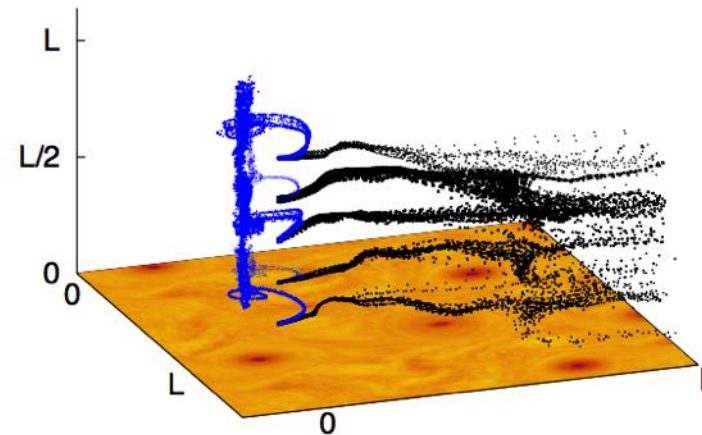
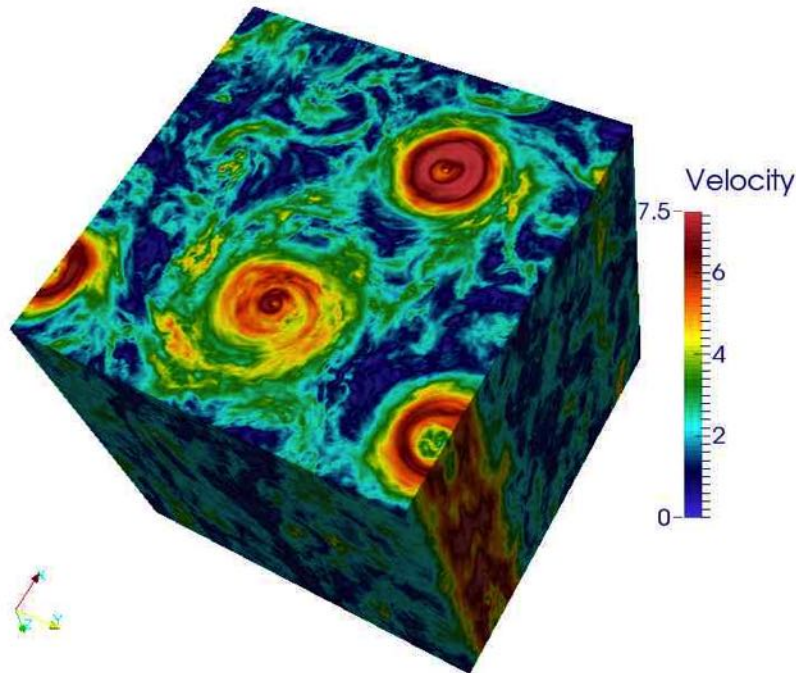


# TURBULENT AT HIGH AND LOW ROTATION RATES: EULERIAN AND LAGRANGIAN STATISTICS



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A.Lanotte (Lecce, Italy)  
S. Musacchio (Nice, France)  
P.Perlekar (Hyderabad, India)



