A NUMERICAL (EXPERIMENTAL) STUDY OF TURBULENCE ON FRACTAL FOURIER SPACES

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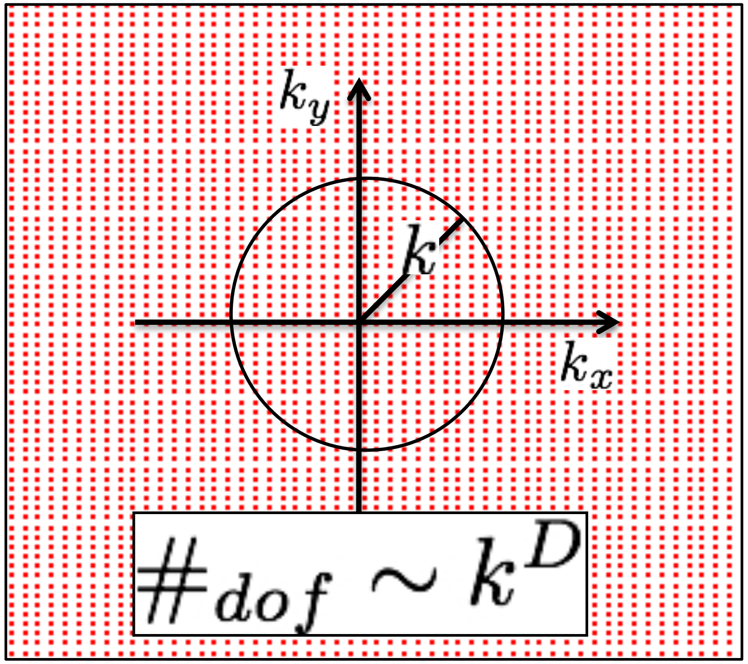
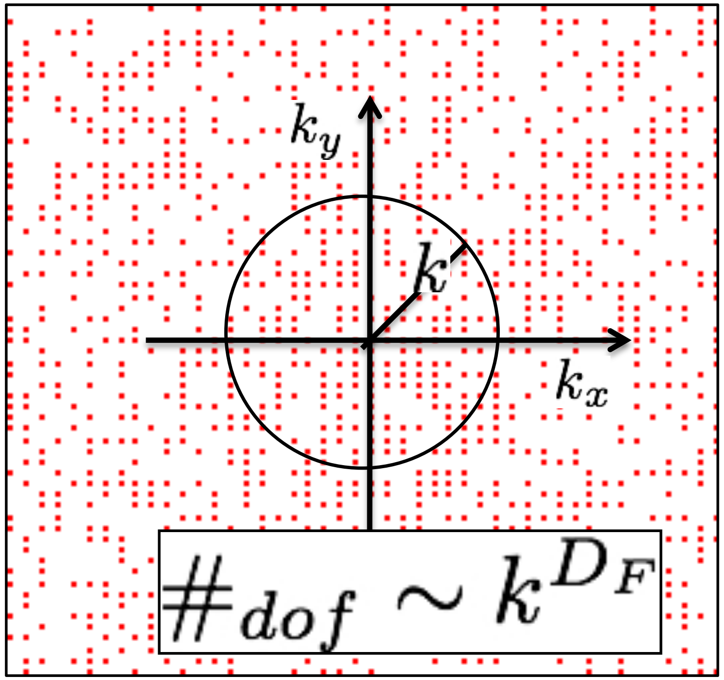
ICMIDS 2015



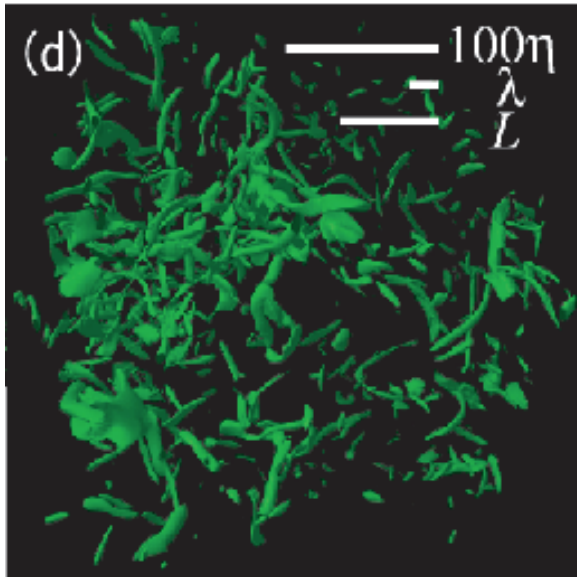


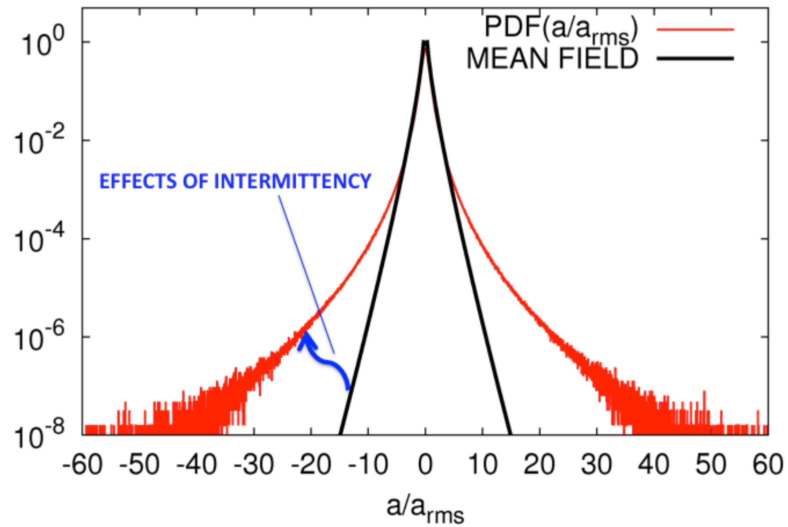
A.S. Lanotte (CNR, Italy)
S. Malapaka (Tor Vergata Univ. Italy)
F. Toschi (TuE, The Netherlands)

3D HOMOGENEOUS AND ISOTROPIC TURBULENCE

$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v}\cdot\partial)\mathbf{v} = -\partial P + \nu \Delta \mathbf{v} + \mathbf{F} \\ \partial \cdot \mathbf{v} = 0 \\ + \; Boundary\ Conditions \end{cases}$$

