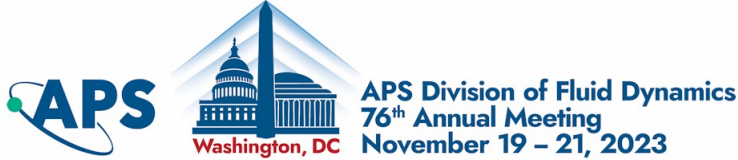# Optimal tracking strategies in a turbulent flow

**Chiara Calascibetta[1], Luca Biferale[1], Francesco Borra[2],**

**Antonio Celani[3], Massimo Cencini[4]**

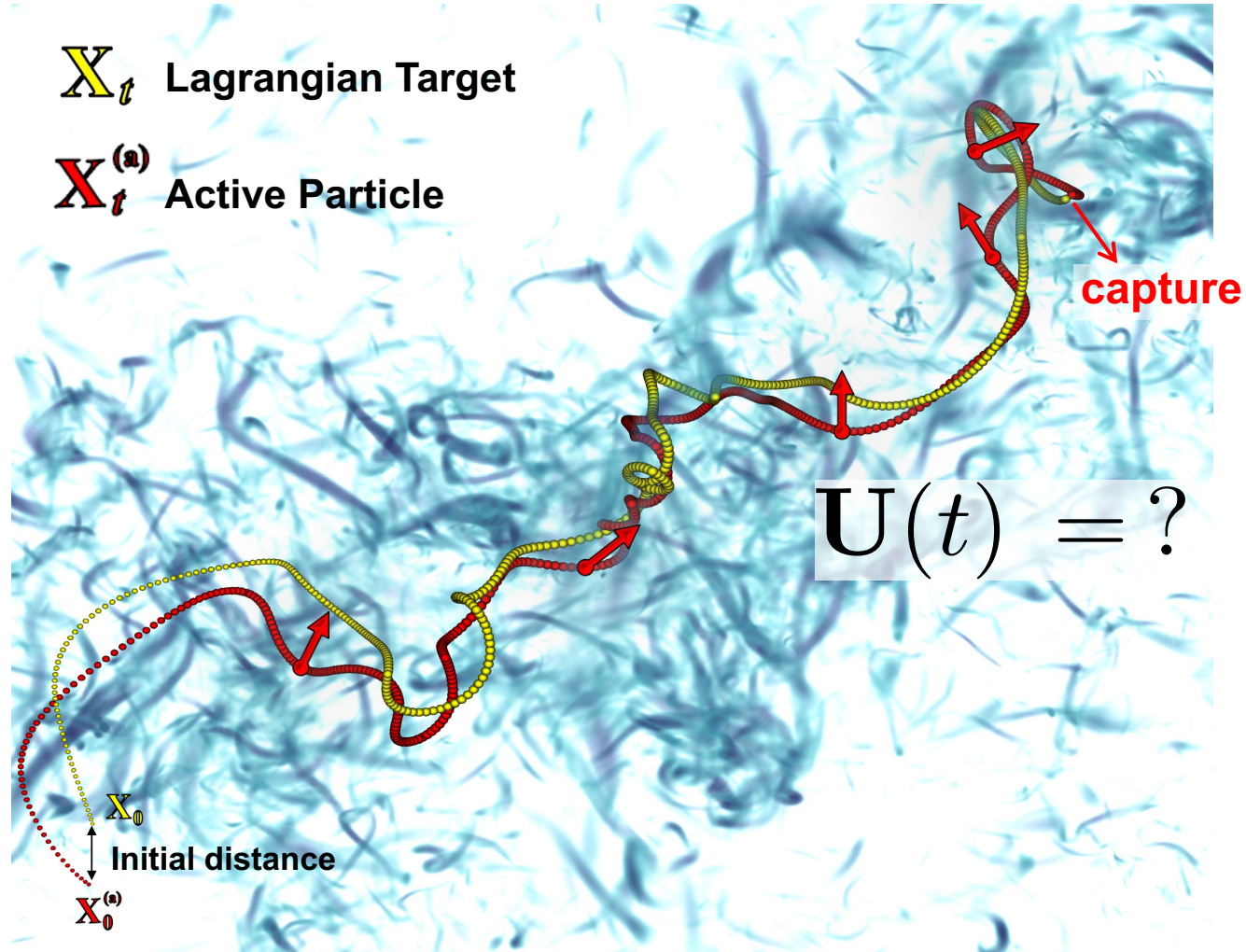[1] Department of Physics and INFN, University of Rome, Tor Vergata (IT)
[2] Laboratoire de physique de l'École normale supérieure, CNRS Paris (FR)
[3] Quantitative Life Sciences, The Abdus Salam International Centre for Theoretical Physics, ICTP (IT)
[4] Istituto dei Sistemi Complessi, CNR and INFN Tor Vergata (IT)
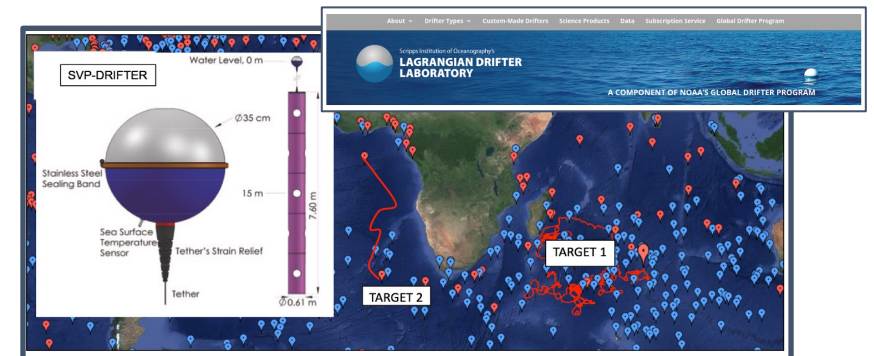
calascibetta@roma2.infn.it