PRACE 09\_2256  
ROTATING TURBULENCE  
2015 – 55MH

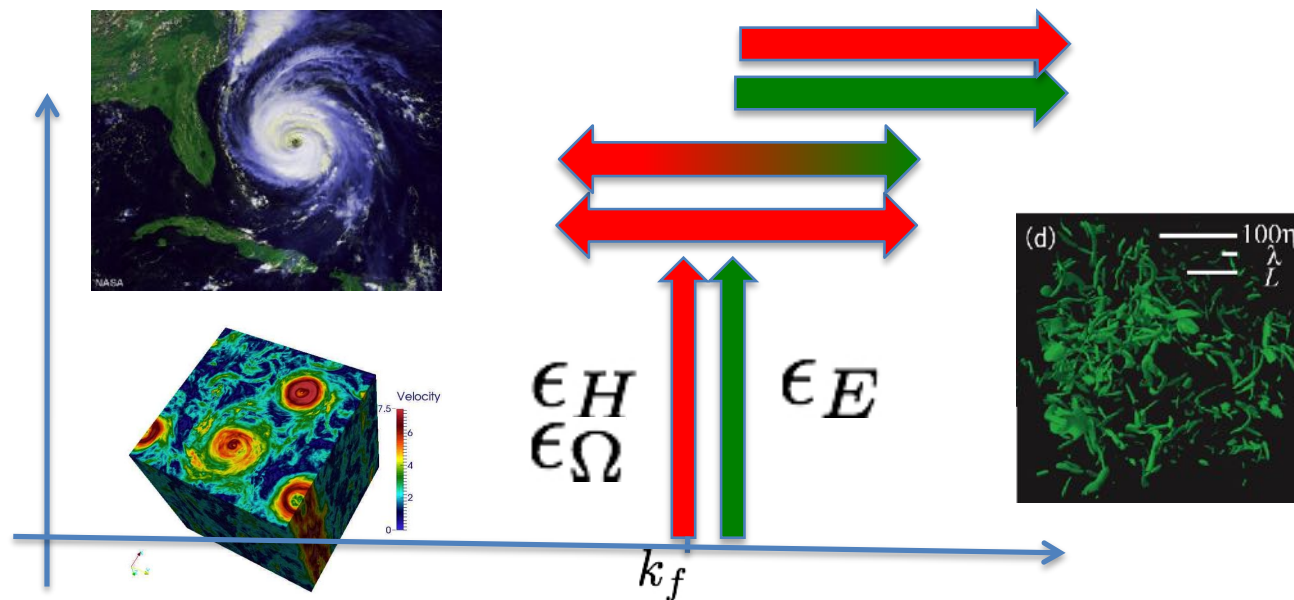
## MOTIVATIONS:

A TALE ABOUT TRANSFER PROPERTIES OF INVISCID CONSERVED QUANTITIES, KINETIC ENERGY, HELICITY ENSTROPY, MAGNETIC HELICITY ETC...

Q1: HOW TO PREDICT THE DIRECTION OF THE TRANSFER (FORWARD/BACKWARD) AND ITS ROBUSTNESS UNDER EXTERNAL PERTURBATION (FORCING/BOUNDARY CONDITIONS)?

Q2: HOW MUCH THE FLUCTUATIONS AROUND THE MEAN TRANSFER ARE INTENSE AND SELF-SIMILAR (INTERMITTENCY AND ANOMALOUS SCALING) ?

**AS A MATTER OF FACT, FOR 3D NAVIER STOKES EQUATIONS, WE DO NOT KNOW HOW TO PREDICT NEITHER THE SIGN OF THE MEAN ENERGY TRANSFER NOR THE INTENSITY OF THE FLUCTUATIONS AROUND IT.**

