Direct and/or inverse energy cascades in TURBULENCE

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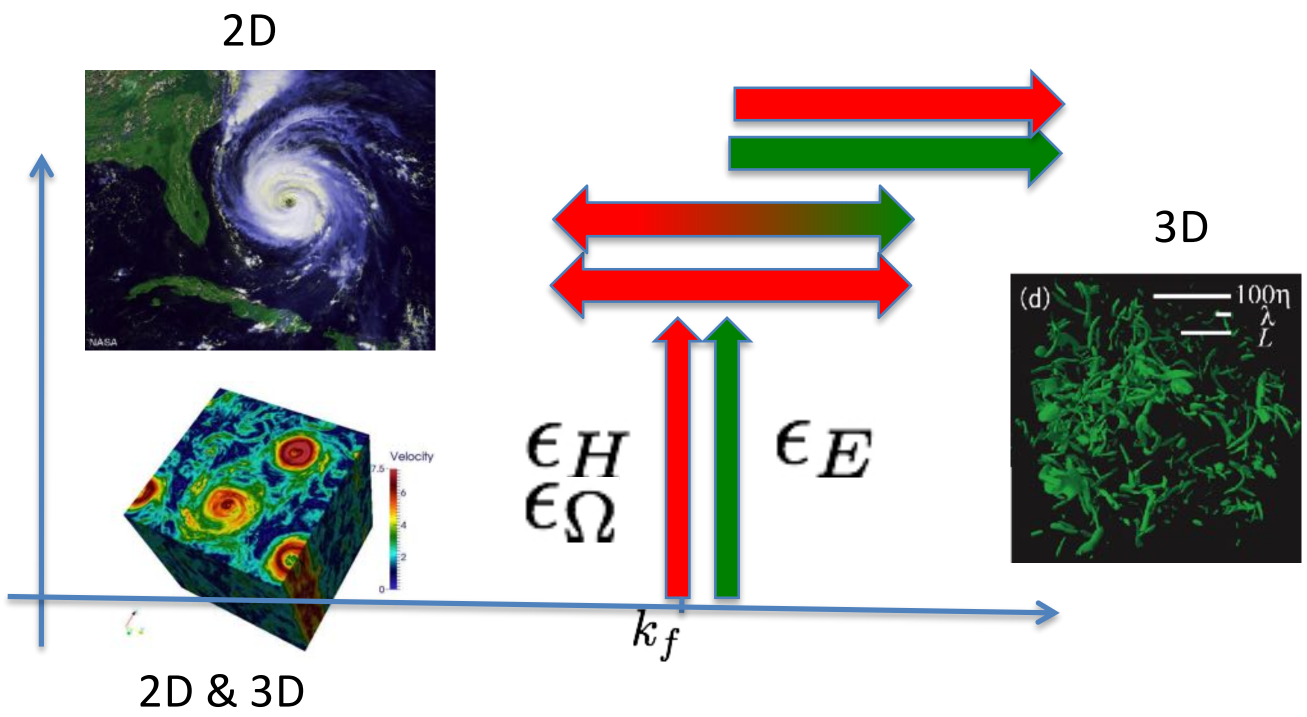
Credits [in order of appearance]: **F. Toschi** (TuE, The Netherlands); **S. Musacchio** (CNRS-Nice, France); **E. Titi** (Weizmann Institute of Science, Israel), **G. Sahoo** (University of Helsinki, Finland); **F. Bonaccorso, M. Buzzicotti** (Univ. of Roma 'Tor Vergata', Italy), **M. Linkmann** (Univ. of Marburg, Germany); **A. Alexakis** (ENS-Paris, France)

MOTIVATIONS:

Q1: HOW TO PREDICT THE DIRECTION OF THE TRANSFER (FORWARD/BACKWARD) AND ITS ROBUSTNESS UNDER EXTERNAL PERTURBATION (FORCING/BOUNDARY CONDITIONS)?

