On the effects of helicity on turbulent flows

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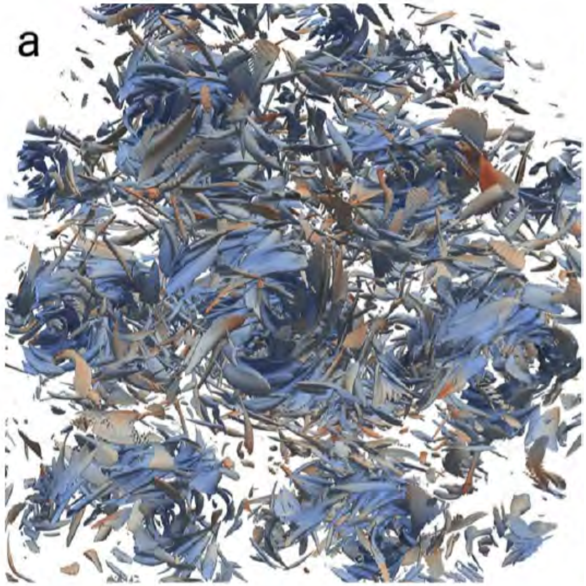
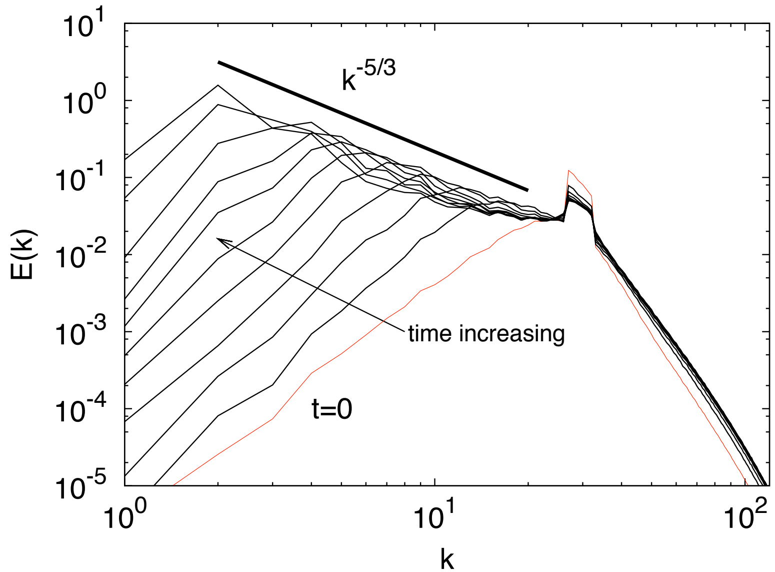




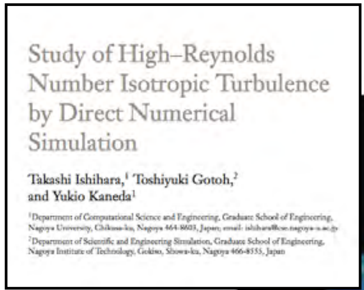
Credits: **S. Musacchio** (CNRS-France); **F. Toschi** (University of Eindhoven, The Netherlands); **E. Titi** (Weizmann Institute of Science, Israel), **F. Bonaccorso** and **G. Sahoo** (U. Tor Vergata, Italy)

HOW TO USE UNCONVENTIONAL NUMERICS TO UNDERSTAND AND MODEL EULERIAN TURBULENCE IN 2D, 3D (AND IN BETWEEN)

$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v} \cdot \partial)\mathbf{v} = -\partial P + \nu \Delta \mathbf{v} + \mathbf{F} \\ \partial \cdot \mathbf{v} = 0 \\ + \, Boundary \ Conditions \end{cases}$$

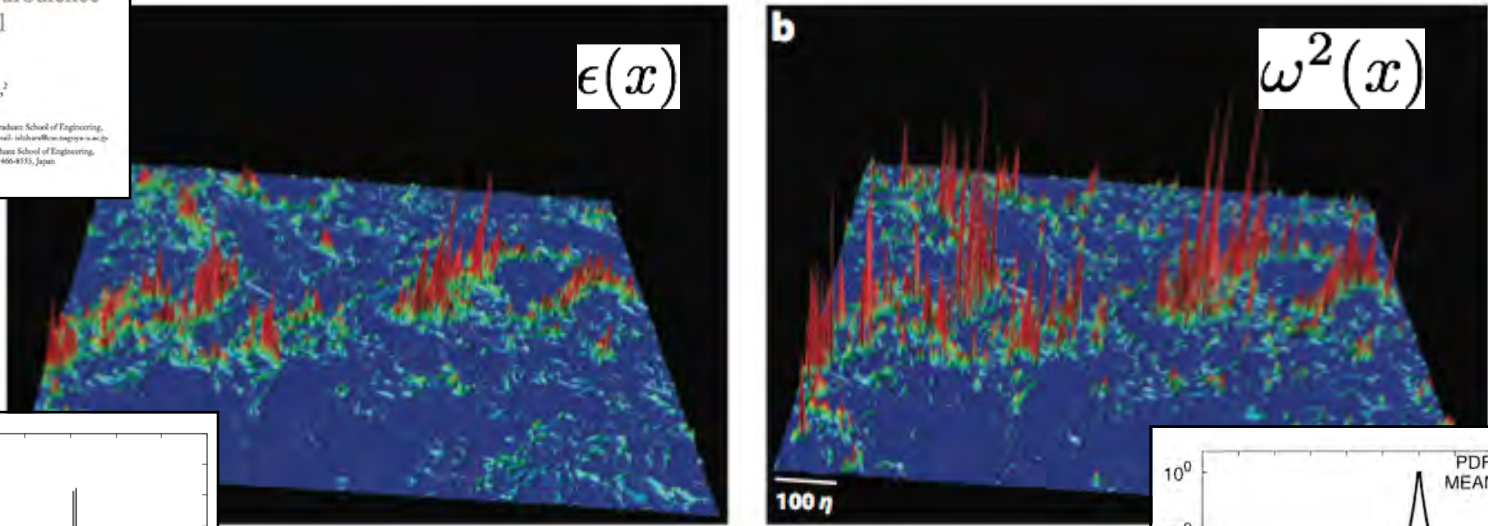
Q: CAN WE DISSECT (AND RECONSTRUCT) NS EQUATIONS TO EXTRACT INTERESTING INFORMATION FROM ITS ELEMENTARY CONSTITUENTS?

SMALL-SCALES INTERMITTENCY

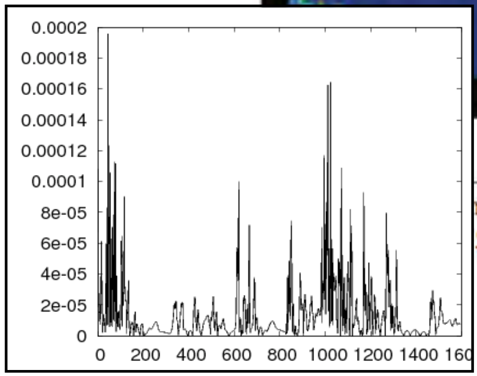

Study of High-Reynolds Number Isotropic Turbulence by Direct Numerical Simulation

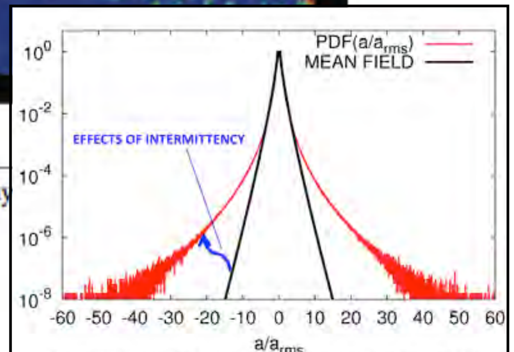
Takashi Ishihara,[1] Toshiyuki Gotoh,[2] and Yukio Kaneda[1]

[1]Department of Computational Science and Engineering, Graduate School of Engineering, Nagoya University, Chikusa-ku, Nagoya 464-8603, Japan; email: ishihara@cse.nagoya-u.ac.jp
[2]Department of Scientific and Engineering Simulation, Graduate School of Engineering, Nagoya Institute of Technology, Gokiso, Showa-ku, Nagoya 466-8555, Japan