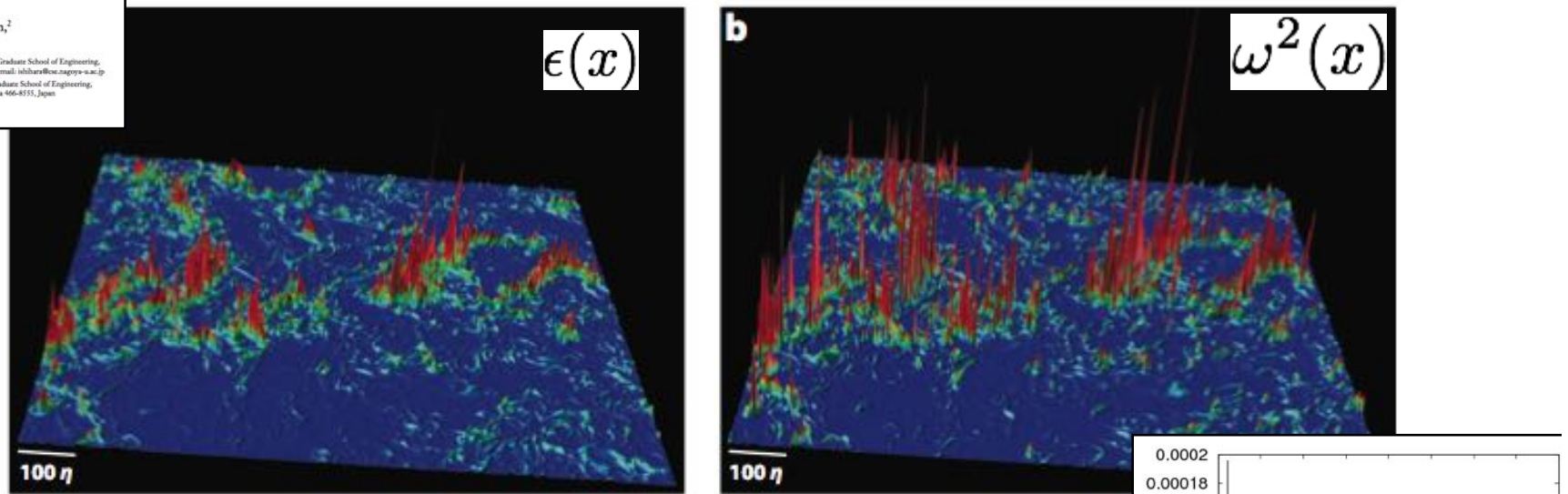


Study of High-Reynolds  
Number Isotropic Turbulence  
by Direct Numerical  
Simulation

Takashi Ishihara,<sup>1</sup> Toshiyuki Gotoh,<sup>2</sup>  
and Yukio Kaneda<sup>1</sup>

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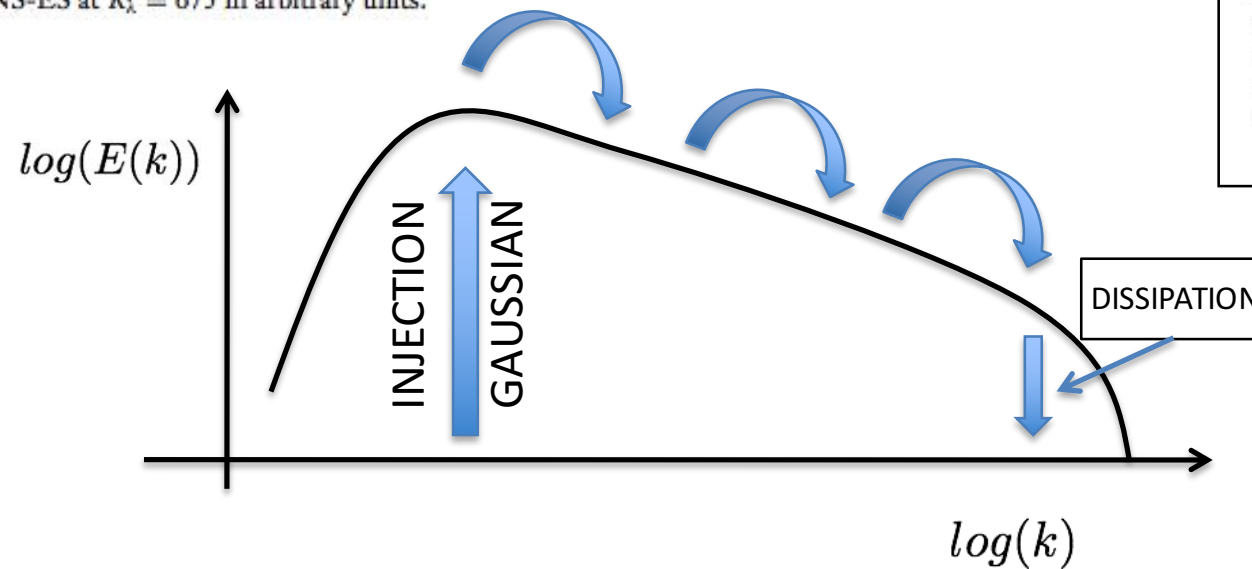
## 3D HOMOGENEOUS AND ISOTROPIC TURBULENCE FLUCTUATIONS: SMALL-SCALES INTERMITTENCY



**Figure 4**

Snapshot of the intensity distributions of (a) the energy-dissipation rate  $\bar{\epsilon} = \epsilon/(2\nu)$  and (b) the enstrophy  $\Omega = \omega^2$  at  $R_\lambda = 675$  in arbitrary units.

