

# ENERGY TRANSFER AND ENERGY DISSIPATION IN TURBULENT FLOWS

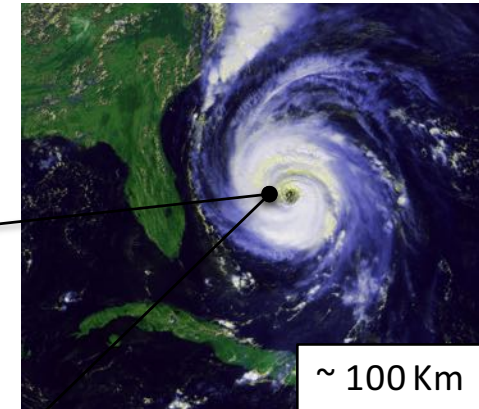
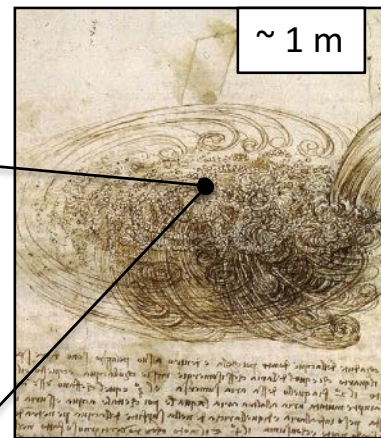
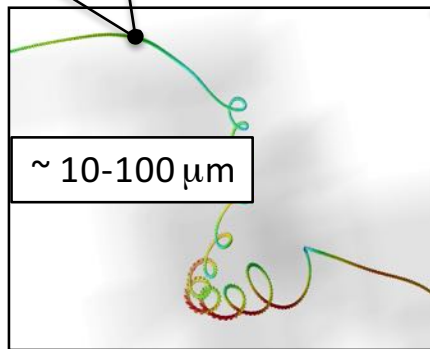
Πάντα ρει (everything flows)

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Xiamen University (March 2018)



MULTISCALE !



WHERE DOES ENERGY GO ?

WHAT CAN WE SAY ABOUT THE STATISTICAL PROPERTIES OF TURBULENT FLOWS AT LARGE/SMALL SCALES ?

### NAVIER-STOKES EQUATIONS:

$$m\vec{a} = \vec{F}$$

$$\underbrace{\partial_t \vec{v}}_{\text{acceleration}} + \underbrace{(\vec{v} \cdot \vec{\partial}) \vec{v}}_{\text{acceleration}} = - \underbrace{\vec{\partial} P}_{\text{pressure}} + \underbrace{\nu \Delta \vec{v}}_{\text{viscosity}} + \underbrace{\vec{f}}_{\text{external forcing}}$$

**Leonardo da Vinci (~ 1500):** “doue la turbolenza de si genera **[injected]**; doue la turbolenza dell aqua si mantiene **[advected]** plugho; doue la turbolenza dell aqua si posa **[dissipated]**”



# NAVIER-STOKES 3D $\leftrightarrow$ 2D

(NASA - Space Flight Center Scientific Visualization Studio)



(Vortices within vortices - APS Gallery of Fluid Motions)

