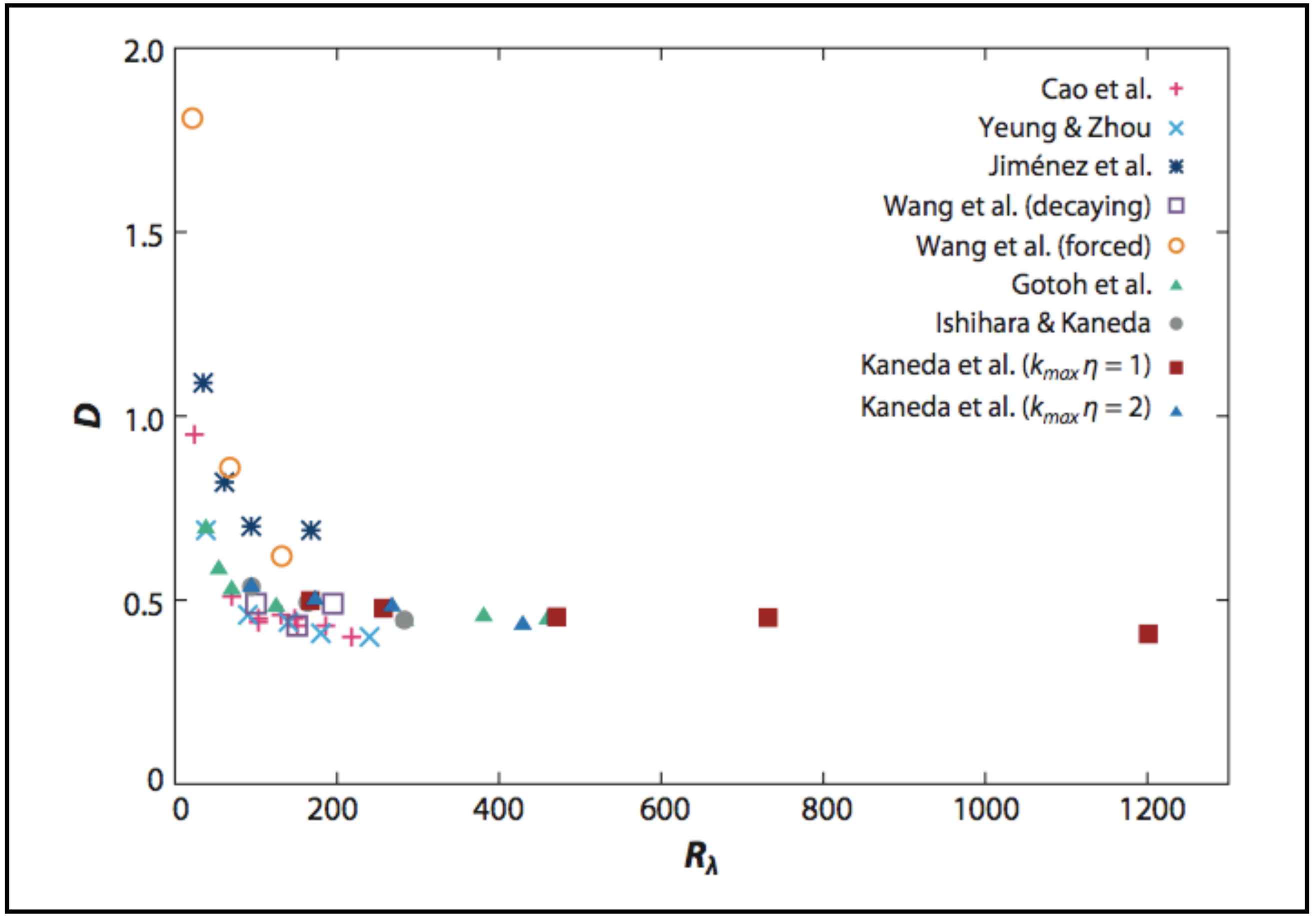
"4/5 law" (K41)

$$S_3(r) = \langle (\delta_{\|} u(r))^3 \rangle = -\frac{4}{5}\epsilon r + 6\nu \partial_r S_2(r) + \ldots \neq 0 \quad \nu \to 0$$

irreversibility in the Lagrangian Frame :
asymmetry of backward/forward two-particles separations

DISSIPATIVE ANOMALY

$$\epsilon = \nu \left\langle (\partial_x v)^2 \right\rangle$$



Can we understand the time arrow looking at turbulence following single fluid elements or at single point observables?

Flight–crash events in turbulence

Haitao Xu[a,b], Alain Pumir[a,b,c], Gregory Falkovich[a,d,e], Eberhard Bodenschatz[a,b,f,g,1], Michael Shats[h], Hua Xia[h], Nicolas Francois[h], and Guido Boffetta[a,i]