arXiv:1505.04777



- MOTIVATION: WHY ROTATING TURBULENT FLOWS ARE IMPORTANT
- DIRECT AND INVERSE ENERGY TRANSFERS (2D-3D PHYSICS)
- OUR DNS (DIFFERENCES WRT PREVIOUS STUDIES)
- EULERIAN STATISTICS (MEAN SPECTRAL PROPERTIES)
- EULERIAN STATISTICS (LARGE FLUCTUATIONS)
- LAGRANGIAN STATISTICS (EFFECTS OF CORIOLIS AND CENTRIFUGAL FORCES)
- LAGRANGIAN STATISTICS (SINGLE PARTICLE DISPERSIONS)
- CONCLUSIONS



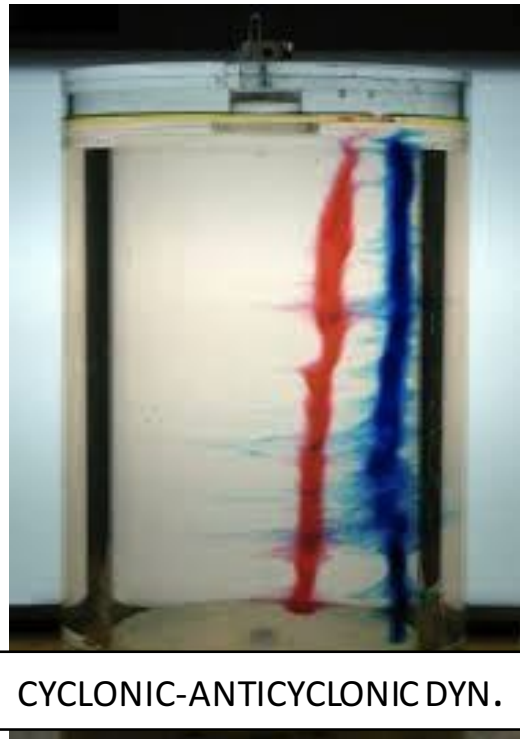
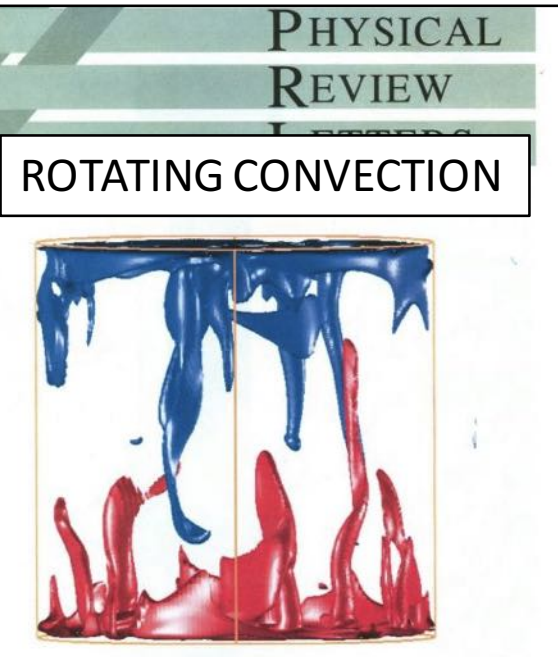
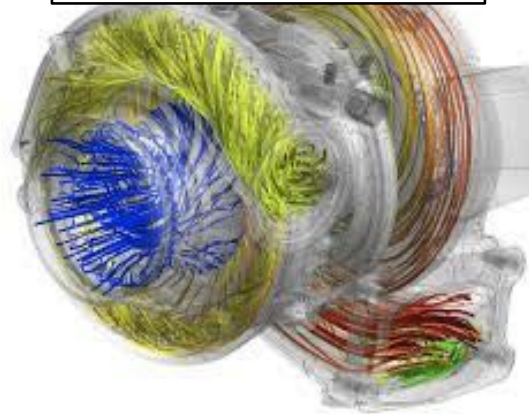
TAYLOR-COUPETTE

ROTATING CONVECTION (+  
STRATIFICATION + MHD)