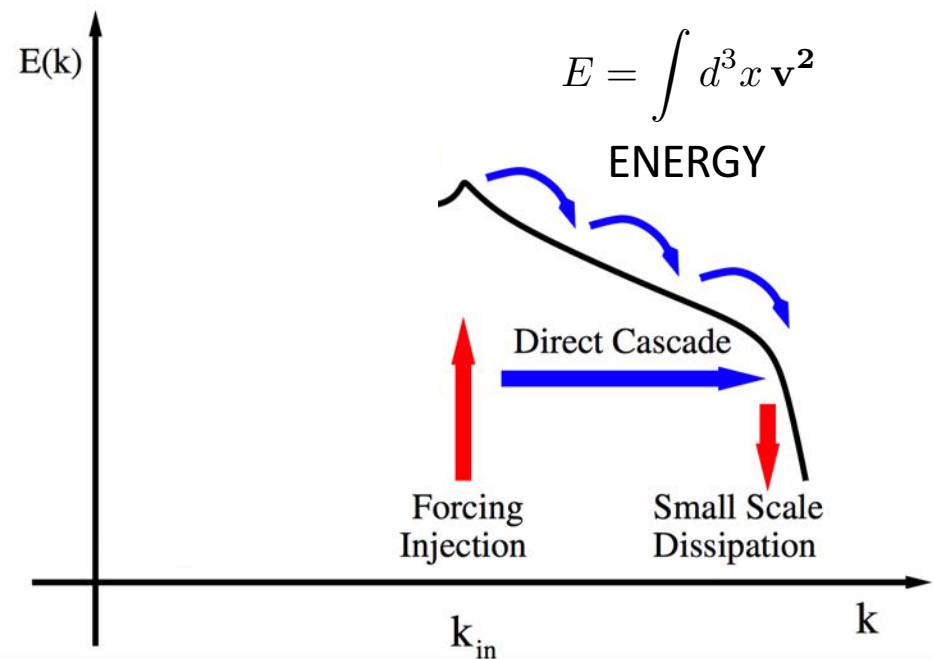
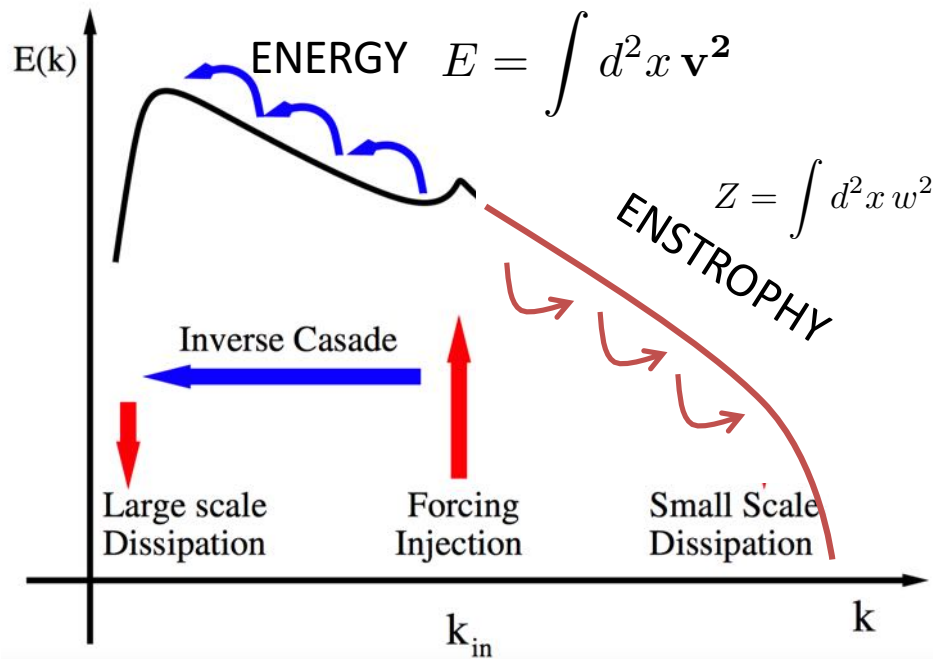
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## Vortices within vortices: hierarchical nature of vortex tubes in turbulence