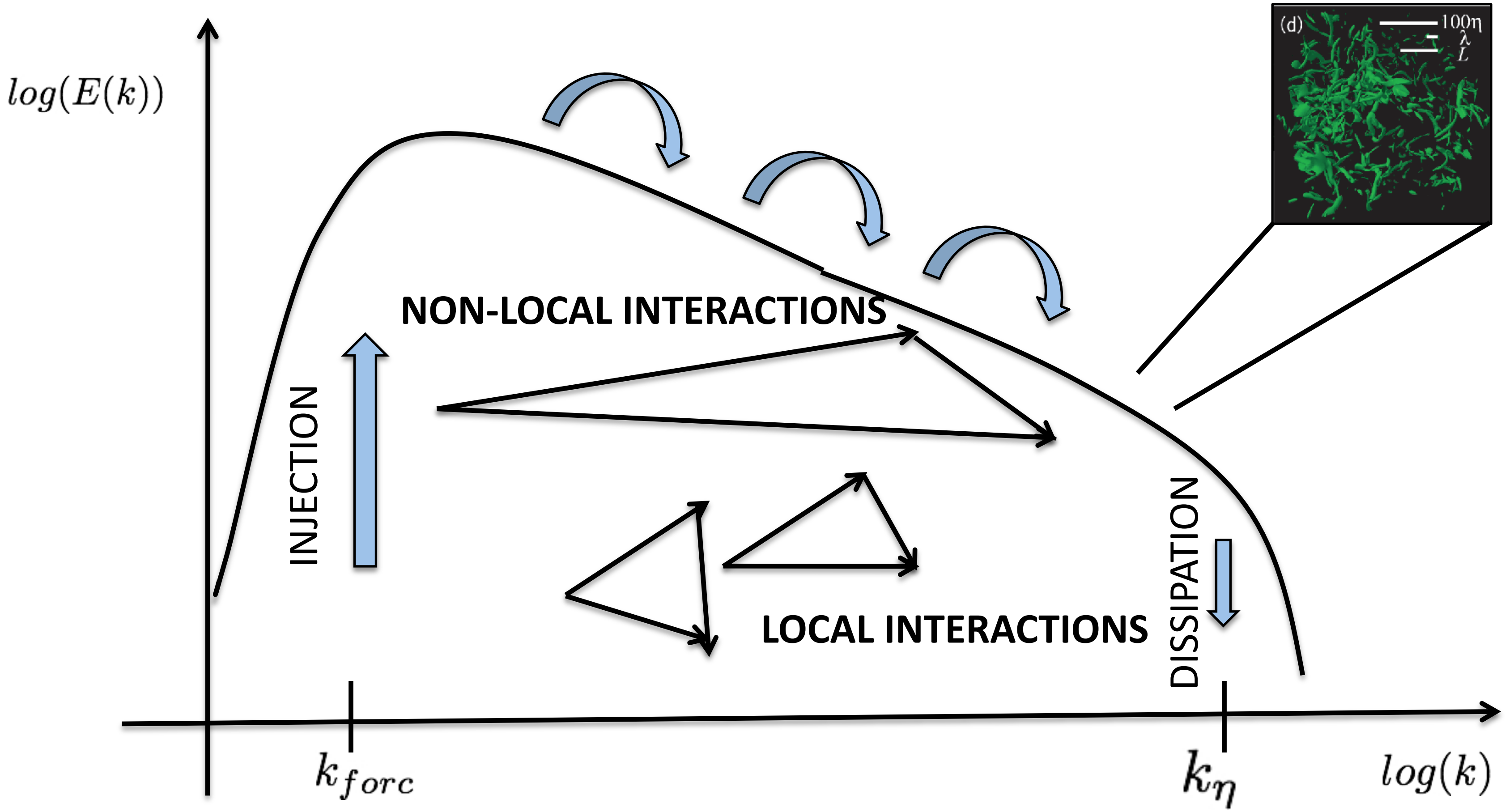
EXPERIMENTS IN-SILICO:
CAN WE ASK QUESTIONS ABOUT THE ENERGY TRANSFER BY DECIMATING INTERACTIONS IN THE NON LINEAR TERM?





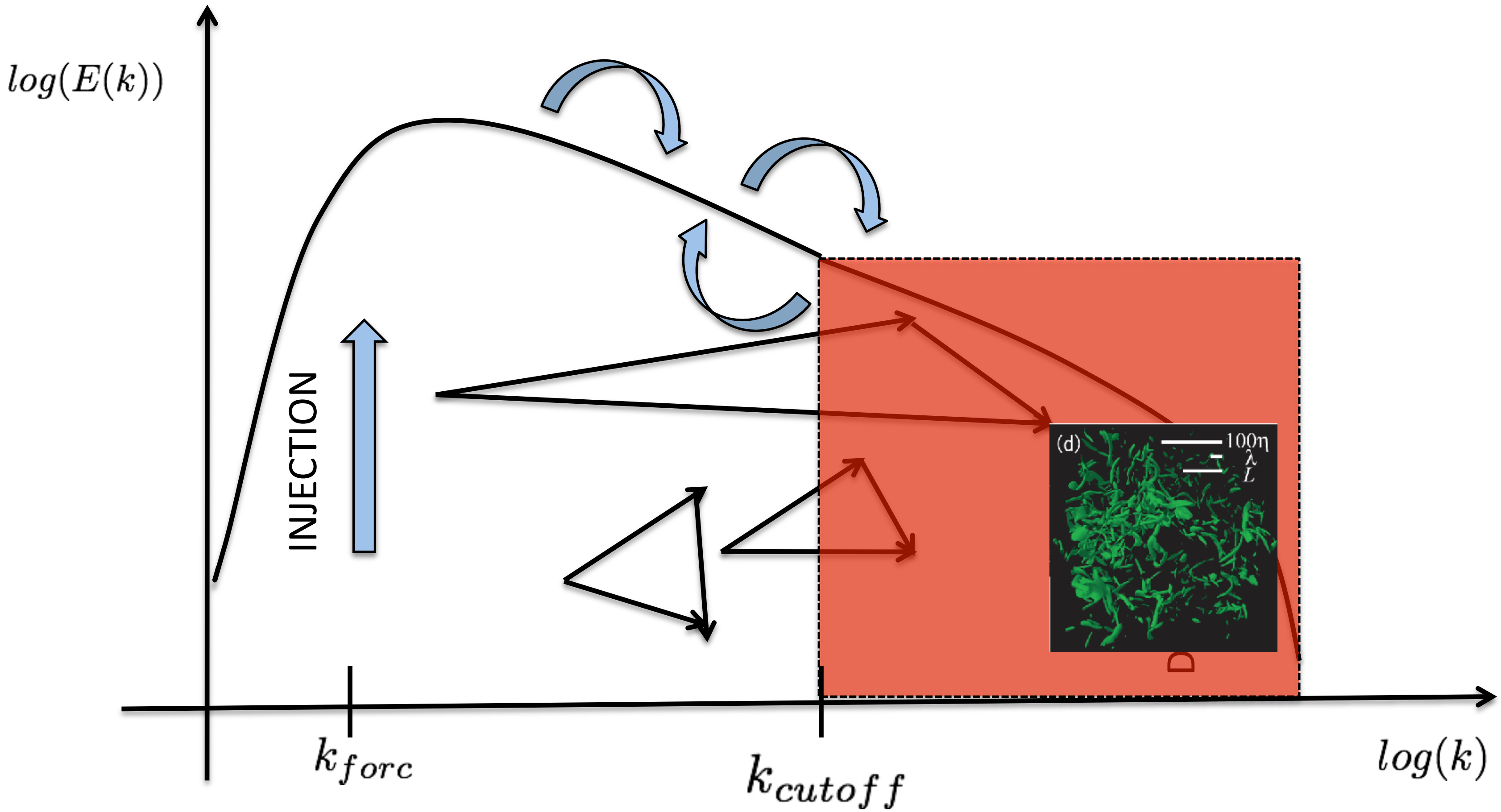
ACCELERATION PROBABILITY DISTRIBUTION FUNCTION (PDF) AT RE ~ 10^5 [Bi04] COMPARED WITH THE PREDICTION FROM MEAN FIELD (KOLMOGOROV THEORY)