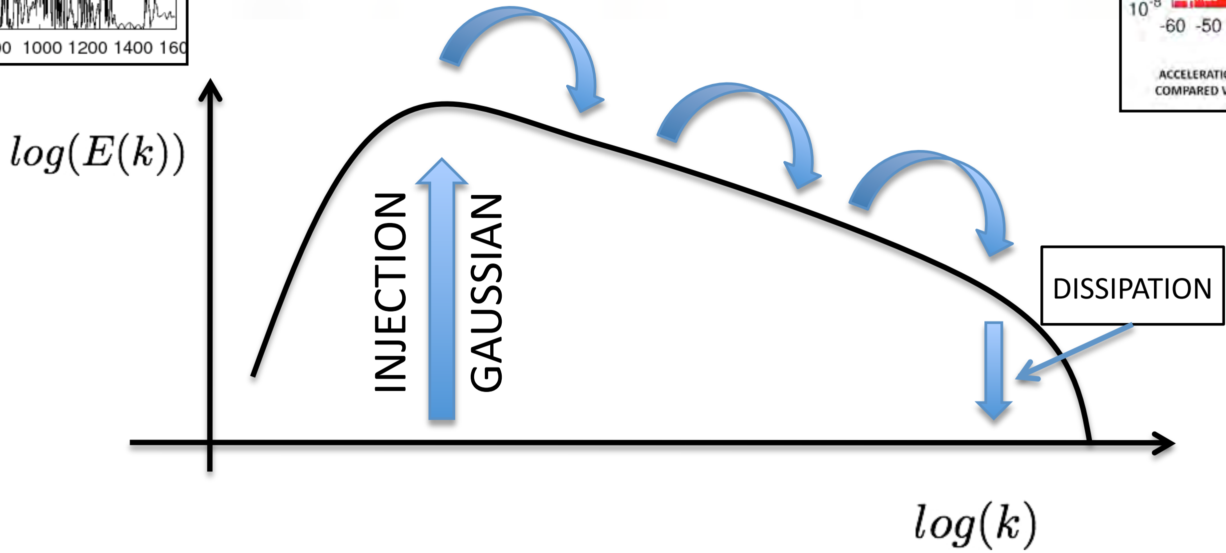


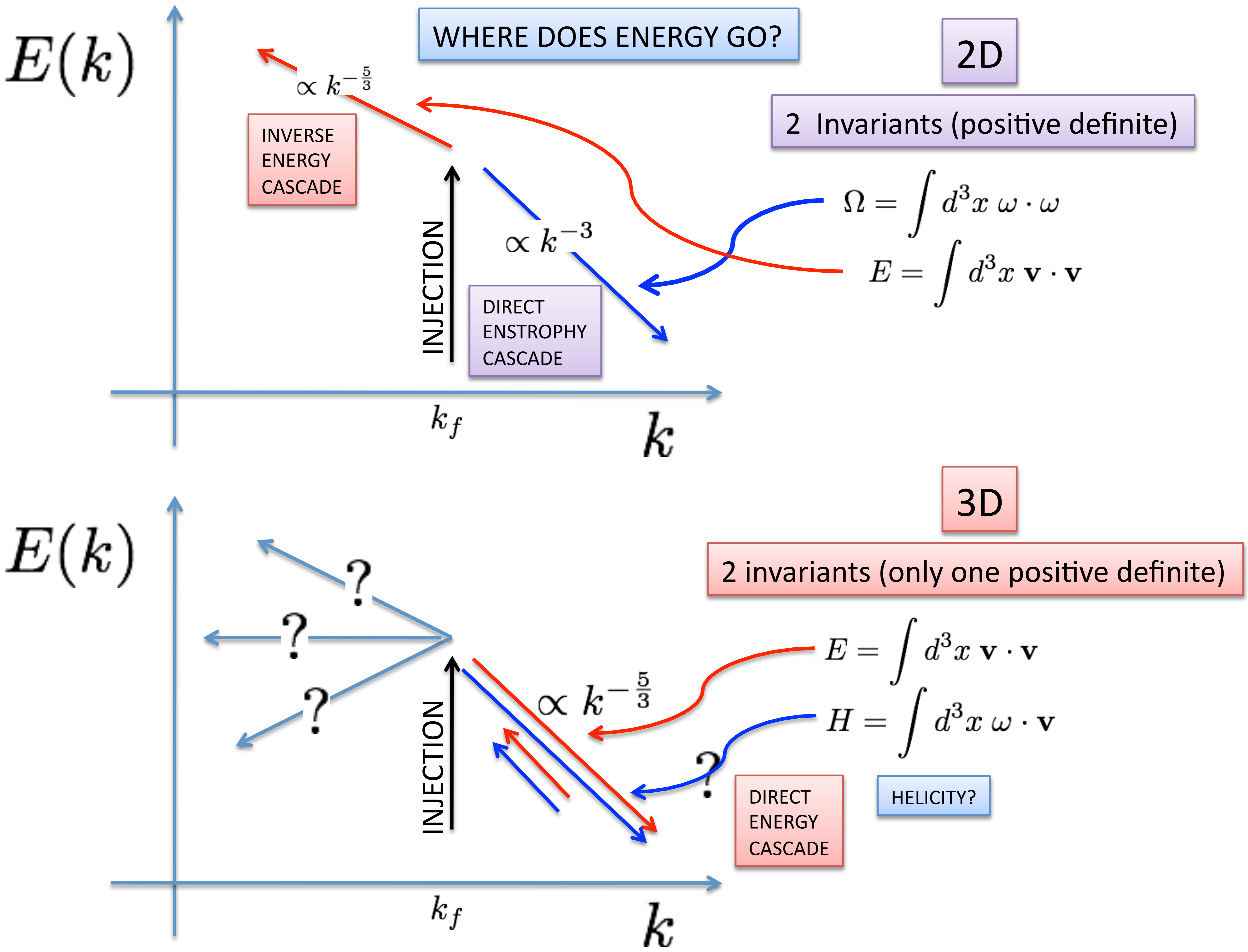
nsity distributions of (*a*) the energy-dissipation rate $\tilde{\varepsilon} = \varepsilon/(2\nu)$ and (*b*) the enstrophy
5 in arbitrary units.





ACCELERATION PROBABILITY DISTRIBUTION FUNCTION (PDF) AT RE ~ 10⁵ [Bi04]
COMPARED WITH THE PREDICTION FROM MEAN FIELD (KOLMOGOROV THEORY)





WHERE DOES ENERGY GO?
2D
2 Invariants (positive definite)
$E(k)$
$\propto k^{-\frac{5}{3}}$
INVERSE ENERGY CASCADE
INJECTION
$\propto k^{-3}$
DIRECT ENSTROPHY CASCADE
$\Omega = \int d^3x\, \omega \cdot \omega$
$E = \int d^3x\, \mathbf{v} \cdot \mathbf{v}$
k_f
k
3D
2 invariants (only one positive definite)
$E(k)$
?
?
?
INJECTION
$\propto k^{-\frac{5}{3}}$
$E = \int d^3x\, \mathbf{v} \cdot \mathbf{v}$
$H = \int d^3x\, \omega \cdot \mathbf{v}$
?
DIRECT ENERGY CASCADE
HELICITY?
k_f
k

PRL 104, 184506 (2010) PHYSICAL REVIEW LETTERS week ending 7 MAY 2010

Turbulence in More than Two and Less than Three Dimensions

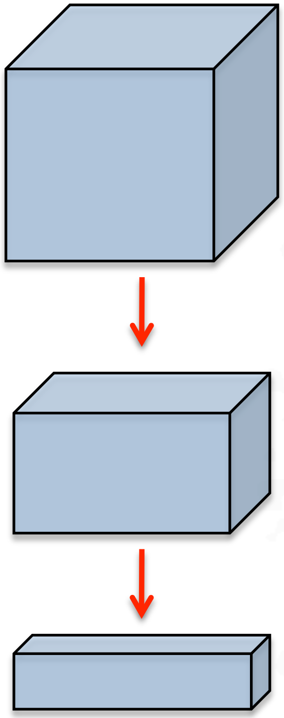
Antonio Celani,[1] Stefano Musacchio,[2,3] and Dario Vincenzi[3]

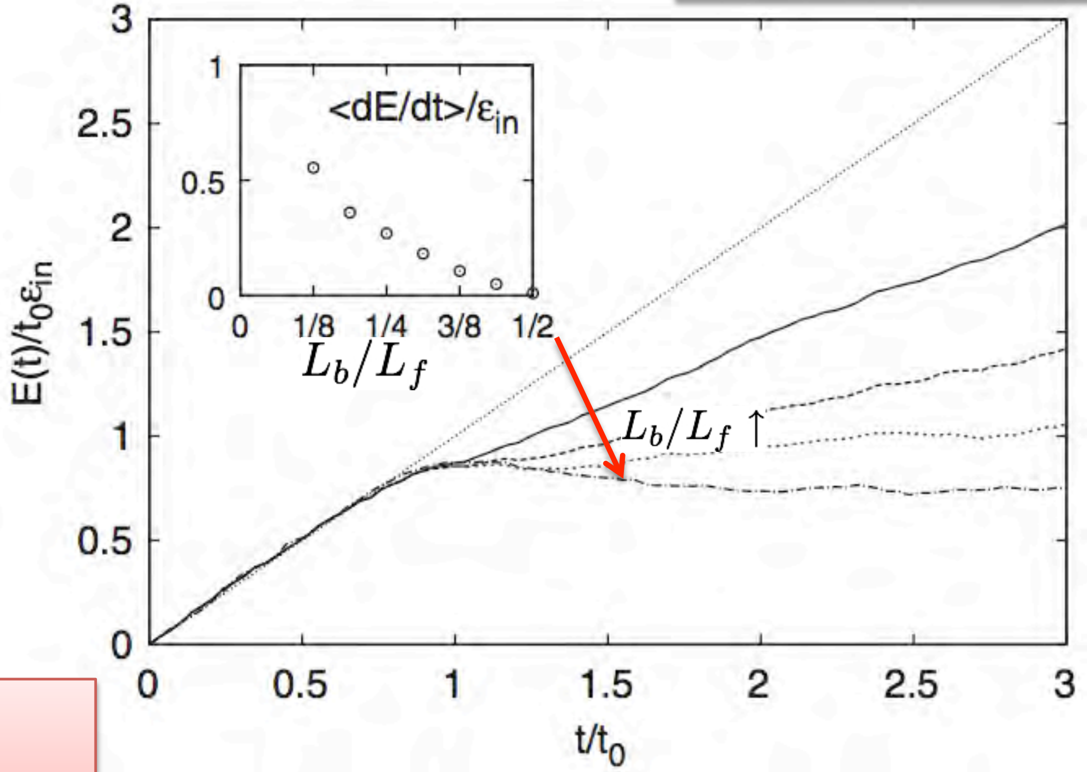
Upscale energy transfer in thick turbulent fluid layers

Nat Phys. 2011

H. Xia[1], D. Byrne[1], G. Falkovich[2] and M. Shats[1]*

CONFINEMENT 3D → 2D





PHASE TRANSITION?

How to switch from 3D to 2D ?