PNAS 2014

See also same authors in different order: PRX 2014

Lagrangian velocity

$$\dot{\mathbf{x}} = \mathbf{v}(t) = \boldsymbol{u}(\mathbf{x}(t), t)$$

Time increments of Lagrangian kinetic energy are negatively skewed

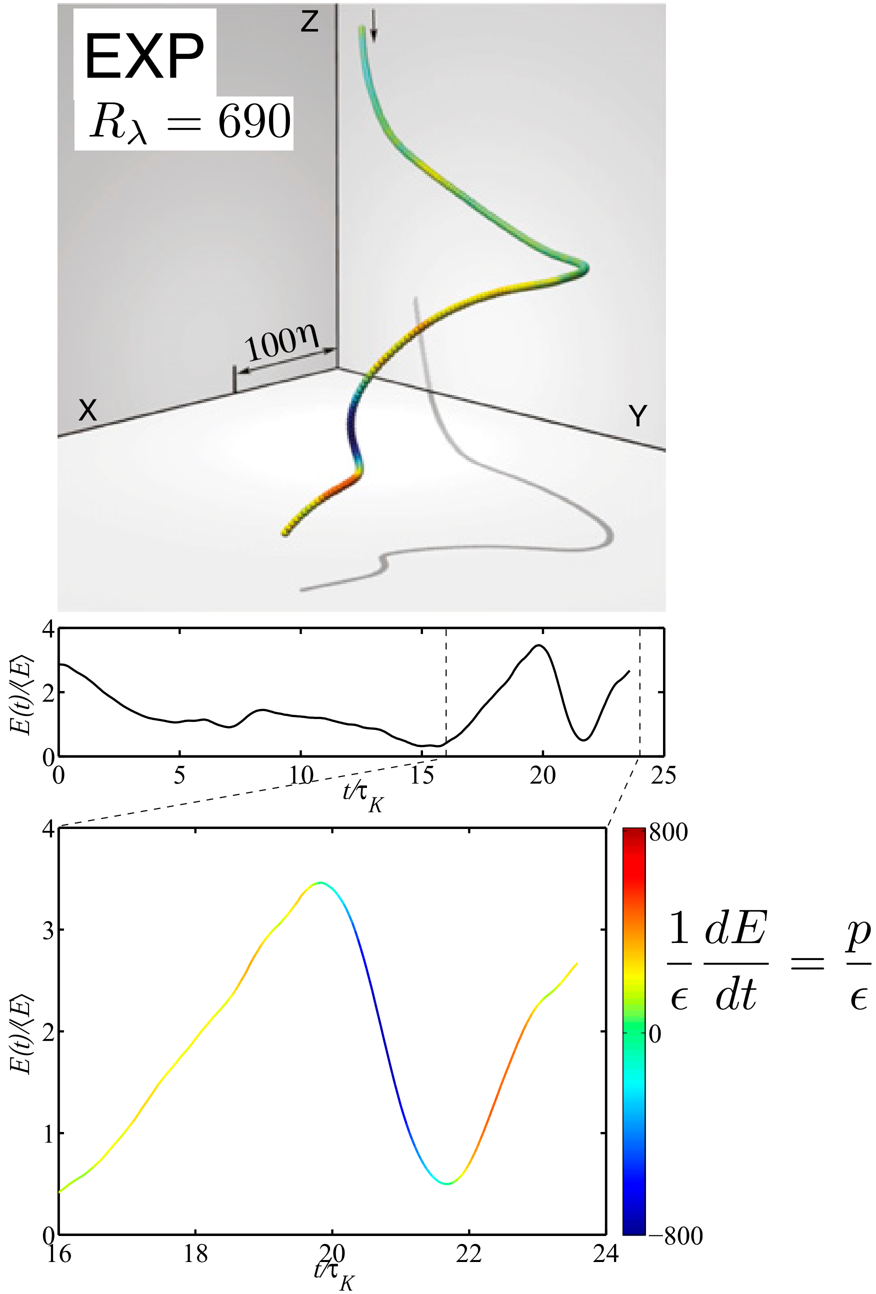
$$E(t) = \frac{1}{2}|\mathbf{v}^2(t)|$$

$$\delta_\tau E = E(t+\tau) - E(t)$$

Tail of pdf $\delta_\tau E$ dominated by events in which energy grows more slowly than it decreases

breaking of detailed-balance

$$P(E \to E + \Delta E) \neq P(E + \Delta E \to E)$$



Lagrangian Power Statistics

Skewness of $\delta_\tau E$ implies skewness of Lagrangian power (single point observable)

$$\frac{dE}{dt} = p = \boldsymbol{v} \cdot \boldsymbol{a} = \boldsymbol{u}(\mathbf{x}(t), t) \cdot (-\boldsymbol{\nabla} P + \nu \Delta \boldsymbol{u} + \boldsymbol{F})$$

$$\langle p \rangle = 0$$ ← by stationarity

$$\langle p^2 \rangle \sim \epsilon^2 Re_\lambda^{4/3}$$

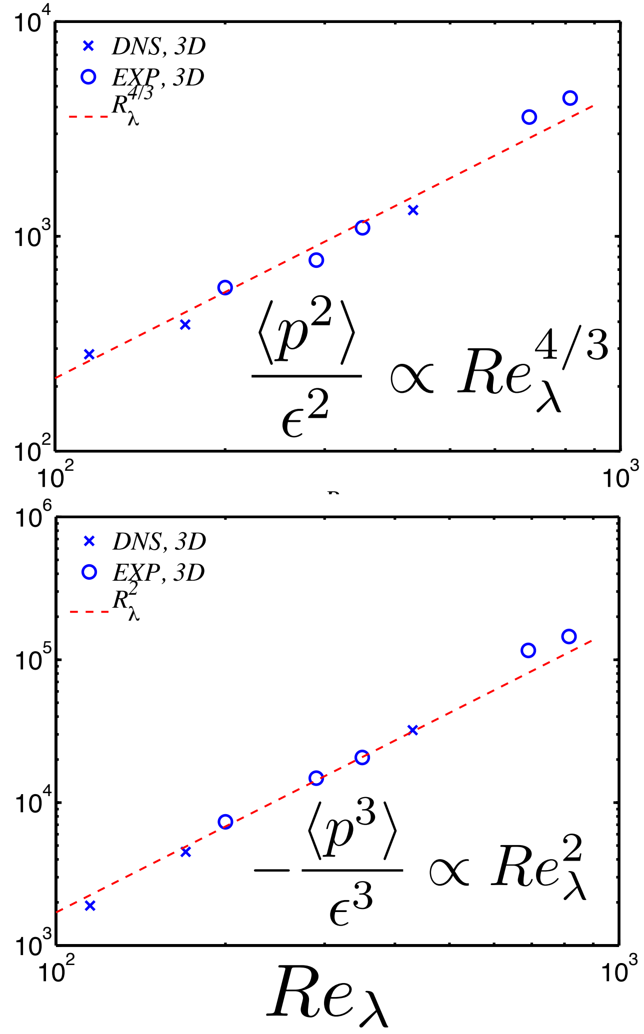
$$\langle p^3 \rangle \sim -\epsilon^3 Re_\lambda^2$$

$$S = \frac{\langle p^3 \rangle}{\langle p^2 \rangle^{3/2}} = const < 0$$

and slow particles in 2D. In 3D, however, it on average slows down slow particles and accelerate fast ones: $\langle -\mathbf{u} \cdot \nabla P | \mathbf{u}^2 \rangle$ is positive and grows with the energy even faster than $\mathbf{u}^2$ for $\mathbf{u}^2 \gtrsim 2\langle \mathbf{u}^2 \rangle$. Our observation of accelerating fast particles, which may suggest a runaway mechanism of the kinetic energy of particles in high Reynolds number flows, points to the importance of pressure forces in understanding fundamental properties of the Navier-Stokes equations in 3D [19–21]. Thus, our results concerning the redistribution of energy between fluid particles, implied by Eq. (4), may shed new light on the very different nature of the dynamics of turbulent flows in 2D and 3D.

A. Pumir et al PRX 2014

H. Xu et al PNAS 2014



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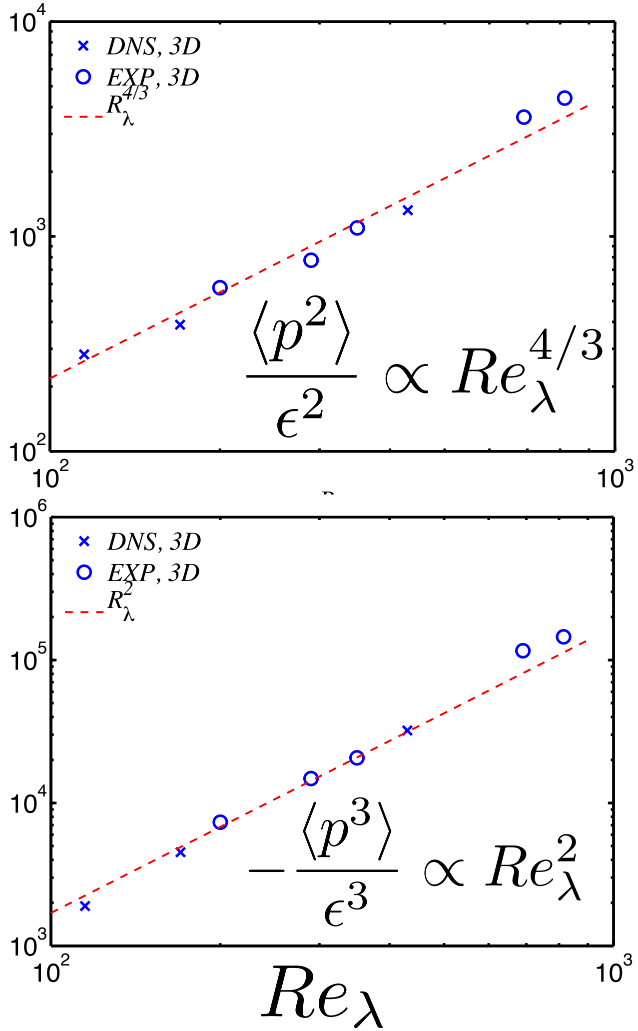


Scaling is anomalous

$$a \approx \frac{u_\eta}{\tau_\eta} \approx \frac{U}{T} Re_\lambda^{1/2} \qquad v \approx U \qquad \epsilon = \frac{U^2}{T}$$

K41 ➡ $p = \mathbf{v}\cdot\mathbf{a} \approx \epsilon Re_\lambda^{1/2}$

How to rationalise the observed scaling behaviour?
Is there a link with Eulerian intermittency?

