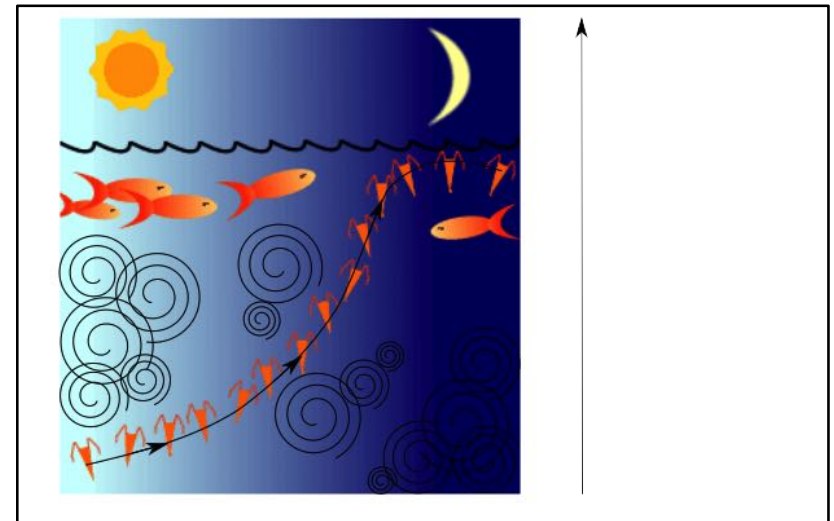


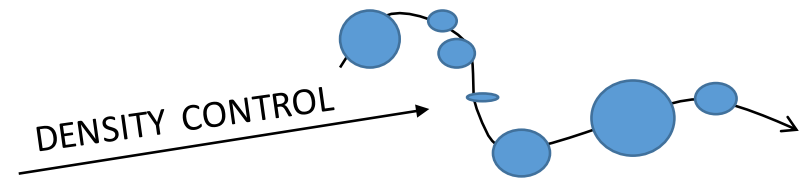
Flow navigation by smart particles via Reinforcement Learning

Luca Biferale
Dept. Physics, INFN & CAST
University of Rome 'Tor Vergata'
biferale@roma2.infn.it
SUSTech June 2018



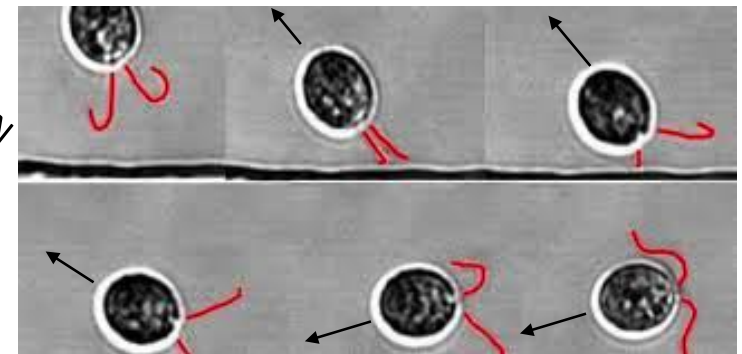
CREDITS: SIMONA COLABRESE (TOR VERGATA UNIV. ROME-IT); ANTONIO CELANI (ICTP TRIESTE-IT); KRISTIAN GUSTAVSSON (GOTHEBORG UNIV. SWEDEN)





- PARTICLES IN COMPLEX FLOWS I: **SMART INERTIAL PARTICLES**
- PARTICLES IN COMPLEX FLOWS II: **SMART MICROSWIMMERS**

SWIMMING DIRECTION CONTROL



- **Flow navigation by smart microswimmers via reinforcement learning**

S Colabrese, K Gustavsson, A Celani, L Biferale
Physical Review Letters 118 (15), 158004, 2017