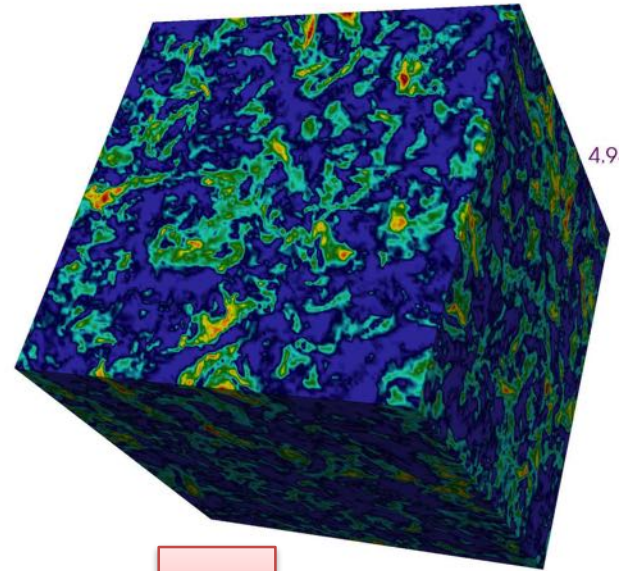
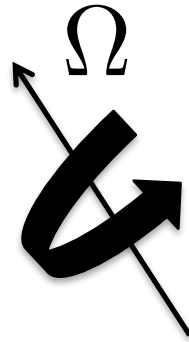
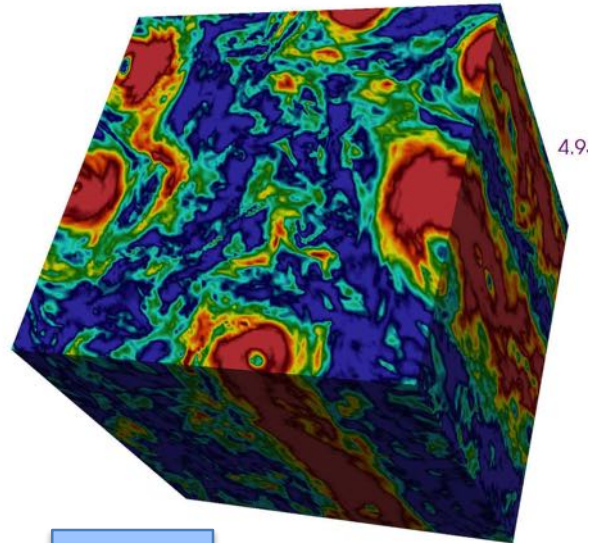
Kai Bürger<sup>1</sup>, Marc Treib<sup>1</sup>, Rüdiger Westermann<sup>1</sup>,  