Multifractal prediction for Lagrangian power

Bridging Lagrangian and Eulerian frames

(M. Borgas 1993, G. Boffetta et al 2002, L. Chevillard et al 2003)

$$\delta v(\tau) \underset{r}{\sim} \delta u(r)$$
$$\tau \sim \frac{r}{\delta u(r)}$$

$$\delta u(r) \sim r^h \longrightarrow \tau \sim \frac{L^h r^{1-h}}{u_L} \sim T\left(\frac{r}{L}\right)^{1-h}$$

$$\eta \delta u(\eta) \sim \nu \qquad a \sim \frac{\delta v(\tau_\eta)}{\tau_\eta} \longrightarrow a \sim \nu^{\frac{2h-1}{1+h}} u_L^{\frac{3}{1+h}} L^{\frac{-3h}{1+h}}$$

(M. Borgas 1993, LB et al 2004)

$$\langle p^q \rangle \sim \langle (a u_L)^q \rangle = \int du_L P(u_L) \int dh P_h(\tau_\eta)(a u_L)^q$$

$$\nu = U L Re_\lambda^2 \qquad P_h(\tau) = \left(\frac{\tau}{T}\right)^{\frac{3-D(h)}{1-h}}$$

$$\langle p^q \rangle \sim \epsilon^q Re_\lambda^{\alpha(q)} \qquad \alpha(q) = \sup_h \left\{ 2 \frac{(1-2h)q - 3 + D(h)}{1+h} \right\}$$

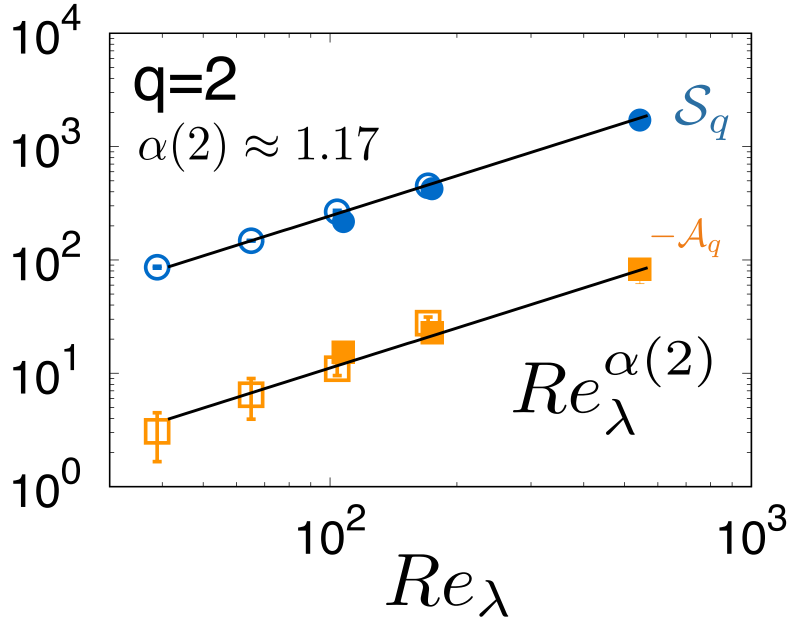
same D(h) as that used for Eulerian statistics

Results from DNS

In order to probe the symmetric and asymmetric components of power statistics we studied

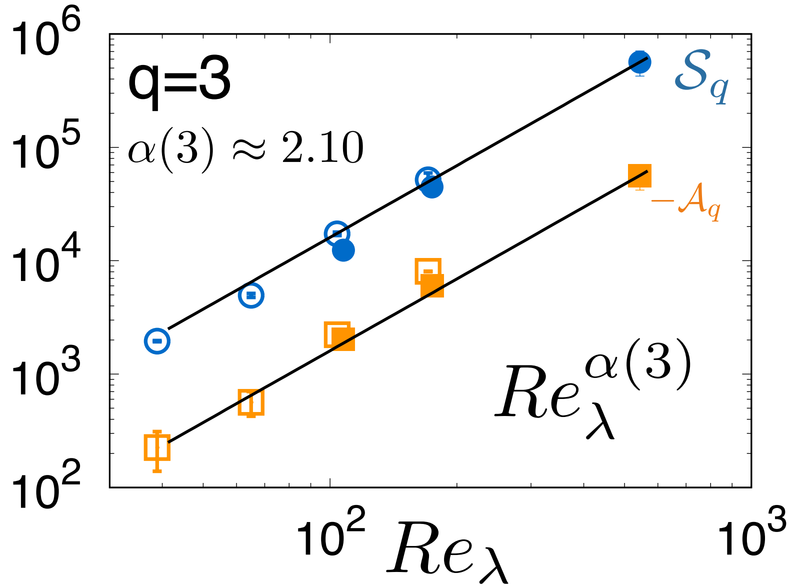
$$\mathcal{S}_q = \frac{\langle |p|^q \rangle}{\epsilon^q}$$

$$\mathcal{A}_q = \frac{\langle |p|^{q-1} p \rangle}{\epsilon^q} \begin{cases} > 0 & q < 1 \\ = 0 & q = 1 \\ < 0 & q > 1 \end{cases}$$



DNS: Pseudo spectral, HIT

Set	N	Re_λ	info
DNS1	2048	544	Gaussian, time-correlated forcing
DNS1	512	176	(B. Sawford, Phys. Fluids A 1991)
DNS1	256	115	AB II order
DNS2	1024	171	Constant input forcing
DNS2	512	104	RK II order
DNS2	256	65	
DNS2	128	38.9	