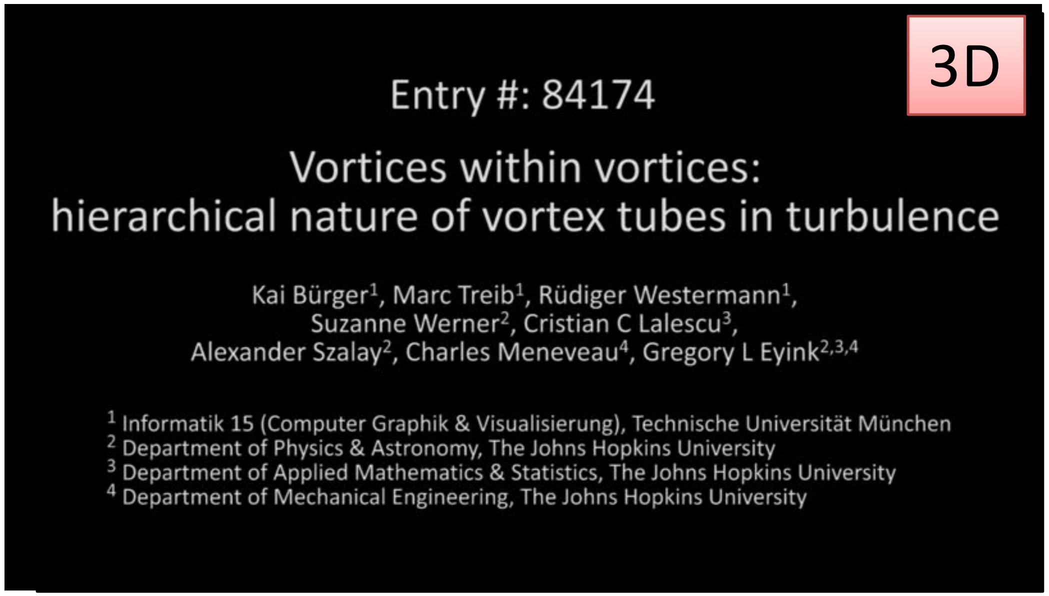
Q2: WHAT ABOUT FLUCTUATIONS AROUND THE MEAN?

AS A MATTER OF FACT, FOR 3D NAVIER STOKES EQUATIONS, WE DO NOT KNOW HOW TO PREDICT NEITHER THE SIGN OF THE MEAN ENERGY TRANSFER NOR THE INTENSITY OF THE FLUCTUATIONS AROUND IT.



NAVIER-STOKES 3D-2D

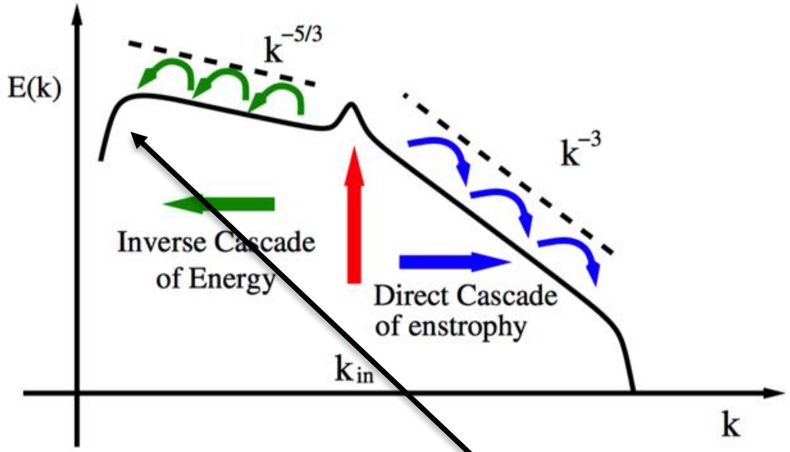
(NASA/Goddard Space Flight Center Scientific Visualization Studio)

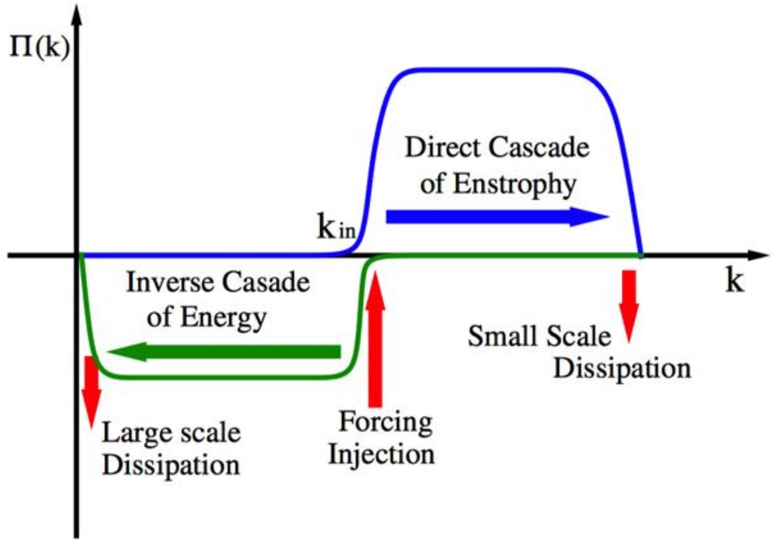


2 invariants (positive definite)