Crossover from Two- to Three-Dimensional Turbulence

Leslie M. Smith
Yale University, New Haven, Connecticut 06520

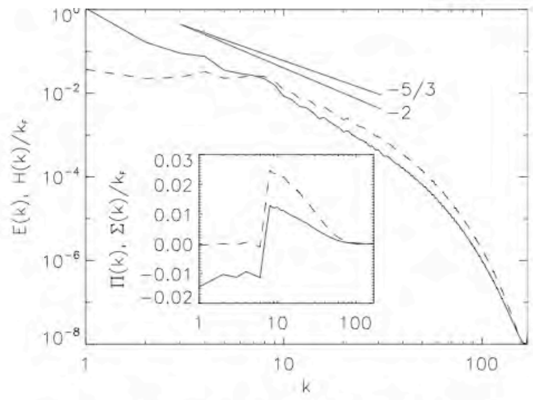
Jeffrey R. Chasnov
The Hong Kong University of Science and Technology, Hong Kong

Fabian Waleffe
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

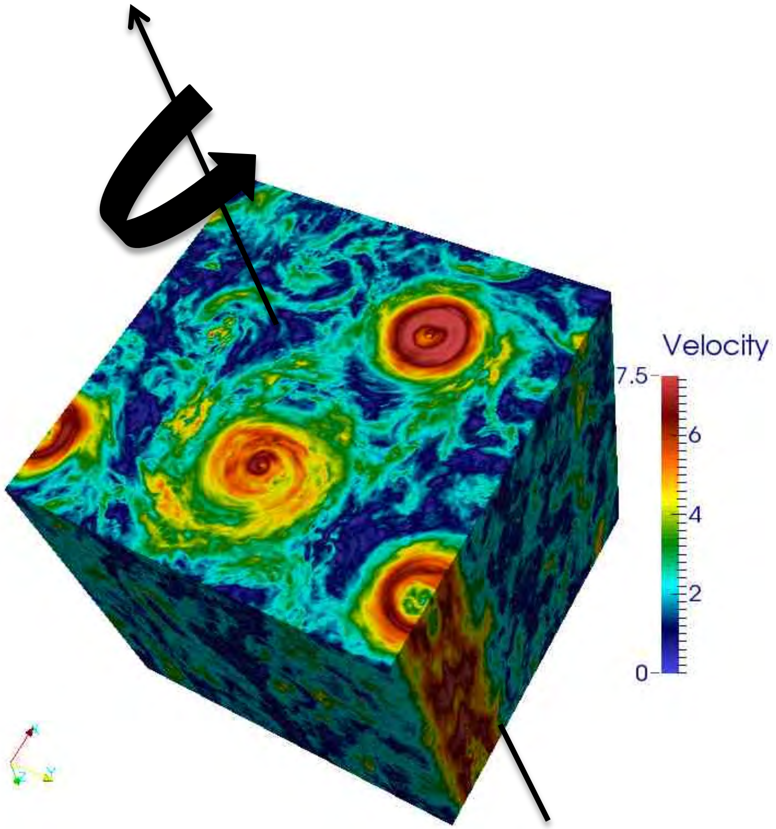
(Received 22 March 1996)

Forced rotating turbulence is simulated within a periodic box of small aspect ratio. Critical parameter values are found for the stability of a 2D inverse cascade of energy in the presence of 3D motions at small scales. There is a critical rotation rate below which 2D forcing leads to an equilibrated 3D state, while for a slightly larger rotation rate, 3D forcing drives a 2D inverse cascade. It is shown that inverse and forward cascades of energy can coexist. This study is relevant to geophysical flows, and contains physics beyond the scope of quasigeostrophic models. [S0031-9007(96)01175-1]

P.D. Mininni and A. Pouquet.
Phys. Rev. E **79,** 026304 (2009)



ROTATION
3D -> 2D



doi: 10.1209/0295-5075/100/44003

Dual non-Kolmogorov cascades in a von Kármán flow

E. Herbert[1], F. Daviaud[1], B. Dubrulle[1], S. Nazarenko[2] and A. Naso[3]

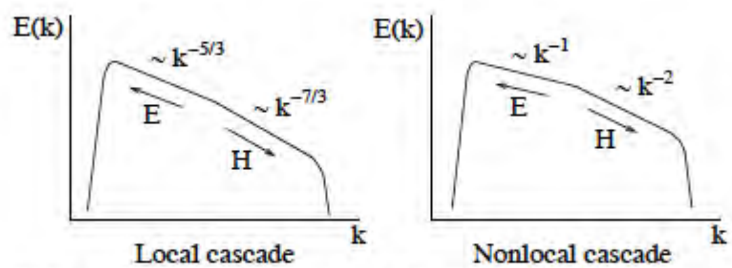


FIG. 1: Summary of the two different possibilities for dual helicity and energy cascades as a function of the wavenumber k in a Beltrami flow. Left : local case; right : non-local case.

ON THE ROLE OF HELICITY IN 3D FORWARD/BACKWARD ENERGY CASCADES

$$H = \int d^3x \, \omega \cdot \mathbf{v}$$

1) **We show that ALL flows in nature posses a class of nonlinear interactions characterized by a backward energy transfer (inverse energy cascade), triggered by the dynamics of Helicity, and that this happens even in fully isotropic, homogeneous 3D turbulence**

2) **Connections to small-scales intermittency**

3) **Connections to regularity of NS equations in 3D**

4) **Extensions to Magnetohydrodynamics**

Helicity and singular structures in fluid dynamics

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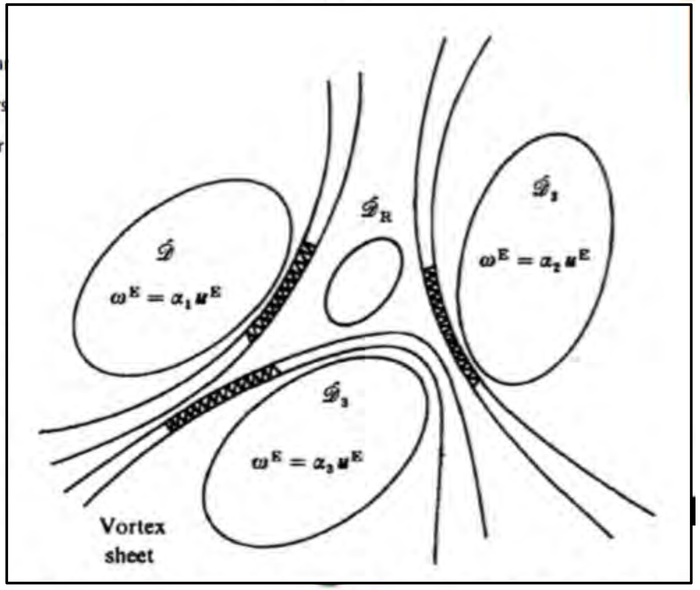
Department of Applied Mathematics and Theoretical Physics, University of Ca

This contribution is part of the special series of Inaugural Articles by members

Contributed by H. Keith Moffatt, January 14, 2014 (sent for review December

Helicity is, like energy, a quadratic invariant of the Euler equations of ideal fluid flow, although, unlike energy, it is not sign definite. In physical terms, it represents the degree of linkage of the vortex lines of a flow, conserved when conditions are such that these vortex lines are frozen in the fluid. Some basic properties of helicity are reviewed, with particular reference to (*i*) its crucial role in the dynamo excitation of magnetic fields in cosmic systems; (*ii*) its bearing on the existence of Euler flows of arbitrarily complex streamline topology; (*iii*) the constraining role of the analogous magnetic helicity in the determination of stable knotted minimum-energy magnetostatic structures; and (*iv*) its role in depleting nonlinearity in the Navier-Stokes equations, with implications for the coherent structures and energy cascade of turbulence. In a final section, some singular phenomena in low Reynolds number flows are briefly described.



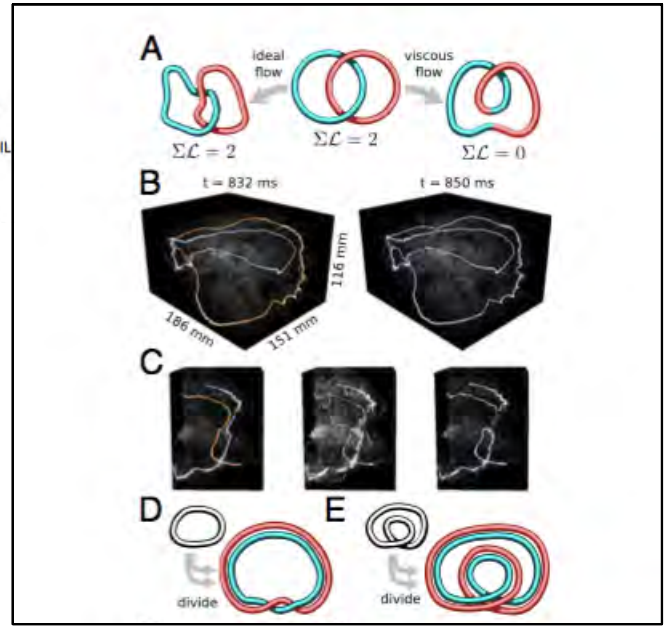
Helicity conservation by flow across scales in reconnecting vortex links and knots

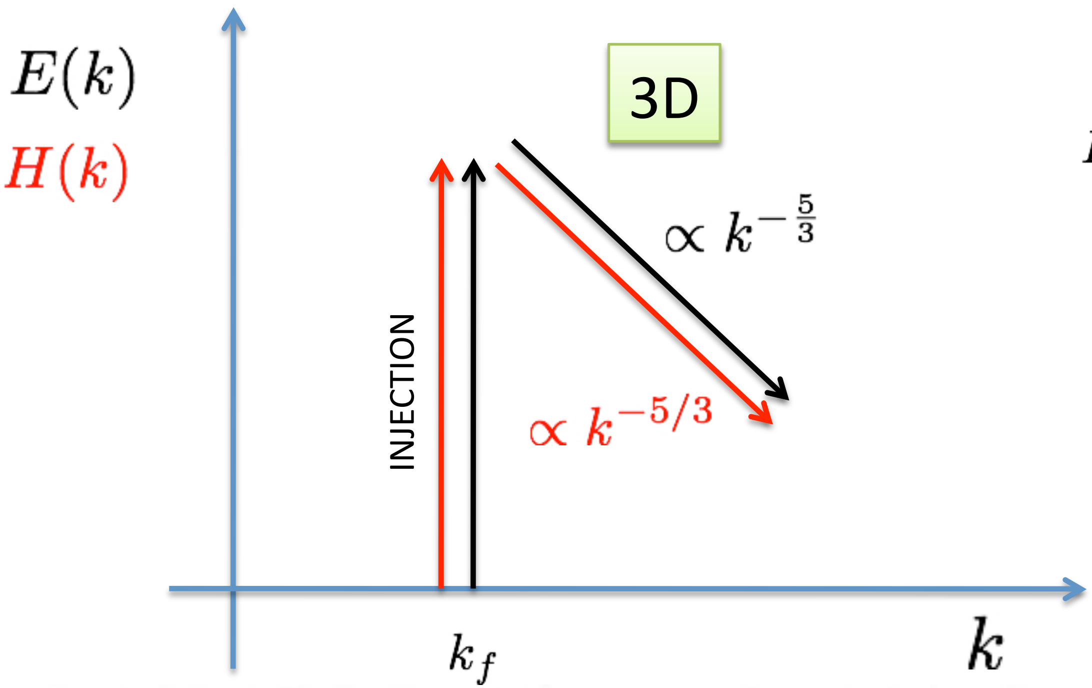
Martin W. Scheeler[a,1,2], Dustin Kleckner[a,1,2], Davide Proment[b], Gordon L. Kindlmann[c], and William T. M. Irvine[a,2]

[a]James Franck Institute, Department of Physics, and [c]Computation Institute, Department of Computer Science, The University of Chicago, Chicago, IL and [b]School of Mathematics, University of East Anglia, Norwich Research Park, Norwich NR4 7TJ, United Kingdom

Edited* by Leo P. Kadanoff, The University of Chicago, Chicago, IL, and approved August 28, 2014 (received for review April 19, 2014)

The conjecture that helicity (or knottedness) is a fundamental conserved quantity has a rich history in fluid mechanics, but the nature of this conservation in the presence of dissipation has proven difficult to resolve. Making use of recent advances, we create vortex knots and links in viscous fluids and simulated superfluids and track their geometry through topology-changing reconnections. We find that the reassociation of vortex lines through a reconnection enables the transfer of helicity from links and knots to helical coils. This process is remarkably efficient, owing to the antiparallel orientation spontaneously adopted by the reconnecting vortices. Using a new method for quantifying the spatial helicity spectrum, we find that the reconnection process can be viewed as transferring helicity between scales, rather than dissipating it. We also infer the presence of geometric deformations that convert helical coils into even smaller scale twist, where it may ultimately be dissipated. Our results suggest that helicity conservation plays an important role in fluids and related fields, even in the presence of dissipation.

