Generative Adversarial Networks to infer velocity component in rotating turbulent flows

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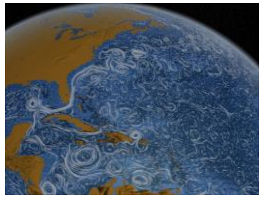
T. Li *et al.*, arXiv:2210.11921 (2022) (accepted by J. Fluid Mech.).
T. Li *et al.*, Eur. Phys. J. E 46, 31 (2023).

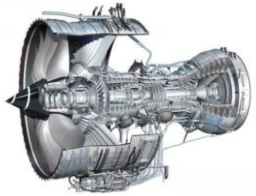
Background

Ocean

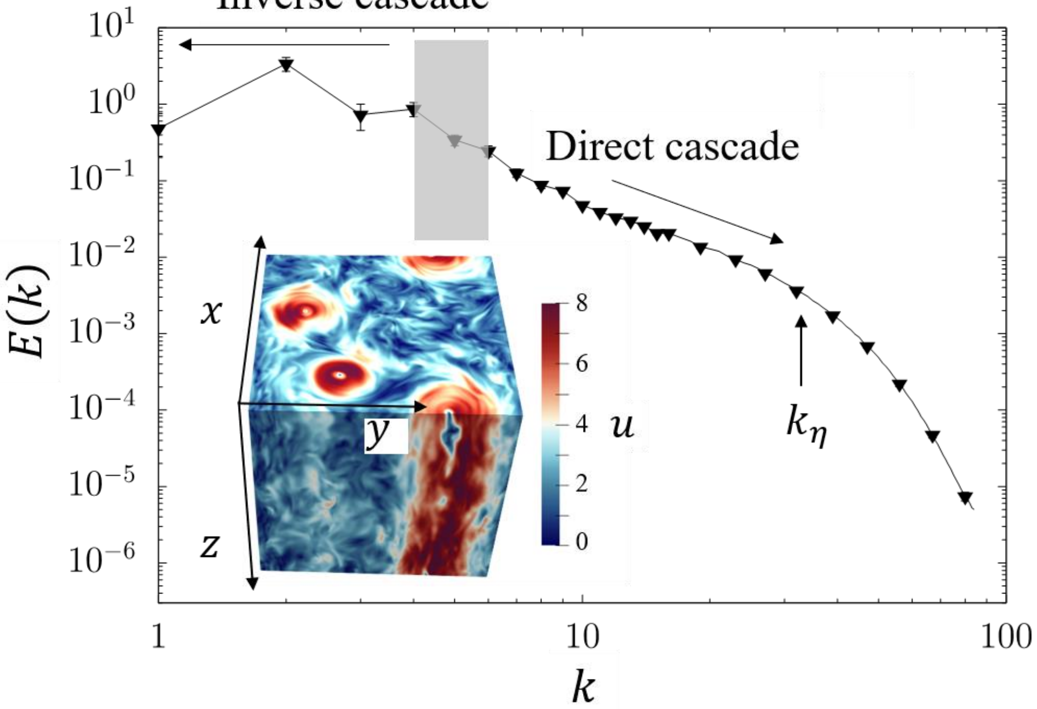


Atmosphere

Turbomachinery



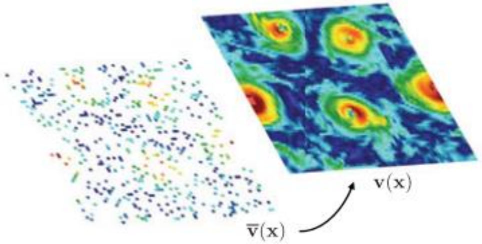
Rotating turbulence



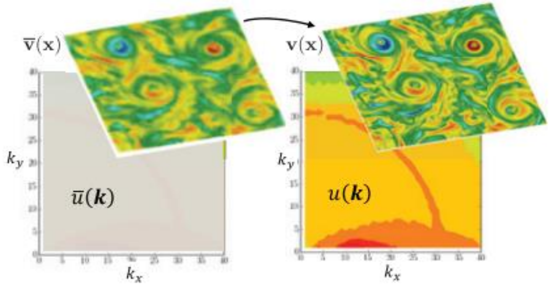
Large-scale vortical structures
Small-scale non-Gaussian fluctuations