L. B. et al *Phys. Rev. Lett.* **93**, 064502, 2004.

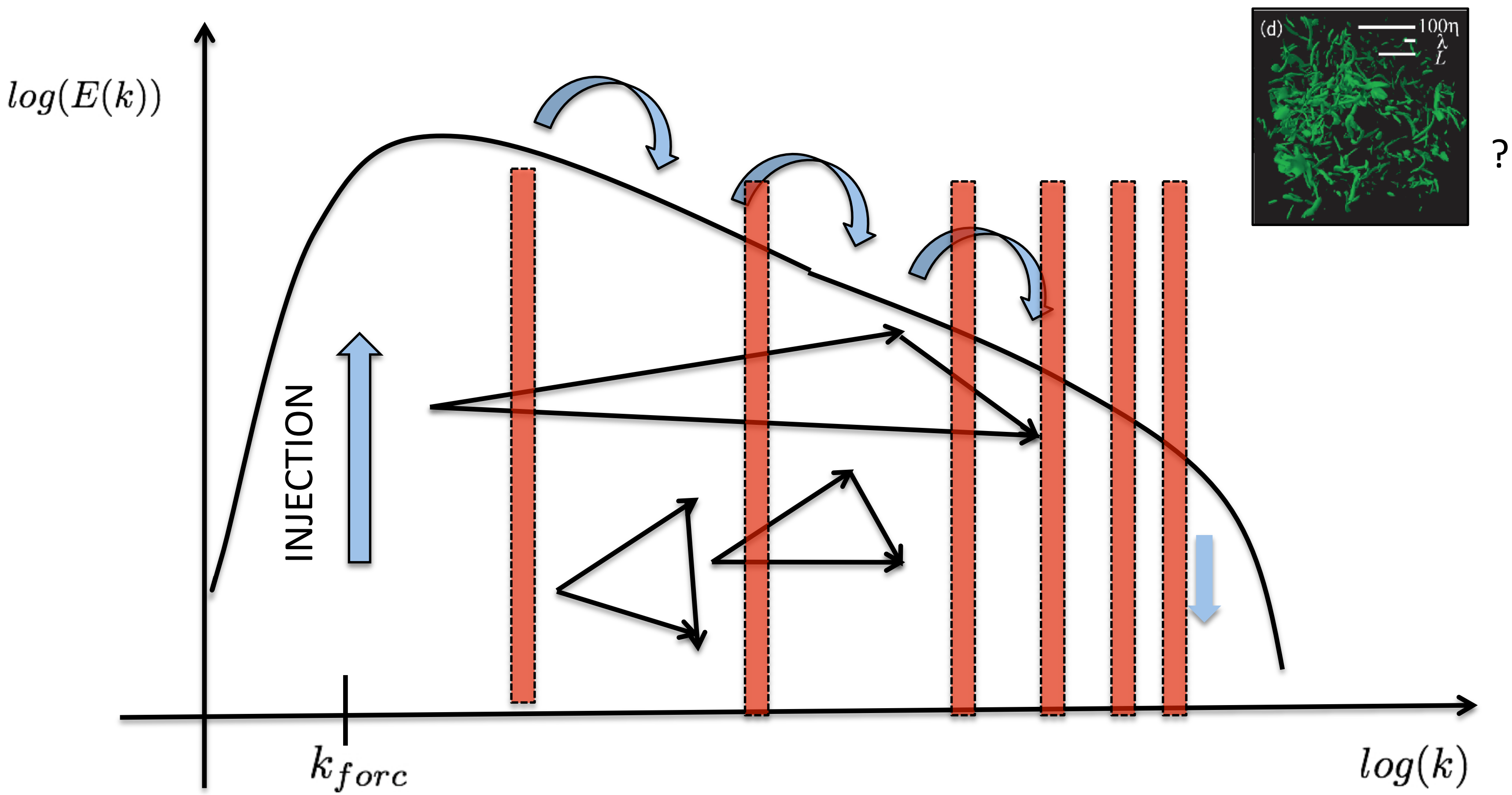


$log(E(k))$
NON-LOCAL INTERACTIONS
INJECTION
LOCAL INTERACTIONS
DISSIPATION
k_{forc}
k_η
$log(k)$
(d)
100η
λ
L

