TUBULENT AT HIGH AND LOW ROTATION RATES: EULERIAN AND LAGRANGIAN STATISTICS

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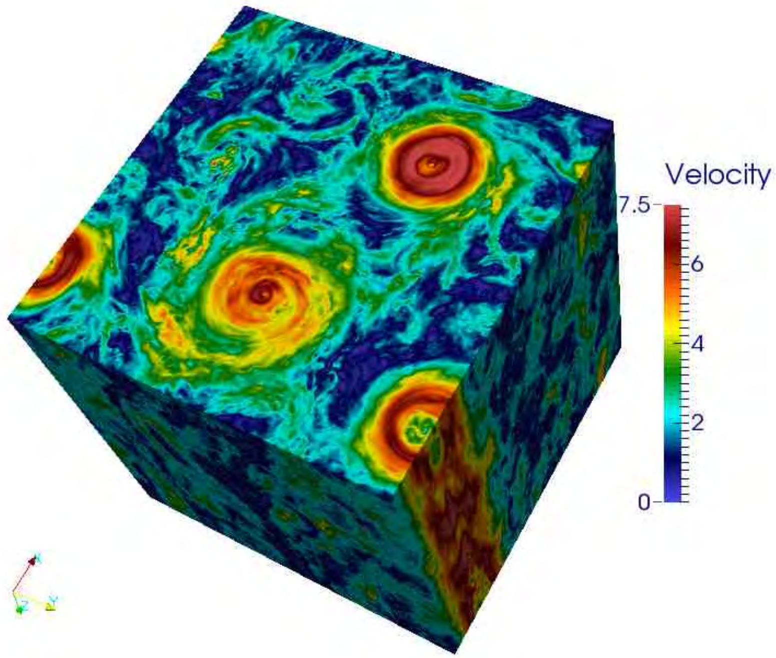
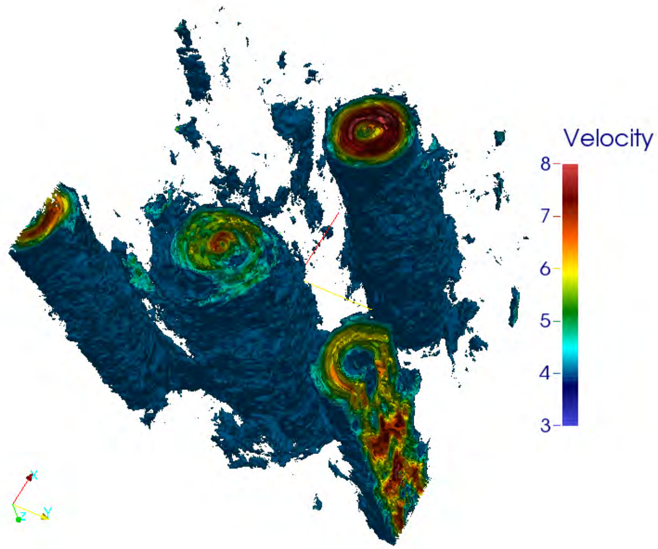
ETC15





F.Bonaccorso, I.Mazzitelli (Rome, Italy)
M.Hinsberg, F. Toschi (Eindhoven, The Netherlands)
A.Lanotte (Lecce, Italy)
S. Musaccio (Nice, France)
P.Perlekar (Hiderabad, India)

Navier Stokes equation in a rotating frame

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -\nabla P + \nu \Delta \mathbf{u} + \mathbf{f}_L$$

$$\nabla \mathbf{u} = 0$$

$\mathbf{\Omega}$ rotation vector

$P = P_0 + \frac{1}{2}||\mathbf{\Omega} \times \mathbf{r}||^2$

$\mathbf{f}_L$ large scale forcing

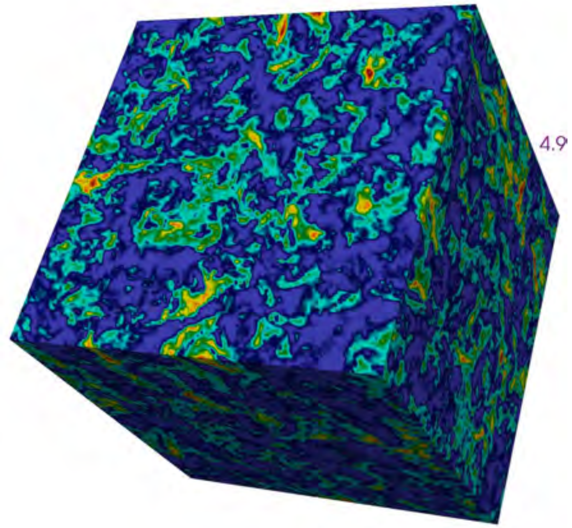
Rossby number

$$\boxed{Ro = \frac{non\, linear\, term}{rotation} \sim \frac{U_0}{\Omega L_0}}$$