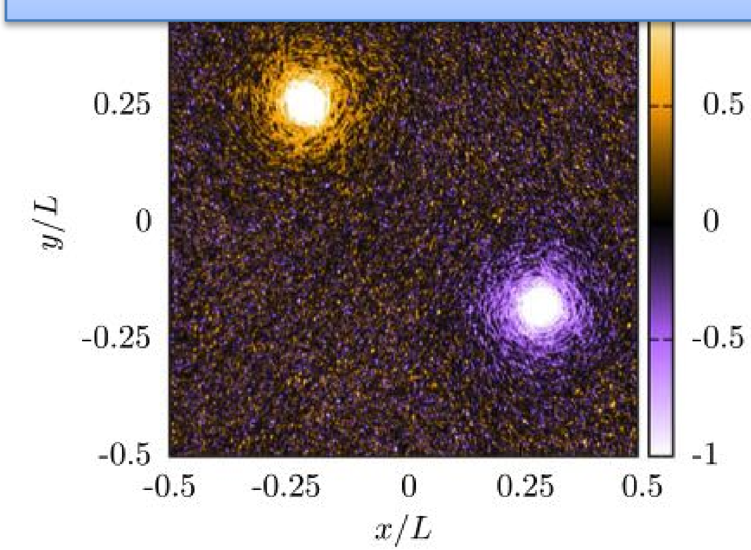
2D

$$\Omega = \int d^3x \, \omega \cdot \omega \quad E = \int d^3x \, \mathbf{v} \cdot \mathbf{v}$$





INVERSE CASCADE + IR CUT-OFF →
→ LARGE SCALE CONDENSATE



J. Laurie et al. PRL 113, 254503 (2014); H. Xia et al. PRL 101, 194504 (2008)

Study of High–Reynolds Number Isotropic Turbulence by Direct Numerical Simulation

Takashi Ishihara,[1] Toshiyuki Gotoh,[2] and Yukio Kaneda[1]

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[2] Department of Scientific and Engineering Simulation, Graduate School of Engineering, Nagoya Institute of Technology, Gokiso, Showa-ku, Nagoya 466-8555, Japan

3D HOMOGENEOUS AND ISOTROPIC TURBULENCE FLUCTUATIONS: SMALL-SCALES INTERMITTENCY

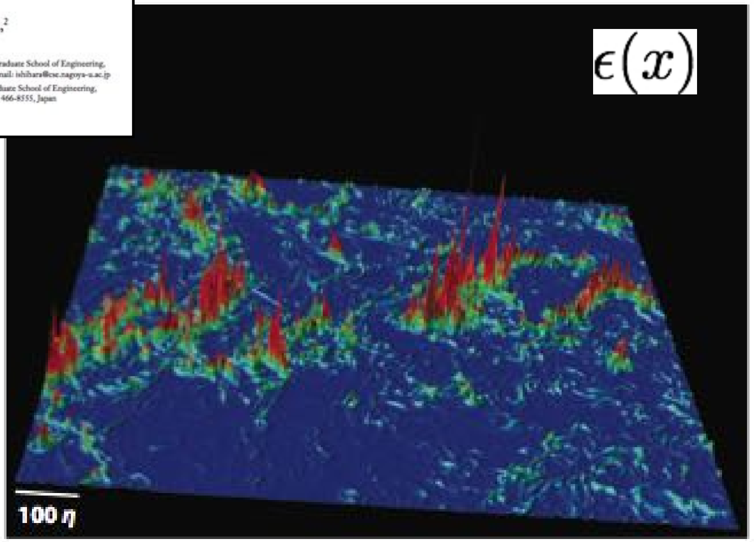


Figure 4

Snapshot of the intensity distributions of (*a*) the energy-dissipation rate $\bar{\varepsilon} = \varepsilon/(2\nu)$ and (*b*) the enstrophy $\Omega = \omega^2/2$ on a cross section in DNS-ES at $R_\lambda = 675$ in arbitrary units.