LARGE EDDY SIMULATION



$$\partial_t \overline{v} = \overline{\overline{v}\partial_x \overline{v}} - \partial_x \overline{P} + \partial_x \Pi_{SG} + \nu \Delta \overline{v} + \overline{f}$$



$$\mathbf{v}^D(\mathbf{x},t) = \mathcal{P}^D \mathbf{v}(\mathbf{x},t) = \sum_{\mathbf{k}\in\mathcal{Z}^3} e^{i\mathbf{k}\cdot\mathbf{x}}\, \gamma_{\mathbf{k}} \mathbf{u}(\mathbf{k},t)\,.$$

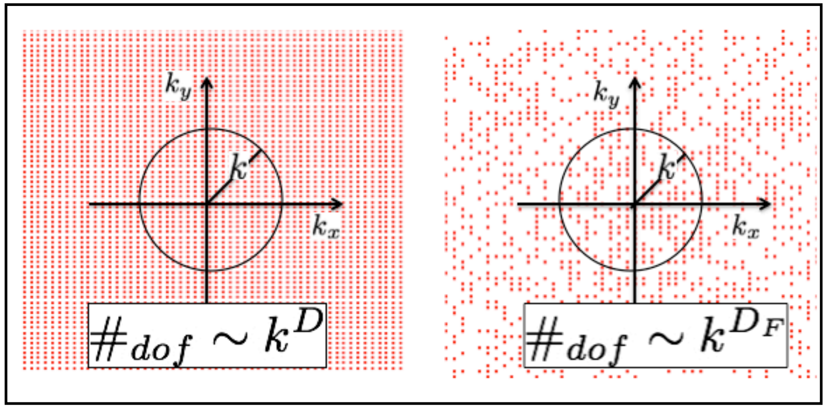
DECIMATED WITH PROBABILITY ~ $1 - k^{D_F - 3}$

SELF-SIMILAR GALERKIN TRUNCATION

U. Frisch, A. Pomyalov, I. Procaccia and S. Ray PRL 2012
S. Grossmann, D. Lohse and A. Reeh, PRL 1996

$$\partial_t \overline{v} = \overline{\overline{v}\partial_x \overline{v}} - \partial_x \overline{P} + \partial_x \Pi_{SG} + \nu \Delta \overline{v} + \overline{f}$$

$$\partial_x \Pi_{SG} = \overline{v \partial_x v} - \overline{\overline{v}\partial_x \overline{v}}$$



HOMOGENEOUS & ISOTROPIC & SELF-SIMILAR (NO EXTERNAL SCALES)
ENERGY & HELICITY INVISCID INVARIANTS
REAL PDE (INFINITE NUMBER OF DEGREES OF FREEDOM)

Turbulence in non-integer dimensions by fractal Fourier decimation

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(Dated: August 8, 2011)

Fractal decimation reduces the effective dimensionality of a flow by keeping only a (randomly chosen) set of Fourier modes whose number in a ball of radius k is proportional to k^D for large k. At the critical dimension $D = 4/3$ there is an equilibrium Gibbs state with a $k^{-5/3}$ spectrum, as in [V. L'vov *et al.*, Phys. Rev. Lett. **89**, 064501 (2002)]. Spectral simulations of fractally decimated two-dimensional turbulence show that the inverse cascade persists below $D = 2$ with a rapidly rising Kolmogorov constant, likely to diverge as $(D - 4/3)^{-2/3}$.

$$E(k) = \frac{k^{D-1}}{\alpha + \beta k^2}; \quad \beta > 0, \quad \alpha > -\beta,$$

$$D = 4/3$$

Enstrophy equipartition <-> 5/3 Kolmogorov spectrum

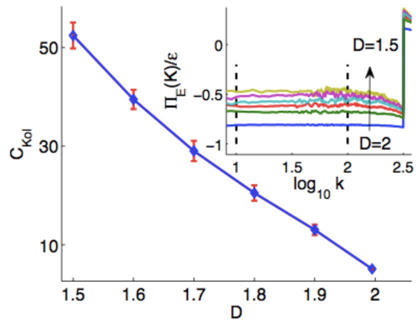

FIG. 3. (Color online) Dependence of the Kolmogorov constant on D. The lowest value, at $D = 2$, is about 5. The inset shows the energy flux normalized by the energy injection ε for the same values of D as in Fig. 2.

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Developed Turbulence: From Full Simulations to Full Mode Reductions