$\mathbf{X}_t$ **Lagrangian Target**

$\mathbf{X}_t^{(a)}$ **Active Particle**

**capture**

$\mathbf{U}(t) = ?$

$\mathbf{X}_0$

**Initial distance**

$\mathbf{X}_0^{(a)}$
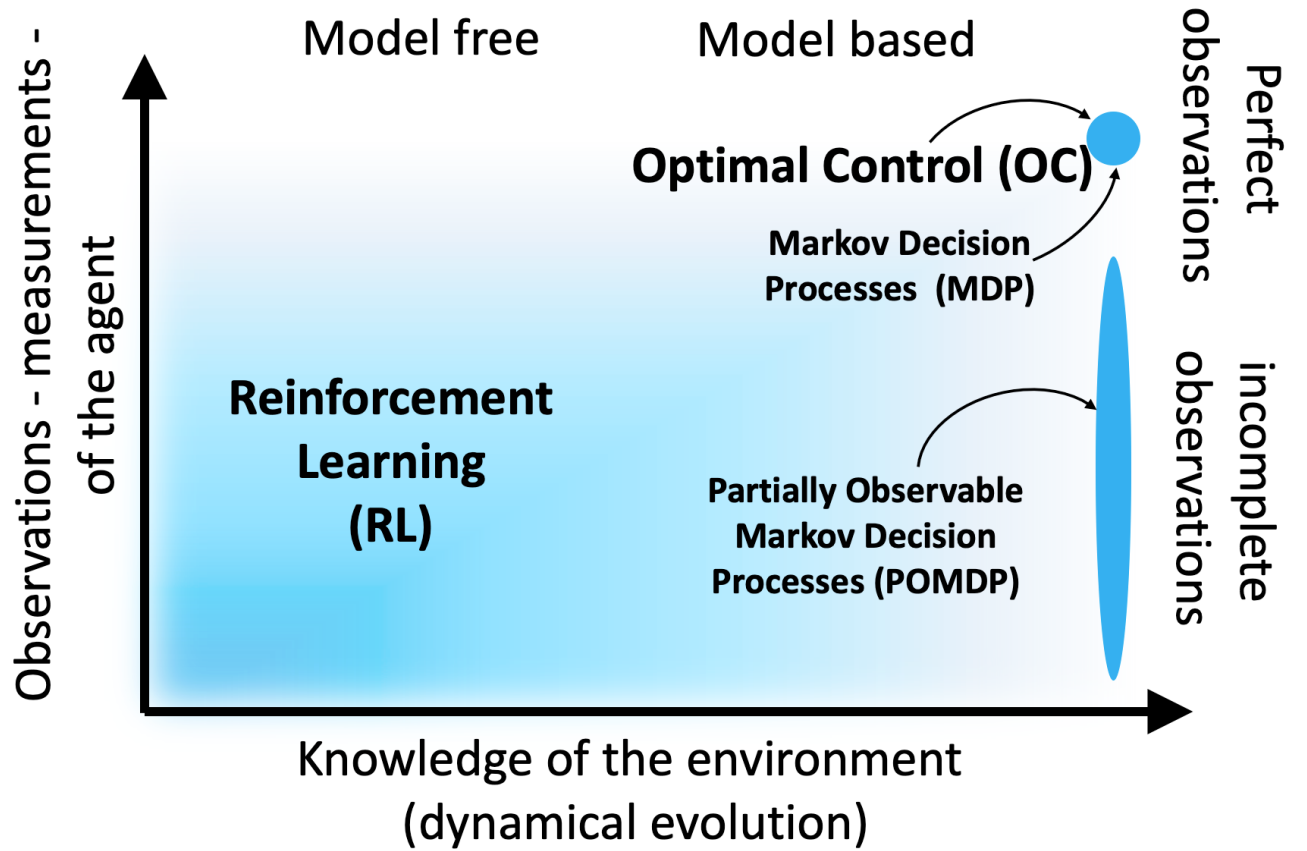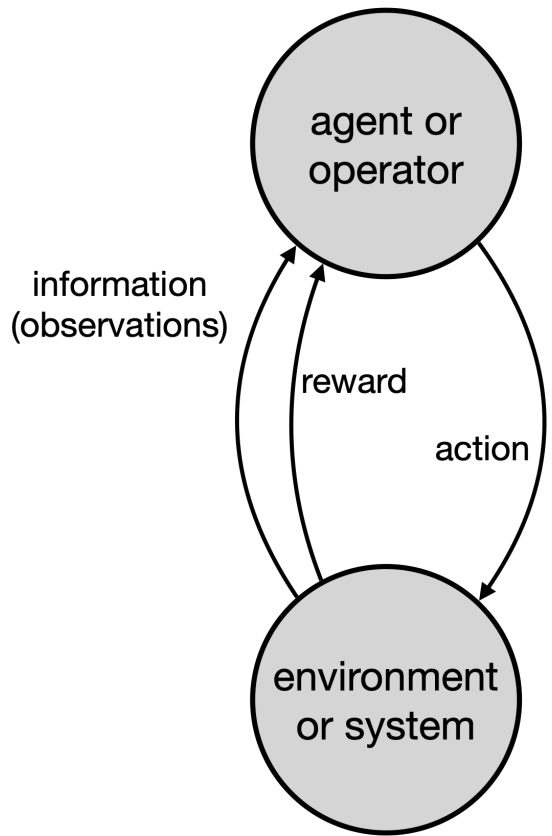
**Time-Dependent 3D Turbulent Flow**

$$
\begin{cases}
\dot{\mathbf{X}}_t = \boldsymbol{v}(\mathbf{X}_t, t) \\
\dot{\mathbf{X}}_t^{(a)} = \boldsymbol{v}(\mathbf{X}_t^{(a)}, t) + \mathbf{U}(t) \\
\mathbf{U}(t) = V_s \hat{\boldsymbol{n}}(t)
\end{cases}
$$