3D

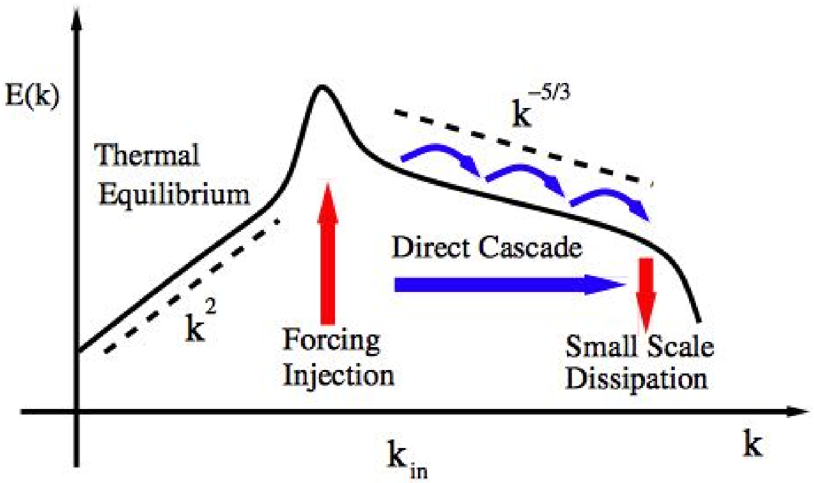
DIRECT ENERGY CASCADE

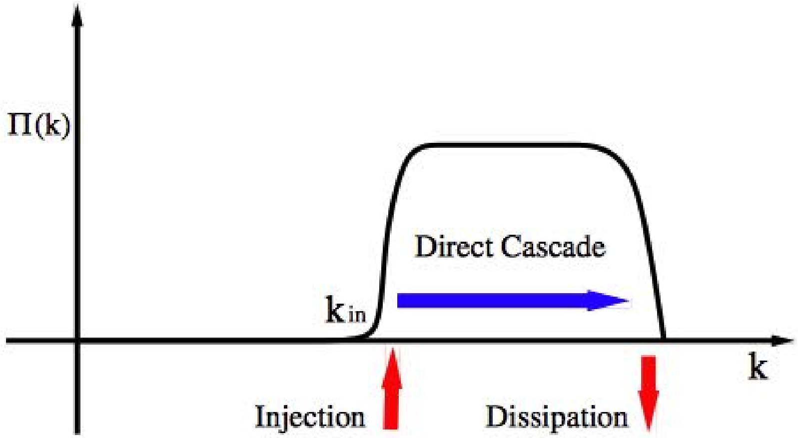
$$E = \int d^3x \, \mathbf{v} \cdot \mathbf{v}$$

$$H = \int d^3x \, \omega \cdot \mathbf{v}$$

HELICITY

2 invariants (only one positive definite)





2D/3D NOT THE END OF THE HISTORY: SPLIT ENERGY CASCADE

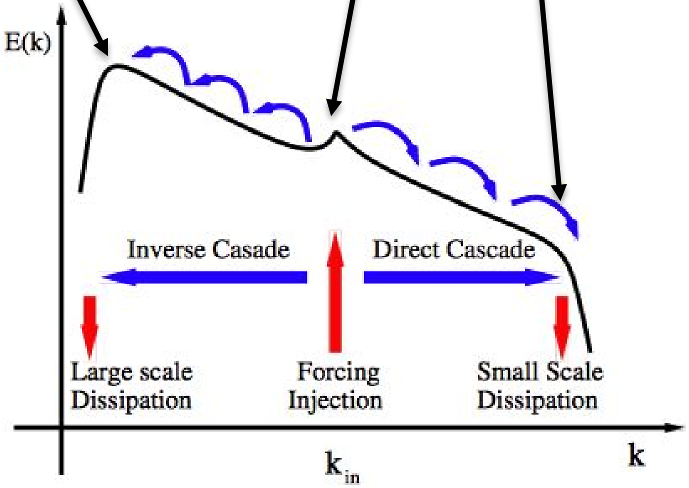
$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nu \Delta \mathbf{u} - \alpha \mathbf{u} + \mathbf{f}.$$