Suzanne Werner<sup>2</sup>, Cristian C Lalescu<sup>3</sup>,  
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3D

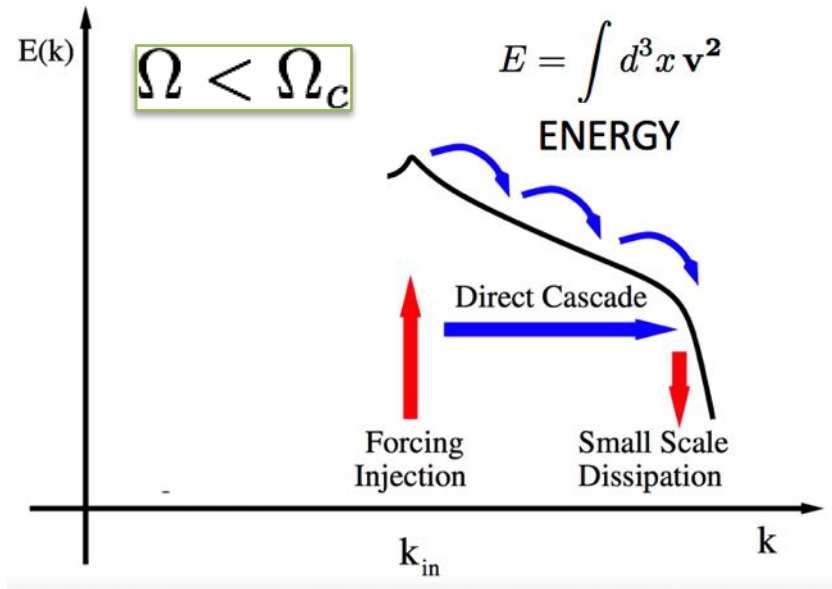
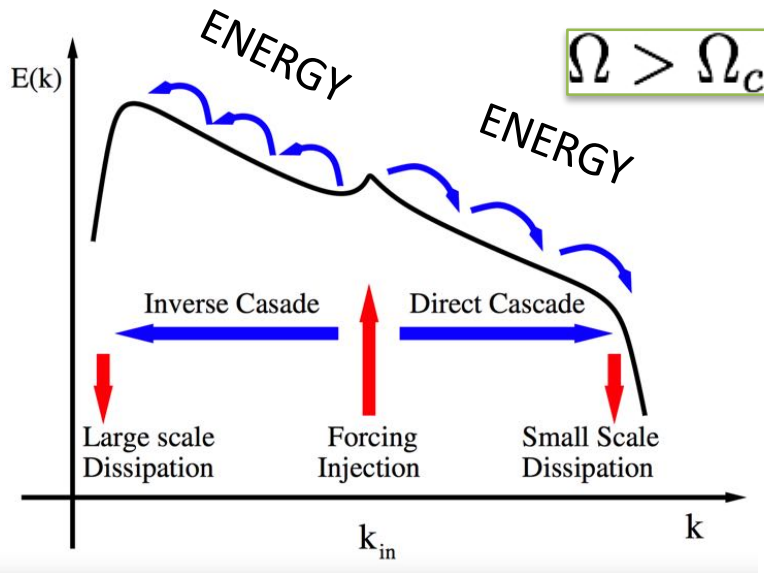