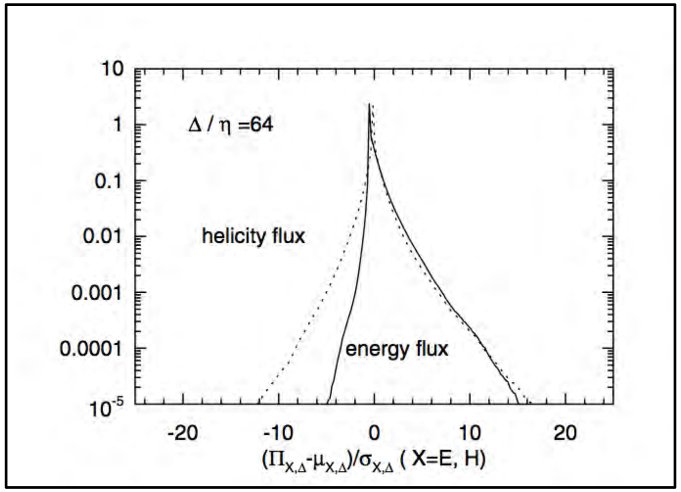




2 invariants

$$E = \int d^3x \, \mathbf{v} \cdot \mathbf{v} \qquad H = \int d^3x \, \omega \cdot \mathbf{v}$$

$$H(k) \propto \eta \epsilon^{-\frac{1}{3}} k^{-\frac{5}{3}}$$
$$E(k) \propto \epsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$$



The joint cascade of energy and helicity in three-dimensional turbulence

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Shiyi Chen
Department of Mechanical Engineering, The Johns Hopkins University, Baltimore, Maryland 21218; Center for Nonlinear Studies and Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545; and Peking University, People's Republic of China

Gregory L. Eyink
Department of Mathematics, University of Arizona, Tucson, Arizona 85721

The role of helicity in three-dimensional turbulence is, in our opinion, still somewhat mysterious. In particular, it is still unclear how energy and helicity dynamics interact in detail. The role of helicity in geophysical flows has been considered[3]—without being fully resolved—while its appearance and influence in engineering applications is still largely unexplored. We hope that this work will be a helpful step in the direction of better understanding the subtle manifestations of helicity in three-dimensional turbulence.

The nature of triad interactions in homogeneous turbulence

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(Received 24 July 1991; accepted 22 October 1991)

$$\boxed{\boldsymbol{u}(\boldsymbol{k}) = u^+(\boldsymbol{k})\boldsymbol{h}^+(\boldsymbol{k}) + u^-(\boldsymbol{k})\boldsymbol{h}^-(\boldsymbol{k})}$$

$$\boldsymbol{h}^{\pm} = \hat{\boldsymbol{\nu}} \times \hat{\boldsymbol{k}} \pm i\hat{\boldsymbol{\nu}}$$

$$\hat{\boldsymbol{\nu}} = \boldsymbol{z} \times \boldsymbol{k}/||\boldsymbol{z} \times \boldsymbol{k}||$$

$$\boxed{i\boldsymbol{k} \times \boldsymbol{h}^{\pm} = \pm k\boldsymbol{h}^{\pm}}$$

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$

$$u^{s_k}(\mathbf{k},t) \quad (s_k = \pm 1)$$

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_p p - s_q q)$$
$$\times [u^{s_p}(\mathbf{p}) u^{s_q}(\mathbf{q})]^* . \qquad (15)$$

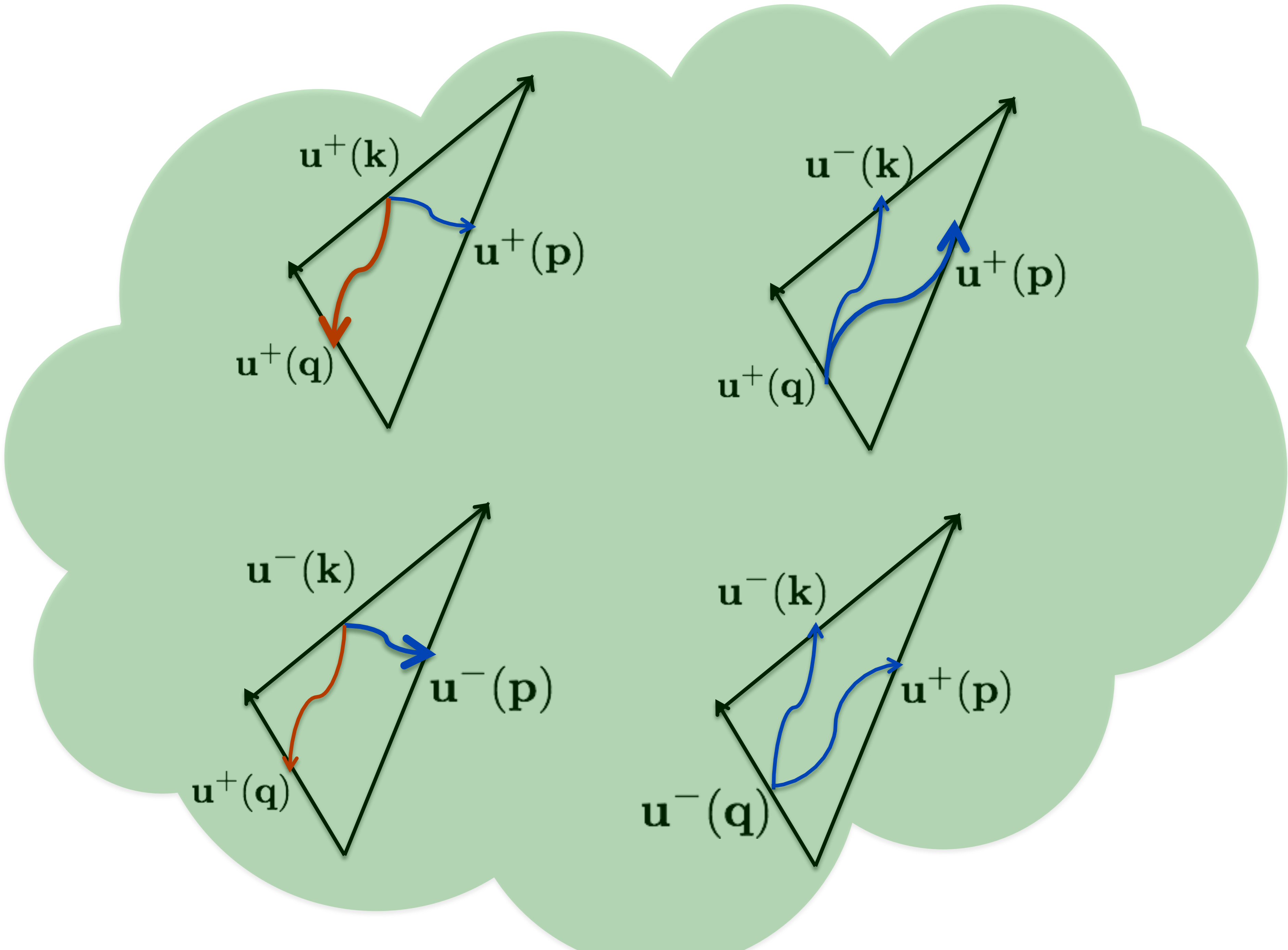
Eight different types of interaction between three modes $u^{s_k}(\mathbf{k})$, $u^{s_p}(\mathbf{p})$, and $u^{s_q}(\mathbf{q})$ with $|\mathbf{k}|<|\mathbf{p}|<|\mathbf{q}|$ are allowed according to the value of the triplet (s_k, s_p, s_q)

$$\dot{u}^{s_k} = r(s_p p - s_q q)\frac{s_k k + s_p p + s_q q}{p}(u^{s_p} u^{s_q})^*,$$

$$\dot{u}^{s_p} = r(s_q q - s_k k)\frac{s_k k + s_p p + s_q q}{p}(u^{s_q} u^{s_k})^*,$$

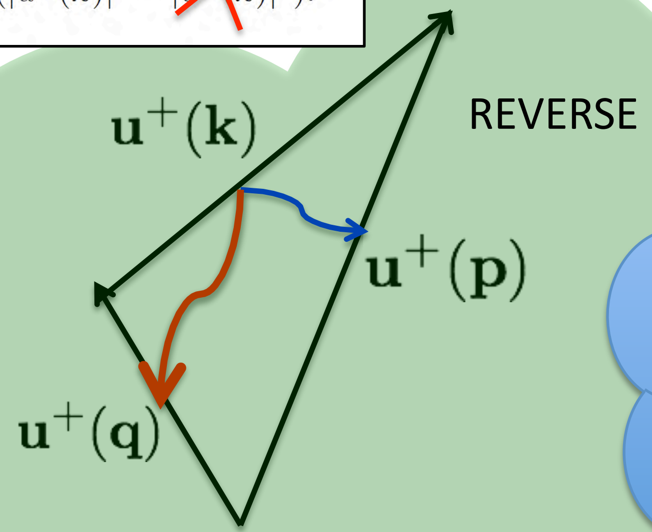
$$\dot{u}^{s_q} = r(s_k k - s_p p)\frac{s_k k + s_p p + s_q q}{p}(u^{s_k} u^{s_p})^*.$$