$$\partial_t \mathcal{E} = \epsilon_{in}(t) - \epsilon_\nu(t) - \epsilon_\alpha(t)$$

$$\epsilon_\alpha(t) = \alpha \langle |\mathbf{u}|^2 \rangle$$

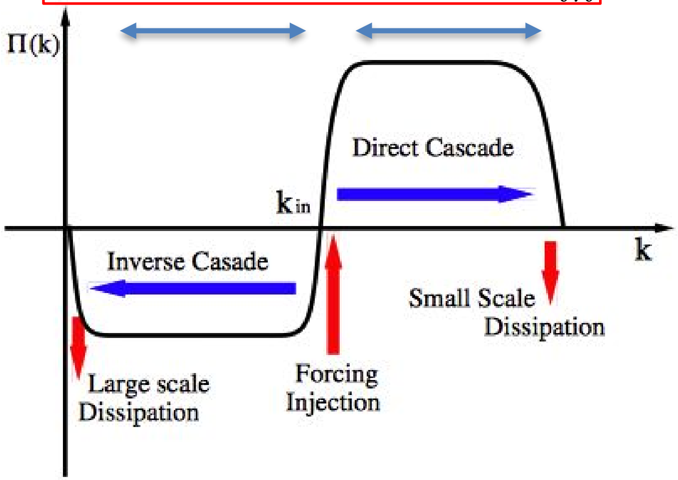
$$\epsilon_{in}(t) = \langle \mathbf{u} \cdot \mathbf{f} \rangle$$

$$\epsilon_\nu(t) = \nu \langle |\nabla \mathbf{u}|^2 \rangle$$



CONTROL PARAMETERS

$$Kr = \frac{u_f k_{in}}{\alpha} \qquad Re = \frac{u_f}{\nu k_{in}}$$



A. Alexakis and L.B "Direct and Inverse cascades in turbulent flows" Phys. Rep 2018 (to appear)

DIRECT/INVERSE CASCADE TRANSITION

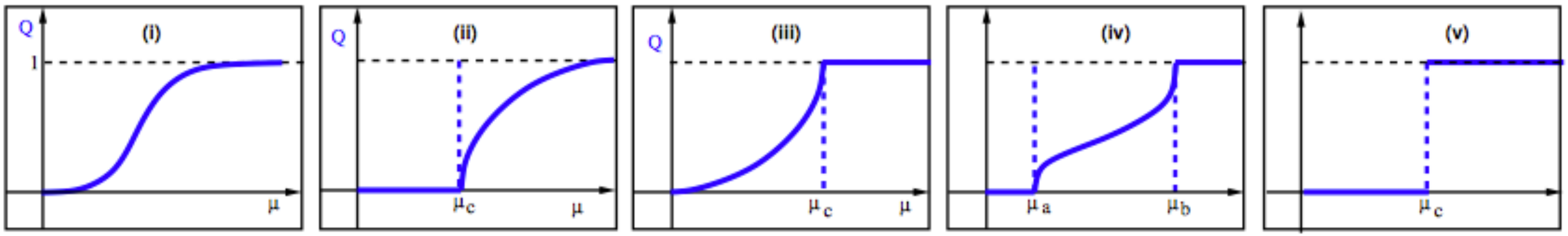
$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla P + \nu\Delta\mathbf{u} - \alpha\mathbf{u} + \mathbf{f}.$$

$$\partial_t \mathcal{E} = \epsilon_{in}(t) - \epsilon_\nu(t) - \epsilon_\alpha(t)$$

$$\overline{\epsilon}_{in} = \overline{\epsilon}_\nu + \overline{\epsilon}_\alpha$$ ← STATIONARY BALANCE

$$D_\nu = \frac{\epsilon_\nu}{\mathcal{E}_{in}{}^{3/2} k_{in}} \quad \text{and} \quad D_\alpha = \frac{\epsilon_\alpha}{\mathcal{E}_{in}{}^{3/2} k_{in}}$$ ← DIMENSIONLESS FLUX

$$\lim_{\substack{Re\to\infty,\\ Kr\to\infty}} \ \lim_{\substack{L\to\infty,\\ k_{max}\to\infty}} \frac{\epsilon_\nu}{\epsilon_{in}} = Q_\nu > 0,$$