Data reconstruction

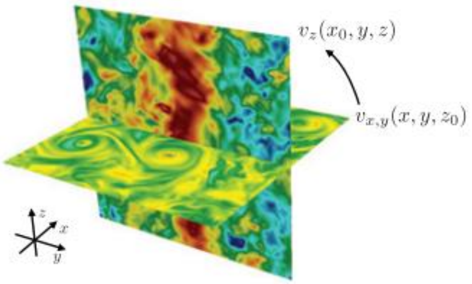
(i) Inpainting



(ii) Super-resolution

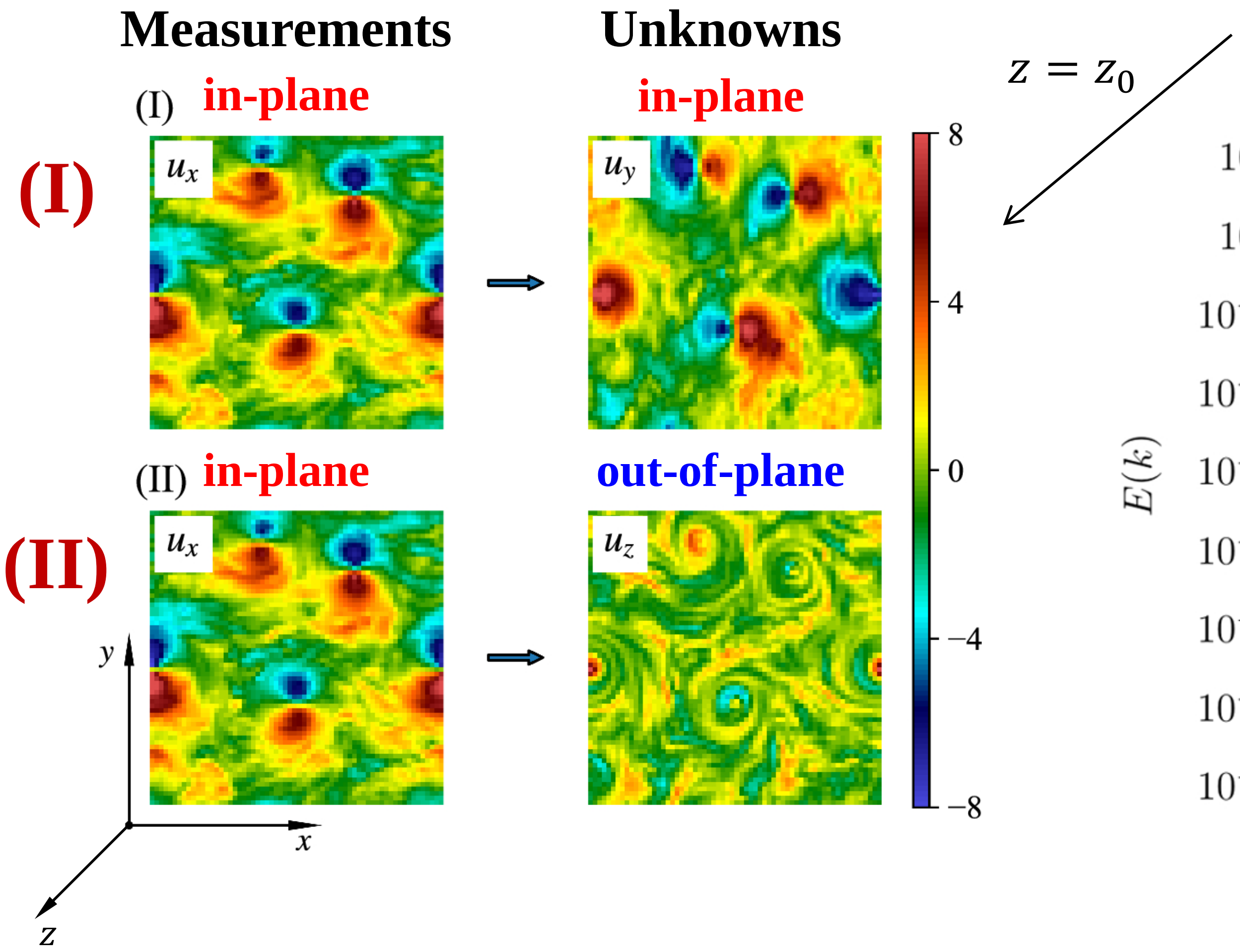


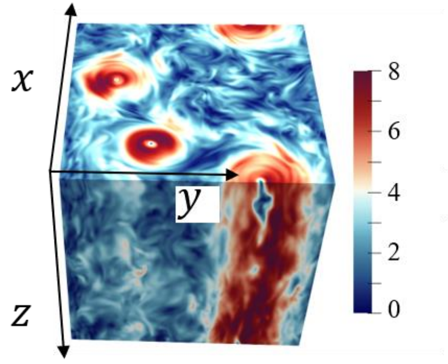
(iii) Inference

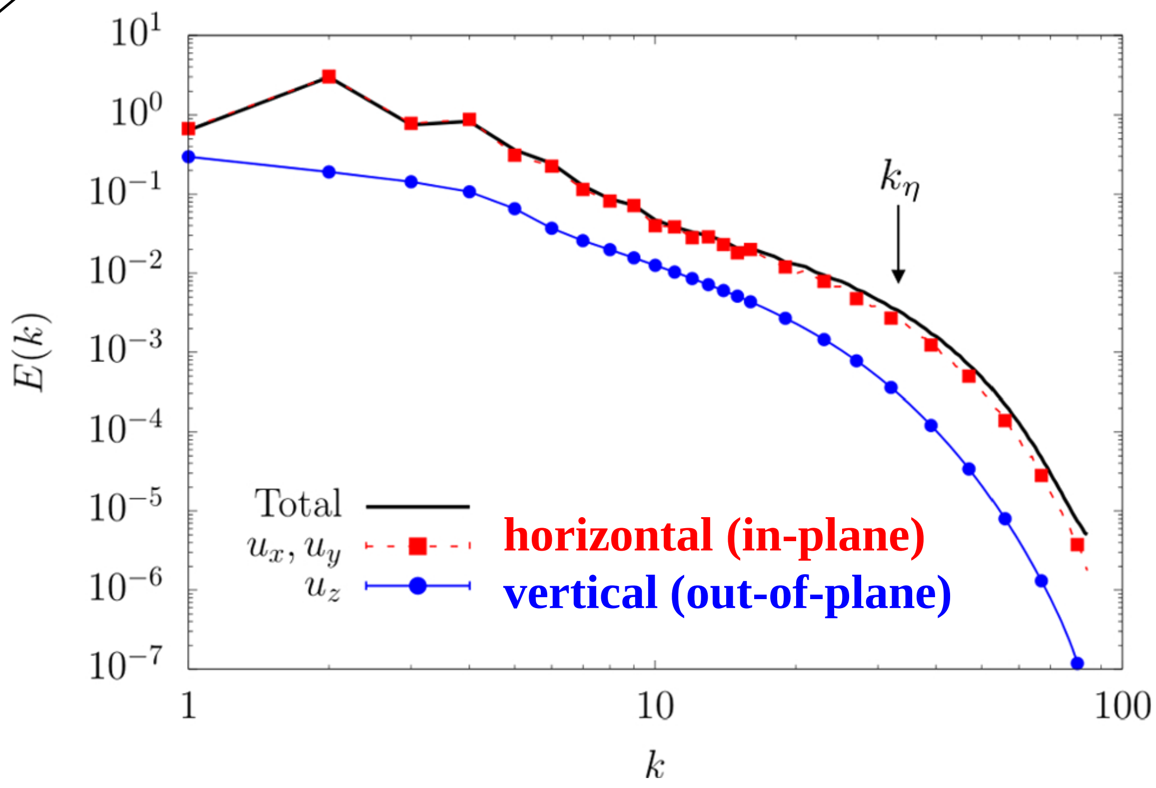


M. Buzzicotti, *Data reconstruction for complex flows using AI: recent progress, obstacles, and perspectives*. Europhysics Letters (2023).