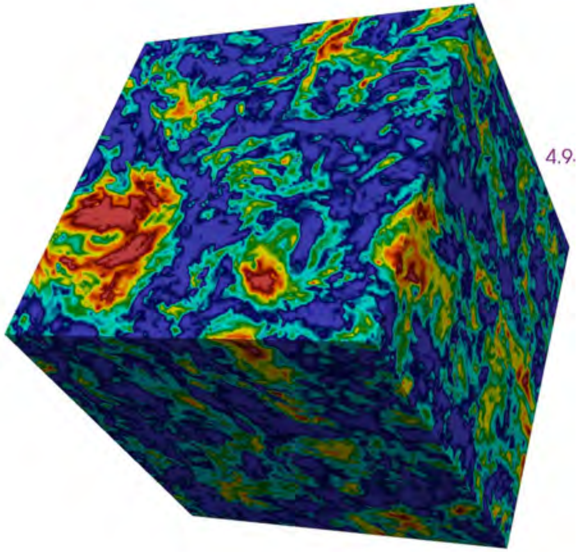
$Ro > Ro_c\,(\lesssim 1) \rightarrow$ direct energy cascade

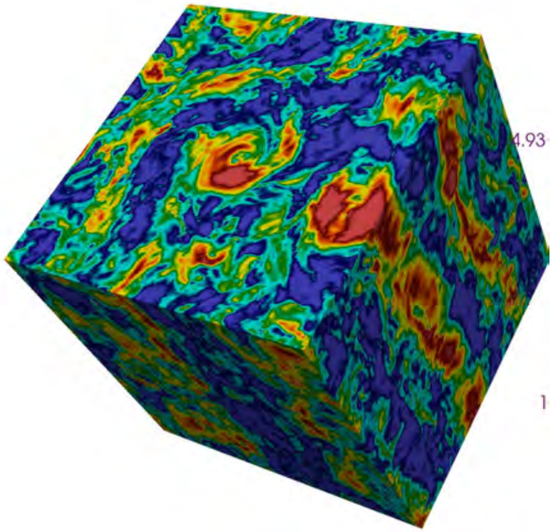
$Ro < Ro_c \qquad \rightarrow$ both direct and inverse cascade