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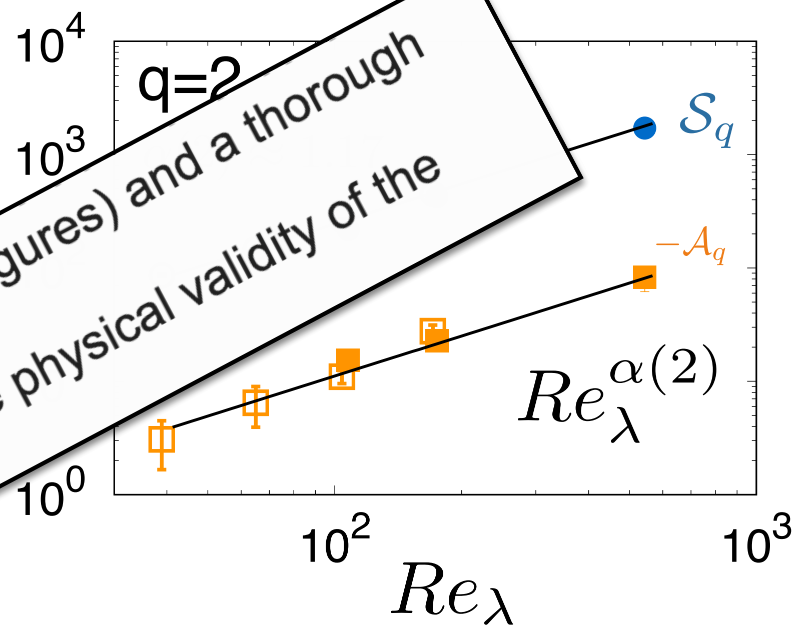
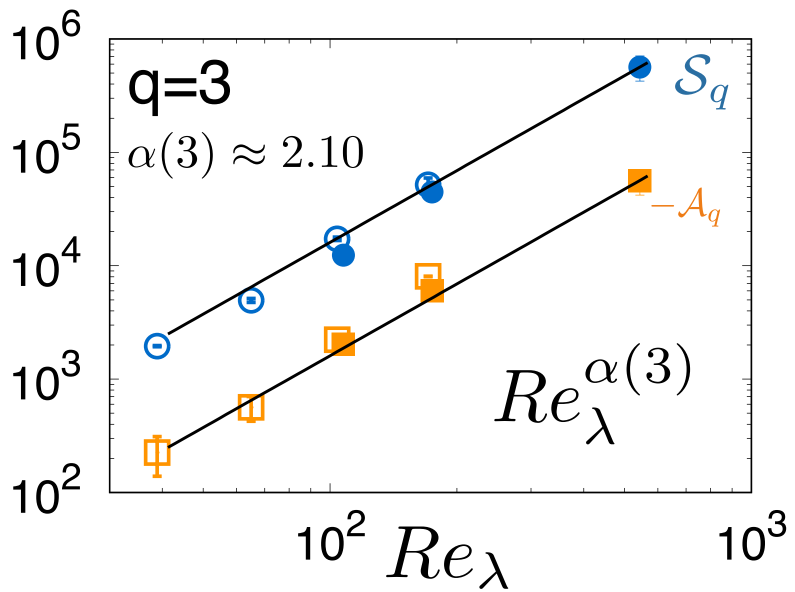
$$A_q = \frac{\langle |p|^{q-1} p \rangle}{\epsilon^q} \; [illegible]$$





DNS: Pseud[illegible]

Set	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible], time-correlated forcing ([illegible] Sawford, Phys. Fluids A 1991) AB II order
DN[illegible]	[illegible]024	171	Constant input forcing
DNS2	512	104	RK II order
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agreement. Error bars (which are lacking in all relevant figures) and a thorough discussion of error measures would only be a first and unavoidable step towards discussing the physical validity of the MF approach to Lagrangian power statistics.

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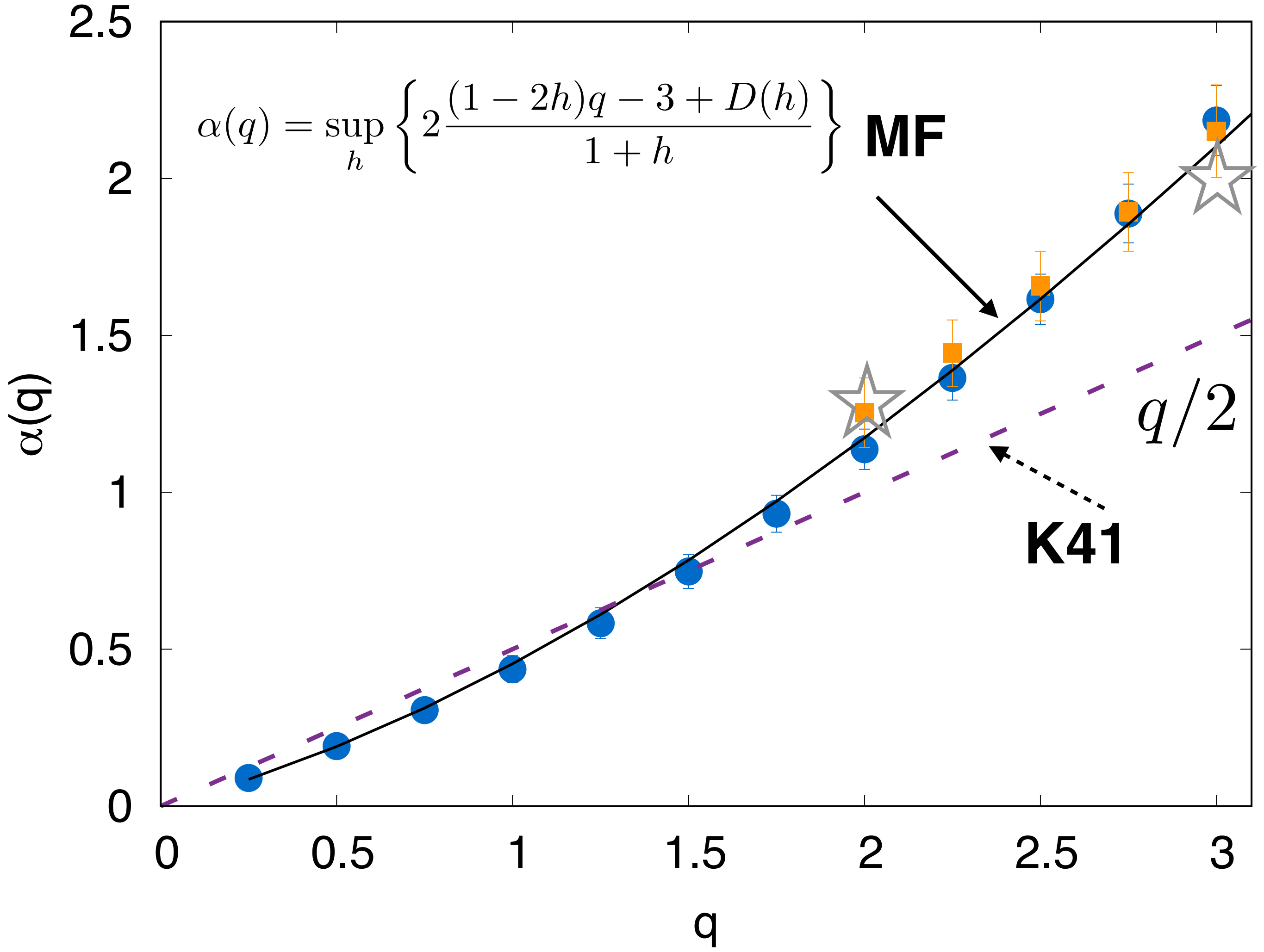


$$\mathcal{A}_q = \frac{\langle |p|^{q-1} p \rangle}{\epsilon^q}$$

$$\mathcal{S}_q = \frac{\langle |p|^q \rangle}{\epsilon^q}$$

☆ =Xu et al PNAS

$$S = \frac{\langle p^3 \rangle}{\langle p^2 \rangle^{3/2}}$$

$$S = \frac{\langle p^3 \rangle}{\langle |p|^3 \rangle}$$

Turbulence in the shell model

NS in Fourier space: $\partial_t \boldsymbol{u}(\boldsymbol{k},t) = -i\boldsymbol{k}\Pi(\boldsymbol{k}) \sum_{\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q}=0} \boldsymbol{u}(\boldsymbol{p},t)\boldsymbol{u}(\boldsymbol{q},t) - \nu k^2 \boldsymbol{u}(\boldsymbol{k},t) + \boldsymbol{F}(\boldsymbol{k},t)$