Problem set-up







L. Biferale, F. Bonaccorso, M. Buzzicotti, P. Clark Di Leoni, *TURB-Rot. A large database of 3D and 2D snapshots from turbulent rotating flows*. arXiv preprint arXiv:2006.07469 (2020).

EPOD inference *(Extended Proper Orthogonal Decomposition)*

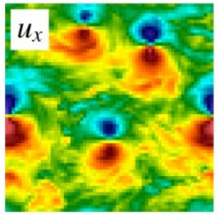
$$\mathcal{R}_S(\boldsymbol{x},\boldsymbol{y}) = \langle \boldsymbol{u}_S(\boldsymbol{x})\boldsymbol{u}_S(\boldsymbol{y})^T \rangle$$

$$\int_\Omega \mathcal{R}_S(\boldsymbol{x},\boldsymbol{y}) \cdot \boldsymbol{\phi}_S^{(n)}(\boldsymbol{y})\mathrm{d}\boldsymbol{y} = \sigma_n \boldsymbol{\phi}_S^{(n)}(\boldsymbol{x})$$

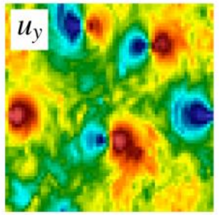
$$\boldsymbol{u}_S(\boldsymbol{x}) = \sum_{n=1}^{N_\Omega} b_S^{(n)} \boldsymbol{\phi}_S^{(n)}(\boldsymbol{x})$$

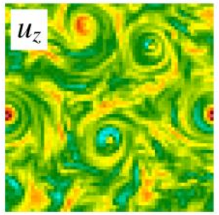
$$\boldsymbol{\phi}_S^{(n)}(\boldsymbol{x}) = \langle b_S^{(n)} \boldsymbol{u}_S(\boldsymbol{x}) \rangle / \sigma_n$$

Measured quantities: $\boldsymbol{u}_S$



Quantities to be inferred: $\boldsymbol{u}_G$





Ω

$$\boldsymbol{\phi}_E^{(n)}(\boldsymbol{x}) = \langle b_S^{(n)} \boldsymbol{u}_G(\boldsymbol{x}) \rangle / \sigma_n$$ **(TRAINING)**

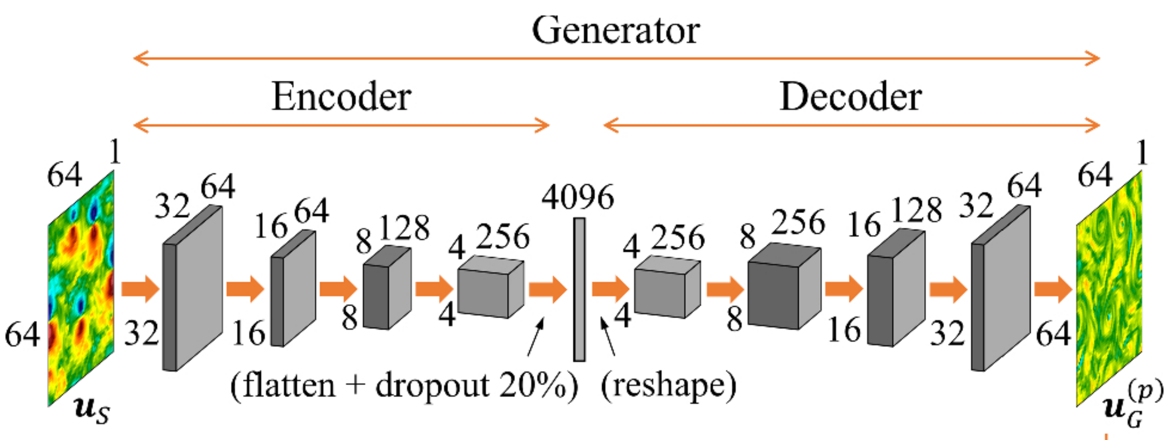
$$\boldsymbol{u}_G^{(p)}(\boldsymbol{x}) = \sum_{n=1}^{N_\Omega} b_S^{(n)} \boldsymbol{\phi}_E^{(n)}(\boldsymbol{x})$$ **(PEDICTION)**

J. Borée, *Extended proper orthogonal decomposition: a tool to analyse correlated events in turbulent flows*. Experiments in fluids (2003) 35.2: 188–192.

$\mathrm{JSD}(P\|Q) = \frac{1}{2}\mathrm{KL}(P\|M) + \frac{1}{2}\mathrm{KL}(Q\|M),\ M = \frac{1}{2}(P+Q)$

$\mathrm{KL}(P\|Q) = \int_{-\infty}^{\infty} P(x)\log(P(x)/Q(x))\mathrm{d}x$

CNN-based inference *with context encoders*



Expression of nonlinearity 😄

Often gives 'blurry' predictions!!! 😅

Loss functions

$$\mathcal{L}_{GEN} = (1-\lambda_{adv})\mathcal{L}_{\mathrm{MSE}} + \lambda_{adv}\mathcal{L}_{adv}$$

$$\mathcal{L}_{\mathrm{MSE}} = \langle \frac{1}{A_\Omega}\int_\Omega \|\boldsymbol{u}_G^{(p)}(\boldsymbol{x}) - \boldsymbol{u}_G^{(t)}(\boldsymbol{x})\|^2 \mathrm{d}\boldsymbol{x}\rangle$$

$$\mathcal{L}_{adv} = \langle \log(1 - D(\boldsymbol{u}_G^{(p)}))\rangle$$

Minimize $\mathrm{JSD}(p_t(\boldsymbol{u}_G)\|p_p(\boldsymbol{u}_G))$

$$\mathcal{L}_{DIS} = \langle \log(D(\boldsymbol{u}_G^{(t)}))\rangle + \langle \log(1 - D(\boldsymbol{u}_G^{(p)}))\rangle$$

M. Buzzicotti, F. Bonaccorso, P. C. Di Leoni, L. Biferale, *Reconstruction of turbulent data with deep generative models for semantic inpainting from TURB-Rot database.* Physical Review Fluids (2021) 6.5: 050503.

