On the preferential sampling of helicity by isotropic helicoids
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Luca Biferale
Dept. of Physics and INFN University of Rome "Tor Vergata"
biferale@roma2.infn.it

CREDITS: K. GUSTAFSSON (U. GOTHEBORG), R. SCATAMACCHIA, F. BONACCORSO (U. TOR VERGATA)

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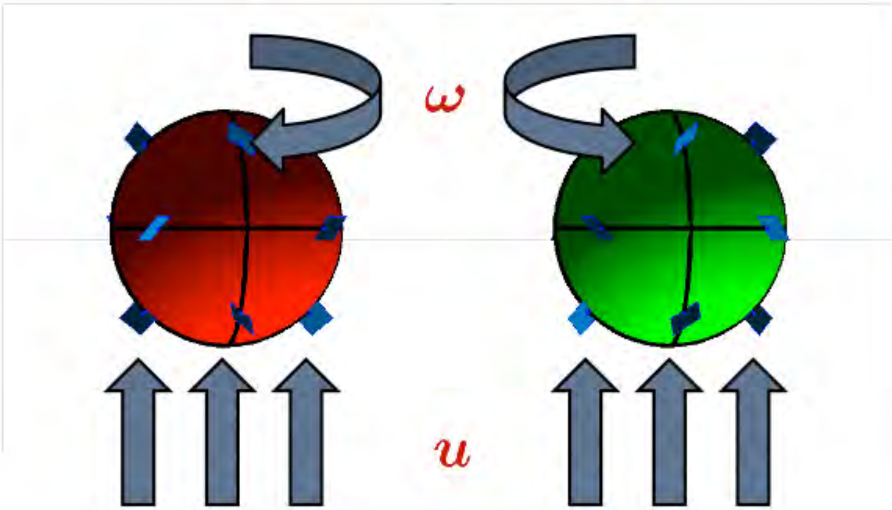
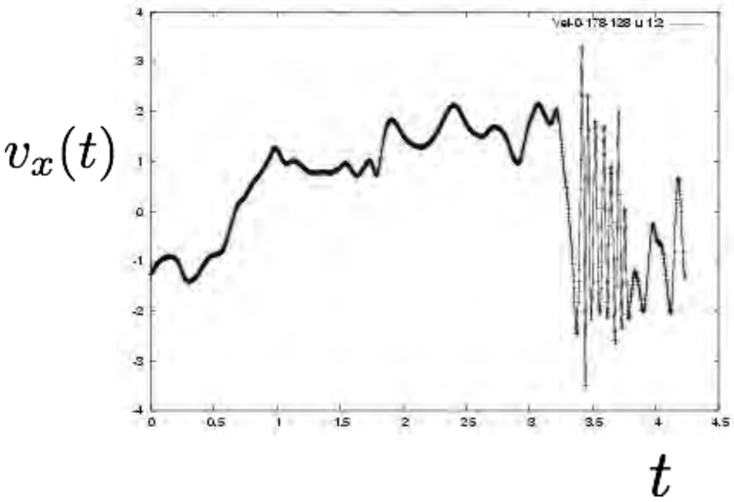
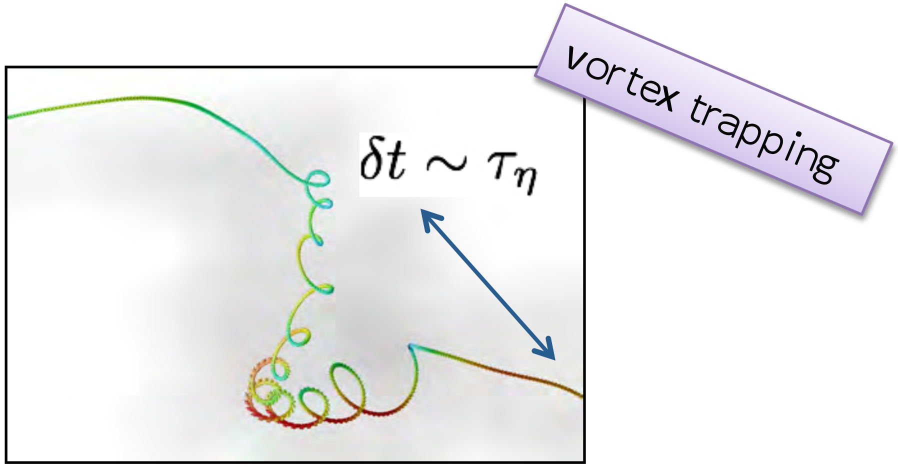


COMPLEX PARTICLES IN COMPLEX FLOWS:

HOW TO ESCAPE/FALL FROM/ON EULERIAN TURBULENT TRAPS?