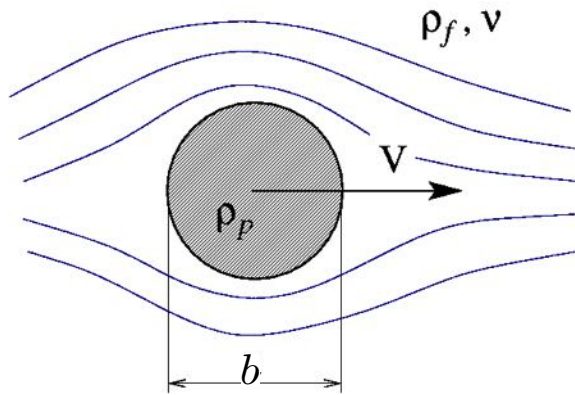
- **Smart Inertial Particles**

S Colabrese, K Gustavsson, A Celani, L Biferale
arXiv preprint arXiv:1711.05853, 2017

- **Finding efficient swimming strategies in a three-dimensional chaotic flow by reinforcement learning**

K Gustavsson, L Biferale, A Celani, S Colabrese
The European Physical Journal E 40 (12), 110, 2017

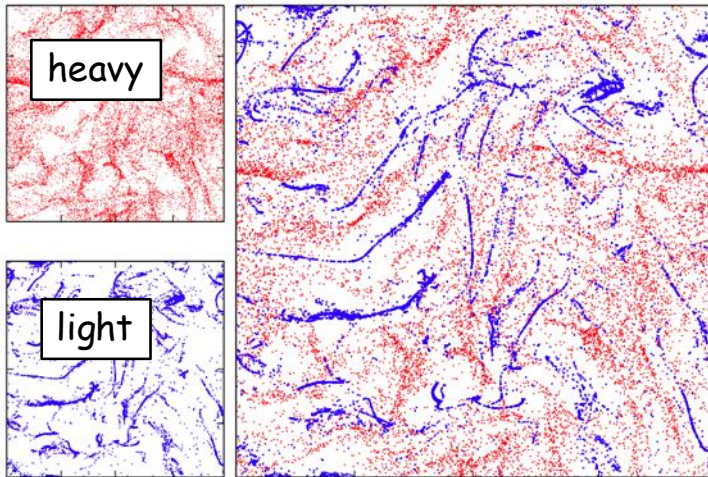
PARTICLES IN COMPLEX FLOWS I: INERTIAL PARTICLES



$$\frac{d\mathbf{X}}{dt} = \mathbf{V}$$

$$\frac{d\mathbf{V}}{dt} = \beta \frac{D\mathbf{u}(\mathbf{X},t)}{Dt} + \frac{\mathbf{u}(\mathbf{X},t) - \mathbf{V}}{\tau}$$

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \partial) \mathbf{u} = -\partial P + \nu \nabla^2 \mathbf{u}$$