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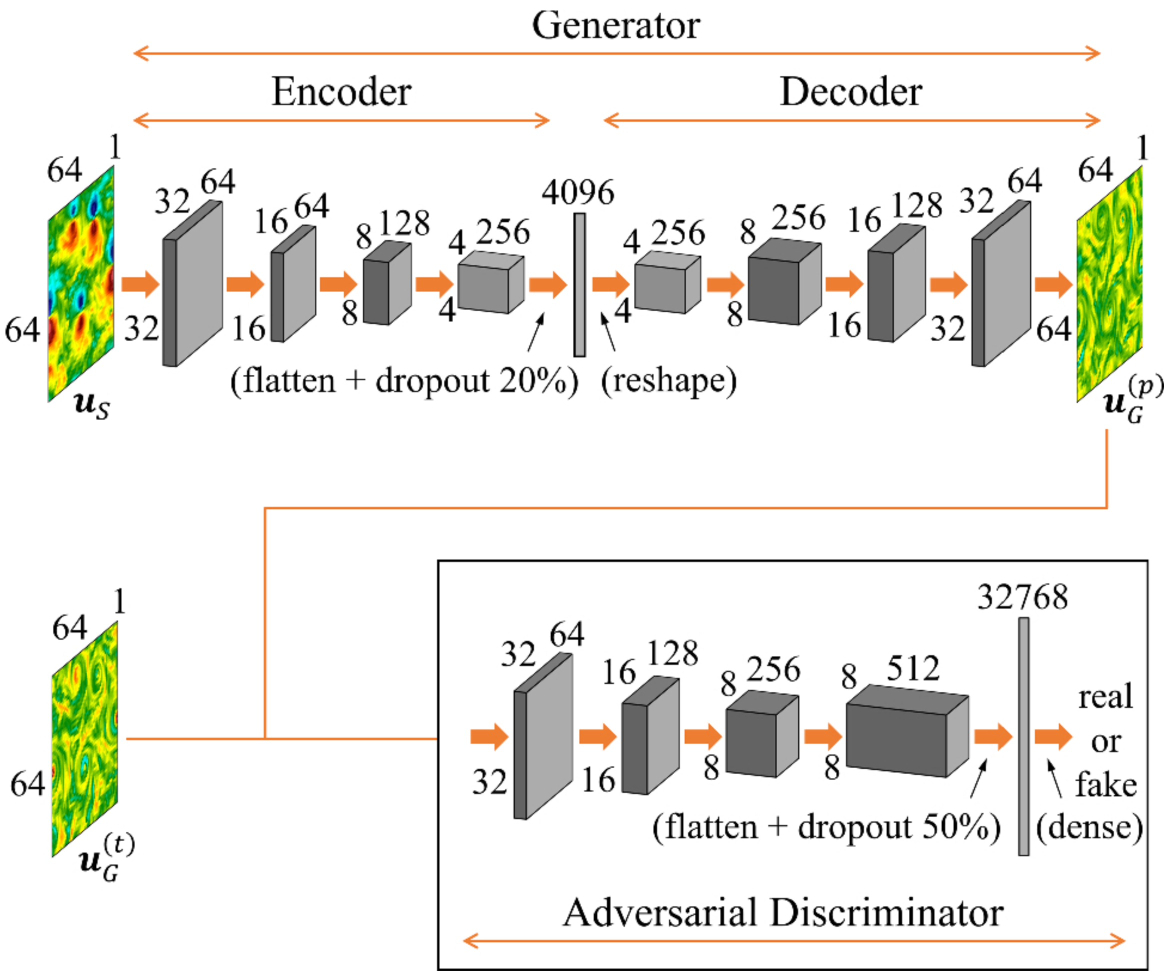
GAN-based inference *with context encoders*



Loss functions

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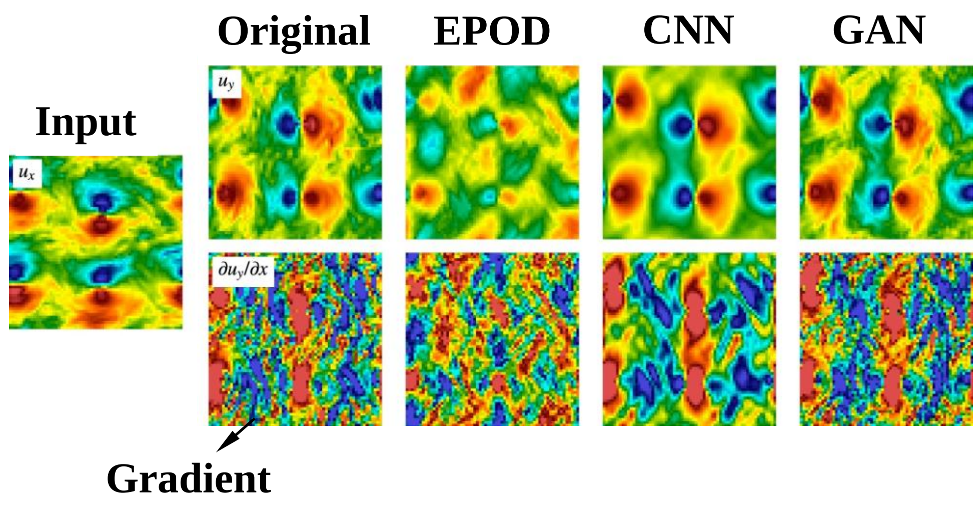
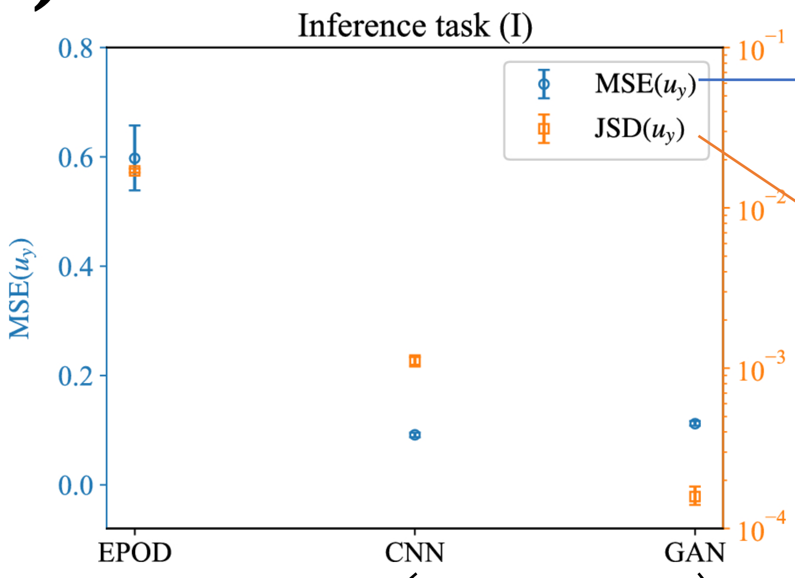
M. Buzzicotti, F. Bonaccorso, P. C. Di Leoni, L. Biferale, *Reconstruction of turbulent data with deep generative models for semantic inpainting from TURB-Rot database.* Physical Review Fluids (2021) 6.5: 050503.