ROTATING RAYLEIGH-TAYLOR

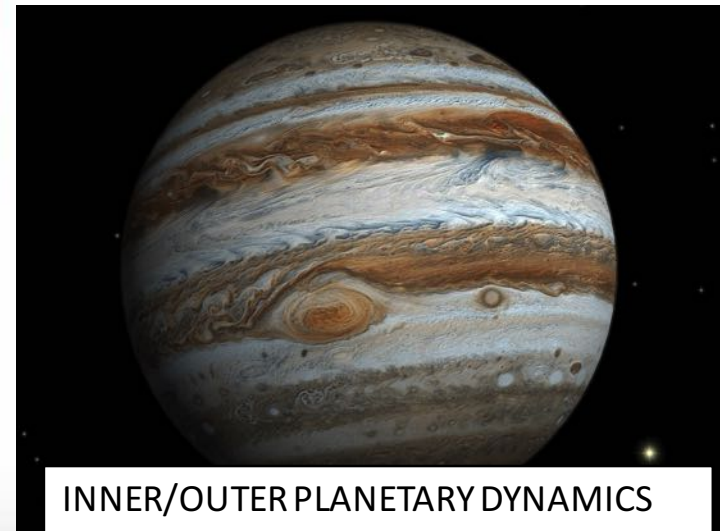
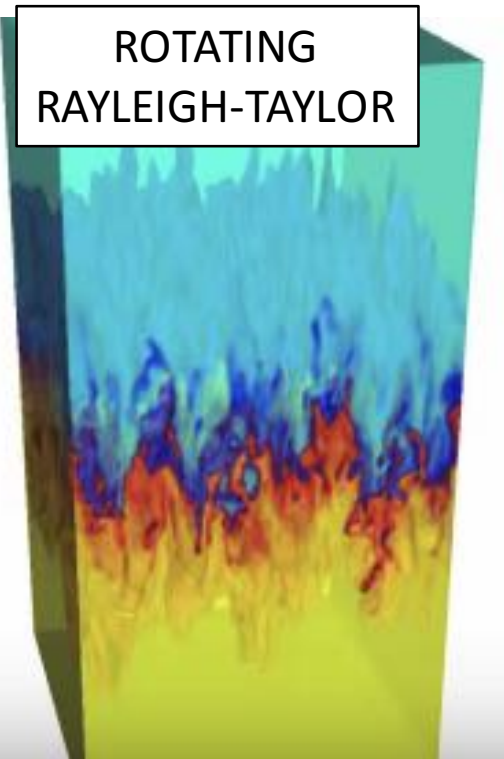
Recent reviews/books by Lohse,  
Boffetta, Cambon, Clercx, Davidson  
etc...

TURBOMACHINERY



CYCLONIC-ANTICYCLONIC DYN.

ROTATING  
RAYLEIGH-TAYLOR



INNER/OUTER PLANETARY DYNAMICS

## NAVIER-STOKES EQS IN A ROTATING FRAME (NO BOUNDARIES)

DNS: A. Pouquet, P. Mininni, A. Alexakis, S. Chen, G. Eyink ....

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \boldsymbol{\Omega} \times \mathbf{v} = \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{F} - \alpha \mathbf{v}$$

$\boldsymbol{\Omega}$  = rotation

$$P = P_0 + \frac{1}{2} |\boldsymbol{\Omega} \times \mathbf{r}|^2$$

$\mathbf{F}$  = large scale Forcing

$\alpha$  = large scale energy sink

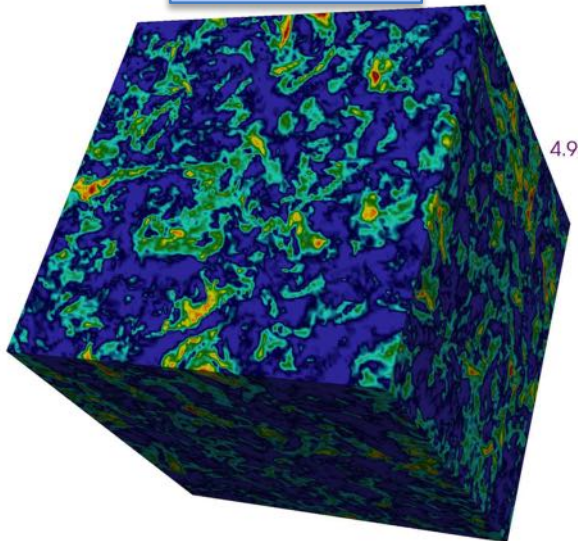
ROSSBY NUMBER  $\sim$  NON-LINEAR/ROTATION