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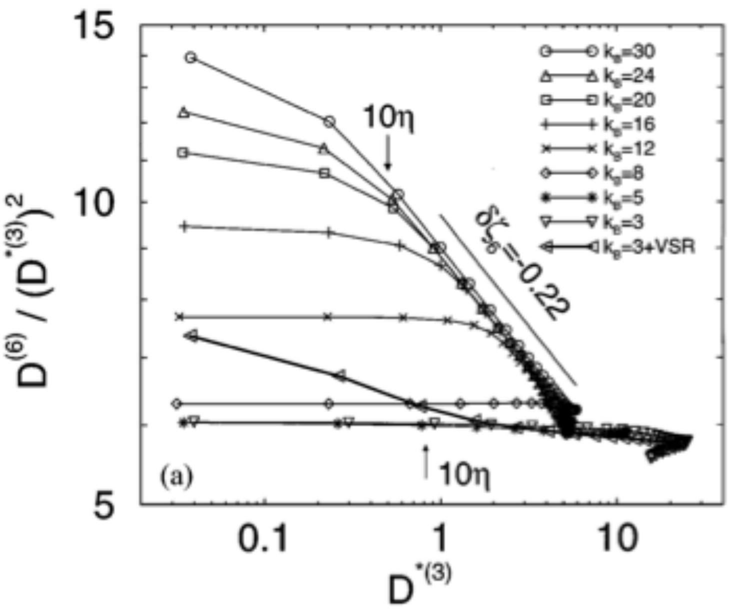
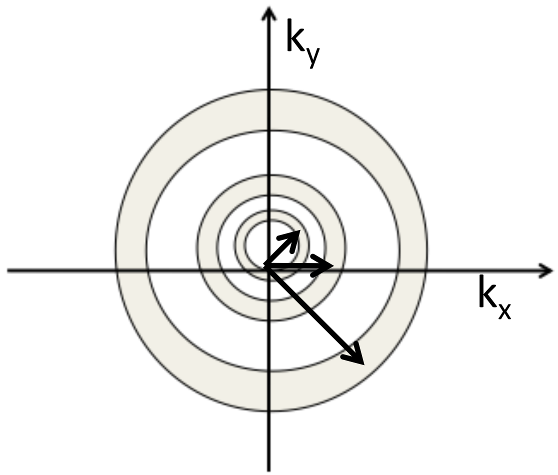
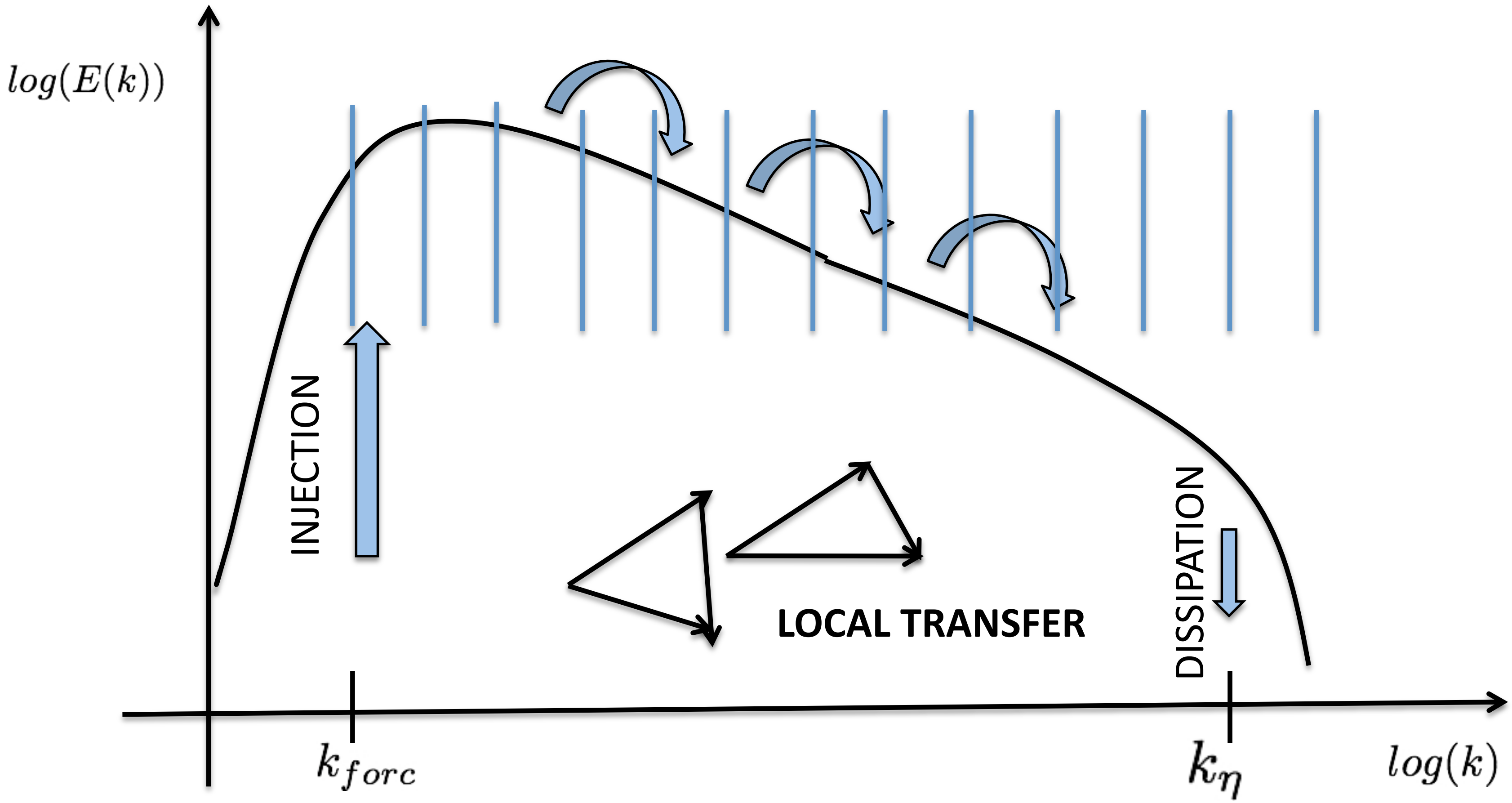
(Received 5 August 1996)

Developed Navier-Stokes turbulence is simulated with varying wave-vector mode reductions. The flatness and the skewness of the velocity derivative depend on the degree of mode reduction. They show a crossover towards the value of the full numerical simulation when the viscous subrange starts to be resolved. The intermittency corrections of the scaling exponents ζ_p of the pth order velocity structure functions seem to depend mainly on the proper resolution of the inertial subrange. *Universal* scaling properties (i.e., independent of the degree of mode reduction) are found for the relative scaling exponents $\rho_{p,q} = (\zeta_p - \zeta_3 p/3)/(\zeta_q - \zeta_3 q/3)$. [S0031-9007(96)01942-4]