Results: Inference task (I)





Point-wise MSE

$\mathrm{MSE}(\boldsymbol{u}_G) = \langle \Delta_{\boldsymbol{u}_G} \rangle / E_{\boldsymbol{u}_G}$

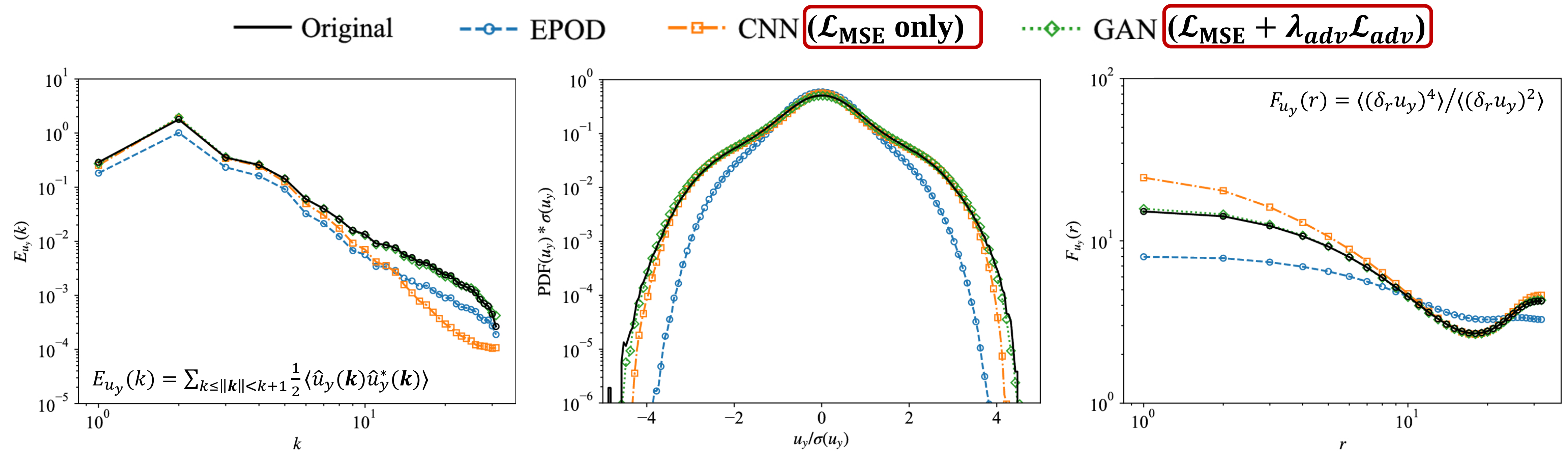
$\Delta_{\boldsymbol{u}_G} = \frac{1}{A_\Omega} \int_\Omega \| \boldsymbol{u}_G^{(p)}(\boldsymbol{x}) - \boldsymbol{u}_G^{(t)}(\boldsymbol{x}) \|^2 \mathrm{d}\boldsymbol{x}$

Statistical distance

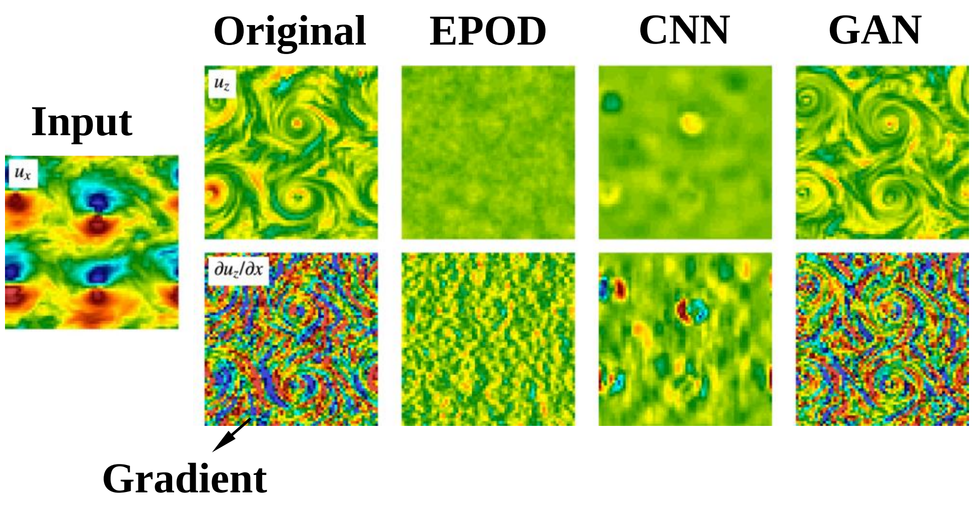
$\mathrm{JSD}(P\|Q) = \frac{1}{2}\mathrm{KL}(P\|M) + \frac{1}{2}\mathrm{KL}(Q\|M),$