PHYSICS OF FLUIDS **17**, 021701 (2005)

Particle trapping in three-dimensional fully developed turbulence

L. Biferale
Dipartimento di Fisica and INFN, Università degli Studi di Roma "Tor Vergata," Via della Ricerca Scientifica 1, 00133 Roma, Italy

G. Boffetta
Dipartimento di Fisica Generale and INFN, Università degli Studi di Torino, Via Pietro Giuria 1, 10125 Torino, Italy

A. Celani
CNRS, INLN, 1361 Route des Lucioles, 06560 Valbonne, France

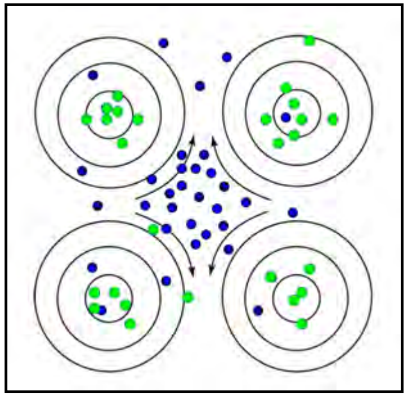
A. Lanotte
CNR-ISAC, Str. Prov. Lecce-Monteroni km. 1200, 73100 Lecce, Italy

F. Toschi
Istituto per le Applicazioni del Calcolo, CNR, Viale del Policlinico 137, 00161 Roma, Italy

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{V}$$

SMALL (POINT-LIKE) INERTIAL PARTICLES

$$\frac{d\boldsymbol{V}}{dt} = \beta \frac{D\boldsymbol{u}(\boldsymbol{X},t)}{Dt} + \frac{\boldsymbol{u}(\boldsymbol{X},t) - \boldsymbol{V}}{St}$$



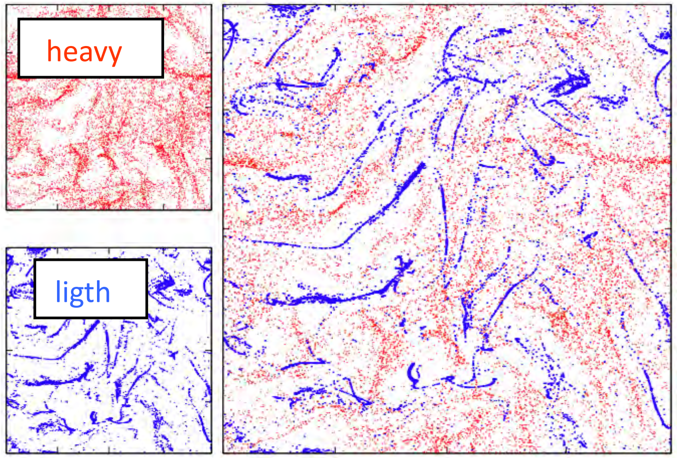
LOCAL COMPRESSIBILITY:

$$\boldsymbol{\nabla} \cdot \boldsymbol{V}(\boldsymbol{x},t) = St(\beta - 1)\boldsymbol{\nabla} \cdot [\boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u}] = St(\beta - 1) Tr A^T A$$