Rossby = 2

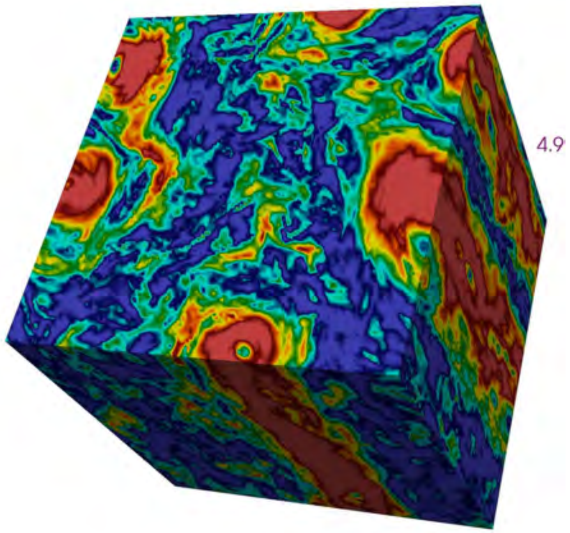


Rossby = 0.8





Rossby = 0.2



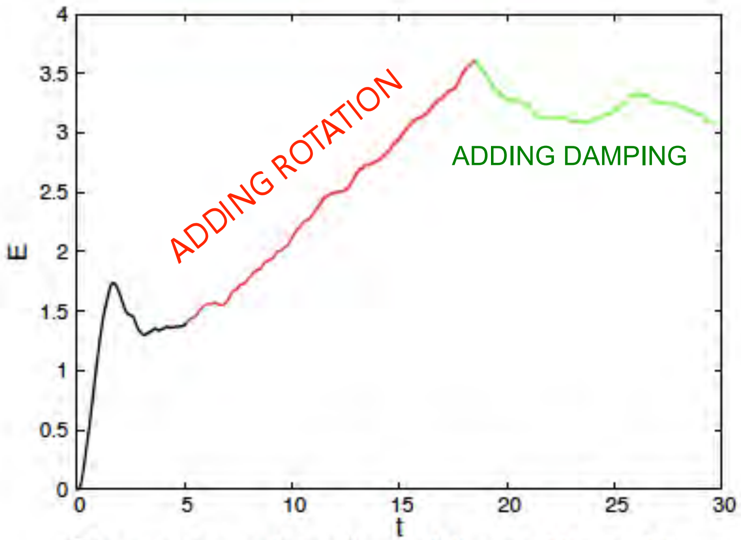
Rossby = 0.1

Navier Stokes equation in a rotating frame

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \; 2\mathbf{\Omega} \times \mathbf{u} = -\nabla P + \nu \Delta \mathbf{u} + \mathbf{f}_L - \boxed{\alpha_L}$$

rotation

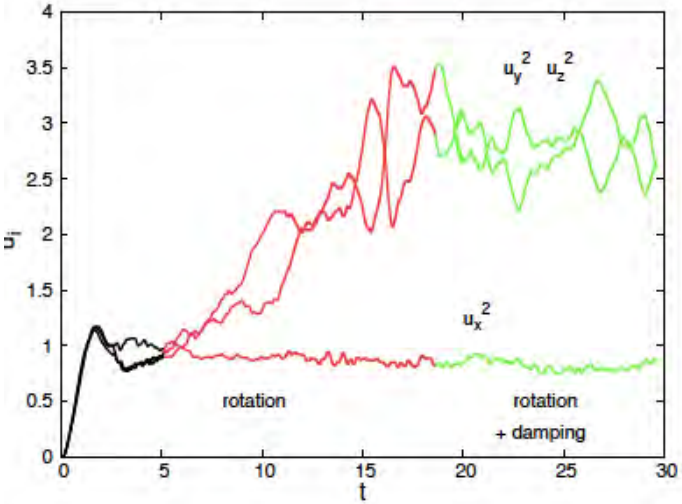
damping



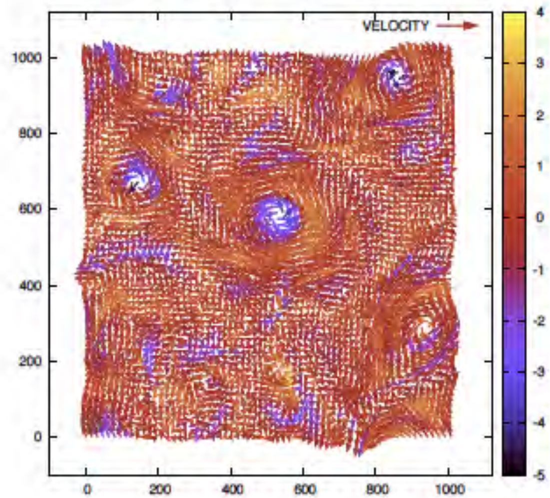
Energy as a function of time, $Ro < Ro_c$

Consequences of anisotropy

- Partial two-dimensionalization of the velocity field
- Cyclone - Anticyclone asymmetry