$$Ro \sim \frac{v_0}{\Omega L_0}$$

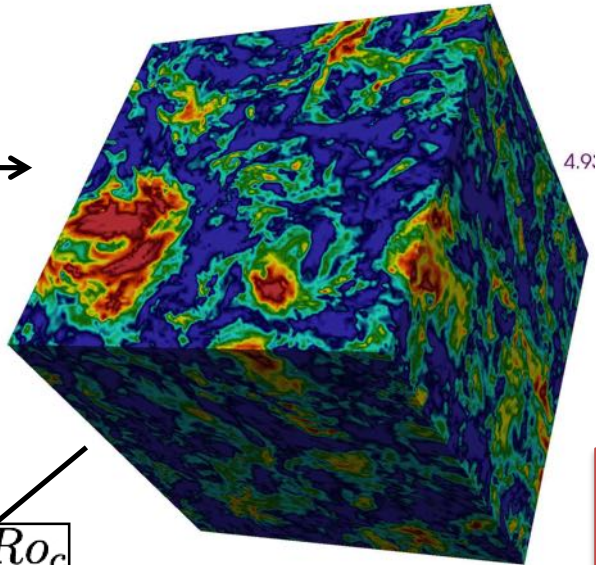
$Ro \geq Ro_c \rightarrow$  FORWARD ENERGY TRANSFER