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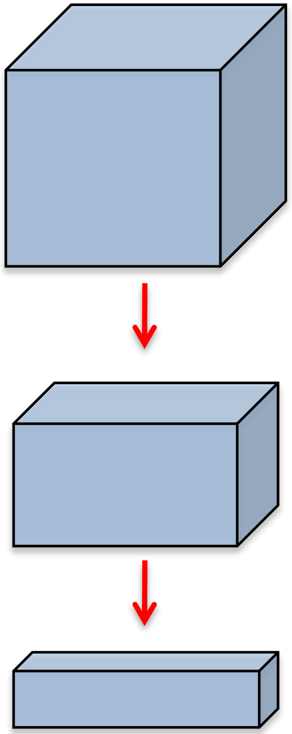
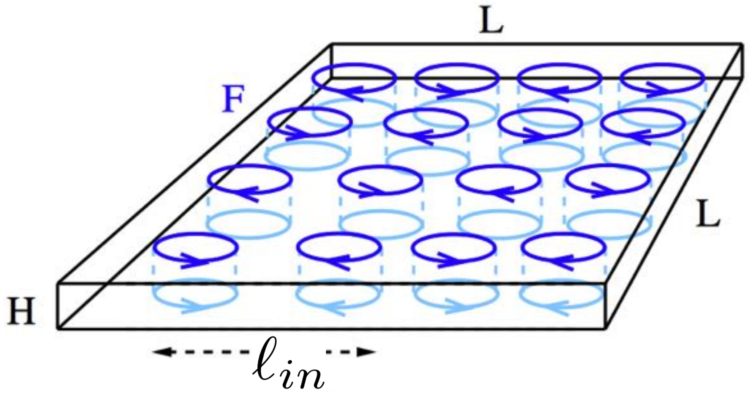
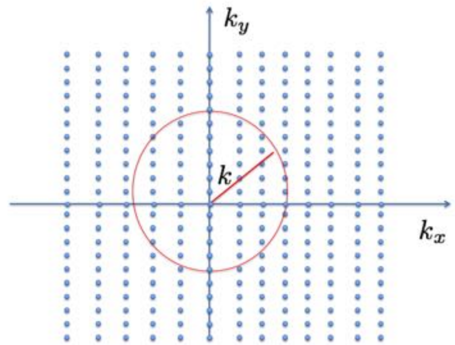
3D ← → 2D: THIN LAYERS

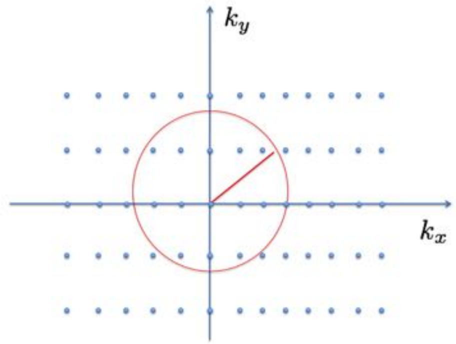




CONTROL PARAMETER

$$\frac{H}{\ell_{in}}$$





$$\partial_t \mathbf{u}_{2D} + \mathbf{u}_{2D} \cdot \nabla \mathbf{u}_{2D} = -\overline{\mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{3D}} - \overline{\nabla P} + \nu \Delta \mathbf{u}_{2D} - \alpha \mathbf{u}_{2D} + \mathbf{f}_{2D},$$

$$\partial_t \mathbf{u}_{3D} + \mathbf{u}_{2D} \cdot \nabla \mathbf{u}_{3D} = -\mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{2D} + (\overline{\mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{3D}} - \mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{3D}) - \nabla P + \nu \Delta \mathbf{u}_{3D} + \mathbf{f}_{3D},$$

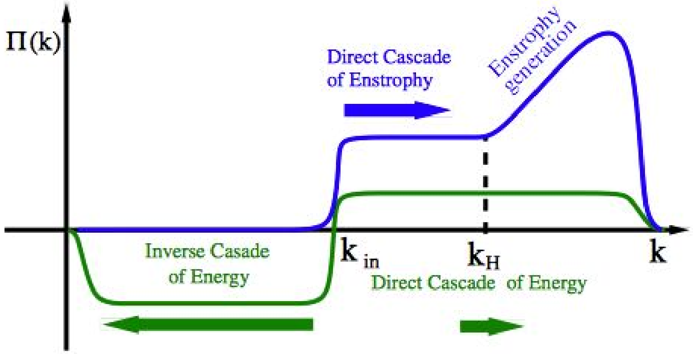
ENERGY BALANCE

$$\partial_t \mathcal{E}_{2D}(t) = \epsilon_{in}^{2D} - \mathcal{T} - \epsilon_\nu^{2D} - \epsilon_\alpha^{2D}$$