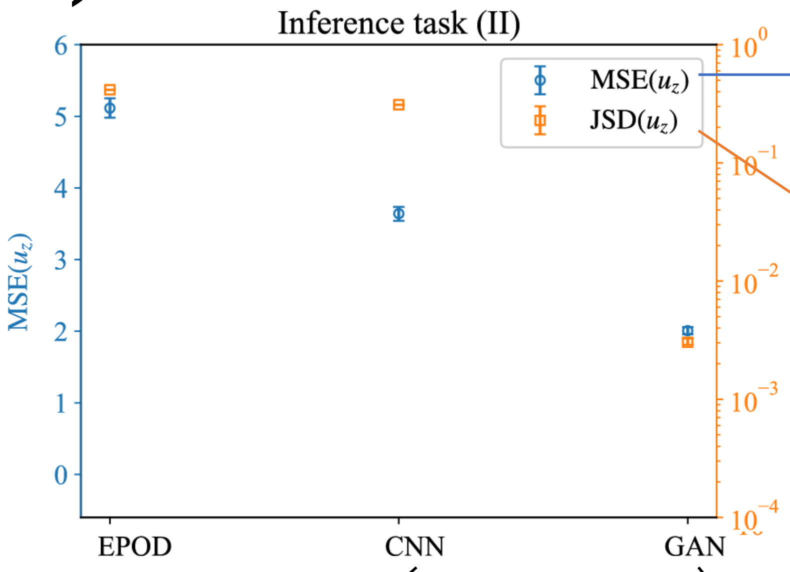
$M = \frac{1}{2}(P + Q)$

$\mathrm{KL}(P\|Q) = \int_{-\infty}^{\infty} P(x) \log(P(x)/Q(x)) \mathrm{d}x$



Results: Inference task (II)





Point-wise MSE

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$$\Delta_{\boldsymbol{u}_G} = \frac{1}{A_\Omega}\int_\Omega \|\boldsymbol{u}_G^{(p)}(\boldsymbol{x}) - \boldsymbol{u}_G^{(t)}(\boldsymbol{x})\|^2 \mathrm{d}\boldsymbol{x}$$

Statistical distance

$$\mathrm{JSD}(P\|Q) = \frac{1}{2}\mathrm{KL}(P\|M) + \frac{1}{2}\mathrm{KL}(Q\|M),$$

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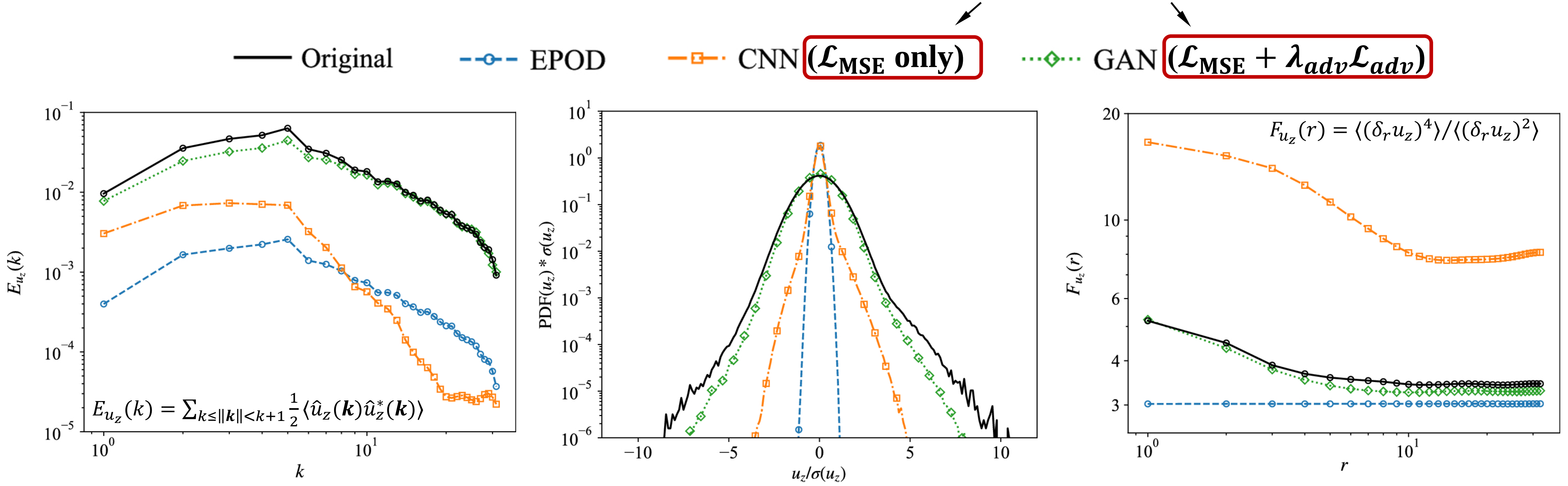

Summary

1.Purpose: Exploring practical geophysical/engineering problem of inferring one velocity component from another in 2D rotating turbulent flows.

2.Methods: Compared linear (EPOD) and nonlinear (CNN & GAN) methods using two tasks with different complexities.

3.Findings:

1. For the task where input and output components are well correlated, EPOD produced meaningful results. Improvements observed using CNN and further refined with GAN.
2. For the task where input and output components are not well correlated, EPOD failed due to low correlation between components. CNN and GAN recognized coherent structures but had limitations.

4.Conclusion: GANs optimize both instantaneous and statistical reconstruction, outperforming EPOD, which only minimizes field variance. GANs deliver more realistic results, albeit at a higher computational cost.