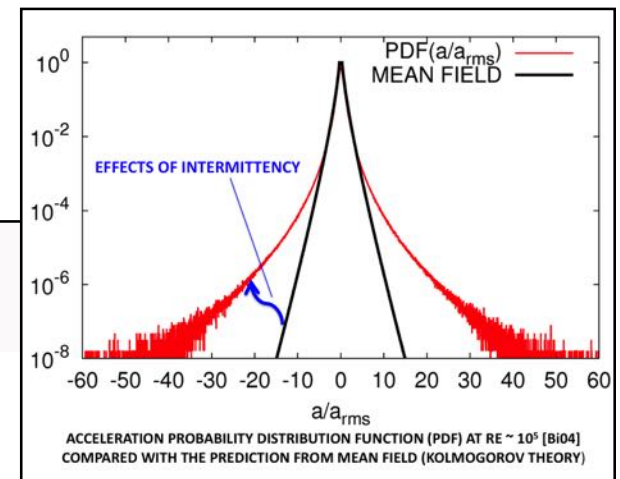
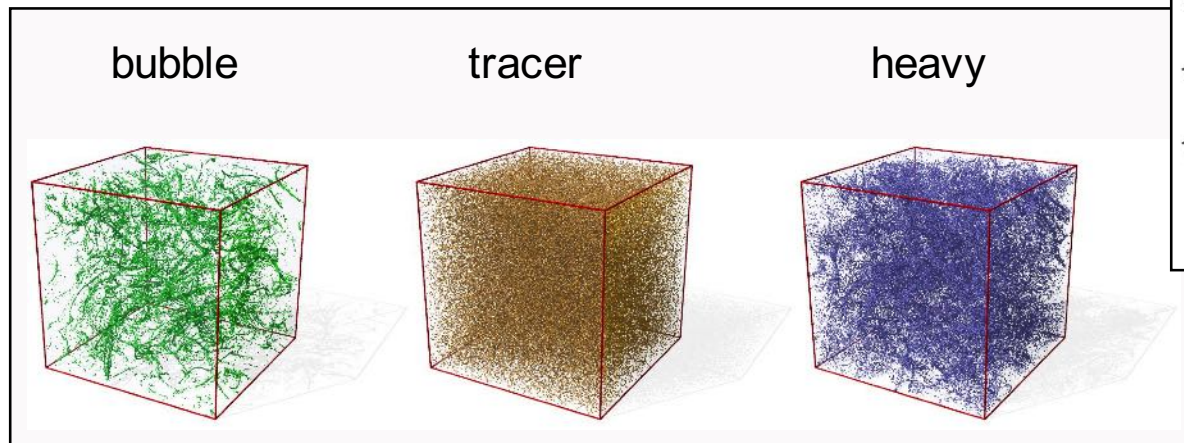
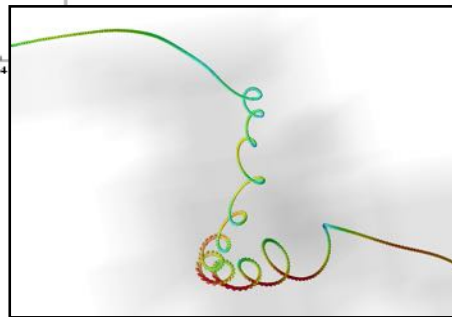
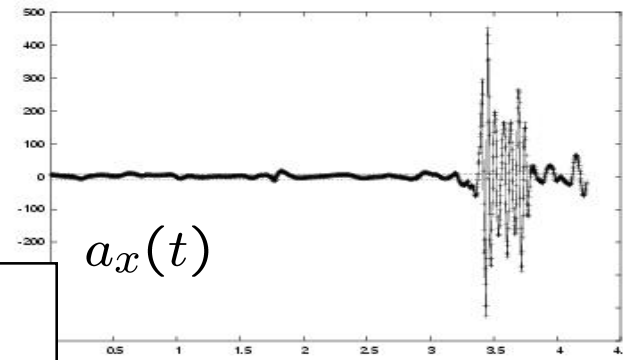
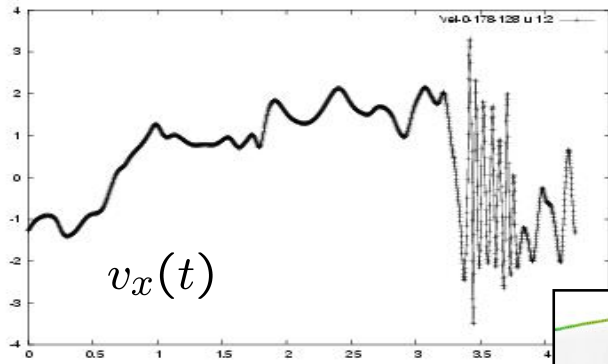
$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$

$$\tau = \frac{b^2}{3\nu\beta}$$

$\beta < 1$ heavy particles
 $\beta > 1$ light particles

Drag: **Stokes Time**

Preferential concentration!
 Light(heavy) particles accumulate
 inside(outside) highly vortical regions