PREFERENTIAL SAMPLING <-> STRAIN/VORTICITY COMPETITION

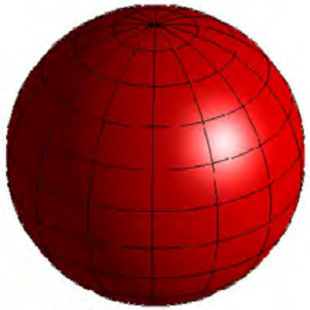
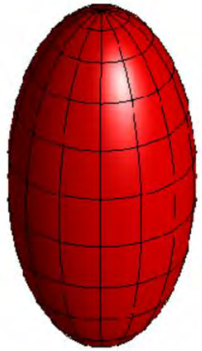
$$\beta < 1 \quad S^2 > \Omega^2 \Longrightarrow \boldsymbol{\nabla} \cdot \boldsymbol{V} < 0$$

$$\beta > 1 \quad \Omega^2 > S^2 \Longrightarrow \boldsymbol{\nabla} \cdot \boldsymbol{V} < 0$$



POSITIVE/NEGATIVE CENTRIFUGE EFFECT

HOW TO CHANGE THE FATE OF A PARTICLE IN COMPLEX FLOWS USING SYMMETRIES

Rotation invariance / Reflection invariance	✓	✗
✓		
✗	**'Isotropic helicoid'** (this talk)	

Example of an isotropic helicoid

Recipe from Lord Kelvin:

"An isotropic helicoid can be made by attaching projecting vanes to the surface of a globe in proper positions; for instance cutting at 45º each, at the middles of the twelve quadrants of any three great circles dividing the globe into eight quadrantal triangles."

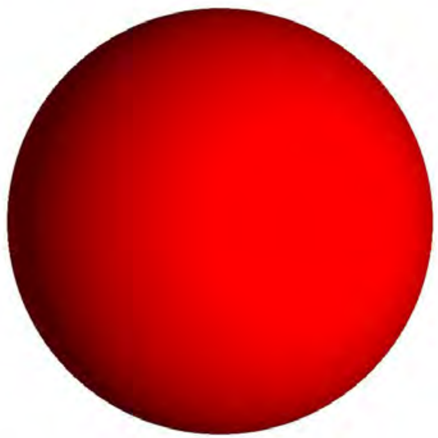
Kelvin, Phil. Mag. **42** (1871)