u_i^2 as a function of time

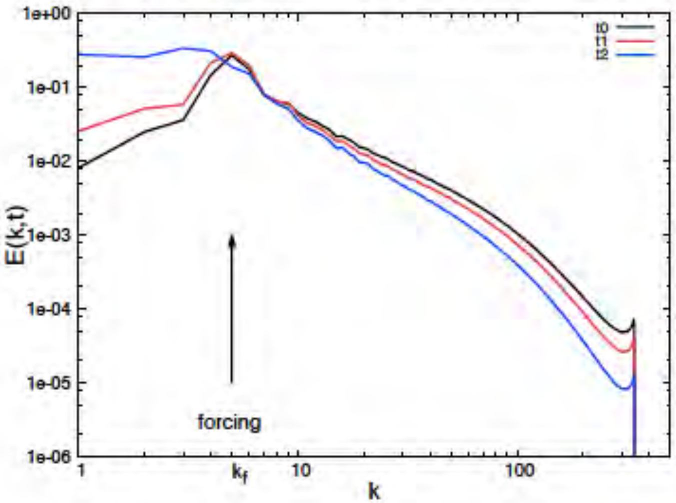


Velocity projected into a plane $\perp$ to Ω

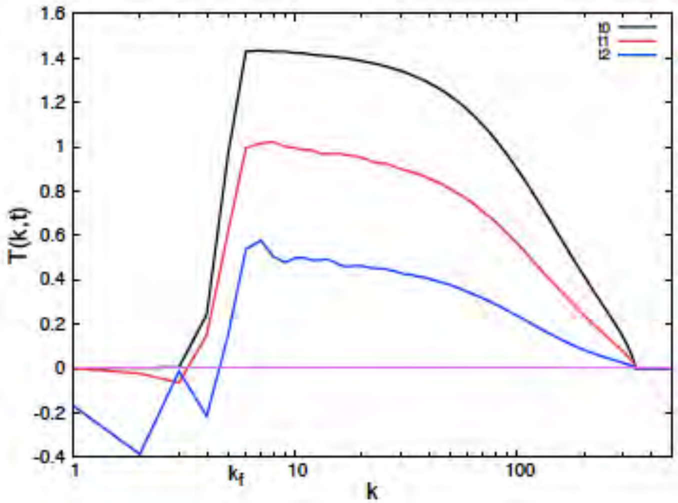
Energy spectrum balance

$$\frac{\partial}{\partial t}E(k,t) = F_L(k,t) - \frac{\partial T(k,t)}{\partial k} - 2\nu k^2 E(k,t)$$

production - spectral transfer - dissipation



Energy spectrum



Energy flux

Energy spectrum scaling

(dimensional argument)

$$\tau_{nl} \sim \varepsilon^{-1/3} k^{-2/3} \qquad \tau_\Omega \sim 1/\Omega$$

At large rotation rates: $\tau_\Omega < \tau_{nl}$

The condition $\tau_\Omega = \tau_{nl}$ fixes the Zeman frequency

$$\boxed{k_\Omega = \left(\frac{\Omega^3}{\varepsilon}\right)^{1/2}}$$

At scales: $k_f < k < k_\Omega$

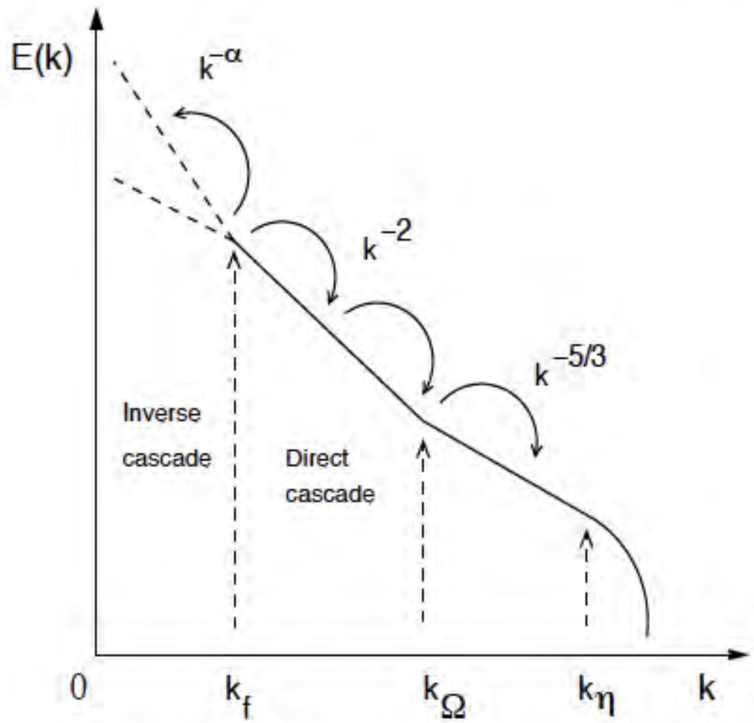
Energy Flux: $\varepsilon = \frac{Energy}{\tau_{tr}}$ with $\tau_{tr} = \frac{\tau_{nl}}{\bar{r}}, \quad \bar{r} = \frac{\tau_\Omega}{\tau_{nl}}$

$\tau_{nl} \simeq 1/(ku(k)) \sim 1/(k^3 E(k))^{1/2}$

$$\varepsilon = \frac{kE(k)}{\tau_{nl}^2/\tau_\Omega} = \frac{E(k)^2 k^4}{\Omega} \Rightarrow \boxed{E(k) \sim k^{-2}}$$

Energy spectrum scaling

- $k > k_\Omega \longrightarrow E(k) \sim \varepsilon^{2/3} k^{-5/3}$
- $k_f < k < k_\Omega \longrightarrow E(k) \sim (\varepsilon\Omega)^{1/2} k^{-2}$
- $k < k_f \longrightarrow E(k) \sim \Omega^2 k^{-3}$ - - - $E(k) \sim k^{-5/3}$ (?)