$Ro \leq Ro_c \rightarrow$  FORWARD & BACKWARD ENERGY TRANSFER

Rossby = 2

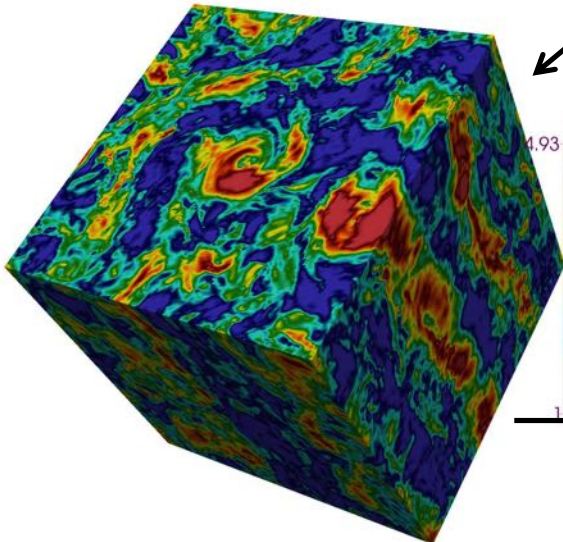


Rossby = 0.8

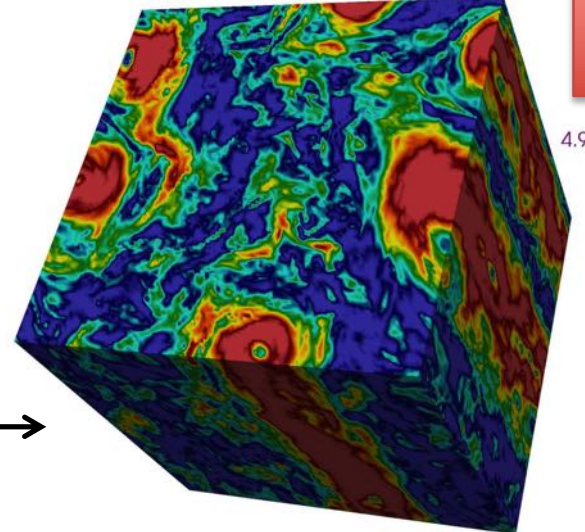


$Ro < Ro_c$

Rossby = 0.2



Rossby = 0.1

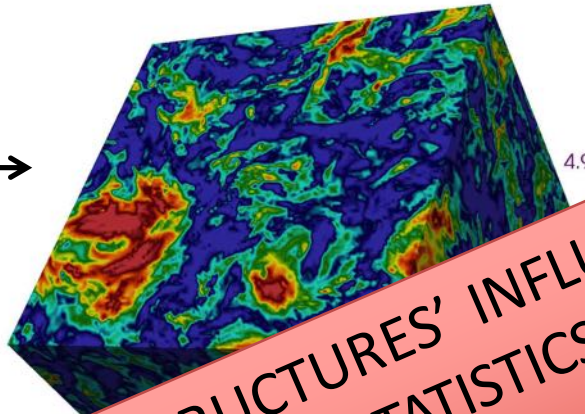
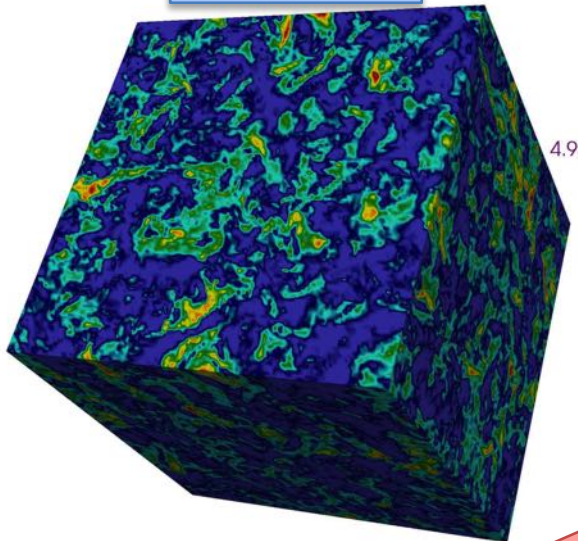


HOMOGENEOUS  
ANISOTROPIC  
2D & 3D PHYSICS  
CHOERENT -STRUCTURES

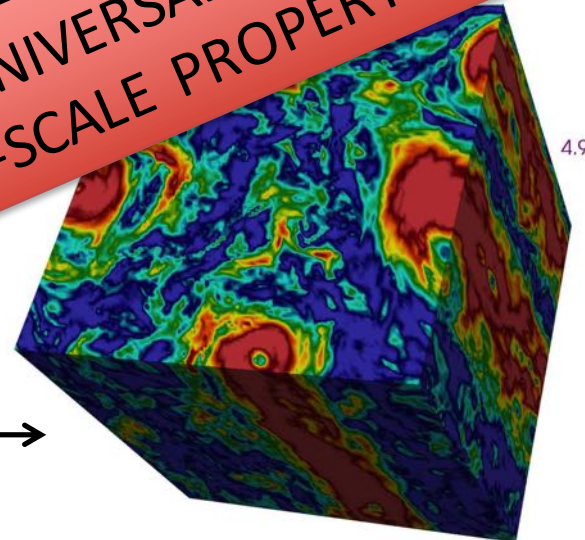
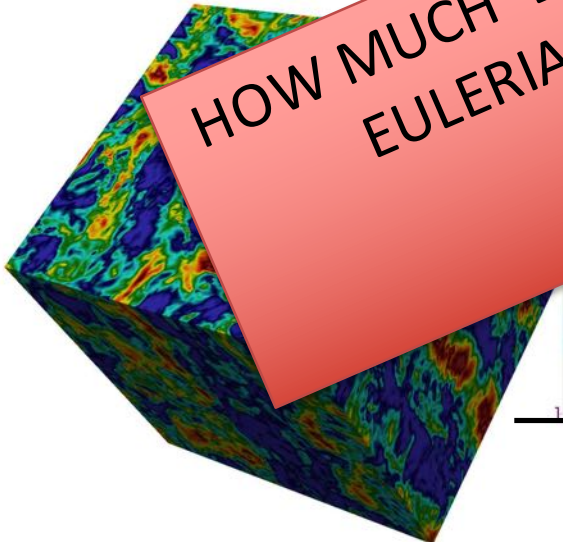


Rossby = 2

Rossby = 0.8



HOW MUCH 'LARGE-SCALE STRUCTURES' INFLUENCE  
EULERIAN AND LAGRANGIAN STATISTICS?  
UNIVERSALITY?  
MULTI-SCALE PROPERTIES?