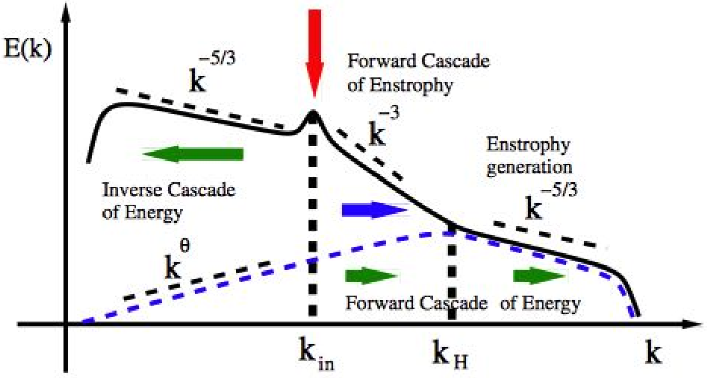
$$\partial_t \mathcal{E}_{3D}(t) = \epsilon_{in}^{3D} + \mathcal{T} - \epsilon_\nu^{3D} - \epsilon_\alpha^{3D}$$

L.M. Smith et al PRL 77, 2467 (1996); A. Celani et al PRL 104, 184605 (2010); H. Xia et al Nature Physics (2011); S. Musacchio and G. Boffetta PoF 29, 111106 (2017)