PACS numbers: 47.27.Eq, 47.11.+j



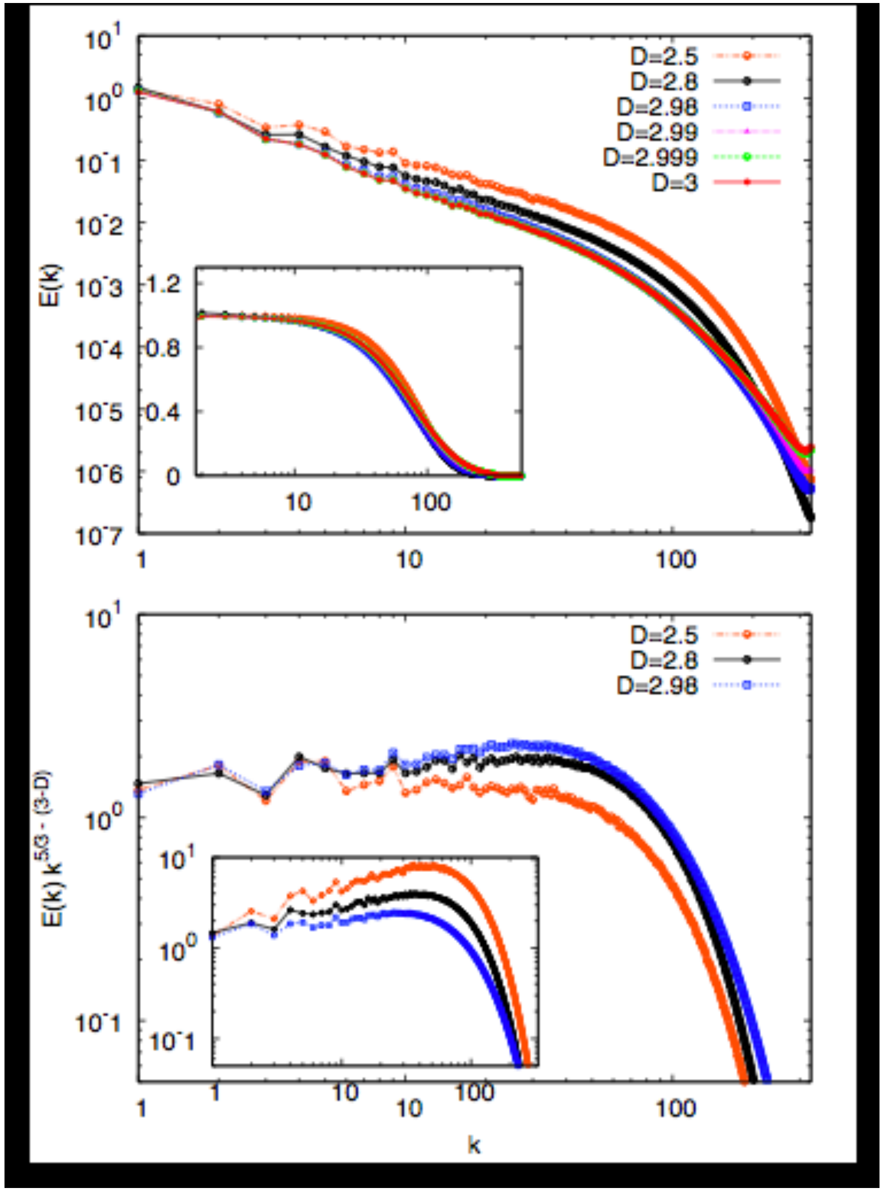




$$\frac{d}{dt}u(k_n) = k_n[a\,u(k_{n+2})u(k_{n+1}) + b\,u(k_{n+1})u(k_{n-1}) + c\,u(k_{n-2})u(k_{n-1})] - \nu k_n^2 u(k_n)$$

Bohr T., Jensen M. H., Paladin G. and Vulpiani A., *Dynamical Systems Approach to Turbulence*, Cambridge, in press (1998)

L.B. Annu. Rev. Fluid. Mech. 35, 441, 2003



A.S. Lanotte, R. Benzi, L.B, S. Malapaka and F. Toschi PRL 2015 (submitted)

$$E^D(k) = \int_{|\mathbf{k}_1|=k} d^3k_1\, \gamma_{\mathbf{k}_1} \int d^3k_2\, \gamma_{\mathbf{k}_2} \langle\, \mathbf{u}(\mathbf{k}_1)\, \mathbf{u}(\mathbf{k}_2)\, \rangle\,.$$

$$\Pi^D(k) = \int_{|\mathbf{k}_1|<k} d^3k_1 \gamma_{\mathbf{k}_1} \int d^3k_2\, d^3k_3\, \gamma_{\mathbf{k}_2}\gamma_{\mathbf{k}_3} S(\mathbf{k}_1|\mathbf{k}_2, \mathbf{k}_3)\,,$$

$$\mathbf{u}(\mathbf{k}) \sim k^{-a}$$

$$\Pi^D(\lambda k) \;\sim\; \lambda^{3D+1-3a}\Pi^D(k).$$

$$a = D + 1/3 \rightarrow E^D(k) \sim E^{K41}(k)k^{3-D}$$

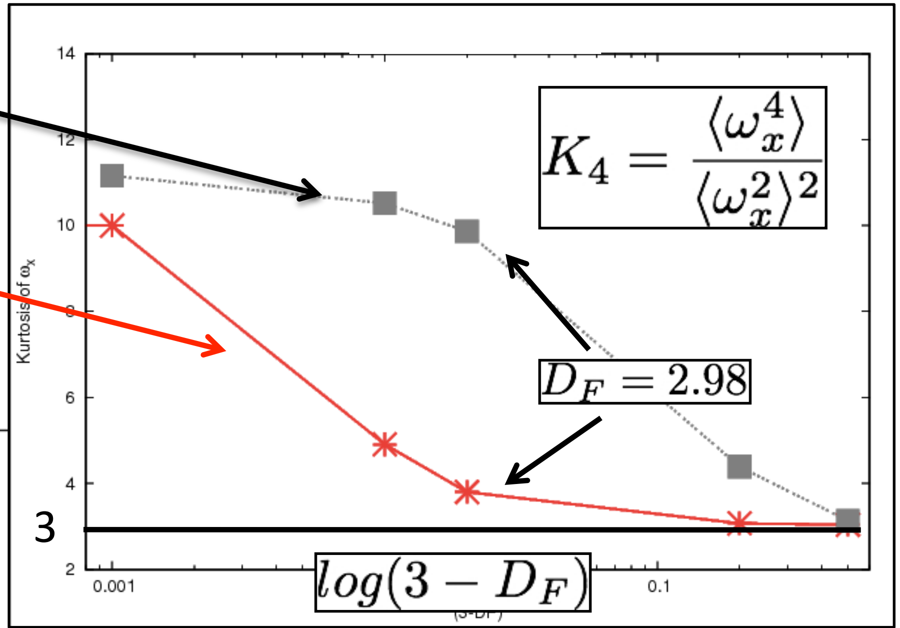
DF	2.5	2.8	2.98	2.99	2.999	3.0
1024^3	X	X	X	X	X	X
2048^3			X	X		