(PHASE) TRANSITIONS IN THE ENERGY TRANSFER:  
ROTATING FLOWS



~ 2D

3D



# COMPLEX FLUID & COMPLEX FLOWS

$$\left\{ \begin{aligned} \partial_t v + v \partial v &= -\partial p + \nu \Delta v + F(B, B) + g\theta + \sum_i c_0(u_i, v) \delta(r - r_i) + f \\ \partial_t \theta + v \cdot \partial \theta &= \chi \partial^2 \theta \\ \partial_t B + v \cdot \partial B &= B \cdot \partial v + \chi \partial^2 B \\ \partial \cdot v &= 0 \end{aligned} \right.$$

+ boundary conditions

$$\frac{du_i(r_i, t)}{dt} = -\rho_f |u_i - v| (u_i - v)$$

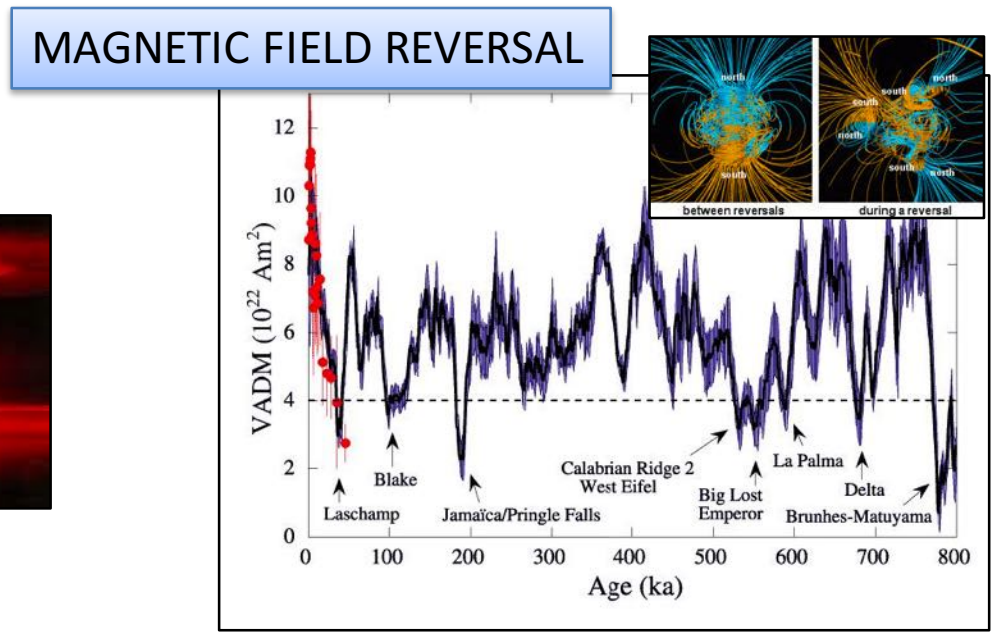
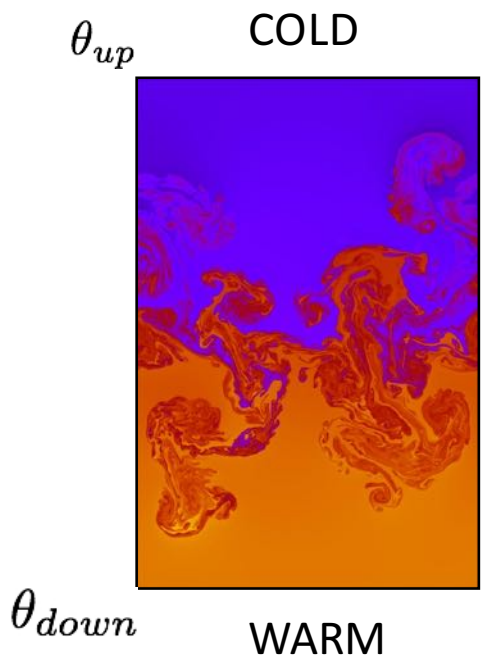
$$+ \rho_f \left( \frac{Dv}{Dt} - \frac{Du_i}{Dt} \right) + (u_i - v) \times \omega$$

control parameter:

$$Re = \frac{l_0 v_0}{\nu}$$

$Re \rightarrow \infty$

**FULLY NON-LINEAR**



# COMPLEX FLUID & COMPLEX FLOWS

$$\left\{ \begin{aligned} \partial_t v + v \partial v &= -\partial p + \nu \Delta v + F(B, B) + g\theta + \sum_i c_0(u_i, v) \delta(r - r_i) + f \\ \partial_t \theta + v \cdot \partial \theta &= \chi \partial^2 \theta \\ \partial_t B + v \cdot \partial B &= B \cdot \partial v + \chi \partial^2 B \\ \partial \cdot v &= 0 \\ &+ \text{boundary conditions} \end{aligned} \right.$$

control parameter:

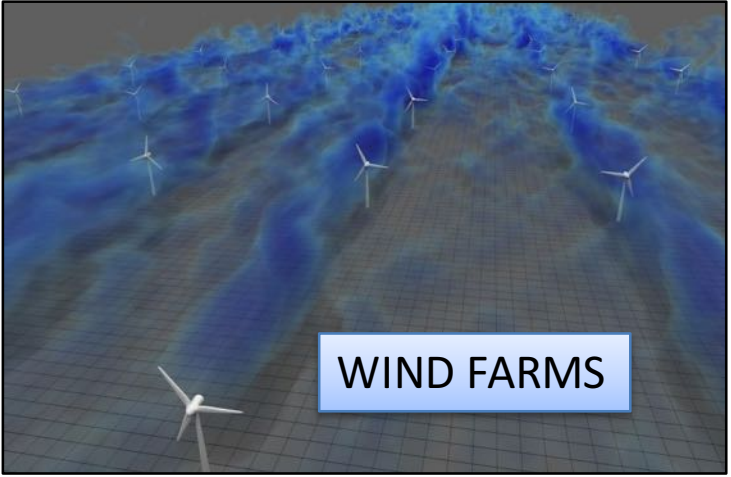
$$Re = \frac{l_0 v_0}{\nu}$$

$$Re \rightarrow \infty$$

FULLY NON-LINEAR

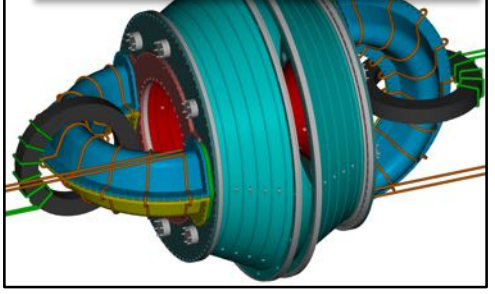
$$\left\{ \begin{aligned} \frac{du_i(r_i, t)}{dt} &= -\rho_f |u_i - v| (u_i - v) \\ + \rho_f \left( \frac{Dv}{Dt} - \frac{Du_i}{Dt} \right) &+ (u_i - v) \times \omega \end{aligned} \right.$$

ROTATING CONVECTION



WIND FARMS

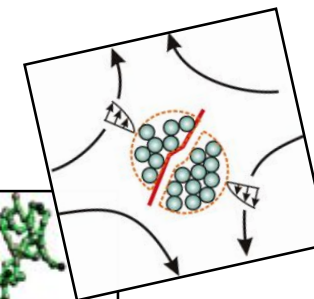
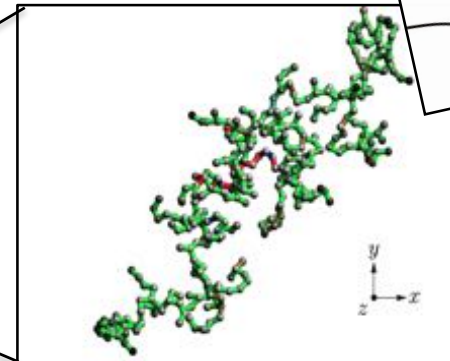
MAGNETIC PLASMA



## COMPLEX FLUID & COMPLEX FLOWS

$$\left\{ \begin{aligned} \partial_t \mathbf{v} + \mathbf{v} \cdot \partial \mathbf{v} &= -\partial P + \nu \partial^2 \mathbf{v} + F(\mathbf{B}, B) + g\theta + \sum_i c_0(\mathbf{u}_i, \mathbf{v}) \delta(\mathbf{r} - \mathbf{r}_i) + \mathbf{f} \\ \partial_t \theta + \mathbf{v} \cdot \partial \theta &= \chi \partial^2 \theta \quad \leftarrow \text{temperature} \\ \partial_t \mathbf{B} + \mathbf{v} \cdot \partial \mathbf{B} &= \mathbf{B} \cdot \partial \mathbf{v} + \chi \partial^2 \mathbf{B} \quad \leftarrow \text{magnetic field} \\ \partial \cdot \mathbf{v} &= 0 \\ &+ \text{boundary conditions} \end{aligned} \right.$$

$$\left\{ \begin{aligned} \frac{d\mathbf{u}_i(\mathbf{r}_i, t)}{dt} &= -\rho_f |\mathbf{u}_i - \mathbf{v}| (\mathbf{u}_i - \mathbf{v}) \\ &+ \rho_f \left( \frac{D\mathbf{v}}{Dt} - \frac{D\mathbf{u}_i}{Dt} \right) + (\mathbf{u}_i - \mathbf{v}) \times \boldsymbol{\omega} \end{aligned} \right. \quad \leftarrow \text{small particles/colloidal aggregates: Stokes drag, added mass, lift force, etc...}$$