THES SIMPLEST (BUT NOT SIMPLER) GENERALISATION OF SPHERICAL HEAVY PARTCILES

Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

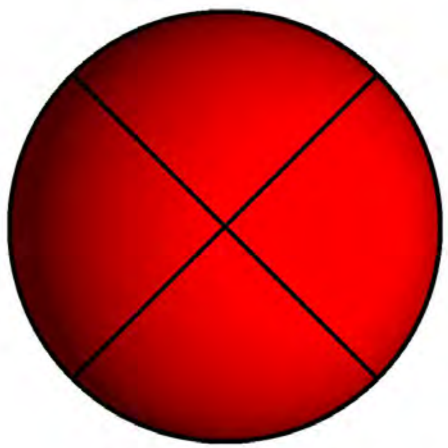
Start with a sphere



Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

✓ Start with a sphere
Draw 3 great circles

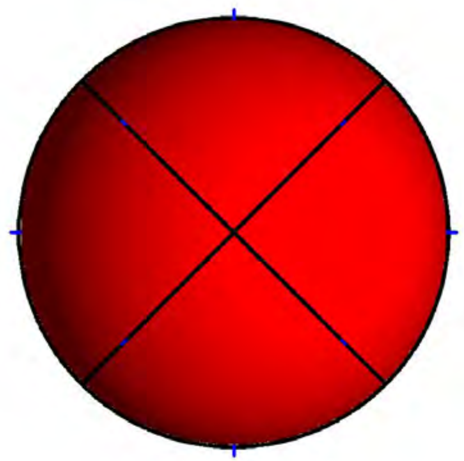


Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

- ✓ Start with a sphere
- ✓ Draw 3 great circles

Identify 12 vane positions at midpoints of quarter-arcs

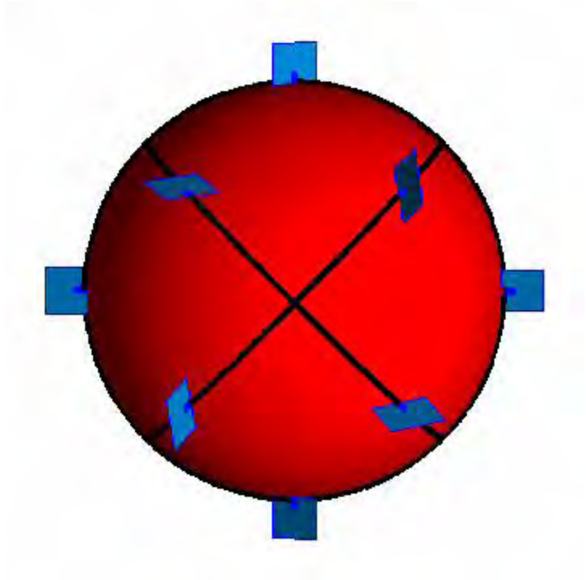


Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

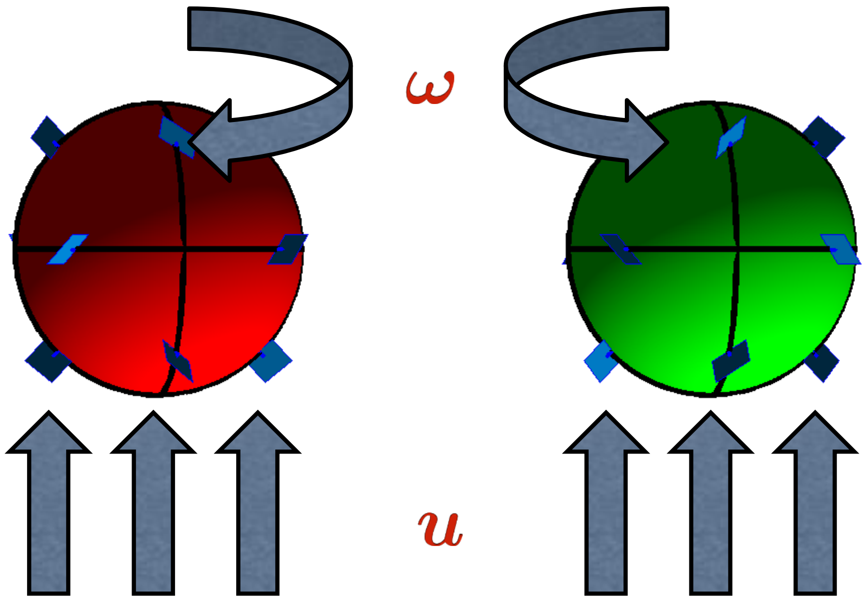
- ✓ Start with a sphere
- ✓ Draw 3 great circles
- ✓ Identify 12 vane positions at midpoints of quarter-arcs

Put a vane on each vane position (45° to arc line)



Chirality

In a constant flow $\boldsymbol{u}$, the isotropic helicoid starts spinning around the flow direction with angular velocity $\boldsymbol{\omega}$.
The spinning direction depends on the chirality of the vanes.



Left handed $C_0 < 0$ *Right handed* $C_0 > 0$

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Motion of an 'isotropic helicoid'

Equations for velocity $\boldsymbol{v}$ and angular velocity $\boldsymbol{\omega}$ for small isotropic helicoid:

Happel & Brenner, Low Reynolds number hydrodynamics (1963)

$$\dot{\boldsymbol{v}} = \frac{1}{\tau_\mathrm{p}}\left[\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v} + \frac{2a}{9}C_0(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega})\right]$$

$$\dot{\boldsymbol{\omega}} = \frac{1}{\tau_\mathrm{p}}\left[\frac{10}{3}(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega}) + \frac{5}{9a}C_0(\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v})\right]$$

Stokes' law translation – rotation coupling (scalar)

$a = \sqrt{5I_0/(2m)}$ Particle 'size' (defined by mass m and moment of inertia I_0)

C_0 Helicoidality

Ratio of rotational and translational inertia fixed to that of sphere

Equations break spatial reflection symmetry ($\boldsymbol{\omega}$ pseudovector)

Dimensionless parameters

Stokes number $\mathrm{St} \equiv \frac{\tau_\mathrm{p}}{\tau_\eta}$ Size $\overline{a} \equiv \frac{a}{\eta}$ Helicoidality C_0

with τ_η and η smallest time- and length scales of flow.

Dynamics may grow indefinitely unless $-\sqrt{27} < C_0 < \sqrt{27}$.