**Small active control** compared to the underling flow!

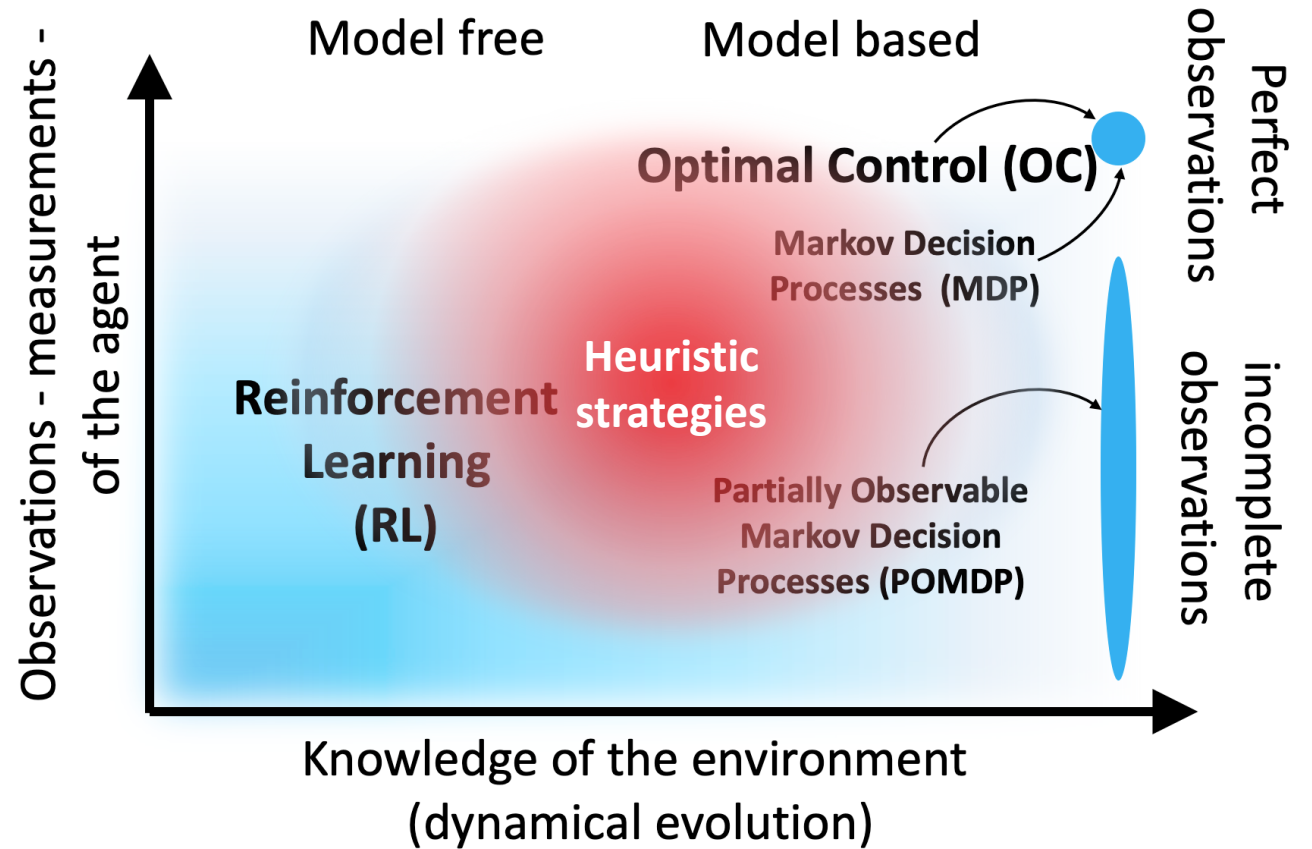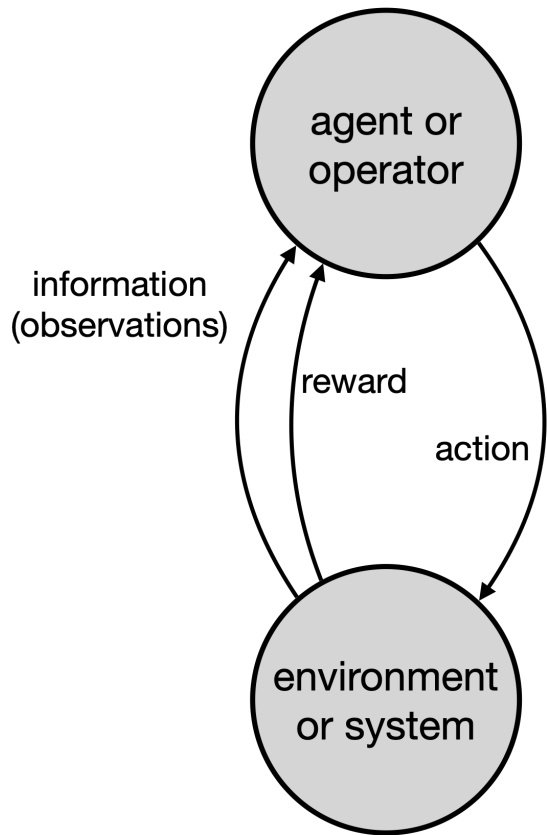**Goal:** **minimize** the **separation/capture** in a finite time horizon

Control Theory

Sutton, R. S., & Barto, A. G. **Reinforcement learning: An introduction**. MIT press (2018).
Bryson, A. E. and Ho, Y. **Applied optimal control: optimization, estimation and control** (New York: Routledge, 1975).