Shell model $\dot{u}_n = ik_n \left(A\lambda u_{n+2}u^*_{n+1} + Bu_{n+1}u^*_{n-1} + C\lambda^{-1}u_{n-1}u_{n-2} \right) - \nu k_n^2 u_n + f_n$

(V. L'vov et al PRE 1998)

Basic ingredients

Physical invariants: A,B,C chosen to preserve Energy &"Helicity" triad by triad

$k_n=k_0 \lambda^n$ logarithmically spaced shells (typically $\lambda=2$) allowing to reach very high Re

1 representative (complex) velocity per shell $u(k_n)=u_n$

Simplifying assumption locality: (u_{n-1},u_n,u_{n+1})

It displays anomalous scaling quantitatively similar to NS-turbulence!!

$r \to k_n^{-1} \qquad \delta_{\|}u(r) \to u_n$

$$S_q(r) = \langle (\delta_{\|}u(r))^q \rangle \sim r^{\zeta_q} \longrightarrow S_q(k_n) = \langle |u_n|^q \rangle \sim k_n^{-\zeta_q}$$

Lagrangian properties & shell model

In the shell model there is no notion of space, no direct way to introduce a Lagrangian frame
But, shell models are intrinsically “Lagrangian”: no sweeping from the large scales

Lagrangian velocity as superposition of fluctuations at all scales

$$v(t) = \sum_n \mathcal{R}\{u_n\}$$

G. Boffetta, F. De Lillo & S. Musacchio PRE 2002

$$E(t) = \frac{1}{2} v^2(t)$$

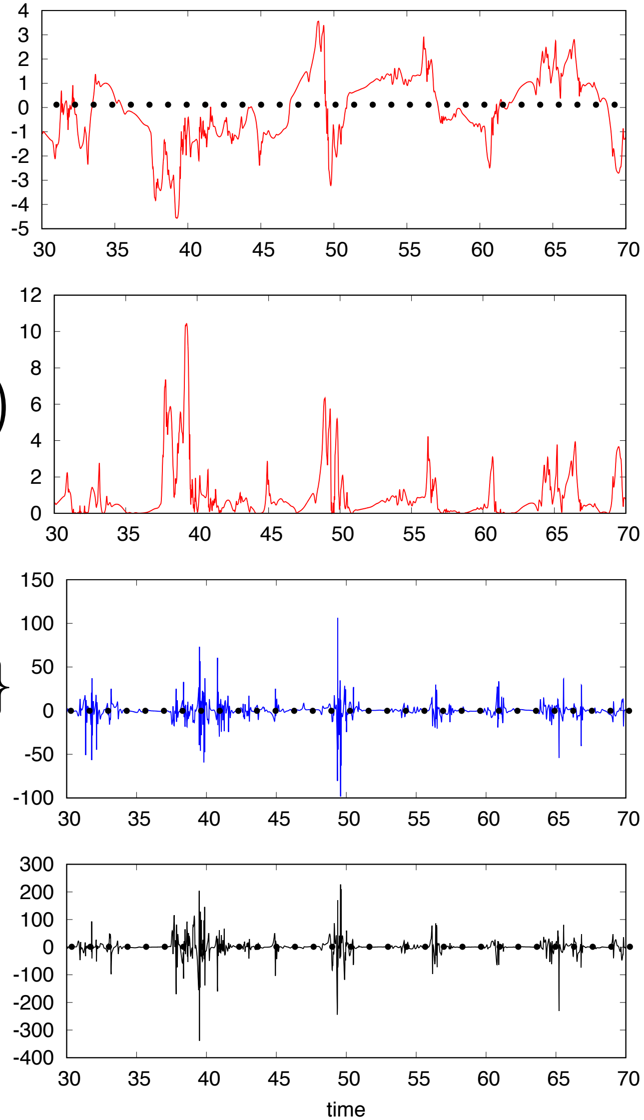
Used to study Lagrangian SF

$$\langle (\delta v(\tau))^q \rangle$$

$$a(t) = \sum_n \mathcal{R}\{\dot{u}_n\}$$

see also
N. Mordant et al PRL 2001
L. Chevillard et al PRL 2003
L.B. et al 2008
A. Arneodo et al PRL 2008