TRIADIC INTERACTION IN THE WHOLE NAVIER_STOKES EQS

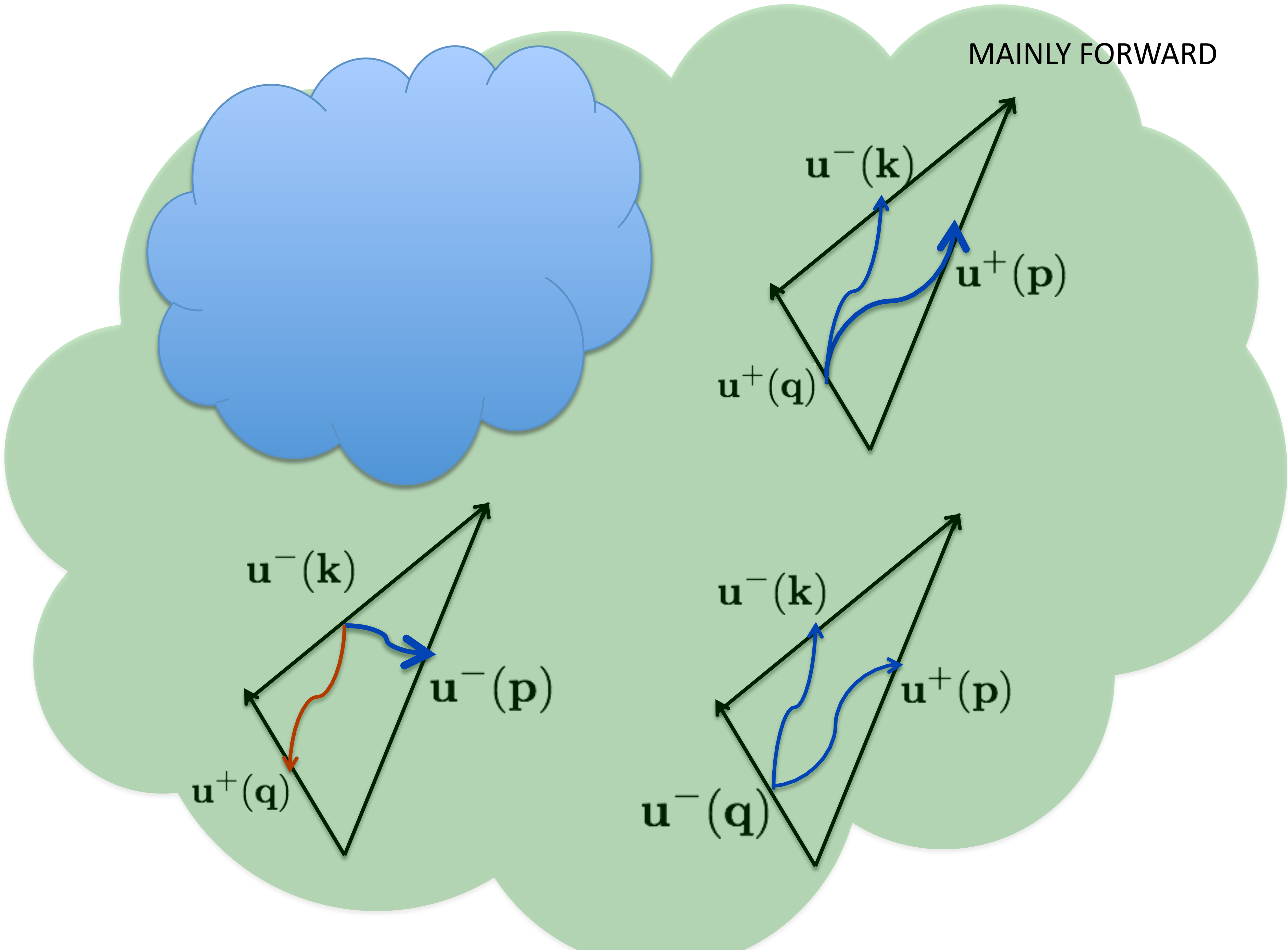


TRIADIC INTERACTION IN DECIMATED NAVIER_STOKES EQS

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + \cancel{|u^-(\boldsymbol{k})|^2}; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - \cancel{|u^-(\boldsymbol{k})|^2}). \end{cases}$$

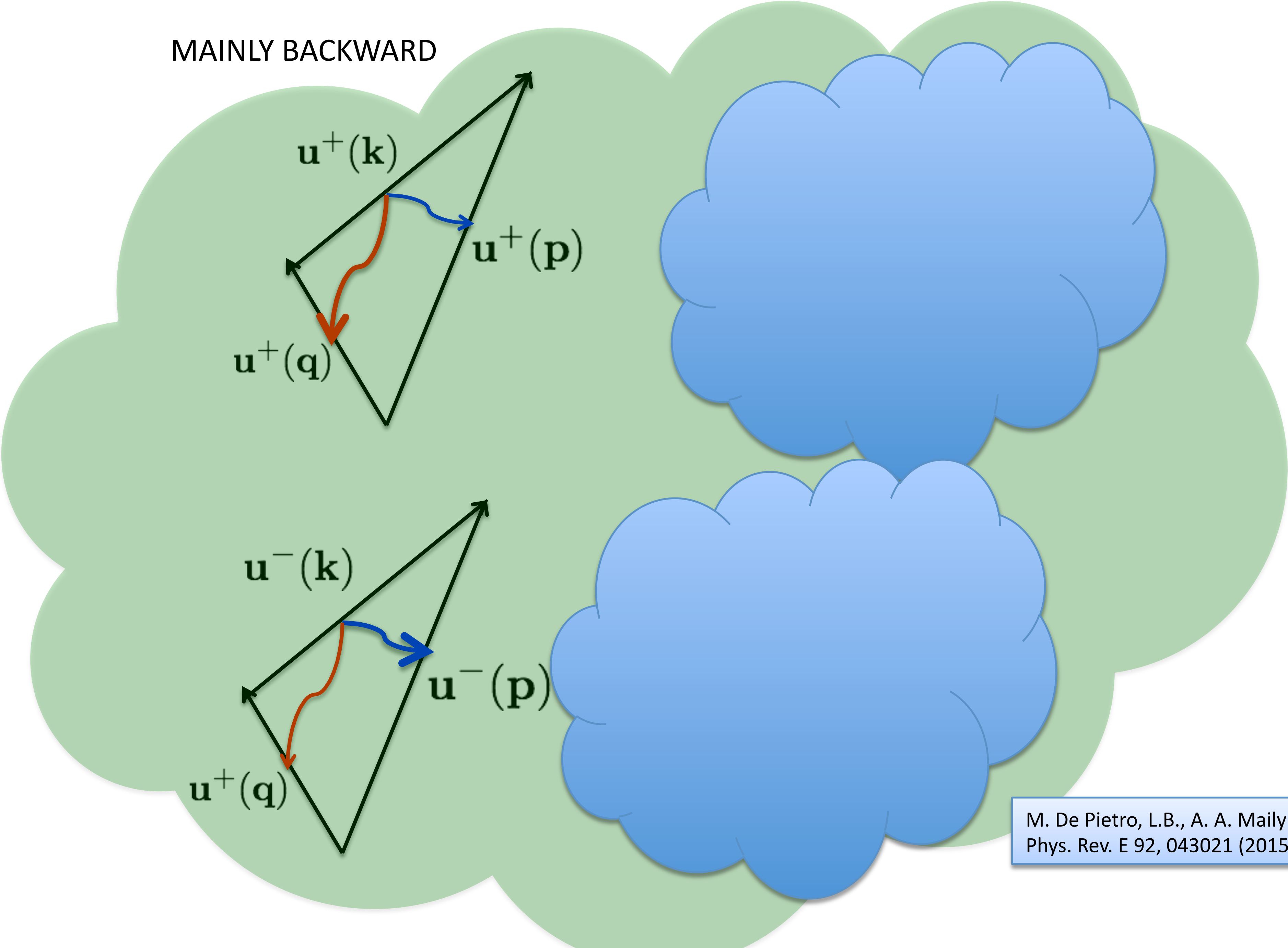


TRIADIC INTERACTION IN DECIMATED NAVIER_STOKES EQS



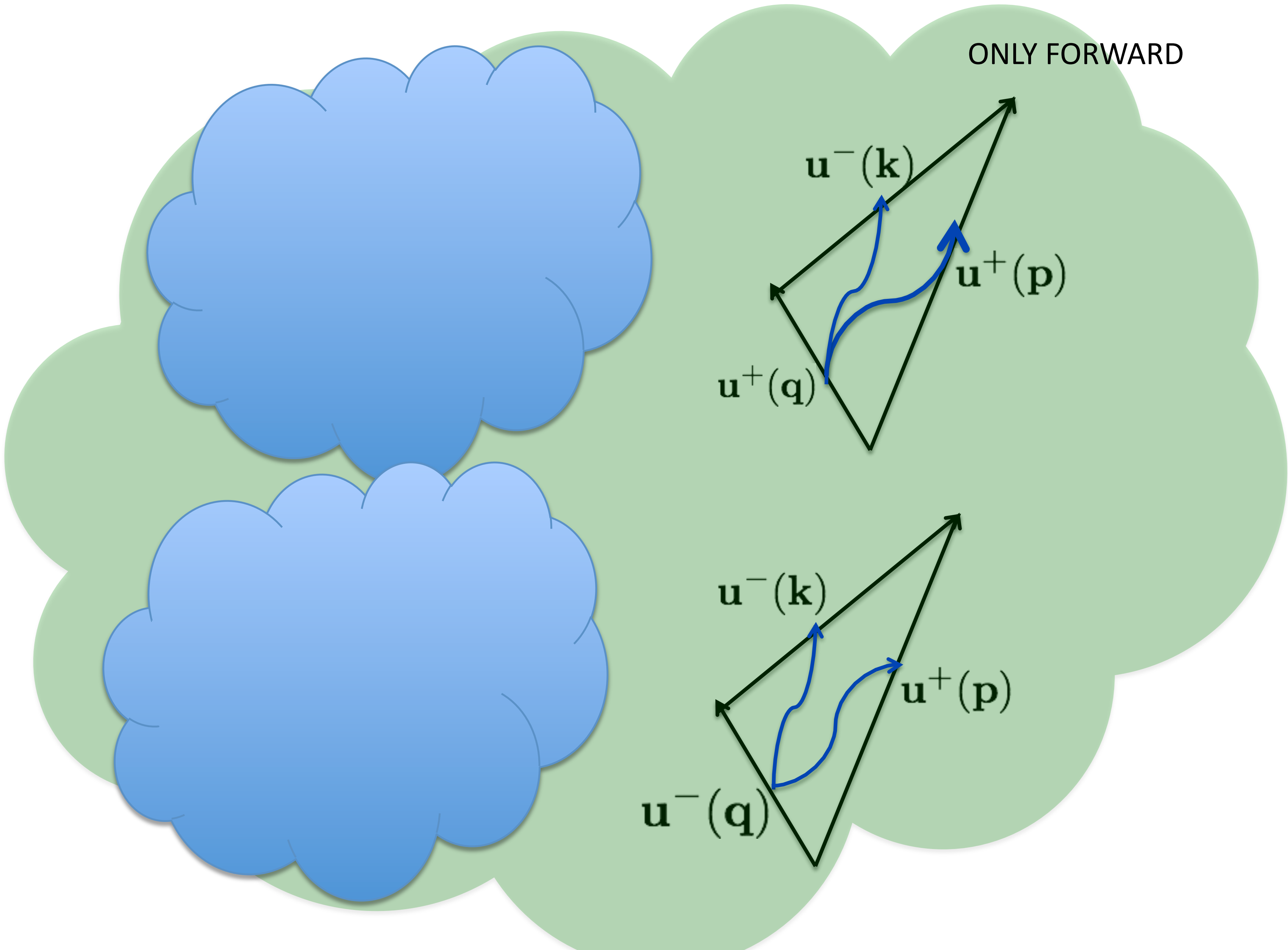
TRIADIC INTERACTION IN DECIMATED NAVIER_STOKES EQS