Smart-Turb ∞ Datasets ▾ Organizations ▾ Help ▾ About ▾ U

Guide for users

What is Smart-TURB? It is a brand new software infrastructure (born June 2020) for
the research community working on turbulence and complex flows with particular
emphasis to collect/standardize and preserve huge datase... ...ig-data and
Machine Learning approaches to fluid mechanics in genera... ...ce, in
particular. It is an easily accessible web platform for high qua...
is to host, standardize and manage a large collecti...
...rimental and numerical data sets from high-end fluid dyn...
...ties and High Performance Computational centers. Smart-...
...ble performances when accessing/uploading/searching data. The...
...munity is asked to contribute, by deploying freely downloadable, accurate an...
...mented dataset for the sake of "reproducibility": The process of documenting
...edures and archiving data so that others can fully reproduce scientific results.
...se contact the administrator for infos about how to upload your dataset. We
...by deploying a first dataset made of 2d and 3d turbulent configurations under
...on TURB-Rot. More will come.

https://smart-turb.roma2.infn.it/

TURB-ROT. A LARGE DATABASE OF 3D AND 2D SNAPSHOTS FROM TURBULENT ROTATING FLOWS

A PREPRINT

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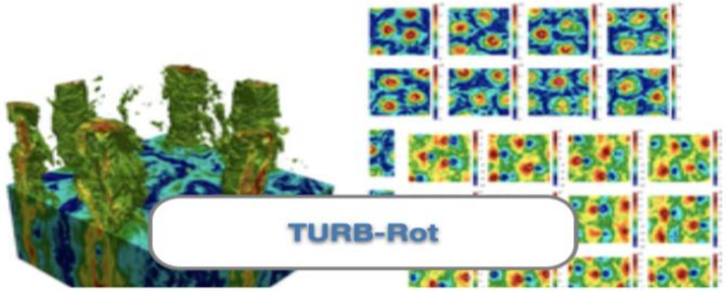
Johns Hopkins University, Baltimore, USA.
pato@jhu.edu

Search for datasets

1
Datasets

TURB-Rot
A large database of 3d and 2d
snapshots from turbulent rotating

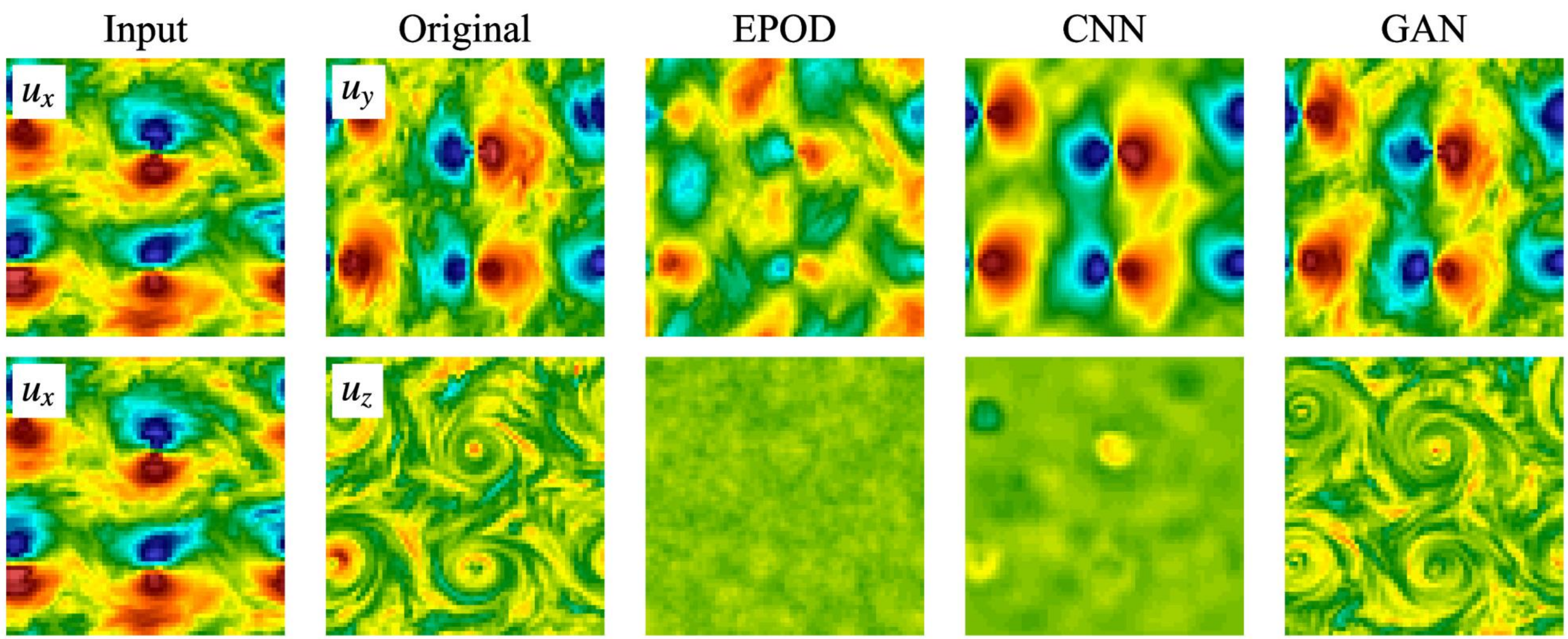


2
Organizations

web_admin
web_admin group

1
member

Thank you! Questions?



T. Li, M. Buzzicotti, L. Biferale, F. Bonaccorso, *Generative adversarial networks to infer velocity components in rotating turbulent flows.* Eur. Phys. J. E 46, 31 (2023).

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Backup slides

Multi-scale prediction error

2D wavelet decomposition (for a $2^N \times 2^N$ grid)

$$u_y(\boldsymbol{x}) = \bar{u}_y + \sum_{j=0}^{N-1} u_y^{(k_j)}(\boldsymbol{x}), \tag{22}$$

where $\bar{u}_y$ is the mean value and

$$u_y^{(k_j)}(\boldsymbol{x}) = \sum_{i_x=0}^{2^j-1} \sum_{i_y=0}^{2^j-1} \sum_{\sigma} c_{j,i_x,i_y}^{(\sigma)} \psi_{j,i_x,i_y}^{(\sigma)}(\boldsymbol{x}) \tag{23}$$

is the wavelet contribution at wave number $k_j = 2^j$, corresponding to the length scale $1/k_j$. Given that $\sigma \in \{x, y, d\}$, $c_{j,i_x,i_y}^{(\sigma)}$ is the wavelet coefficient and

$$\begin{aligned} \psi_{j,k_x,k_y}^{(x)}(x,y) &= \psi_{j,k_x}(x)\phi_{j,k_y}(y), \\ \psi_{j,k_x,k_y}^{(y)}(x,y) &= \phi_{j,k_x}(x)\psi_{j,k_y}(y), \\ \psi_{j,k_x,k_y}^{(d)}(x,y) &= \psi_{j,k_x}(x)\psi_{j,k_y}(y), \end{aligned} \tag{24}$$