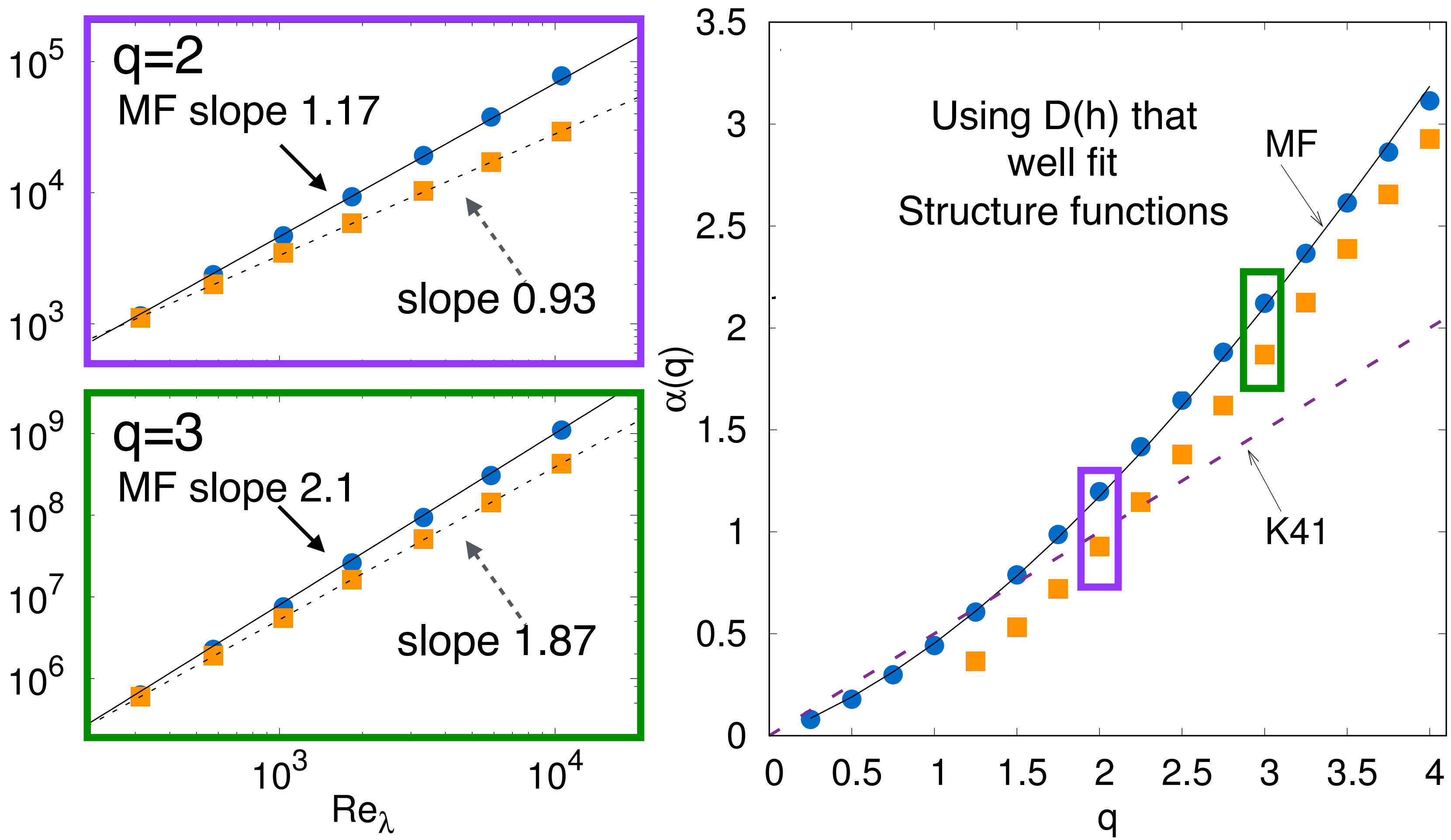
$$p(t) = a(t)v(t)$$



Lagrangian power statistics in the shell model

$$\mathcal{S}_q = \frac{\langle |p|^q \rangle}{\epsilon^q}$$

$$\mathcal{A}_q = \frac{\langle |p|^{q-1} p \rangle}{\epsilon^q}$$



The result is confirmed by using 3 different forcings:
constant, time-correlated smooth & non-smooth in time

Shell Model: multifractal model predicts well the symmetric component while the asymmetric components is sub-leading!

TIME IRREVERSIBILITY IN REVERSIBLE SHELL MODELS

SUBMITTED TO EPJE (L.B. M. Cencini, G. Boffetta and M. De Pietro)

Following Gallavotti's chaotic hypothesis

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \Delta \boldsymbol{u} + \boldsymbol{F}$$

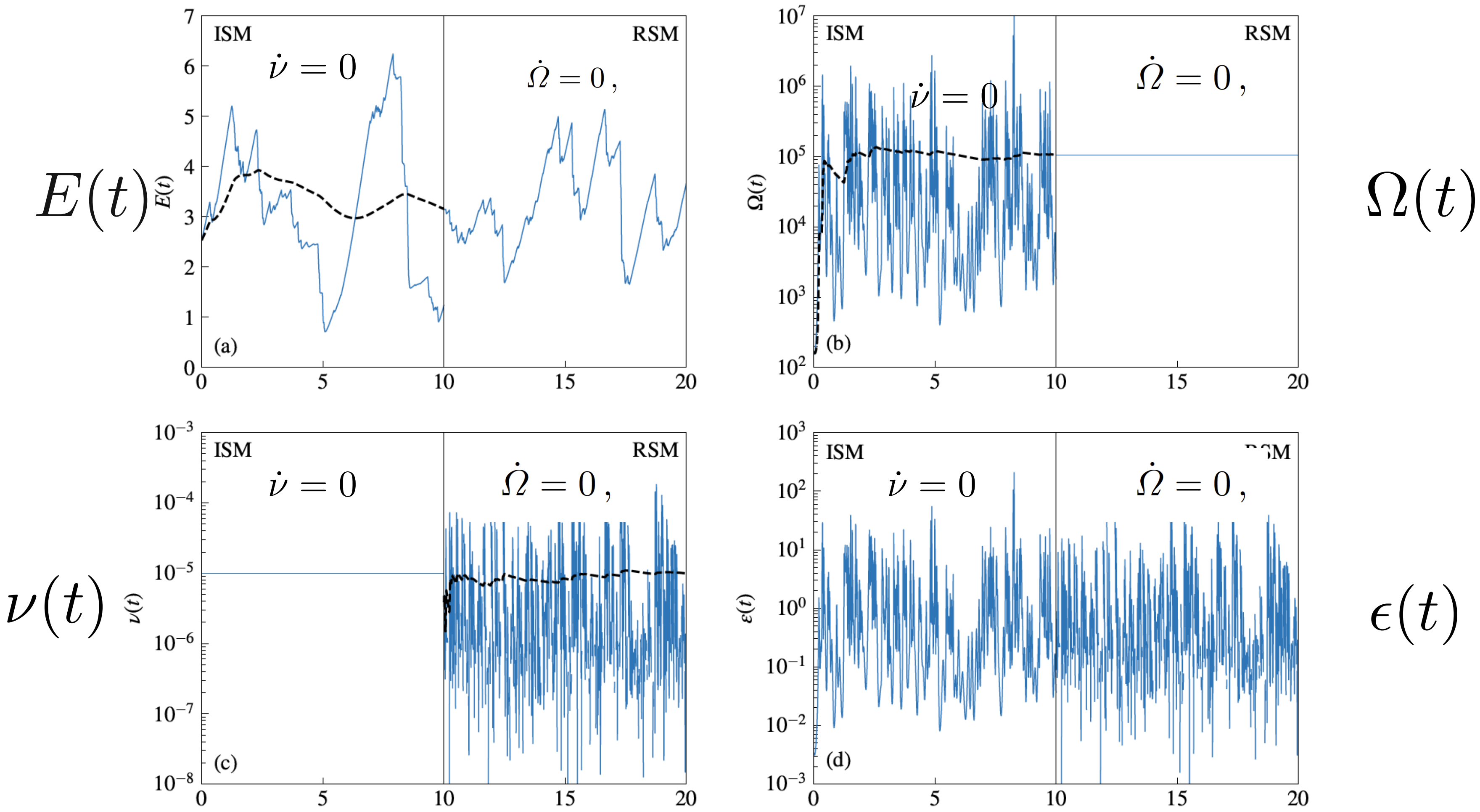
$$\nu \to \nu_R(t)$$

$$\dot{E} = 0 \qquad \nu_R(t) = -\frac{\int d^3x\, \mathbf{F}\, \mathbf{u}}{\int d^3x\, \mathbf{u}\Delta\mathbf{u}}$$

$$\dot{\Omega} = 0\,, \qquad \nu_R(t) = -\frac{\int d^3x\, \mathbf{w}(\nabla \times \mathbf{F}) + \int d^3x\, \mathbf{w}(\nabla \times (\mathbf{u} \times \mathbf{w}))}{\int d^3x\, \mathbf{w}\Delta\mathbf{w}}$$