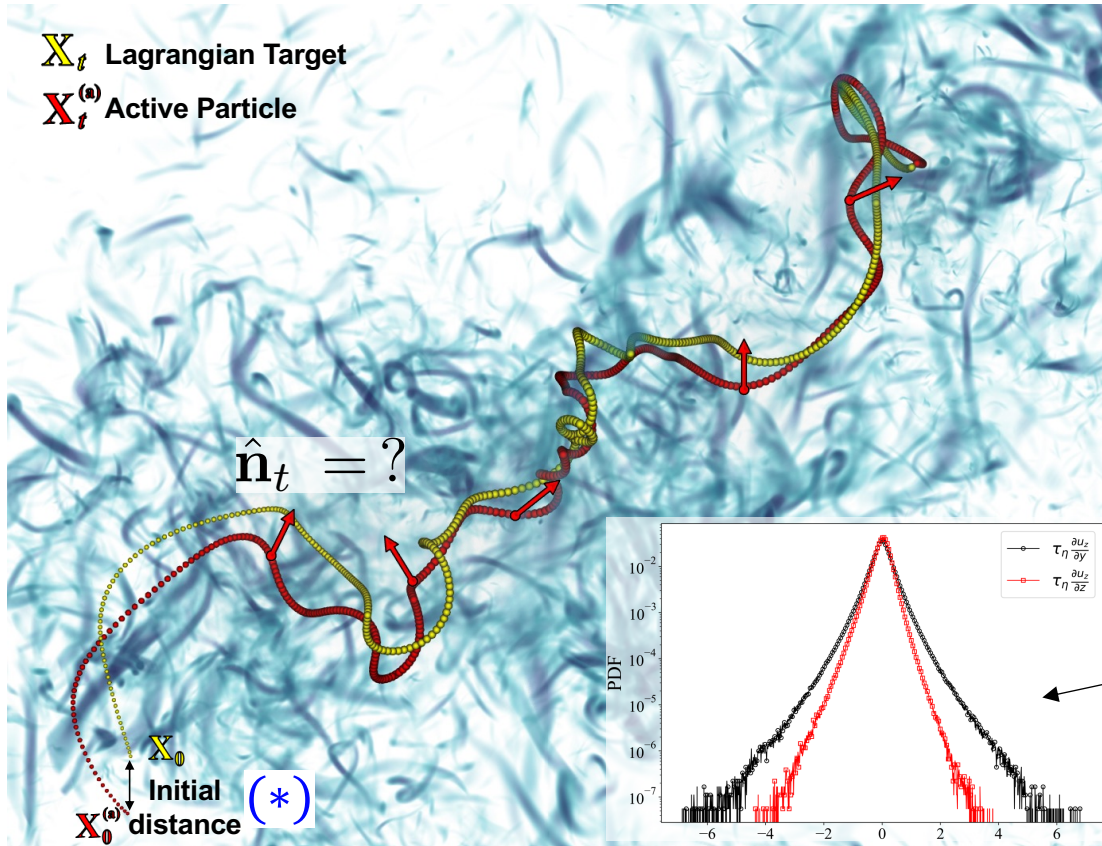https://hdl.handle.net/20.500.11767/84114
Sutton, R. S., & Barto, A. G. **Reinforcement learning: An introduction**. MIT press (2018).
Bryson, A. E. and Ho, Y. **Applied optimal control: optimization, estimation and control** (New York: Routledge, 1975).

$$\boldsymbol{R}_t = \mathbf{X}_t^{(a)} - \mathbf{X}_t$$



$\mathbf{X}_t$ Lagrangian Target

$\mathbf{X}_t^{(a)}$ Active Particle

$\hat{\mathbf{n}}_t = ?$

$\mathbf{X}_0$

Initial distance $(*)$

$\mathbf{X}_0^{(a)}$

$(*)$ Small scales $\|\boldsymbol{R}_{t_0}\| < \eta$

$$\begin{cases} \dot{\mathbf{X}}_t = \boldsymbol{v}(\mathbf{X}_t, t) \\ \dot{\mathbf{X}}_t^{(a)} = \boldsymbol{v}(\mathbf{X}_t^{(a)}, t) + \mathbf{U}(t) \\ \mathbf{U}(t) = V_s \hat{\boldsymbol{n}}(t) \end{cases}$$

$$\boldsymbol{v}(\mathbf{X}_t^{(a)}, t) \simeq \boldsymbol{v}(\mathbf{X}_t, t) + \nabla \boldsymbol{v}_t \boldsymbol{R}_t$$

$$\dot{\boldsymbol{R}}_t = \nabla \boldsymbol{v}_t \boldsymbol{R}_t + \mathbf{U}(t)$$

| Tools: | | |
|---|---|---|
| **(1) Optimal Control theory** | **(2) Heuristic control strategies** | **(3) Reinforcement Learning** |
| Model based problem (full knowledge required). | Reactive controls based on local-only in time cues. | Model free tool. Some past memory can be considered. |

Biferale, L., Bonaccorso, F., Buzzicotti, M., & <u>Calascibetta, C</u>. (2023). **TURB-Lagr. A database of 3d Lagrangian trajectories in homogeneous and isotropic turbulence**. (arXiv:2303.08662)

- **Model based** and analytical tool (Perfect knowledge required)

- **Constrained minimization problem**

Minimize $\quad J = \|\boldsymbol{R}_{t_f}\|^2 + c \int_{t_0}^{t_f} dt\, \theta(\|\boldsymbol{R}_t\|^2 - \|\boldsymbol{R}_c\|^2)$

$\downarrow$

*capture's distance*

$$\|\boldsymbol{R}_c\| = \frac{\|\boldsymbol{R}_{t_0}\|}{100}$$

Imposing: $\begin{cases} \dot{\boldsymbol{R}}_t = \nabla \boldsymbol{v}_t \boldsymbol{R}_t + \mathbf{U}(t)\,, \\ \boldsymbol{R}_{t_0} = \text{ given}\,, \\ \mathbf{U}(t) = V_s \hat{\boldsymbol{n}}(t) \\ \|\hat{\boldsymbol{n}}(t)\| = 1\,. \end{cases}$

$\lambda = \text{lyapunov exponent}$

$\|\boldsymbol{R}_{t_0}\| \sim \dfrac{V_s}{\lambda}\quad$ border of controllability

<span style="color:red">Minimize trajectories' separation</span>

<span style="color:blue">Minimize time of arrival at the desired distance</span>

**Euler-Lagrange equations**

**Calascibetta, C.**, Biferale, L., Borra, F., Celani, A. and Cencini, M. **Optimal tracking strategies in a turbulent flow.** *Commun. Phys.* **6**, 256 (2023).