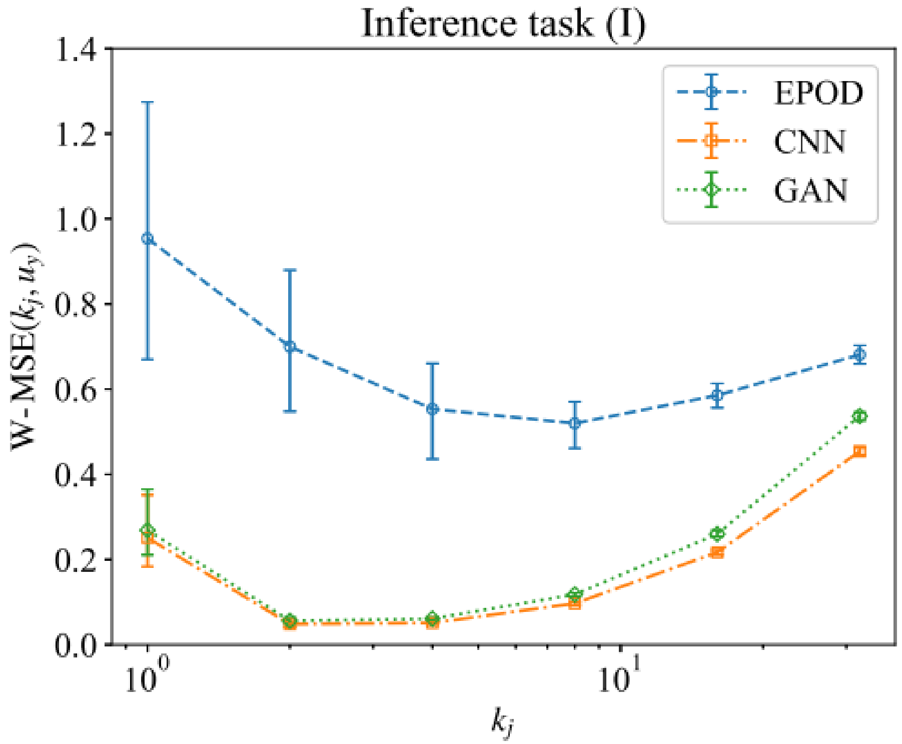
where $\phi(\cdot)$ and $\psi(\cdot)$ are the Haar scaling function and associated wavelet, respectively. To measure

$$\mathrm{MSE}(\boldsymbol{u}_G) = \langle \Delta_{\boldsymbol{u}_G} \rangle / E_{\boldsymbol{u}_G}$$

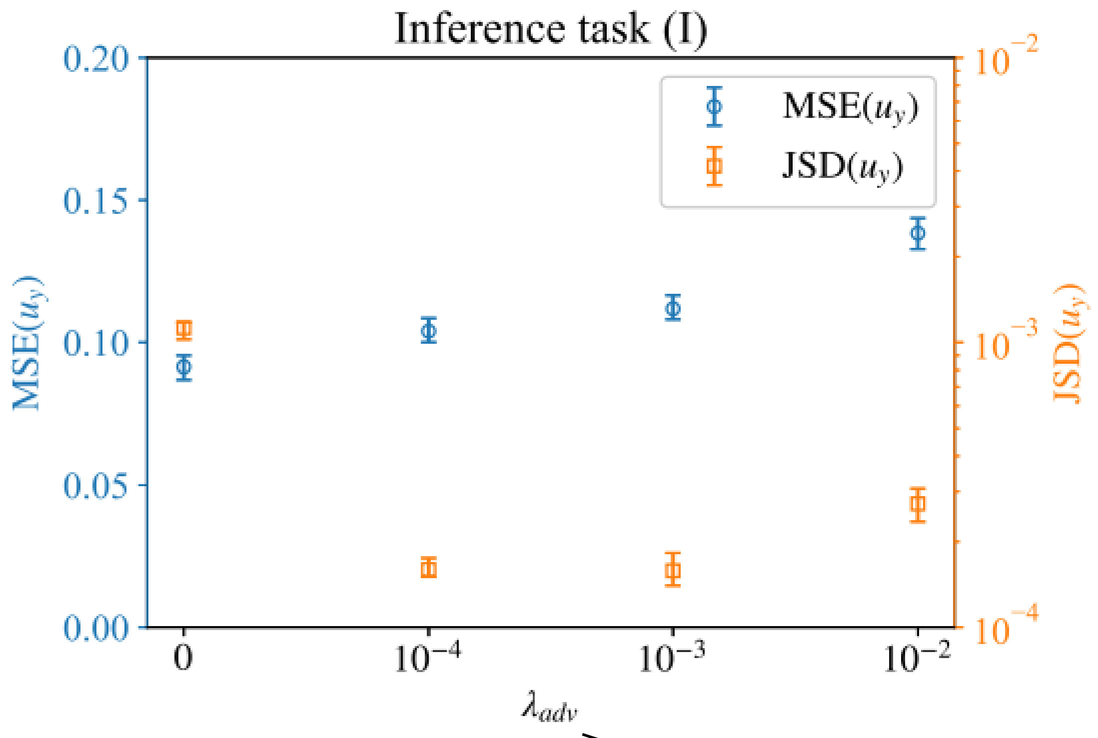
$$\Delta_{\boldsymbol{u}_G} = \frac{1}{A_\Omega} \int_\Omega \|\boldsymbol{u}_G^{(p)}(\boldsymbol{x}) - \boldsymbol{u}_G^{(t)}(\boldsymbol{x})\|^2 \mathrm{d}\boldsymbol{x}$$

Wavelet mean squared error (W-MSE)

$$\text{W-MSE}(k_j, u_y) = \mathrm{MSE}(u_y^{(k_j)})$$



Dependency on adversarial ratios



$$\mathcal{L}_{GEN} = (1 - \lambda_{adv})\mathcal{L}_{\mathrm{MSE}} + \lambda_{adv}\mathcal{L}_{adv}$$