2D < ---------------- > 3D

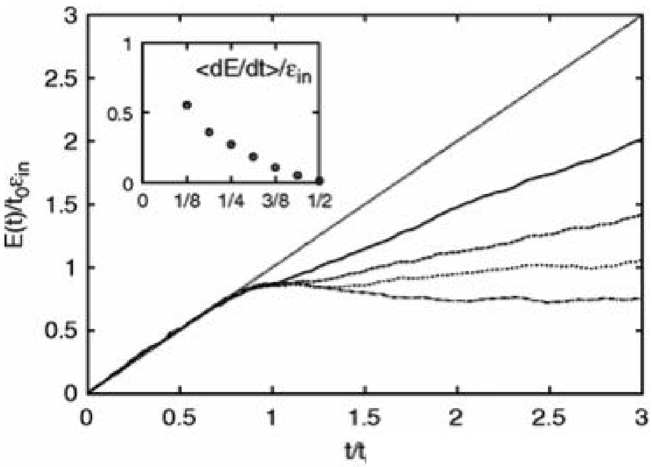


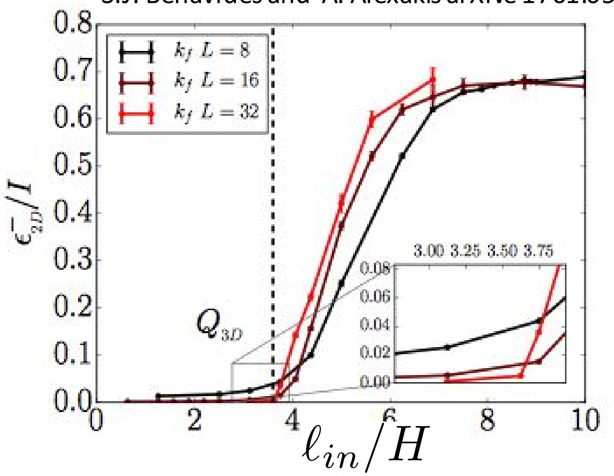
2D < ---------------- > 3D

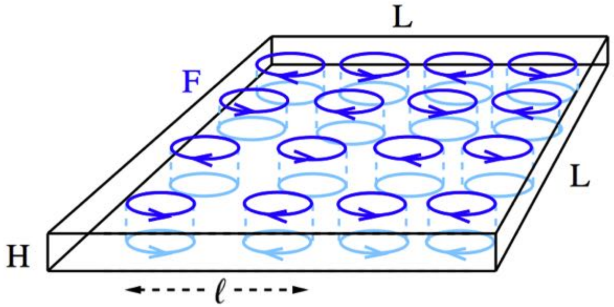


A. Celani et al PRL 104, 184605 (2010)

S.J. Benavides and A. Alexakis arXive 1701.05162







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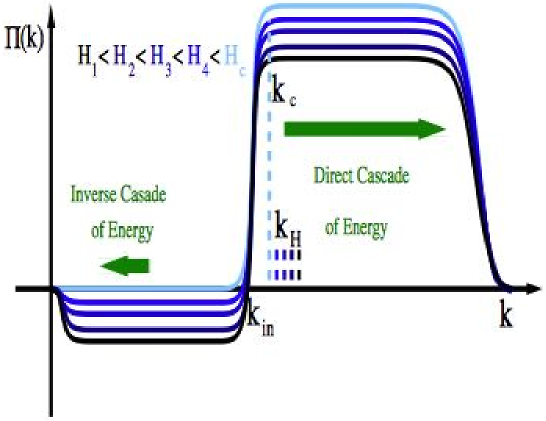
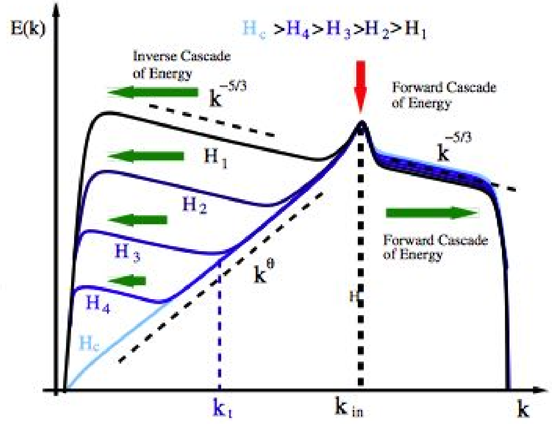
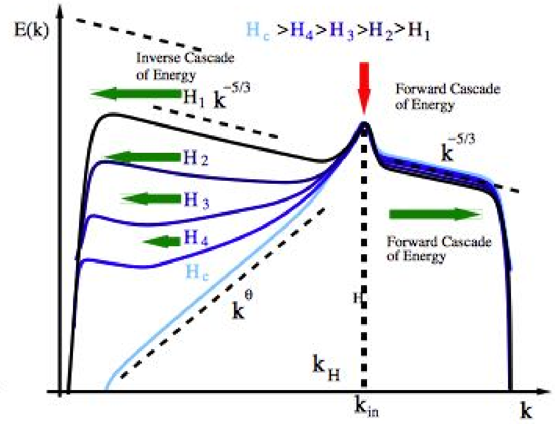
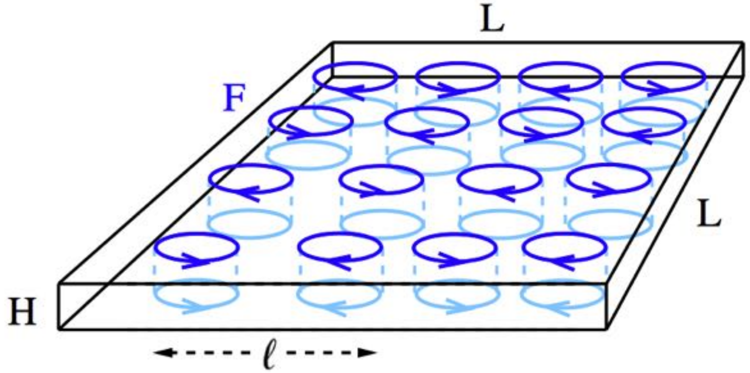
$$\lim H \to H_c$$


Π(k)
H1<H2<H3<H4<Hc
kc
Direct Cascade
of Energy
Inverse Casade
of Energy
kH
kin
k


E(k)
Hc >H4>H3>H2>H1
Inverse Cascade
of Energy
k−5/3
Forward Cascade
of Energy
H1
H2
H3
H4
Hc
kθ
k1
kin
k


E(k)
Hc >H4>H3>H2>H1
Inverse Cascade
of Energy
H1 k−5/3
Forward Cascade
of Energy
H2
H3
H4
Hc
kθ
kH
kin
k


L
F
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