**Pure Pursuit (PP):** constantly chooses the direction that points towards the moving target, $\hat{\boldsymbol{n}}(t) = -\hat{\boldsymbol{R}}_t$

**Surfing Control (SC)\*:**  - constant gradients for a time $\tau_s$ (free parameter);
- maximization of the searcher displacement along the $\boldsymbol{R}_t$ direction;
- good for slowly varying $\boldsymbol{R}_t$ (i.e., at large scales)

$$\hat{\boldsymbol{n}}(t) = -\frac{\left[e^{(\tau_s - t)\nabla \boldsymbol{v}_{t_0}}\right]^{\mathrm{T}} \cdot \hat{\boldsymbol{R}}_{t_0}}{\left\|\left[e^{(\tau_s - t)\nabla \boldsymbol{v}_{t_0}}\right]^{\mathrm{T}} \cdot \hat{\boldsymbol{R}}_{t_0}\right\|}$$

**Perturbative Optimal Control (PO):**  - 0$^{\text{th}}$ order OC with constant gradients for a time $\tau_p$ (free parameter) ;
- valid at small scales

$$\hat{\boldsymbol{n}}(t) = -\frac{\left[e^{(\tau_p - t)\nabla \boldsymbol{v}_{t_0}}\right]^{\mathrm{T}} \cdot e^{(\nabla \boldsymbol{v})_{t_0}\tau_p} \cdot \hat{\boldsymbol{R}}_{t_0}}{\left\|\left[e^{(\tau_p - t)\nabla \boldsymbol{v}_{t_0}}\right]^{\mathrm{T}} \cdot e^{(\nabla \boldsymbol{v})_{t_0}\tau_p} \cdot \hat{\boldsymbol{R}}_{t_0}\right\|}$$

**\***Monthiller, Rémi, et al. **Surfing on Turbulence: A Strategy for Planktonic Navigation**. *Phys. Rev. Lett.* **129**, *064502* (2022)

**Calascibetta, C.**, Biferale, L., Borra, F., Celani, A. and Cencini, M. **Optimal tracking strategies in a turbulent flow.** *Commun. Phys.* **6**, 256 (2023).

Optimal Control vs heuristic strategies: **capture time statistics**

$$\dot{\boldsymbol{R}}_t = \nabla \boldsymbol{v}_t \boldsymbol{R}_t + \mathbf{U}(t)$$

$t_c = $ **Capture time**: (time of arrival at the desired distance)

**PDF of normalized capture time**

**Calascibetta, C.**, Biferale, L., Borra, F., Celani, A. and Cencini, M. **Optimal tracking strategies in a turbulent flow.** *Commun. Phys.* **6**, 256 (2023).
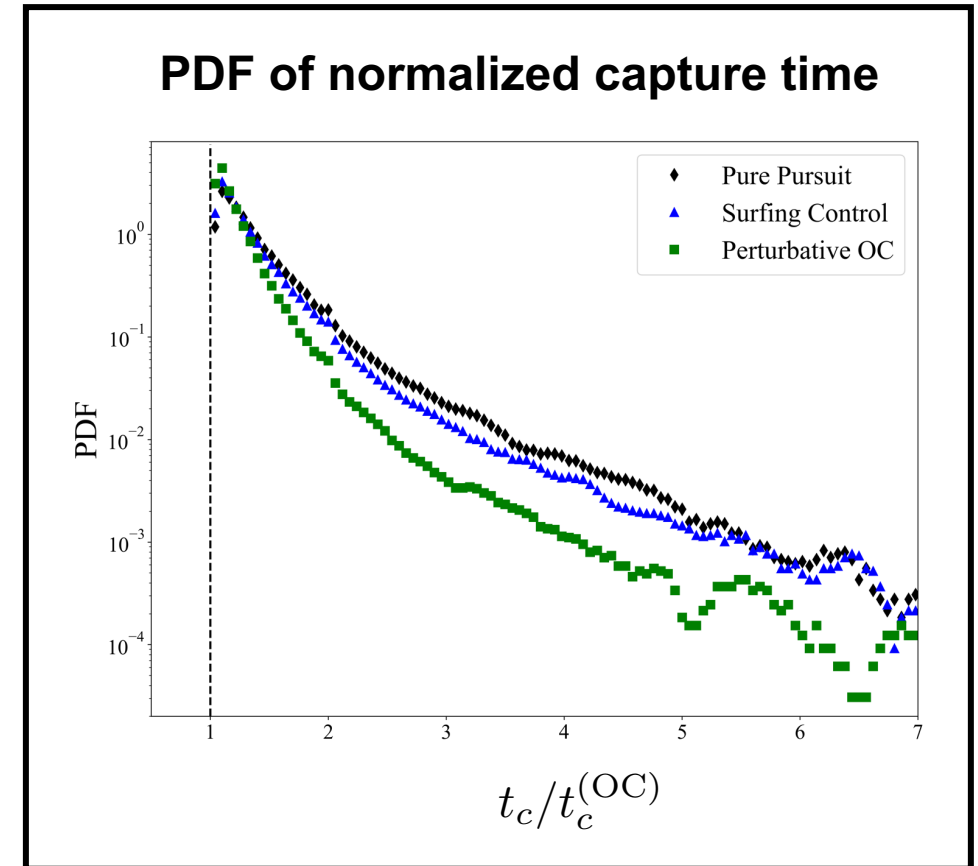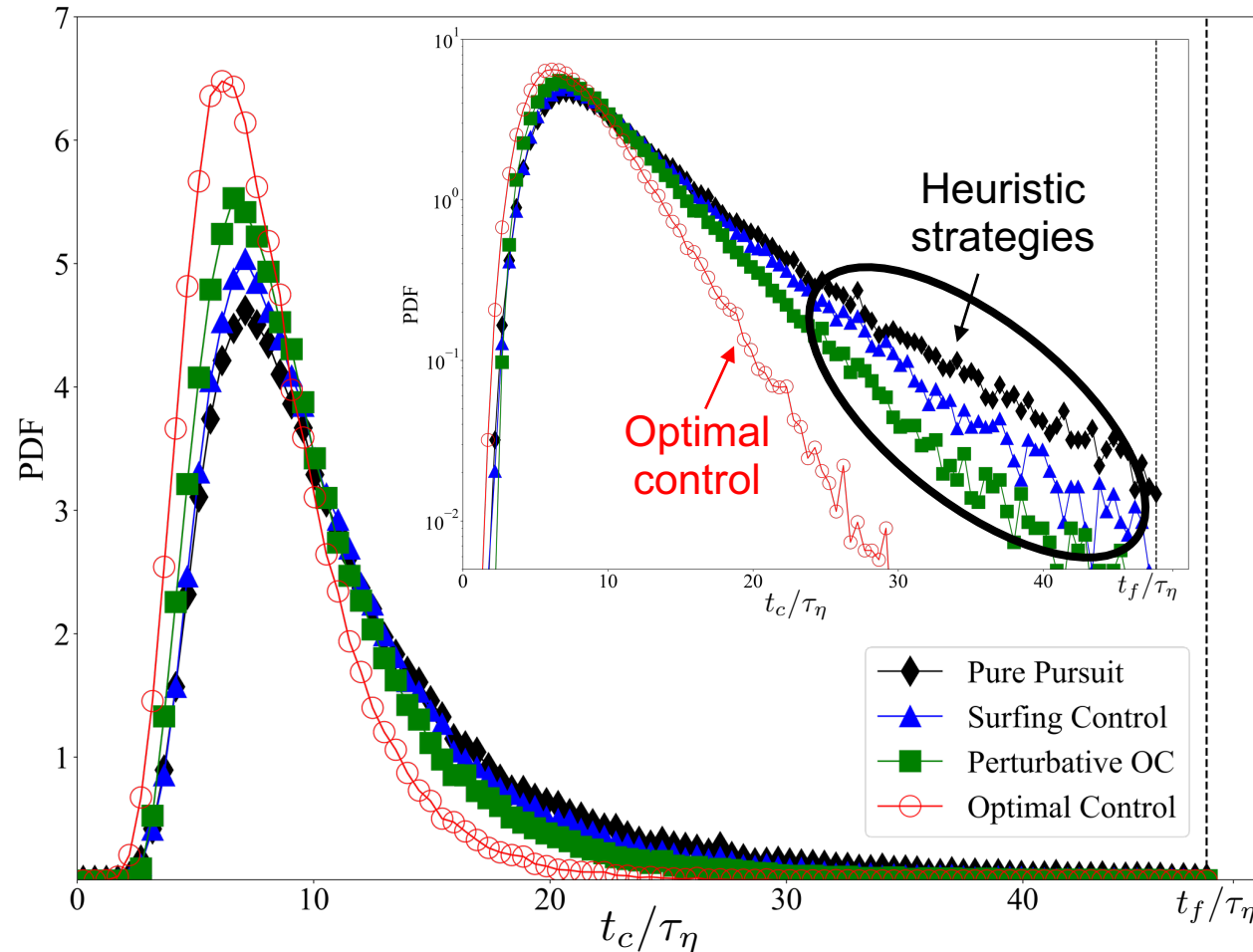
$t_c^{(OC)}$ = Capture time for the unperturbed OC trajectory    $R_{t_c^{(OC)}}$ = Distance of perturbed trajectories
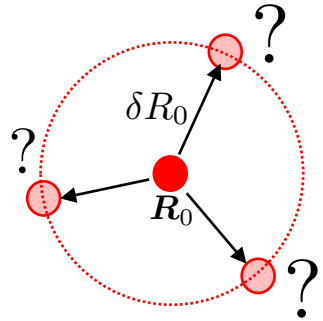
**OC is stable under perturbations of the initial conditions!!!**