St and $\overline{a}$ constrained by particle density higher than that of the fluid and geometrical size must be smaller than η .

Simulations and theory is done using a random single-scale flow characterised by the Kubo number

$$\mathrm{Ku} \equiv \frac{u_0 \tau_\eta}{\eta}$$

with u_0 typical speed of flow.

Clustering at small St

Expand compressibility of particle-velocity field $\boldsymbol{\nabla}\cdot\boldsymbol{v}$ in small $\mathrm{St}\sim\tau_\mathrm{p}$

$$\boldsymbol{\nabla}\cdot\boldsymbol{v} = -\frac{27}{27-C_0^2}\tau_\mathrm{p}\left[\mathrm{Tr}(\boldsymbol{\nabla u}^\mathrm{T}\boldsymbol{\nabla u}^\mathrm{T}) - \frac{1}{15}\mathrm{aC_0Tr}(\boldsymbol{\nabla u}^\mathrm{T}\boldsymbol{\nabla\Omega}^\mathrm{T})\right]$$

Centrifuge effect with modified amplitude
Maxey, J. Fluid Mech. **174** (1987)

Term due to parity breaking of system

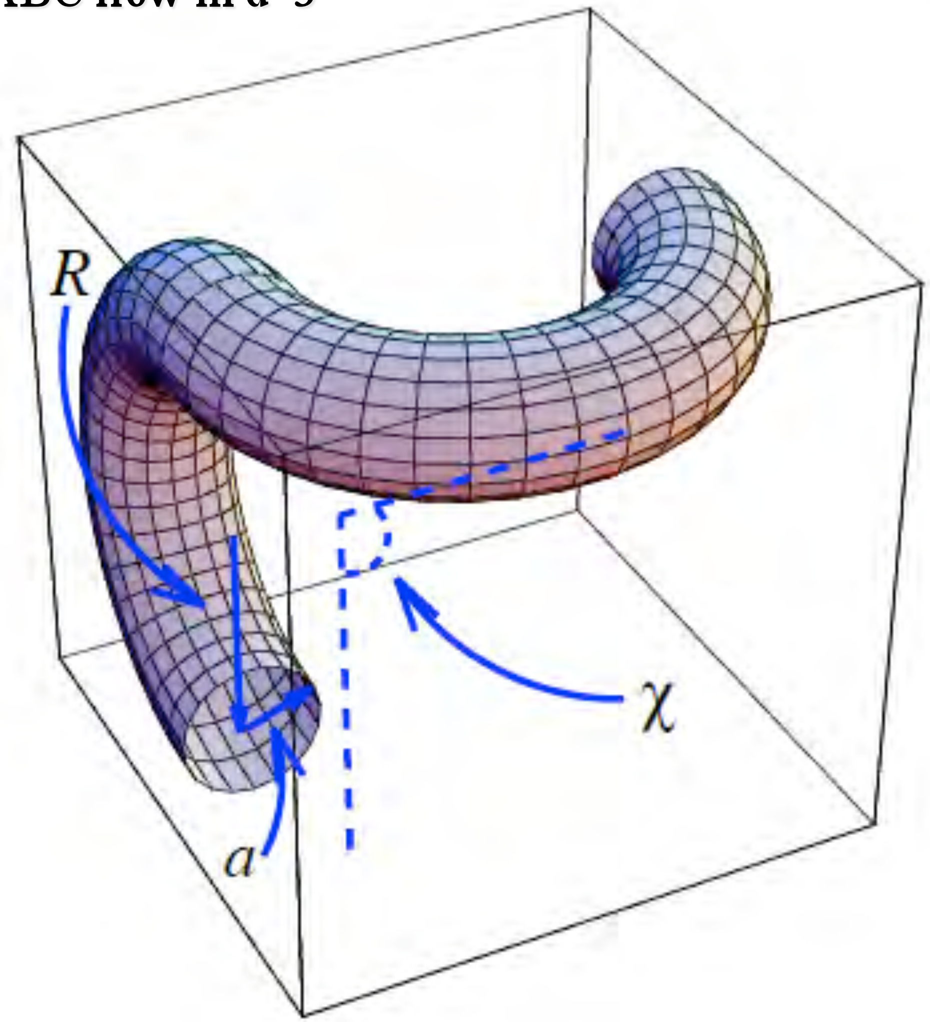
Reflection-invariant systems have $\langle\mathrm{Tr}(\boldsymbol{\nabla u}^\mathrm{T}\boldsymbol{\nabla\Omega}^\mathrm{T})\rangle = 0$