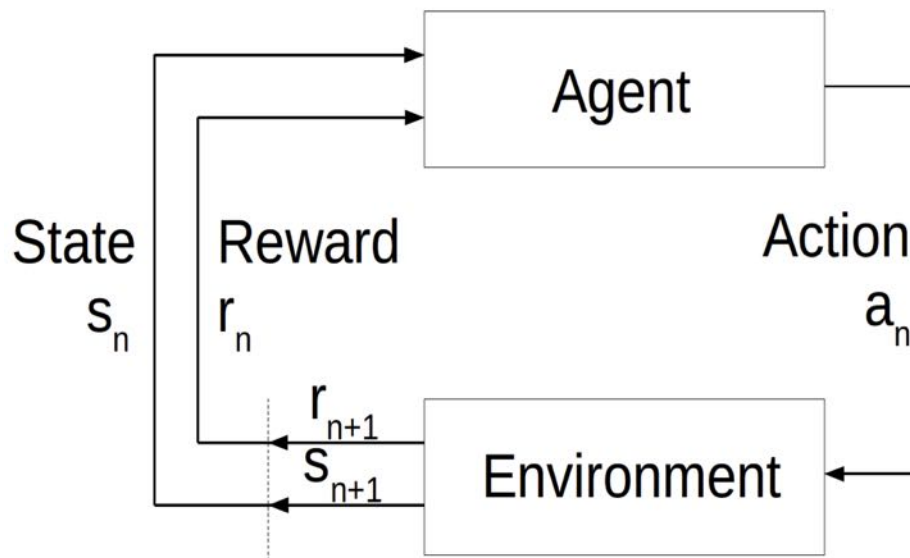
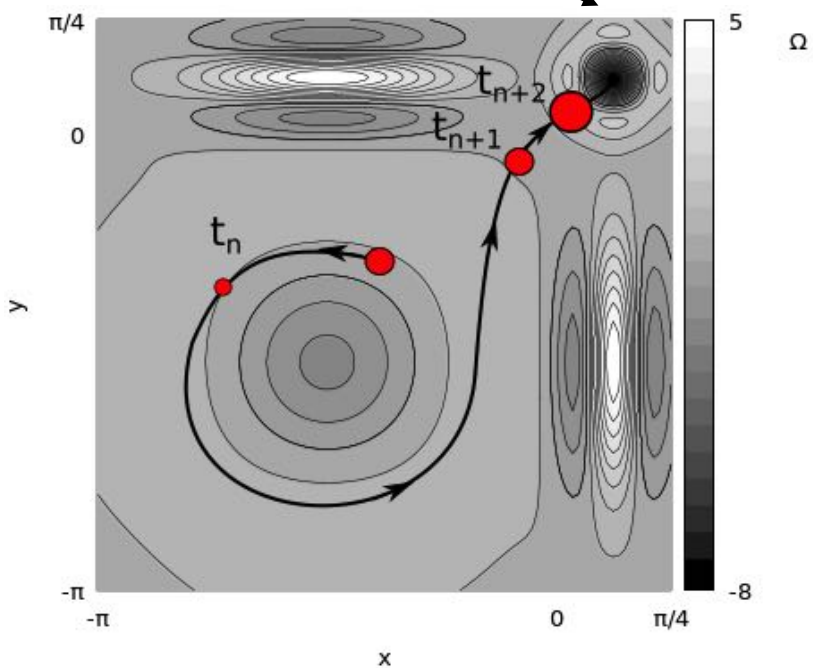
Particle trapping in three-dimensional fully developed turbulence