1. The target trajectory becomes an attractor for the controlled dynamics.
2. Larger capture times are connected to more chaotic trajectories.

FTLE: $\gamma_{t_c^{(OC)}} = 1/t_c^{(OC)} \ln(R_{t_c^{(OC)}}/R_0)$

**Calascibetta, C.**, Biferale, L., Borra, F., Celani, A. and Cencini, M. **Optimal tracking strategies in a turbulent flow.** *Commun. Phys.* **6**, 256 (2023).
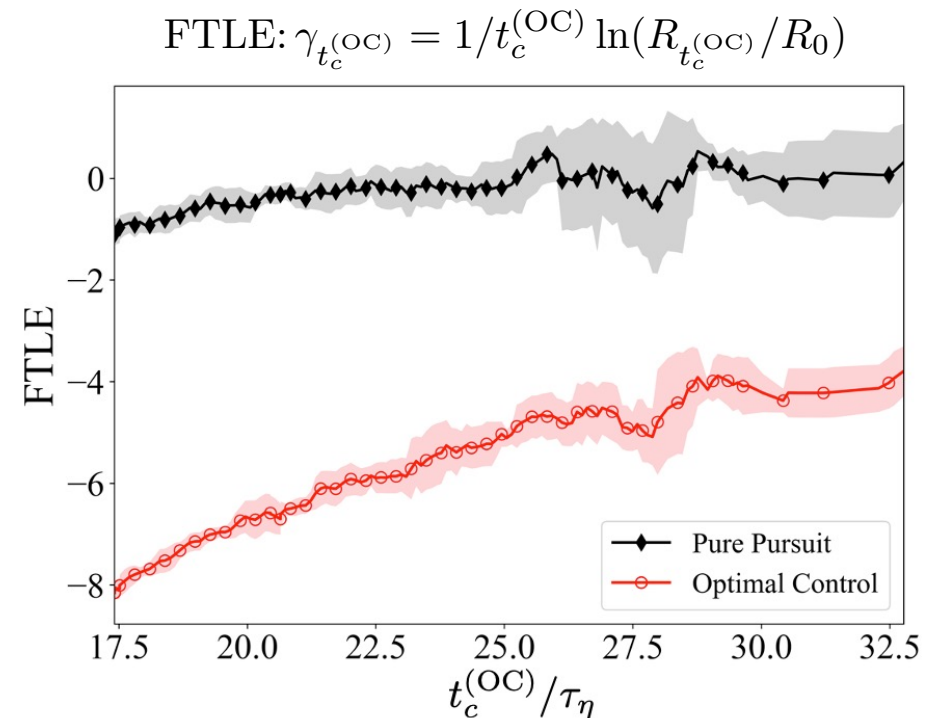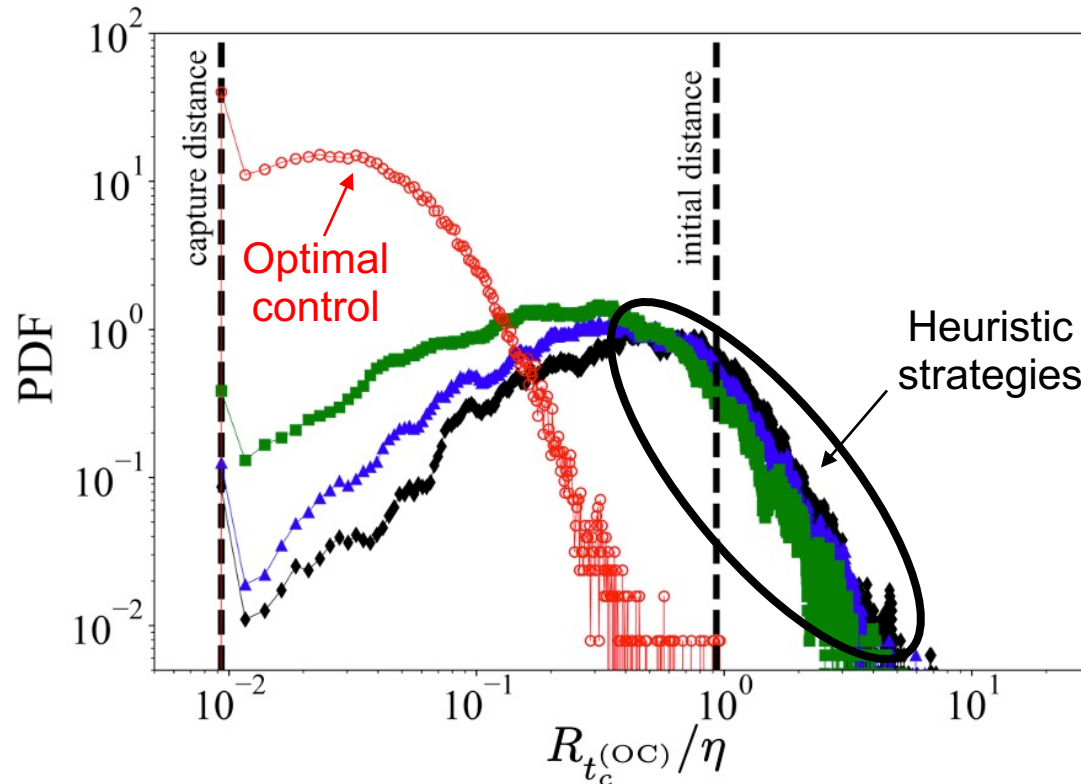
## pros & cons