MAINLY BACKWARD



M. De Pietro, L.B., A. A. Mailybaev
Phys. Rev. E 92, 043021 (2015)

TRIADIC INTERACTION IN DECIMATED NAVIER_STOKES EQS



ONLY REVERSE

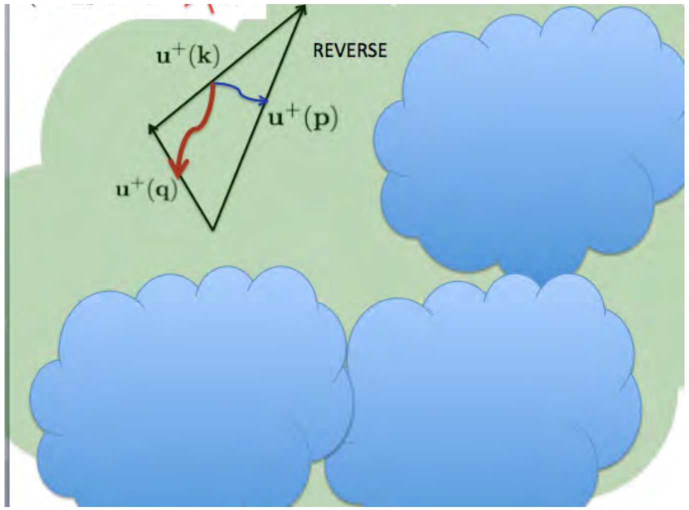
$$\mathcal{P}^{\pm} \equiv \frac{\boldsymbol{h}^{\pm} \otimes \overline{\boldsymbol{h}^{\pm}}}{\overline{\boldsymbol{h}^{\pm}} \cdot \boldsymbol{h}^{\pm}}. \qquad \boldsymbol{v}^{\pm}(\boldsymbol{x}) \equiv \sum_{\boldsymbol{k}} \mathcal{P}^{\pm} \boldsymbol{u}(\boldsymbol{k});$$

$$\boxed{\boldsymbol{u}(\boldsymbol{k}) = u^{+}(\boldsymbol{k})\boldsymbol{h}^{+}(\boldsymbol{k}) + \cancel{u^{-}(\boldsymbol{k})\boldsymbol{h}^{-}(\boldsymbol{k})}}$$

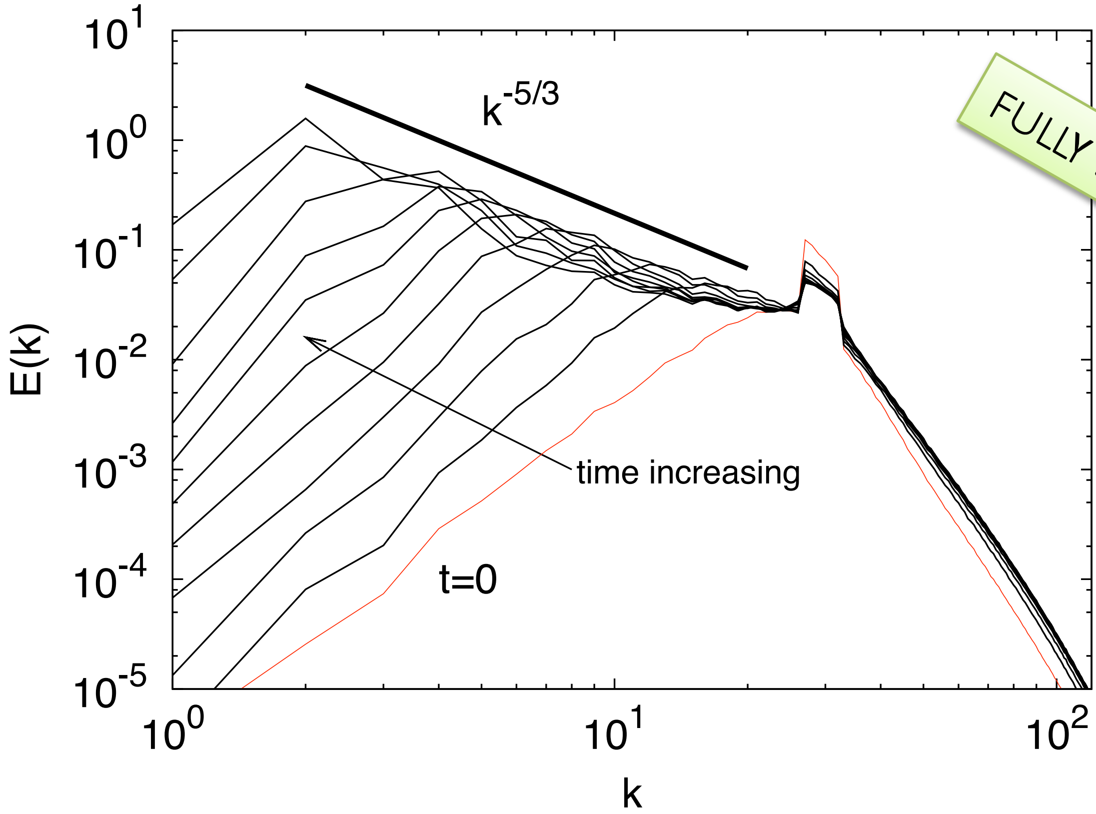
LOCAL BELTRAMIZATION (IN FOURIER)

$$\partial_t v^{+} + \mathcal{P}^{+} B[v^{+}, v^{+}] = \nu \Delta v^{+} + \mathbf{f}^{+}$$

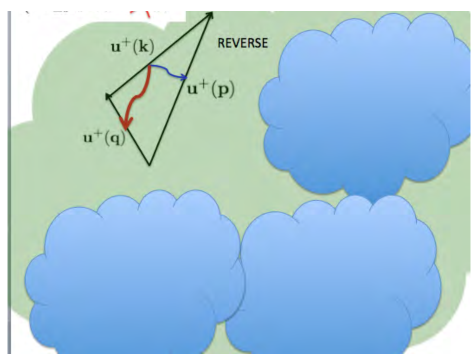
decimated-NSE



$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + \cancel{|u^-(\boldsymbol{k})|^2}; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - \cancel{|u^-(\boldsymbol{k})|^2}). \end{cases}$$