L.B., G Boffetta, A Celani, A Lanotte, F Toschi

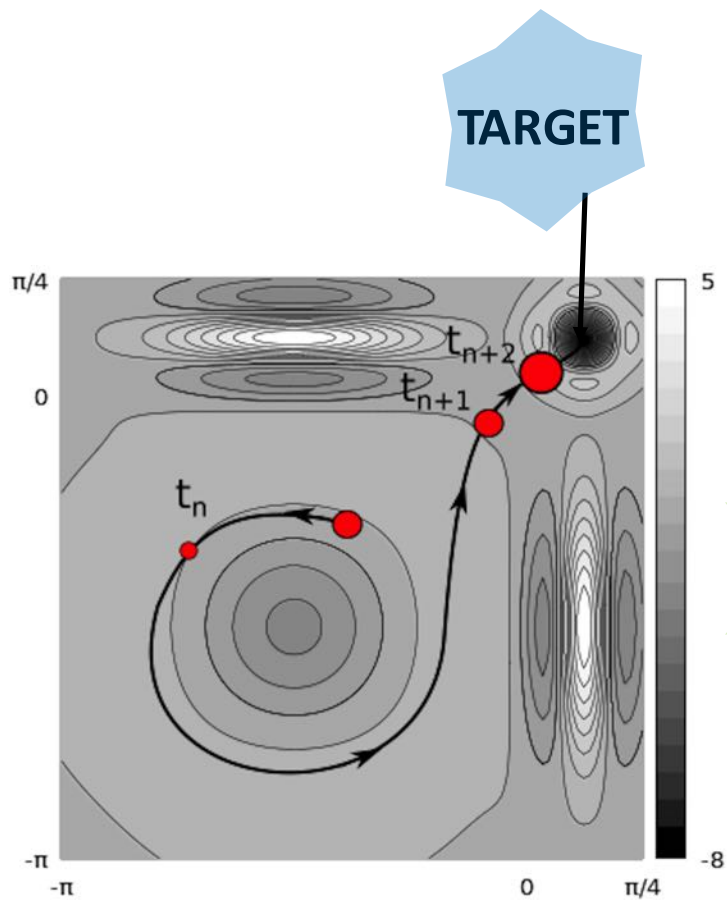
Physics of Fluids 17 (2), 021701

POLICY $\pi : \mathcal{S} \rightarrow \mathcal{a}$

TARGET



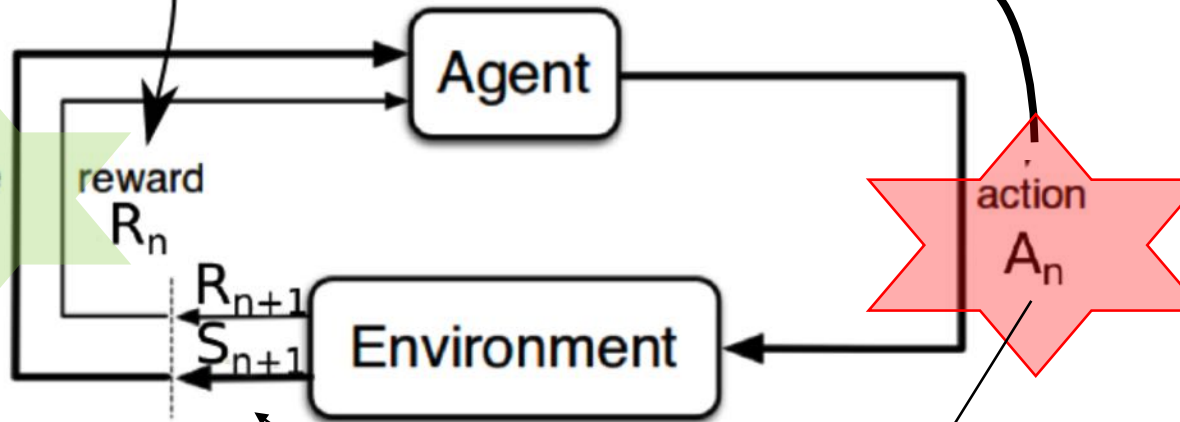
Reinforcement learning is a framework to find a good (optimal) POLICY for achieving given long-term tasks. It is widely used in artificial intelligence and machine learning. It is based on the interaction between a decision-maker (in our case the inertial particle) and the environment. The decision maker can change its behaviour in response to inputs from the system. By trial and error the decision maker progressively learns how to behave optimally



Smart inertial particle

$$R = \Omega^3$$

state S_n



ρ (densities)

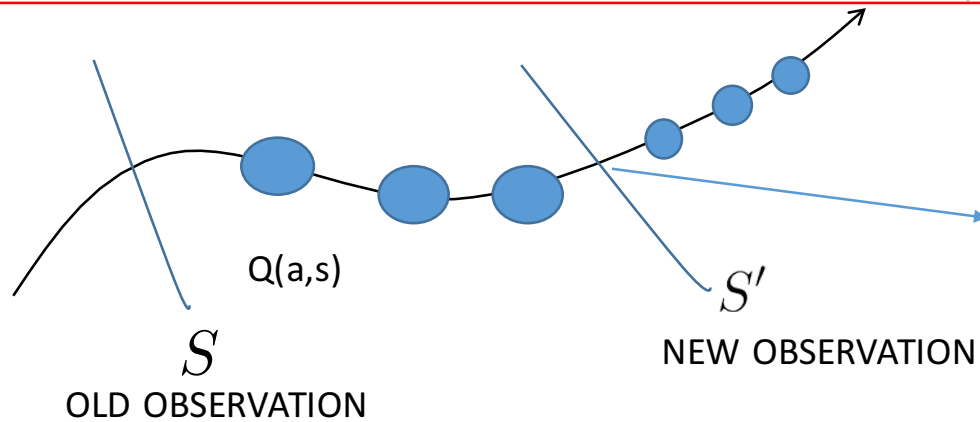
OBSERVATION:
DISCRETIZED VORTICITY LEVELS

$$\frac{d\mathbf{X}}{dt} = \mathbf{V}$$

$$\frac{d\mathbf{V}}{dt} = \beta \frac{D\mathbf{u}(\mathbf{X}, t)}{Dt} + \frac{\mathbf{u}(\mathbf{X}, t) - \mathbf{V}}{\tau}$$