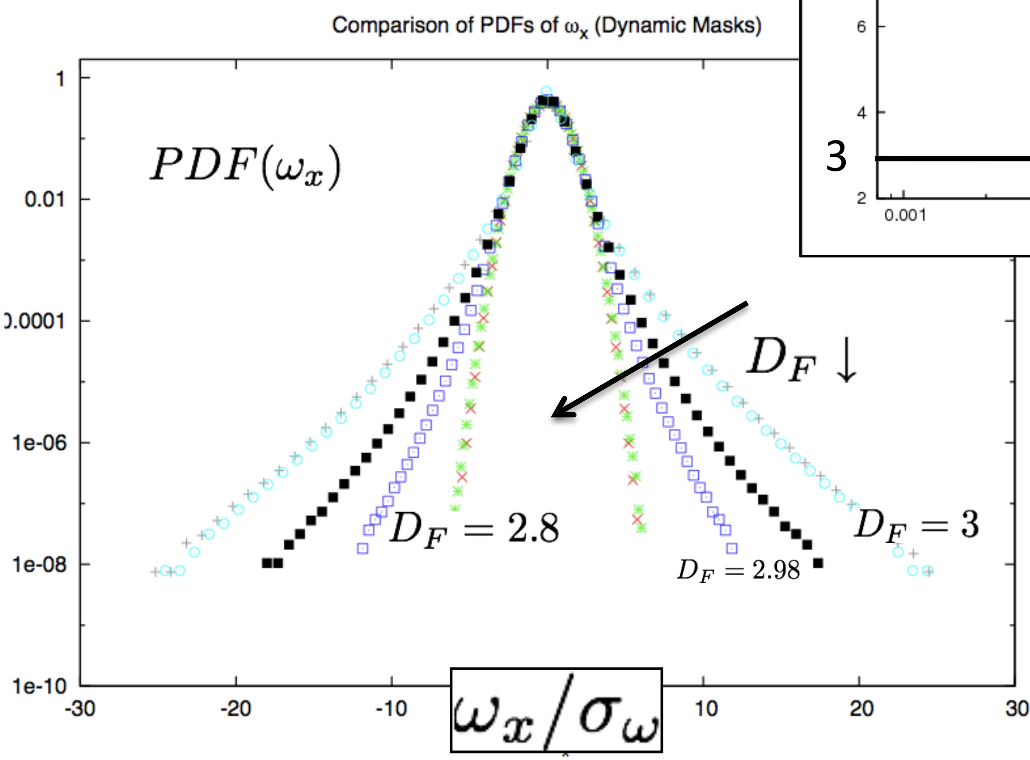
DF	2.5	2.8	2.98	2.99	2.999	3.0
1024^3	**3%**	**25%**	**87%**	**93%**	**99%**	**100%**

PDF OF VORTICITY AT CHANGING FRACTAL DIMENSION

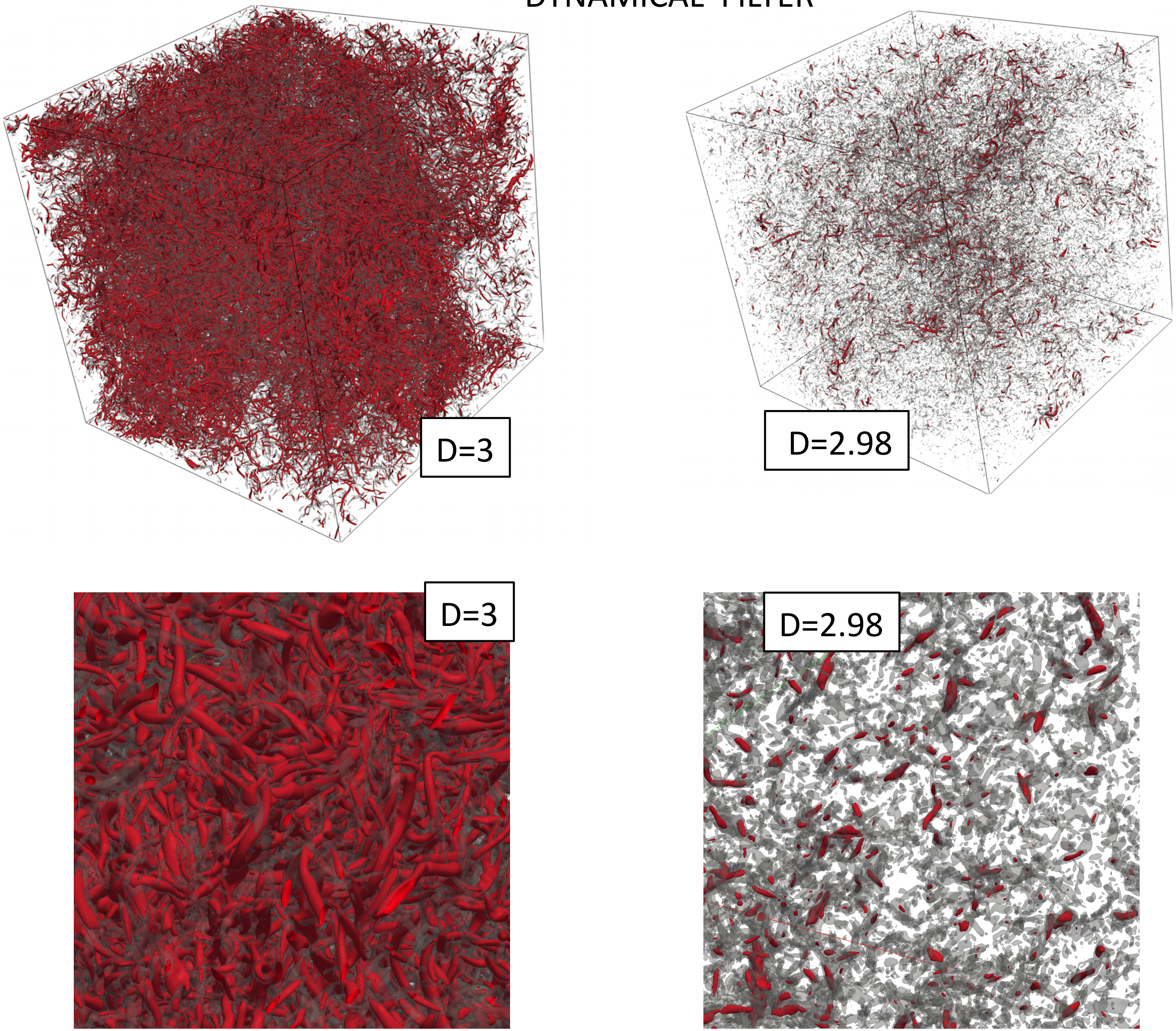
$$\begin{cases} \partial_t \mathbf{v} = B(\mathbf{v},\mathbf{v}) + \Delta \mathbf{v} + \mathbf{f} \\ \mathbf{v} \to P^{D_F} \mathbf{v} \end{cases}$$

$$\partial_t \mathbf{v}^{D_F} = P^{D_F} B(\mathbf{v}^{D_F}, \mathbf{v}^{D_F}) + \Delta \mathbf{v}^{D_F} + \mathbf{f}^{D_F}$$