FULLY 3D AND ISOTORPIC

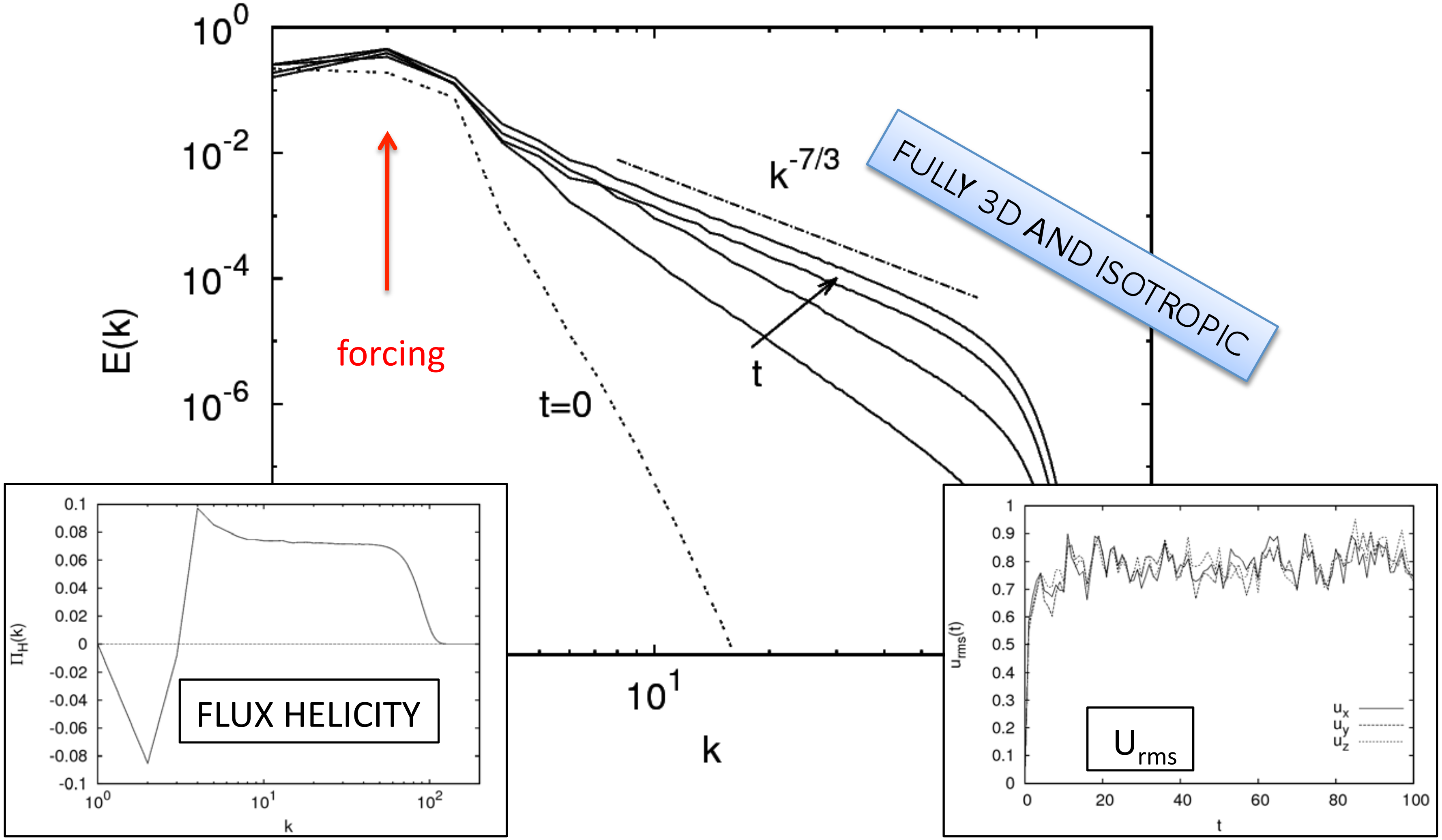


L.B., S. MUSACCHIO & F. TOSCHI Phys. Rev. Lett. 108 164501, 2012.

LARGE SCALES FORCING: DIRECT HELICITY CASCADE

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + \cancel{|u^-(\boldsymbol{k})|^2}; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 \cancel{- |u^-(\boldsymbol{k})|^2}). \end{cases}$$

L. B., S. Musacchio and F. Toschi
J. Fluid Mech. 730, 309 (2013)



ESISTENCE AND UNIQUENESS OF WEAK SOLUTIONS OF THE HELICAL-DECIMATED NSE

$$\begin{cases} \partial_t \boldsymbol{v}^+ = \mathcal{P}^+(-\boldsymbol{v}^+ \cdot \boldsymbol{\nabla} \boldsymbol{v}^+ - \boldsymbol{\nabla} p^+) + \nu \Delta \boldsymbol{v}^+ + \boldsymbol{f}^+ \\ \nabla \cdot \boldsymbol{v}^+ = 0 \end{cases}$$

HILBERT-NORM COINCIDES WITH THE SIGN-DEFINITE HELICTY

$$||g||_{H^{1/2}} = \sum_{\boldsymbol{k}} k|g(k)|^2$$

CONSERVATION HELICITY: NEW APRIORI BOUND ON THE VELOCITY

$$\frac{1}{2}\partial_t \sum_{\boldsymbol{k}} k|u^+(\boldsymbol{k},t)|^2 + \frac{\nu}{2}\sum_{\boldsymbol{k}} k^3|u^+(\boldsymbol{k},t)|^2 \le \frac{1}{2\nu}\sum_{\boldsymbol{k}} |f^+(\boldsymbol{k})|^2 k^{-1}.$$

$$\frac{1}{2}\partial_t ||v^+||^2_{H^{\frac{1}{2}}} + \frac{\nu}{2}||v^+||^2_{H^{\frac{3}{2}}} \le \frac{1}{2\nu}\sum_{\boldsymbol{k}} |f^+(\boldsymbol{k})|^2 k^{-1}.$$

$$\boxed{v^+ \in L^\infty_t H^{\frac{1}{2}}_x; \qquad \sqrt{\nu} v^+ \in L^2_t H^{\frac{3}{2}}_x}$$

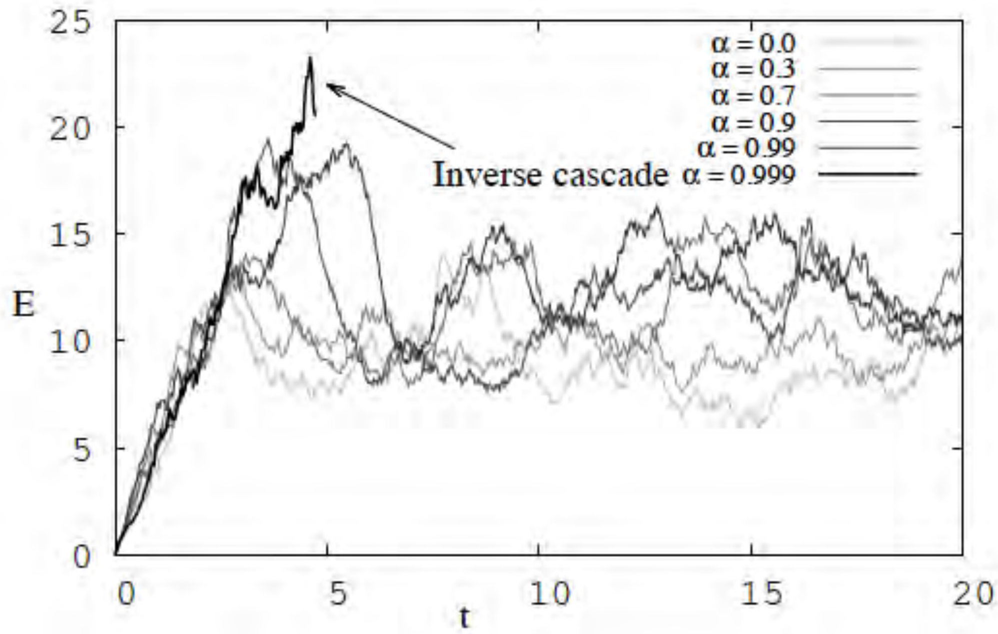
L.B. E. TITI, J. Stat. Phys. submitted (2012)

FROM NS TO FULLY-HELICAL

$$\boldsymbol{u}^\alpha(\boldsymbol{x}) \equiv D^\alpha \boldsymbol{u}(\boldsymbol{x}) \equiv \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\boldsymbol{x}} \, \mathcal{D}^\alpha_{\boldsymbol{k}} \boldsymbol{u}_{\boldsymbol{k}}, \qquad (4)$$

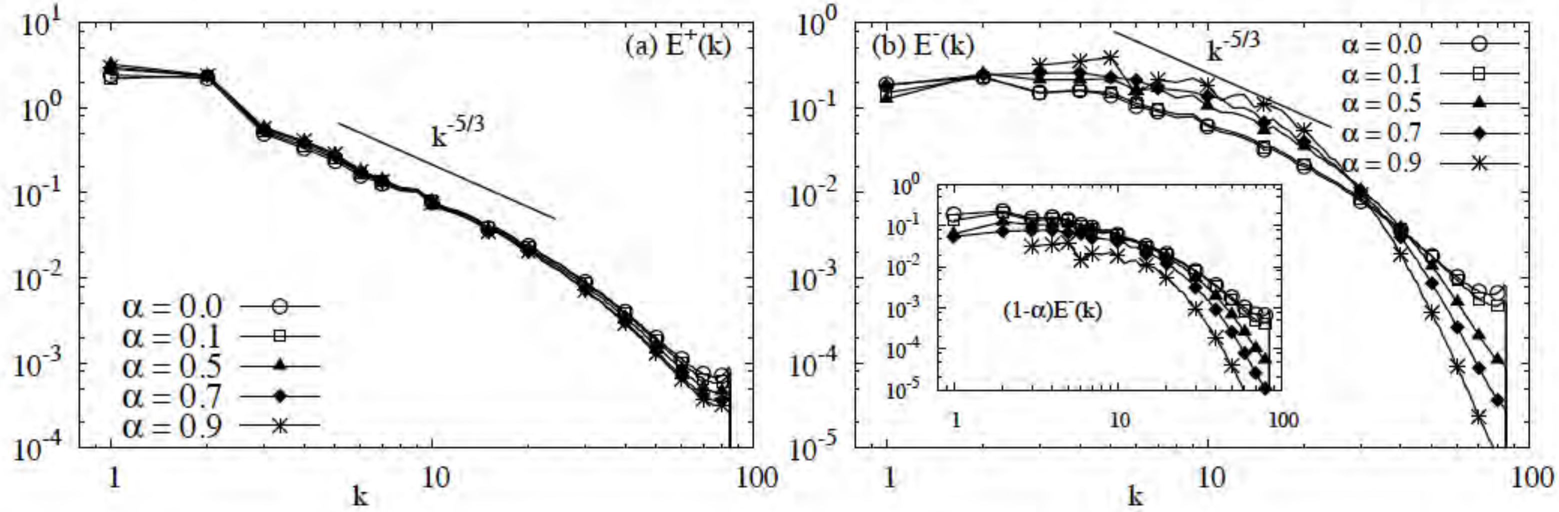
where $\mathcal{D}^\alpha_{\boldsymbol{k}} \equiv (1-\gamma^\alpha_{\boldsymbol{k}}) + \gamma^\alpha_{\boldsymbol{k}} \mathcal{P}^+_{\boldsymbol{k}}$ and $\gamma^\alpha_{\boldsymbol{k}} = 1$ with probability α or $\gamma^\alpha_{\boldsymbol{k}} = 0$ with probability $1-\alpha$. The α-decimated Navier-Stokes equations (α-NSE) are

$$\partial_t \boldsymbol{u}^\alpha = D^\alpha[-\boldsymbol{u}^\alpha \cdot \boldsymbol{\nabla} \boldsymbol{u}^\alpha - \boldsymbol{\nabla} p^\alpha] + \nu \Delta \boldsymbol{u}^\alpha, \qquad (5)$$