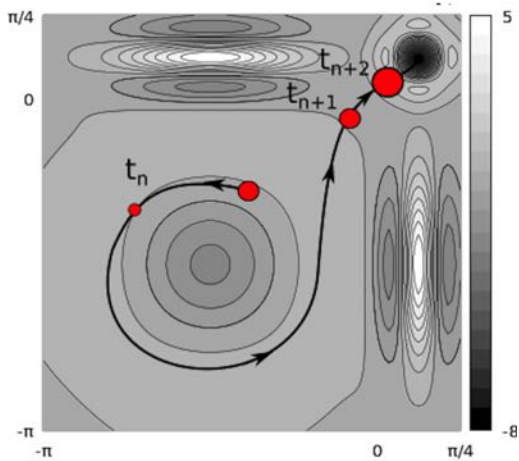
$$\pi_n : S_i \rightarrow a_j$$

$$Q_n(s_i, a_j) = R_n + \gamma R_{n+1} + \gamma^2 R_{n+2} + \gamma^3 R_{n+3} + \dots = \sum_{t=n}^{\infty} \gamma^t R_t$$



$$Q_n(s, a) = R_n + \gamma Q_{n+1}(s', a')$$

$$Q(s, a) \leftarrow Q(s, a) + \alpha [R' + \gamma \max_{a'} Q(s', a') - Q(s, a)]$$



$$\pi_n \rightarrow \pi_{n+1} \rightarrow \dots \rightarrow \pi_{opt}$$

1-step Q-LEARNING ALGORITHM

QUALITY MATRIX AT STEP $n \rightarrow Q_n(a_j, s_i)$

EXPECTED DISCOUNTED FUTURE RETURN IF ACTION a_j is taken after observation of state s_i

$$Q_n(s_i, a_j) = R_n + \gamma R_{n+1} + \gamma^2 R_{n+2} + \gamma^3 R_{n+3} + \dots = \sum_{t=n}^{\infty} \gamma^t R_t$$

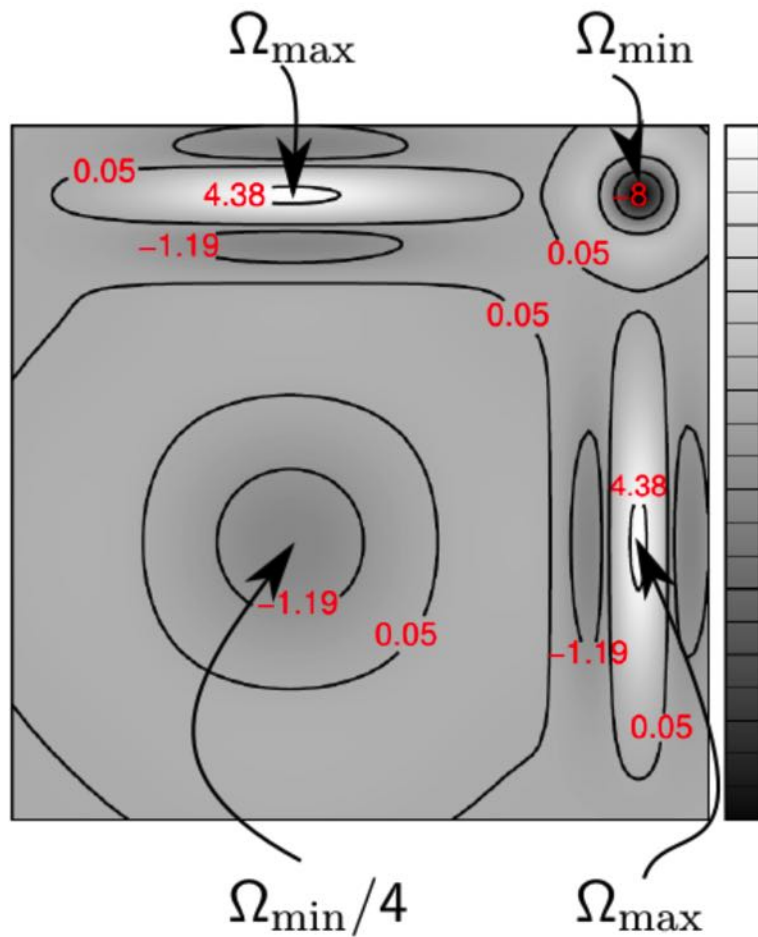
MYOPIC $\rightarrow \gamma = 0$
FAR-SIGHTED $\rightarrow \gamma = 1$