Isotropic helicoids violate that relation $\langle\mathrm{Tr}(\boldsymbol{\nabla u}^\mathrm{T}\boldsymbol{\nabla\Omega}^\mathrm{T})\rangle \propto \tau_\mathrm{p}\mathrm{C}_0$

$\Rightarrow$ In a parity-invariant isotropic flow clustering does not depend on sign of C_0

Eulerian smooth but Lagrangian non-trivial

ABC flow in d=3



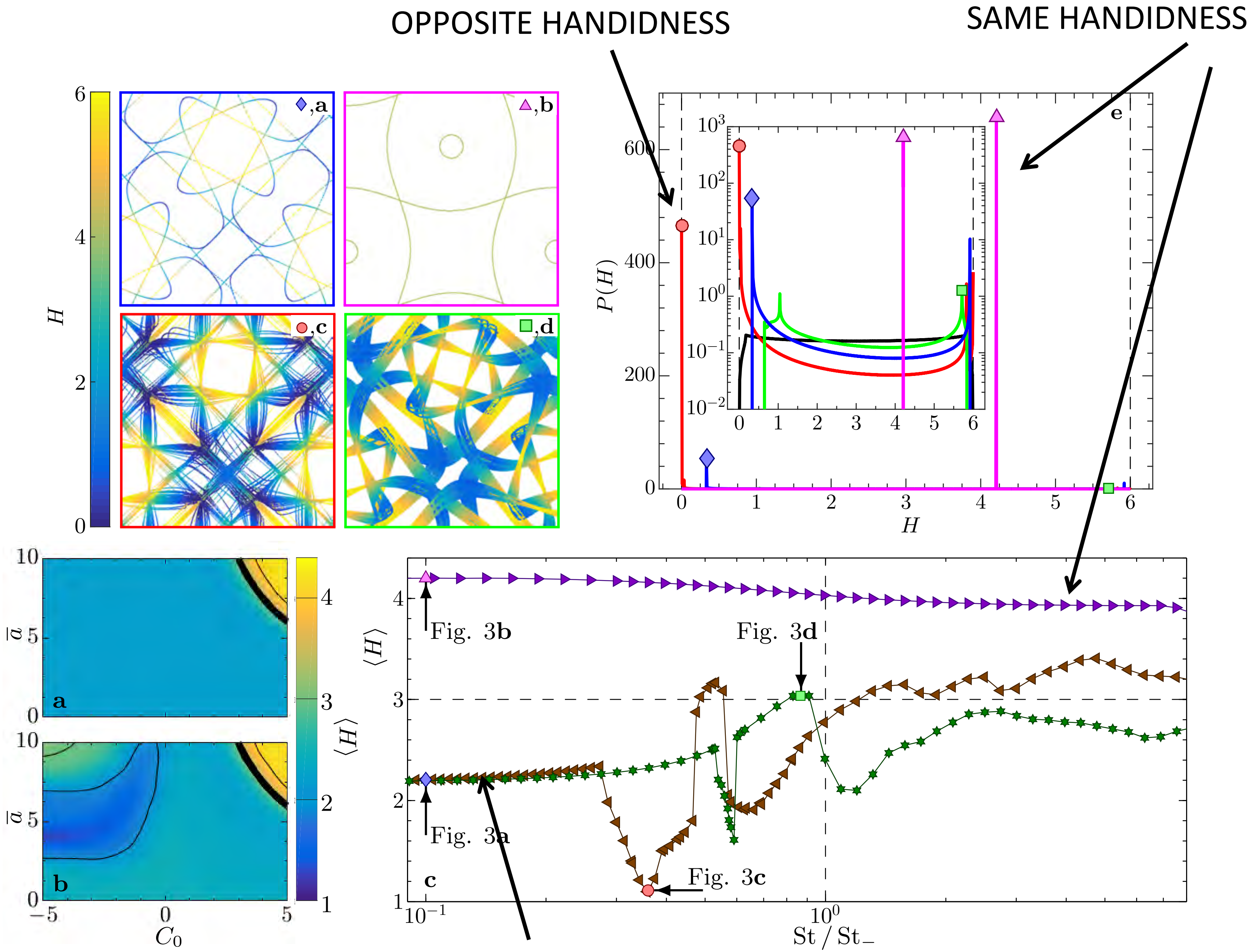
$$\dot{x} = A \sin z + C \cos y,$$
$$\dot{y} = B \sin x + A \cos z,$$
$$\dot{z} = C \sin y + B \cos x.$$