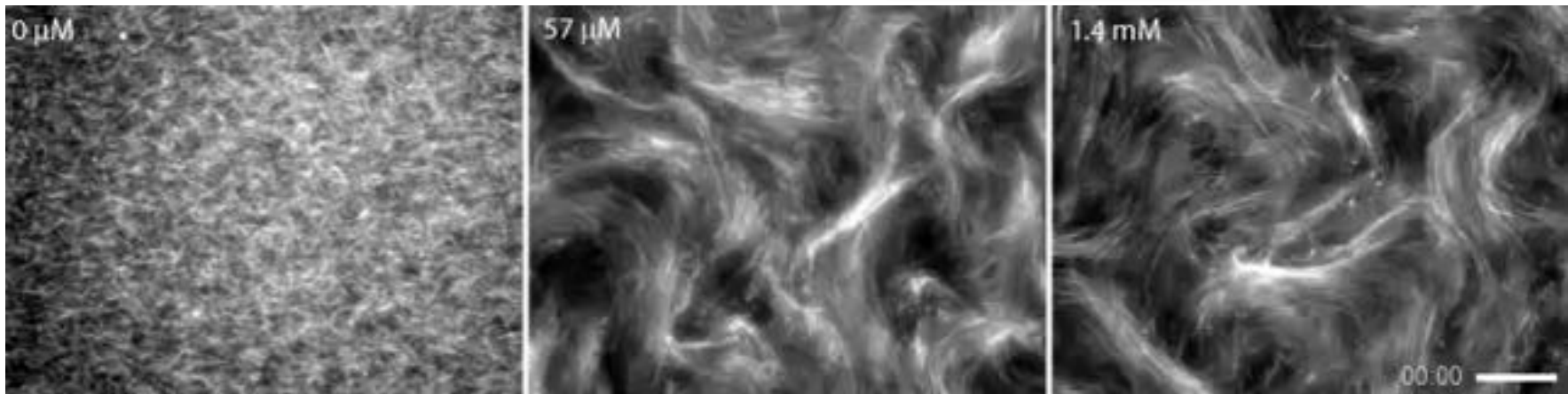
+ Stokesian dynamics

COAGULATION/FRAGMENTATION OF COLLOIDAL AGGREGATES IN TURBULENT FLOWS

## COMPLEX FLUID & COMPLEX FLOWS

$$\left\{ \begin{array}{l} \partial_t v + v \cdot \partial v = -\partial P + \nu \partial^2 v + F(B, B) + g\theta + \sum_i c_0(u_i, v) \delta(r - r_i) + f(V_{active}) \\ \partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta \quad \leftarrow \text{temperature} \\ \partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B \quad \leftarrow \text{magnetic field} \\ \partial \cdot v = 0 \\ + \text{boundary conditions} \\ \frac{du_i(r_i, t)}{dt} = -\rho_f |u_i - v| (u_i - v) \\ + \rho_f \left( \frac{Dv}{Dt} - \frac{Du_i}{Dt} \right) + (u_i - v) \times \omega \end{array} \right.$$

ACTIVE MATTER



Sanchez et al Nature 2012 "Microtubules activated by Kinesin Motor Proteins"



$$\left\{ \begin{array}{l} \partial_t v + v \partial v = -\partial p + \nu \Delta v \\ \partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta \\ \partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B \\ \partial \cdot v = 0 \\ + \text{boundary conditions} \\ \frac{du_i(\mathbf{r}_i, t)}{dt} = -\rho_f |u_i - v| (u_i - v) \\ + \rho_f \left( \frac{Dv}{Dt} - \frac{Du_i}{Dt} \right) + (u_i - v) \times \omega \end{array} \right. + F(B, B) + g\theta + \sum_i c_0(u_i, v) \delta(\mathbf{r} - \mathbf{r}_i) + f$$

control parameter:

$$Re = \frac{l_0 v_0}{\nu}$$

$$\left\{ \begin{array}{l} Re \rightarrow \infty \\ \nu \rightarrow 0 \end{array} \right.$$

Too many turbulences? NO! -> UNIVERSALITY

ALL TURBULENT FLOWS RECOVER ISOTROPY AND HOMOGENEITY (AT SCALES SMALL ENOUGH)

- Homogeneous & Isotropic Turbulence
- Fully periodic 3D domain
- Gaussian delta-correlated forcing
- Incompressible

Homogeneous and Isotropic turbulence: the (UNSOLVED) hydrogen atom of fluid dynamics

WHY STILL UNSOLVED?  
(EQUATIONS ARE KNOWN SINCE 250 YEARS AGO!)