Rossby = 0.2

Rossby = 0.1



## OUR DNS DATA-BASE (EULERIAN + LAGRANGIAN)

### NEW FEATURES:

- 1) IDEAL FORCING MECHANISM (AS NEUTRAL AS POSSIBLE: ISOTROPIC; NON HELICAL, **TIME-COLORED**) + **LARGE SCALE FRICTION**
- 2) UNPRECEDENTED NUMERICAL RESOLUTION/SCALE SEPARATION (**UP TO 4096<sup>3</sup>**)
- 3) LAGRANGIAN STATISTICS (MILLIONS OF **TRACERS** AND **INERTIAL PARTICLES**)

$N$	$\Omega$	$\nu$	$\epsilon$	$\epsilon_f$	$u_0$	$\eta/dx$	$\tau_\eta/dt$	$Re_\lambda$	$Ro$	$f_0$	$\tau_f$	$T_0$	$\alpha$
1024	4	$7 \times 10^{-4}$	1.2	1.2	1.05	0.67	120	150	0.78	0.02	0.023	0.17	0.0
1024	10	$6 \times 10^{-4}$	0.46	0.59	1.6	0.76	294	580	0.24	0.02	0.023	0.25	0.1
2048	4	$2.8 \times 10^{-4}$	1.2	1.2	1.05	0.67	380	230	0.76	0.02	0.023	0.17	0.0
2048	10	$2.2 \times 10^{-4}$	0.45	0.64	1.7	0.72	550	1170	0.25	0.02	0.023	0.3	0.1
4096	10	$1 \times 10^{-4}$	0.46	0.65	1.7	0.78	1010	1600	0.25	0.02	0.023	0.3	0.1

TABLE I: Eulerian dynamics parameters.  $N$ : number of collocation points per spatial direction;  $\Omega$ : rotation rate;  $\nu$ : kinematic viscosity;  $\epsilon = \nu \int d^3x \sum_{ij} (\nabla_i u_j)^2$ : viscous energy dissipation;  $\epsilon_f = \int d^3x \sum_i f_i u_i$ : energy injection;  $u_0 = 1/3 \int d^3x \sum_i u_i^2$ : mean kinetic energy;  $\eta = (\nu^3/\epsilon)^{1/4}$ : Kolmogorov dissipative scale;  $dx = L_0/N$ : numerical grid spacing;  $L_0 = 2\pi$ : box size;  $\tau_\eta = (\nu/\epsilon)^{1/2}$ : Kolmogorov dissipative time;  $Re_\lambda = (u_0\lambda)/\nu$ : Reynolds number based on the Taylor micro-scale;  $\lambda = (15\nu u_0^2/\epsilon)^{1/2}$ : Taylor micro-scale;  $Ro = (\epsilon_f k_f)^{1/3}/\Omega$ : Rossby number defined in terms of the energy injection properties, where  $k_f = 5$  is the wavenumber where the forcing is acting;  $f_0$ : intensity of the Ornstein-Uhlenbeck forcing;  $\tau_f$ : decorrelation time of the forcing;  $T_0 = u_0/L_0$ : Eulerian large-scale eddy turn over time;  $\alpha$ : coefficient of the damping term  $\alpha \Delta^{-1} \mathbf{u}$ .

↑  
MAX RESOLUTION

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \boldsymbol{\Omega} \times \mathbf{v} = \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{F} - \alpha \mathbf{v}$$

# DIMENSIONAL PHENOMENOLOGY

- $k > k_\Omega$   $\rightarrow E(k) \sim \varepsilon^{2/3} k^{-5/3}$
- $k_f < k < k_\Omega$   $\rightarrow E(k) \sim (\varepsilon \Omega)^{1/2} k^{-2} \leftrightarrow \tau_{tr}(k) \sim \frac{\tau_{nl}(k)^2}{\tau_\Omega}$
- $k < k_f$   $\rightarrow E(k) \sim \Omega^2 k^{-3} \leftrightarrow E(k) \sim k^{-5/3}$  ?

## TWO TIME SCALES

$$\tau_{nl}(k) \sim \varepsilon^{-\frac{1}{3}} k^{-\frac{2}{3}}$$

$$\tau_\Omega \sim 1/\Omega$$

$$k_\Omega = \left( \frac{\Omega^3}{\varepsilon} \right)^{1/2}$$

ZEEMAN SCALE