$$\mathbf{v} \parallel \omega$$

Exact stationaty solution of Euler equation

$$\partial_i v_i \propto -\mathrm{Tr}[\mathrm{A}^2](27 - \frac{9\bar{a}C_0}{10}).$$

HELICOIDS MIGHT BEHAVE AS LIGHT OR HEAVY PARTICLES !!!

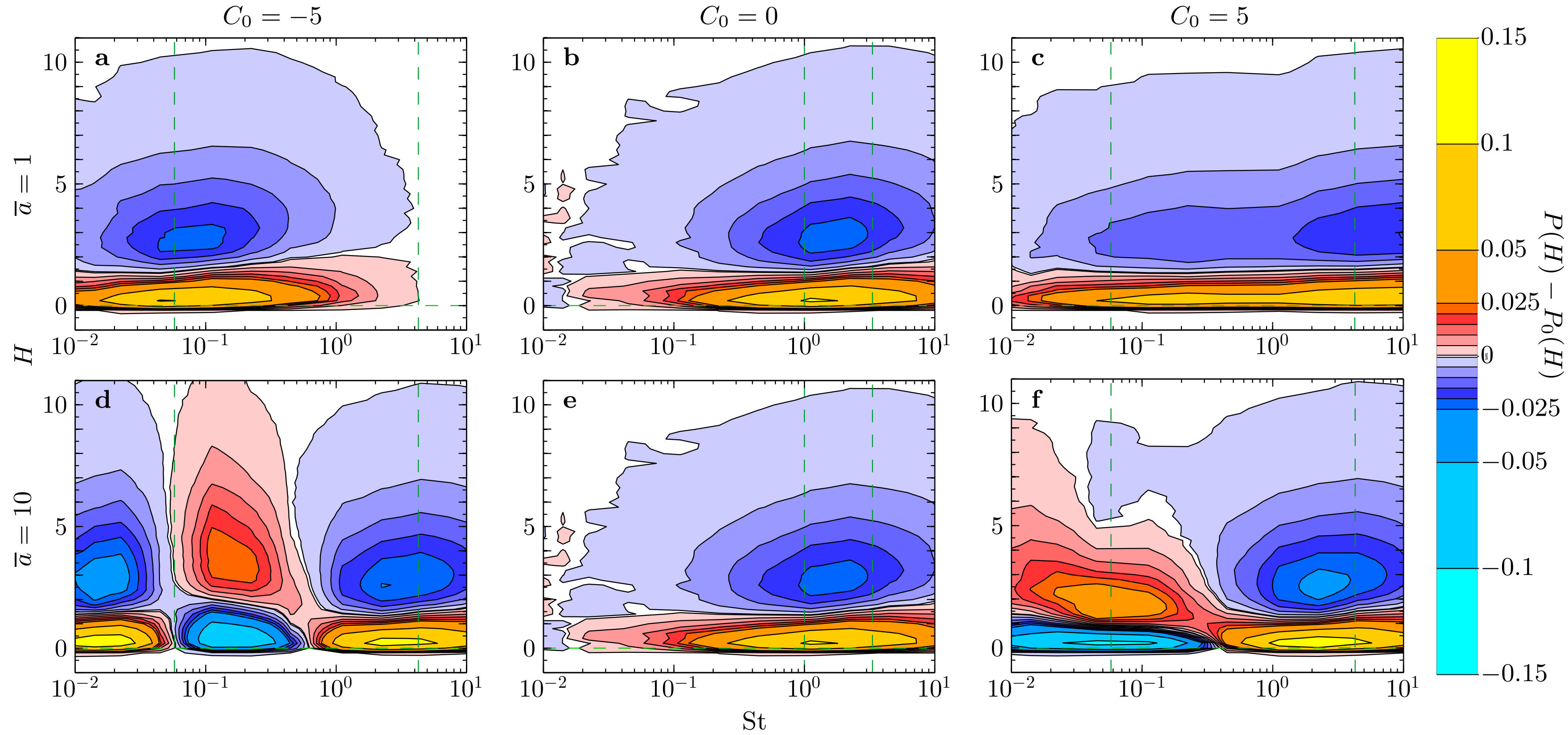


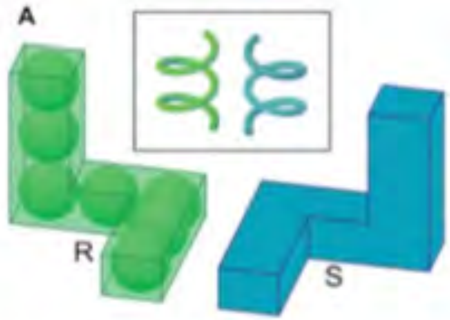
OPPOSITE HANDIDNESS

$$\partial_i v_i \propto -\mathrm{Tr}[\mathbb{A}^2](27 - \frac{9\bar{a}C_0}{10}).$$

STOCHASTIC HELICAL FLOW

$$P_{\mathrm{M}_H}(H) = \frac{|H| \exp\left[\frac{\alpha H\,\mathrm{M}_H}{\beta+\gamma\,\mathrm{M}_H^2}\right] K_1\left[\frac{\delta|H|}{\beta+\gamma\,\mathrm{M}_H^2}\right]}{\sqrt{\beta+\gamma\,\mathrm{M}_H^2}}$$





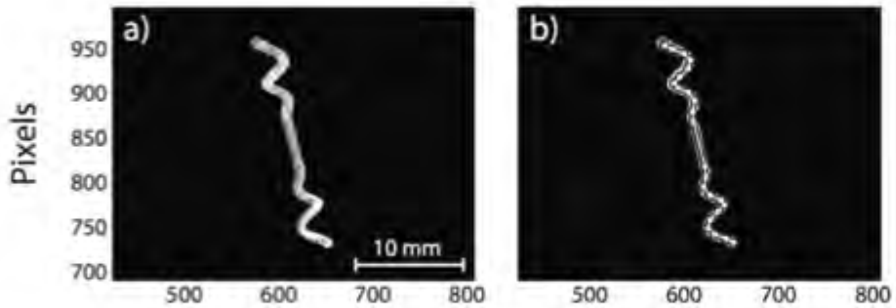
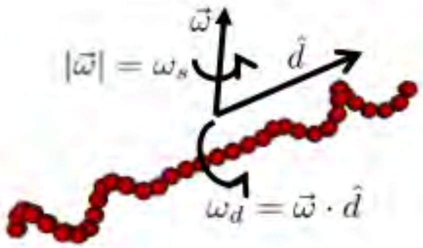


Figure 5: An example of a complex particle, a "strain probe" is basically a chiral-dipole and is sensitive to the local strain level in turbulence. This type of 3D printed particles have been designed and tracked for both position and orientation in turbulent flows by means of optical techniques [20], [24]. Similarly shaped particles were studied numerically by means of Stokesian dynamics simulations [25] (see Figure 6).



S. Kramel, S. Tympel, F. Toschi, and G. A. Voth, "Preferential rotation of chiral dipoles in isotropic turbulence" Phys. Rev. Lett 117 154501 (2016)

Conclusions

Isotropic helicoids are rotation invariant particles which break reflection invariance (two chiralities)

Coupling between translational and rotational degrees of freedom changes dynamics compared to spherical particles (modified clustering, preferential sampling etc.)

The two chiralities may show different dynamics if the particle size is not too small and flow is persistent

Flows with broken parity invariance increase the differences in the dynamics of the two particles

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