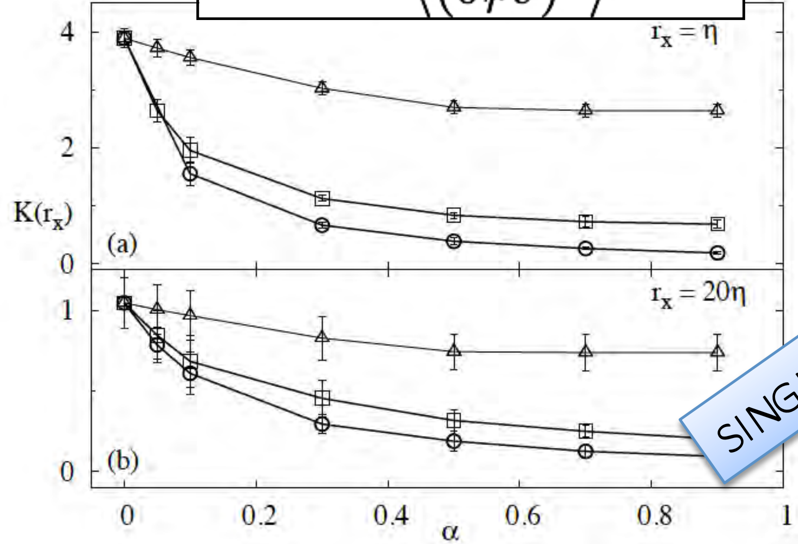


G. Sahoo, F. Bonaccorso and L. B. Phys. Rev. E (2015)

NEGATIVE (OPPOSITE SIGN) HELICAL MODES -> CATALYZER



$$K(r) = \frac{\langle(\delta_r v)^4\rangle}{\langle(\delta_r v)^2\rangle^2} - 3$$

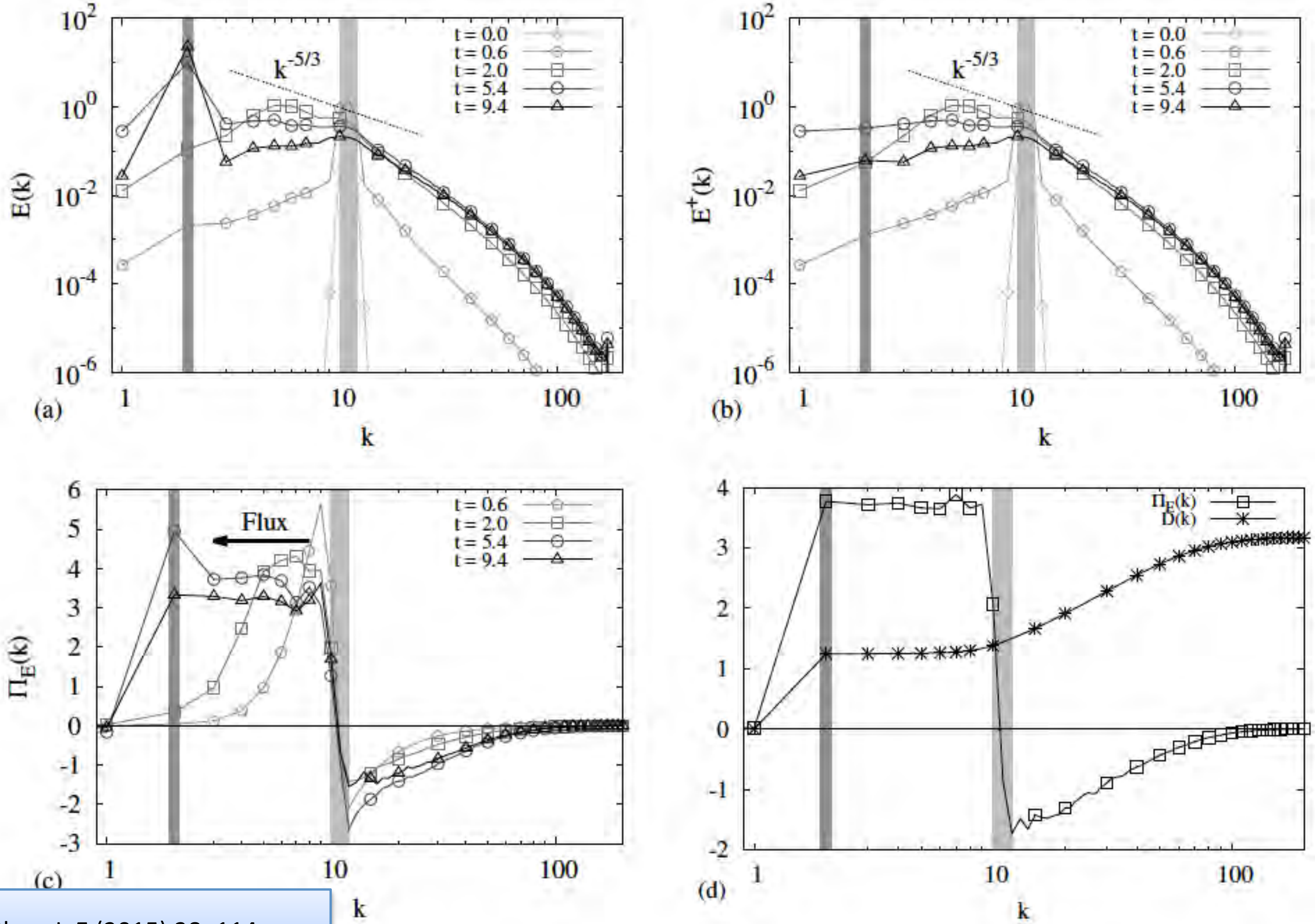


SINGULAR EFFECT ON INTERMITTENCY

G. Sahoo, F. Bonaccorso and L. B.
Phys. Rev. E (2015)

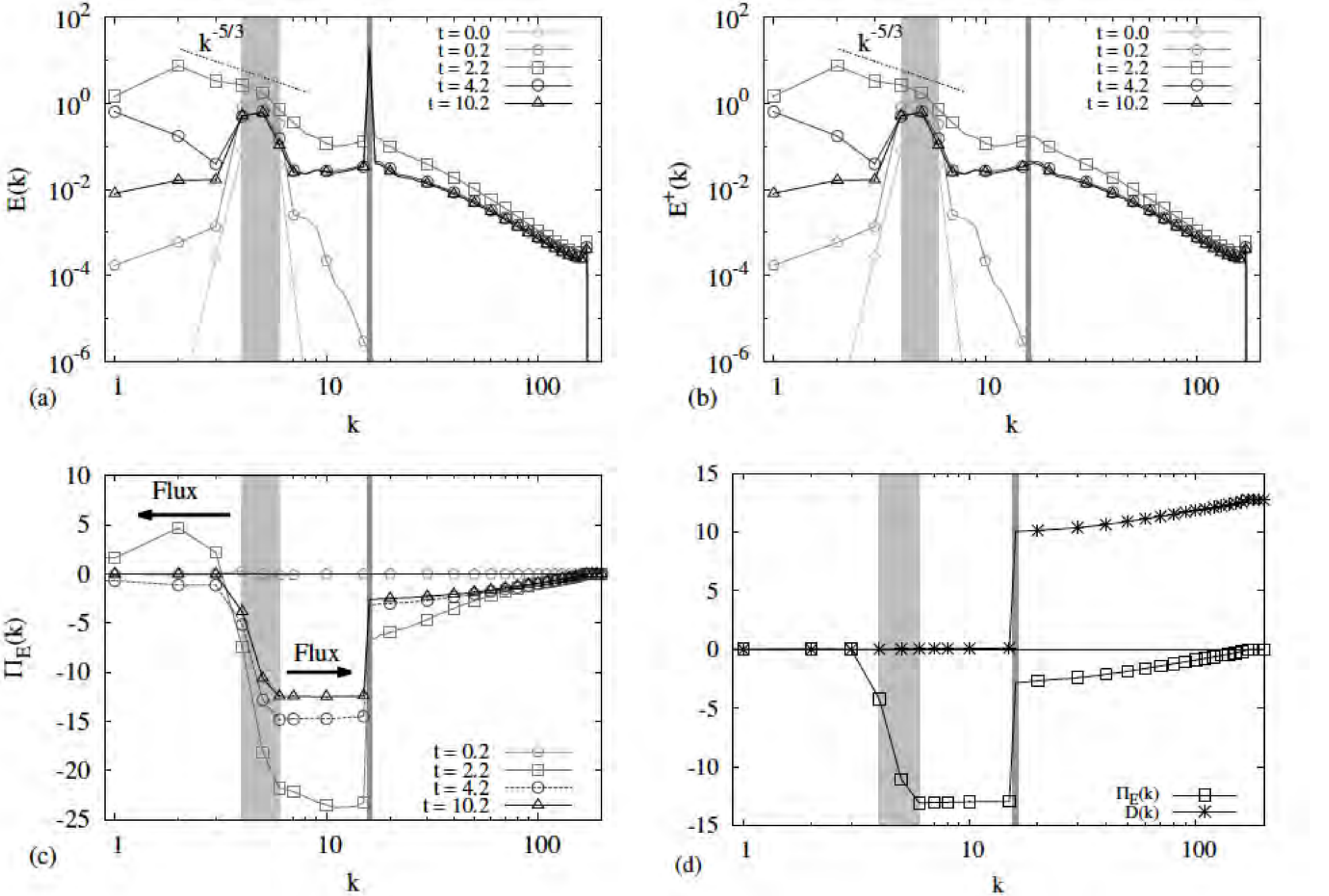
TRIAD-BY-TRAID BACKWARD -> HELICAL CONDENSATE ON THE MINORITY MODES

$$u'(x) \equiv \mathcal{D}_m u(x) \equiv \sum_{k} e^{ikx} \left[(1-\gamma_k) + \gamma_k \mathcal{P}_k^+\right] \hat{u}_k,$$



G. Sahoo, L.B. Eur. Phys. J. E (2015) 38: 114.

TRIAD-BY-TRAID FORWARD



•ALL 3D FLOWS IN NATURE POSSES A SUBSET OF INTERACTIONS RESPONSIBLE OF INVERSE (QUASI-GAUSSIAN) ENERGY CASCADE AND FORWARD (NON-GAUSSIAN) HELICITY CASCADE

•SUCH DECIMATED NS EQUATIONS ARE ‘MORE’ REGULAR THAN THE WHOLE SYSTEM: EXISTENCE AND UNIQUENESS OF SOLUTIONS CAN BE PROVED

•EXACT EQUATIONS (à la Karman-Howart) FOR THRID ORDER CORRELATION FUNCTIONS CAN BE DERIVED

• MINORITY MODES WITH (OPPOSITE) HELICITY SIGNS -> CATALYZER FOR THE FORWARD ENERGY TRASFER

• INTERMITTECY STRONLGY SENSITIVE TO HELICAL MODE REDUCTION

•WHAT ABOUT SIMILAR GAMES FOR MHD?