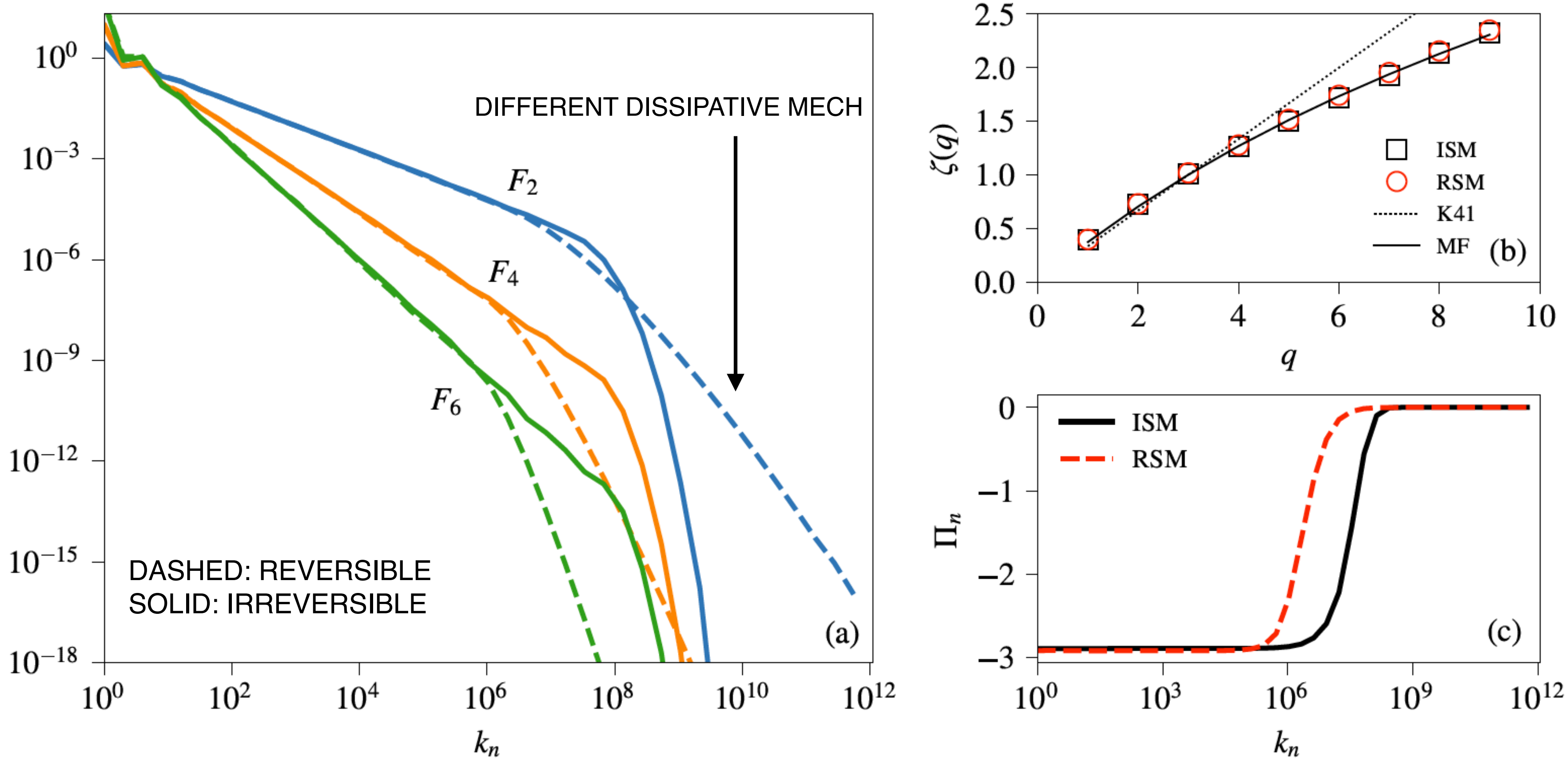
$$\Omega = \sum_n k_n^2 |u_n|^2 \,,$$

$$\epsilon = \nu \, \Omega$$

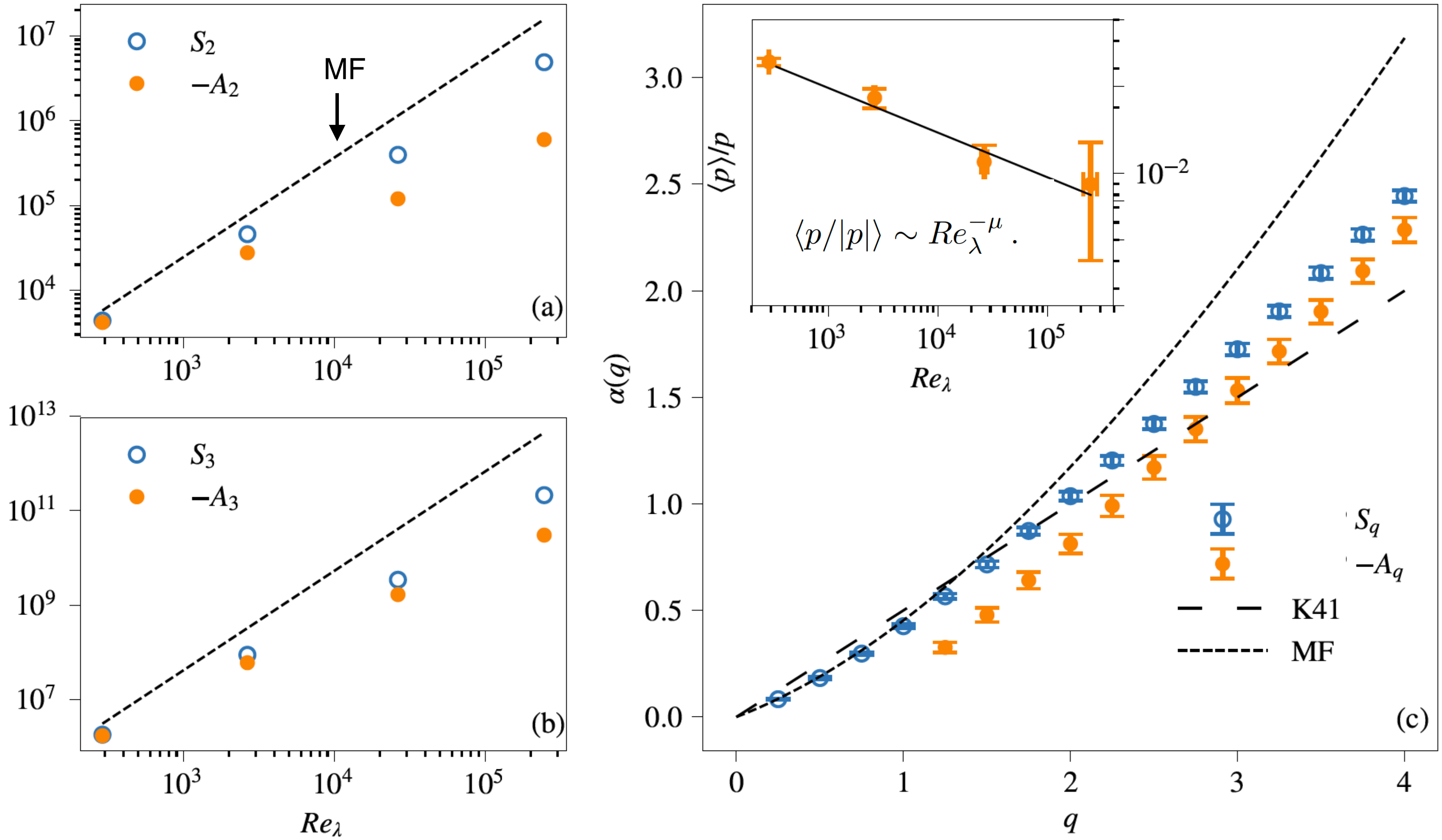


STRUCTURE FUNCTIONS: REVERSIBLE VS IRREVERSIBLE

$$F_q(k_n) = \langle |u_n|^q \rangle \sim k_n^{-\zeta(q)} \, .$$



POWER STATISTICS: REVERSIBLE SHELL MODEL



Conclusions

See arXiv:1707.08837 [physics.flu-dyn]