### Optimal Control (*)

**+** It is optimized
**-** It is model based and needs perfect information (fully history) from the environment
**+** It is resilient to disturbances on the agent starting conditions
**-** It is difficult to consider a decision time in the control variable

### Heuristic strategies (*)

**-** They are not optimized and display worse performances wrt OC
**+** They need only partial information (reactive strategies)
**+** They can be applied also with a discrete decision time

### Reinforcement Learning (**) (not shown) ⟶

**+** It is optimized
**+** It is model free
**+** It needs partial information (past memory can be included)
**-** It is data-hungry
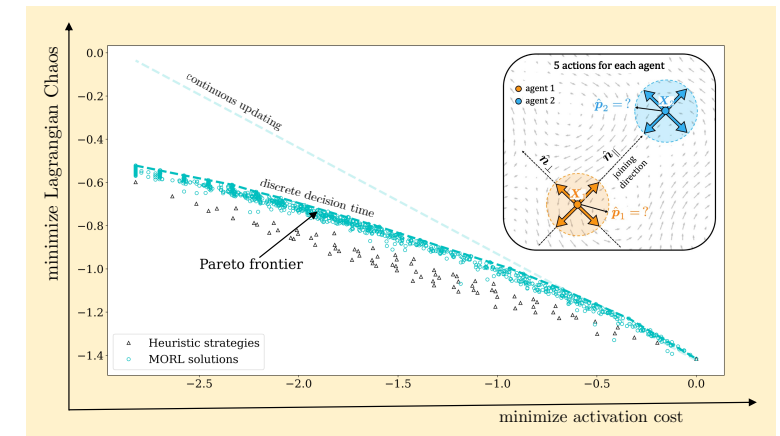
(*) **Calascibetta, C.**, Biferale, L., Borra, F., Celani, A. and Cencini, M. **Optimal tracking strategies in a turbulent flow.** *Commun. Phys.* **6**, 256 (2023).