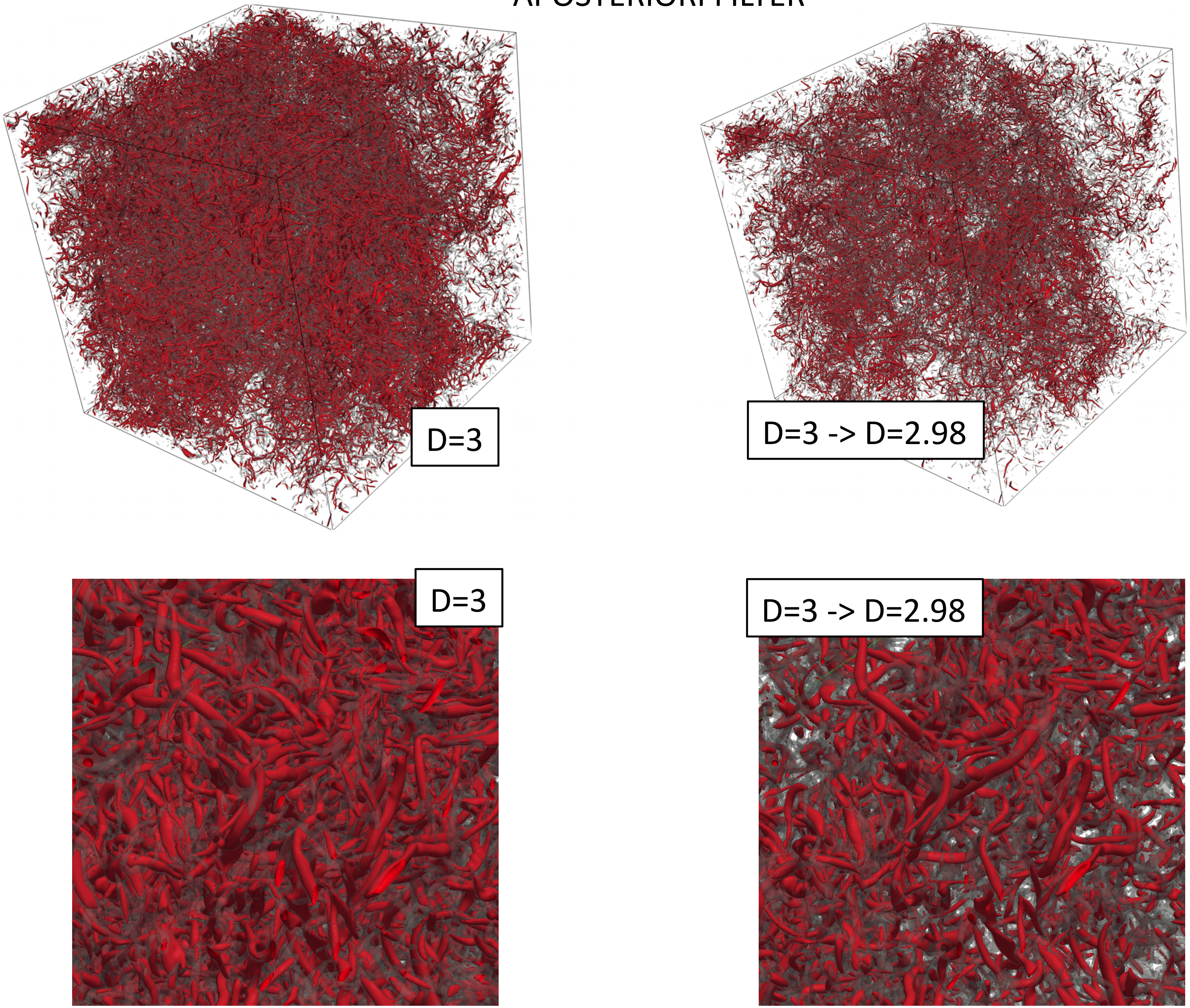




DYNAMICAL FILTER



APOSTERIORI FILTER



CONCLUSIONS

FRACTAL DECIMATION: MILDEST REMOVAL OF DEGREE OF FREEDOM

HOMOGENEOUS & ISOTROPIC & SELF SIMILAR

\+ QUANTIFY IMPORTANCE OF LOCAL VS NON-LOCAL TRIADIC INTERACTIONS

+/- QUANTIFY IMPORTANCE OF $\#_{DOF}$ FOR VORTEX STRETCHING

\+ CORRECTION IN THE MEAN RESPONSE (SPECTRUM) PROPORTIONAL TO 3-D_F: YOU CAN HAVE A LITTLE CHANGE IN THE SPECTRAL PROPERTIES AND STILL GAINING IN THE $\#_{DOF}$

\+ CORRECTION TO FLUCTUATIONS: HUGE. SMALL SCALE VORTICITY IS STRONGLY SENSITIVE TO DECIMATION. "CHOERENT" SMALL-SCALE STRUCTURES FEEL GLOBAL CORRELATIONS ACROSS SCALES IN FOURIER: BAD NEWS FOR MODELING PEOPLE

\+ HOW TO BRING INTERMITTENCY BACK TO NS EQUATIONS?

$$\partial_t \overline{v} = \overline{\overline{v}\partial_x \overline{v}} - \partial_x \overline{P} + \partial_x \Pi_{SG} + \nu \Delta \overline{v} + \overline{f}$$

\- WE STILL MISS A CLEAR DEFINITION OF INTERMITTENCY IN FOURIER SPACE

\+ POTENTIALLY IMPROVABLE BY USING SPARSE FAST FOURIER TRANSFORM

\+ CONCEPTUALLY DIFFERENT FROM KINEMATIC SIMULATIONS

\+ WHAT ABOUT LAGRANGIAN DYNAMICS?