GREEDY POLICY AT STEP n :

$$\pi_n : a = \underset{a'}{\operatorname{arg\,max}} Q_n(a', s)$$

$$\begin{array}{l} s_1 \\ s_2 \\ s_3 \end{array} \begin{bmatrix} 1.2 & 0.3 & 0.1 \\ 2.2 & 4.3 & 10.1 \\ 2.0 & 8.1 & 2.0 \end{bmatrix} \begin{array}{l} s_1 \xrightarrow{\pi_n} a_1 \\ s_2 \xrightarrow{\pi_n} a_3 \\ s_3 \xrightarrow{\pi_n} a_2 \end{array}$$

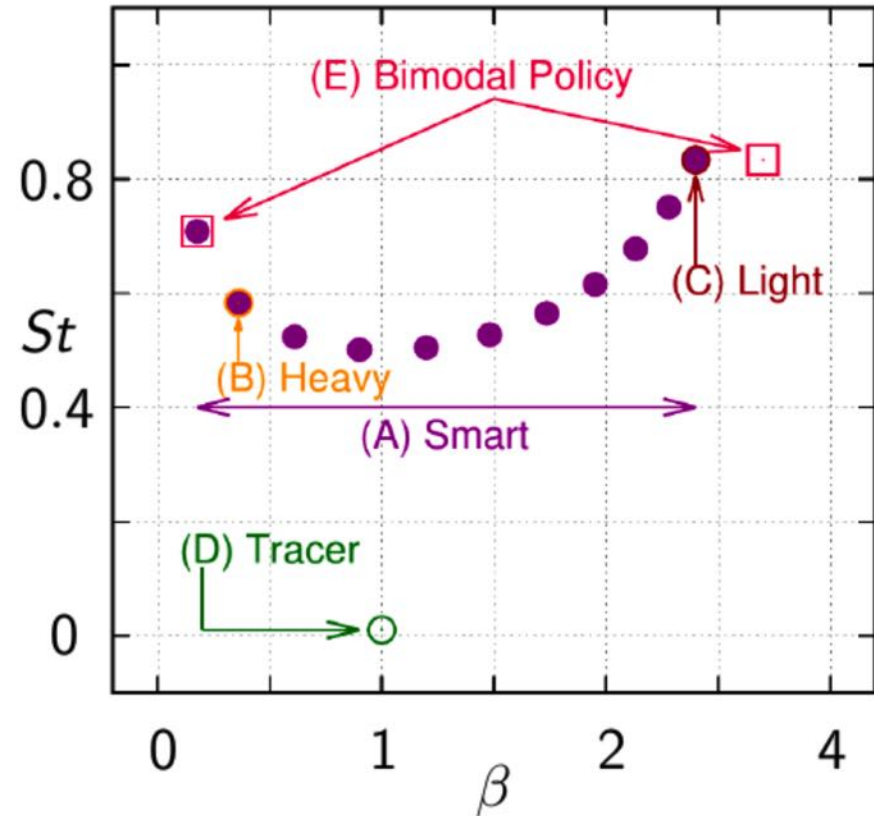
$a_1 \quad a_2 \quad a_3$



$$Ns = 21$$

$$Na = 11$$

CHANGING RADIUS b_n



$$\beta \rightarrow \beta(b_n)$$

$$St \rightarrow St(b_n)$$

Comparison with
naive behaviors

Learning gain $\tilde{\Sigma}(E) = \sqrt[3]{\frac{\sum_{n=1}^N R_n}{N}}$