(**) **Calascibetta, C.**, Biferale, L., Borra, F., Celani, A. and Cencini, M. **Taming Lagrangian Chaos with multi-objective reinforcement learning.** *Eur. Phys. J. E* **46**, 9 (2023).
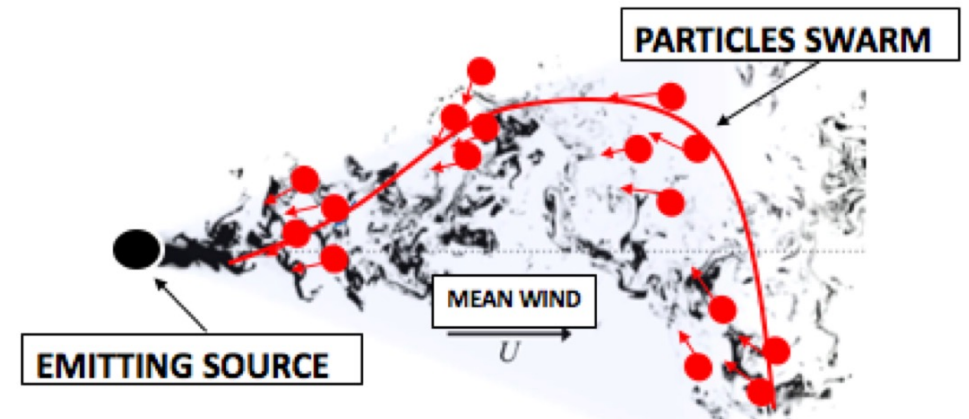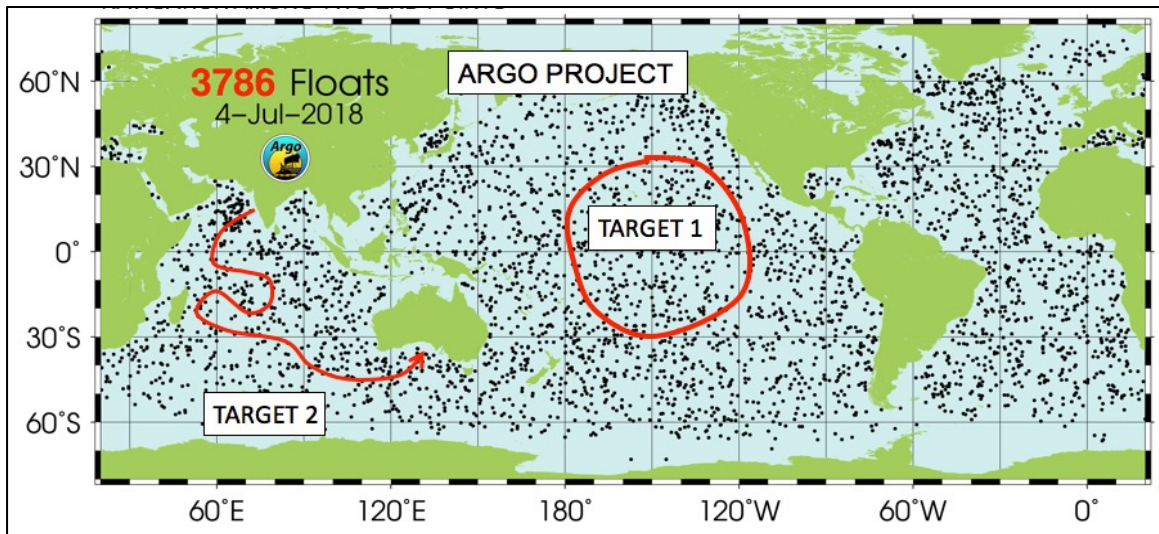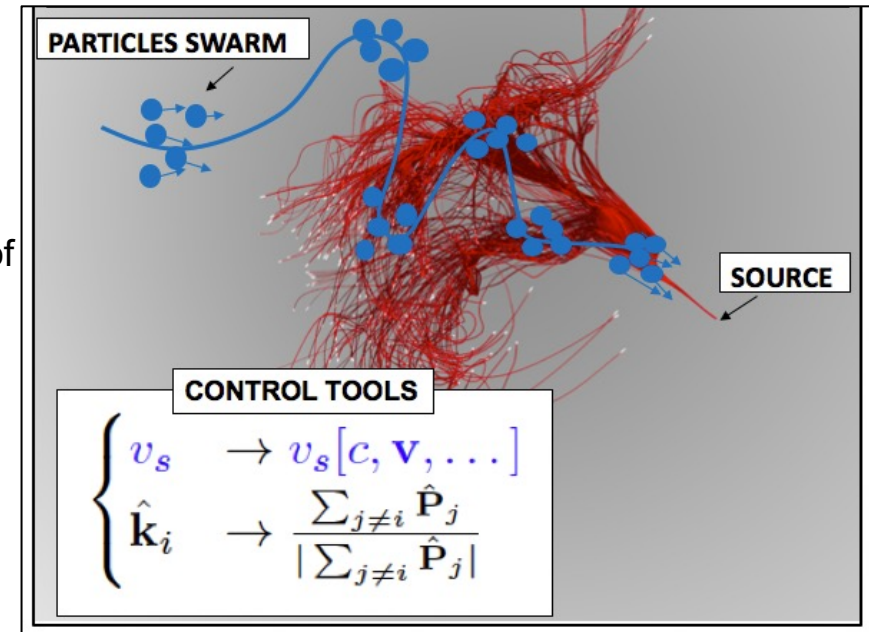
# Conclusions

**Open questions:**
1. How to fill the gap between heuristic strategies and optimal control performances?
2. How to control a multi-agent system to minimize turbulent dispersion at larger range of scales (i.e., beyond the linear regime)?
3. Are the agents able to collaborate with each-other during the navigation?

**Tools:**
We can develop new heuristic controls with short-term memory
We can use RL to control autonomous swimmer to reach complex goals
We can apply OC theory as a benchmark to test the heuristic/data-driven solutions.



PARTICLES SWARM
SOURCE

CONTROL TOOLS

$$\begin{cases} v_s & \to v_s[c, \mathbf{v}, \dots] \\ \hat{\mathbf{k}}_i & \to \dfrac{\sum_{j \neq i} \hat{\mathbf{P}}_j}{|\sum_{j \neq i} \hat{\mathbf{P}}_j|} \end{cases}$$



ARGO PROJECT
3786 Floats
4–Jul–2018
TARGET 1
TARGET 2



PARTICLES SWARM
MEAN WIND $U$
EMITTING SOURCE

# References

- **Calascibetta**, Biferale, Borra, Celani & Cencini. **Comm. Phys. 6, 256 (2023).** Optimal tracking strategies in a turbulent flow.

- **Calascibetta**, Biferale, Borra, Celani & Cencini. **Eur. Phys. J. E 46, 9 (2023).** Taming Lagrangian Chaos with multi-objective reinforcement learning.

- Biferale, Bonaccorso, Buzzicotti & **Calascibetta**. (2023). **TURB-Lagr. A database of 3d Lagrangian trajectories in homogeneous and isotropic turbulence**. arXiv preprint arXiv:2303.08662.

- Borra, Biferale, Cencini & Celani. **Phys. Rev. Fluids 7, 023103 (2022).** Reinforcement learning for pursuit and evasion of microswimmers at low Reynolds number.

- Biferale, Bonaccorso, Buzzicotti, Clark Di Leoni & Gustavsson. **Chaos: An Interdisciplinary Journal of Nonlinear Science. 2019 Oct 24;29(10):103138.** Zermelo's problem: Optimal point-to-point navigation in 2D turbulent flows using reinforcement learning.

- Buzzicotti, Biferale, Bonaccorso, Clark Di Leoni & Gustavsson. **Springer, Cham, (2021)**. Optimal Control of Point-to-Point Navigation in Turbulent Time Dependent Flows Using Reinforcement Learning.

- Colabrese, Gustavsson, Celani, Biferale. **Phys. Rev. Fluids 3**, **084301, (2018)**. Smart Inertial Particles.

- Colabrese, Gustavsson, Celani, Biferale. **Phys. Rev. Letters 118 (15), 158004, (2017)**. Flow navigation by smart microswimmers via reinforcement learning.

https://smart-turb.roma2.infn.it/