Scaling of symmetric components of Lagrangian power statistics is linked to Eulerian intermittency and can be rationalised within the multifractal formalism in both NS and SM turbulence

For NS turbulence MF seems to be able to catch also the scaling of statistical asymmetries in the range of explored Reynolds numbers

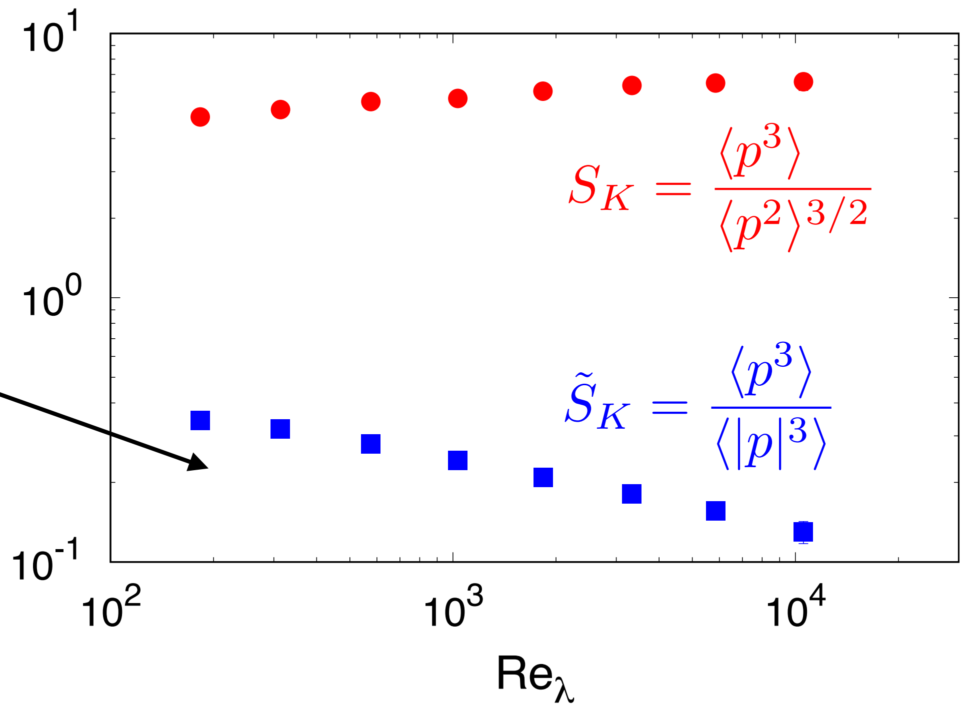
Is the latter property confirmed also at larger Re?

Why important:

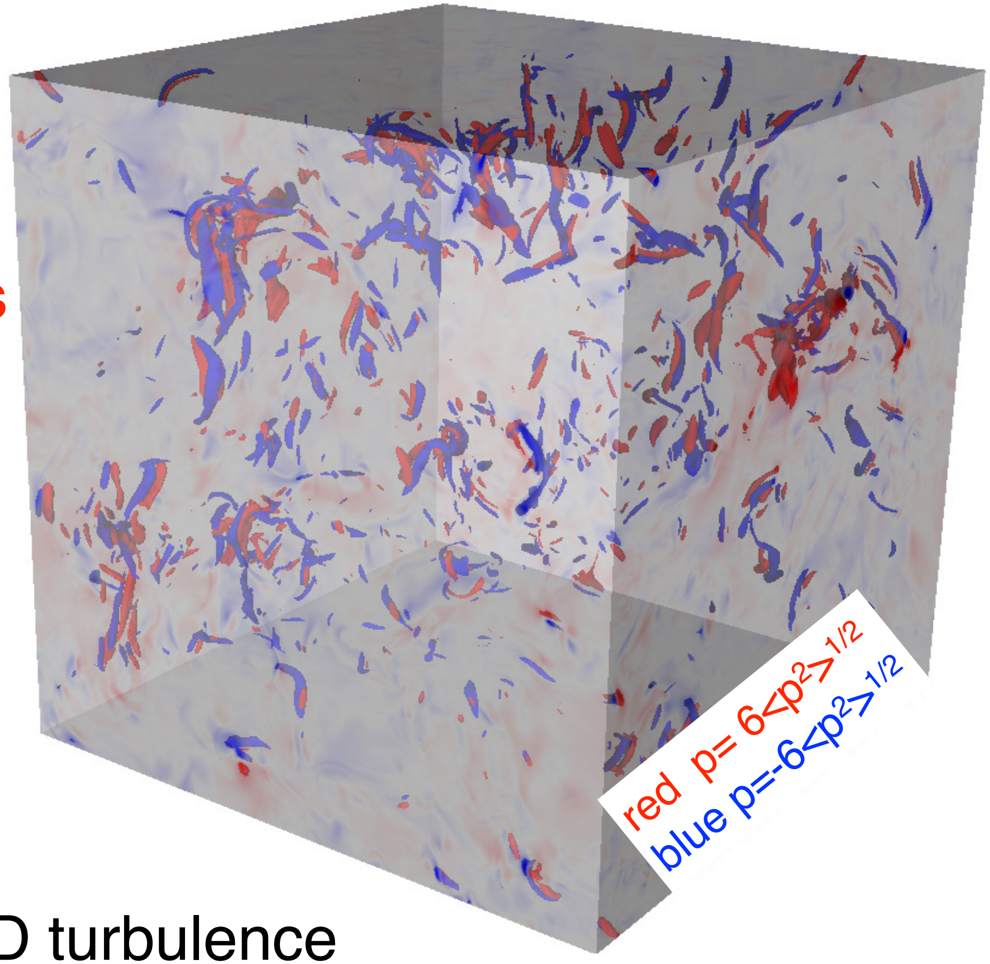
Statistical recovery of time reversal symmetry

Something similar for statistical recovery of isotropy (LB & M. Vergassola PoF 2001)

Shell model asymmetric/symmetric scaling



Open questions



Spatial properties

The spatial structure of power displays interesting (dipole-like) features worth of further investigations

$$p(\boldsymbol{x},t) = \boldsymbol{u}(\boldsymbol{x},t)\cdot\boldsymbol{a}(\boldsymbol{x},t)$$

2D turbulence

$$\langle p\rangle = 0$$
$$\langle p^2\rangle \sim \epsilon^2 Re_\lambda^{4/3}$$
$$\langle p^3\rangle \sim -\epsilon^3 Re_\lambda^2$$

observed also in 2D turbulence in the inverse cascade

(Xu et al PNAS 2014)

Inverse cascade is not anomalous and multifractal formalism cannot be applied, origin of the observed scaling behaviour?