Some details on the simulations

N	Ω	ν	ε	u_0	η/dx	τ_η/dt	k_Ω	Re_λ	Ro
1024	4	$7\cdot10^{-4}$	1.2	1.04	0.67	120	7	150	0.79
	10	$6\cdot10^{-4}$	0.55	1.6	0.73	330	43	580	0.24
2048	4	$2.8\cdot10^{-4}$	1.2	1.05	0.67	370	7	230	0.78
	10	$2.2\cdot10^{-4}$	0.45	1.7	0.72	550	47	1160	0.25
4096	10	$8\cdot10^{-5}$	0.46	1.7	0.67	670	46	1900	0.25

$$Re_\lambda = \frac{u_0\lambda}{\nu} \qquad Ro = \frac{(\varepsilon_f k_f^2)^{1/3}}{\Omega}$$

$$u_0^2 = (u_x^2 + u_y^2 + u_z^2)/3$$

$$\lambda^2 = \left(\frac{15\nu u_0^2}{\varepsilon}\right)$$

NEW ! forcing $\mathbf{f}_L \to$ OU process peaked on $k_f = 5$ (Sawford, Phys. Fluids A 91) NEW !

Particle equation of motion

NEW !

NEW !

$$\frac{d\mathbf{v}}{dt} = \beta\frac{D\mathbf{u}}{Dt} - \frac{1}{\tau_p}(\mathbf{v}-\mathbf{u}) + 2(\mathbf{v}-\beta\mathbf{u})\times\mathbf{\Omega} - (1-\beta)\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})$$

added mass + pressure — Stokes drag — Coriolis — Centrifugal force

$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}, \qquad \tau_p = \frac{a^2}{3\nu\beta}$$

$\beta < 1 \rightarrow$ heavy particles $\rho_p > \rho_f$

$\beta > 1 \rightarrow$ light particles $\rho_p < \rho_f$

Stokes number: $St = \frac{\tau_p}{\tau_\eta}$

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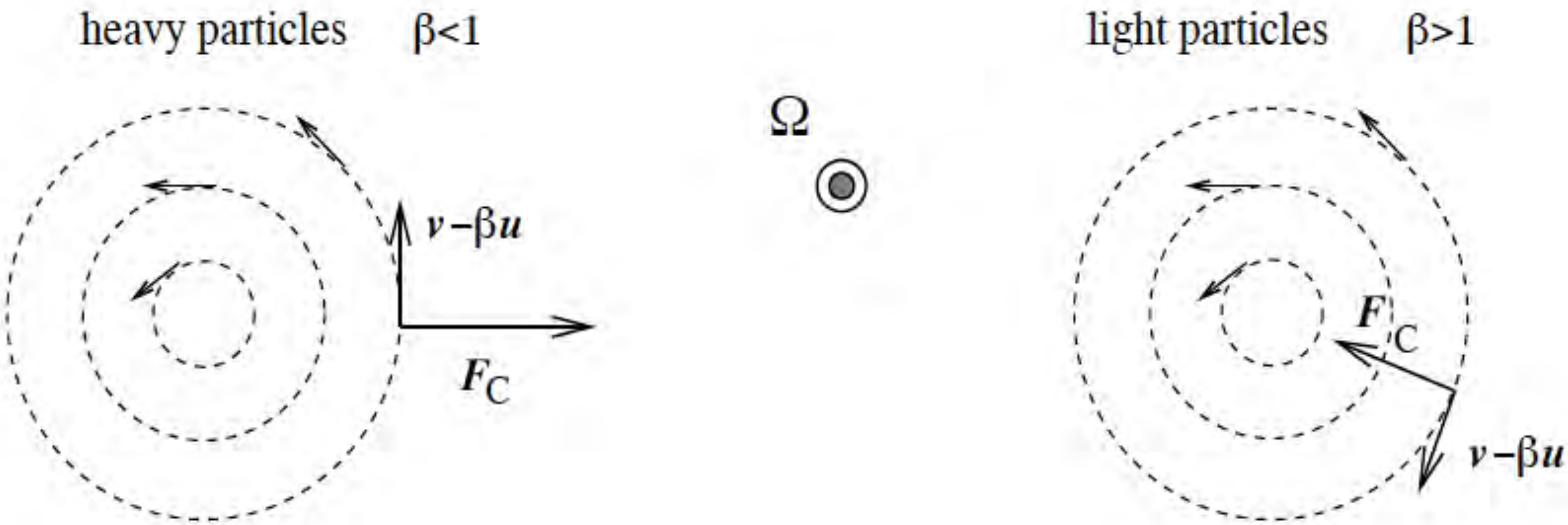
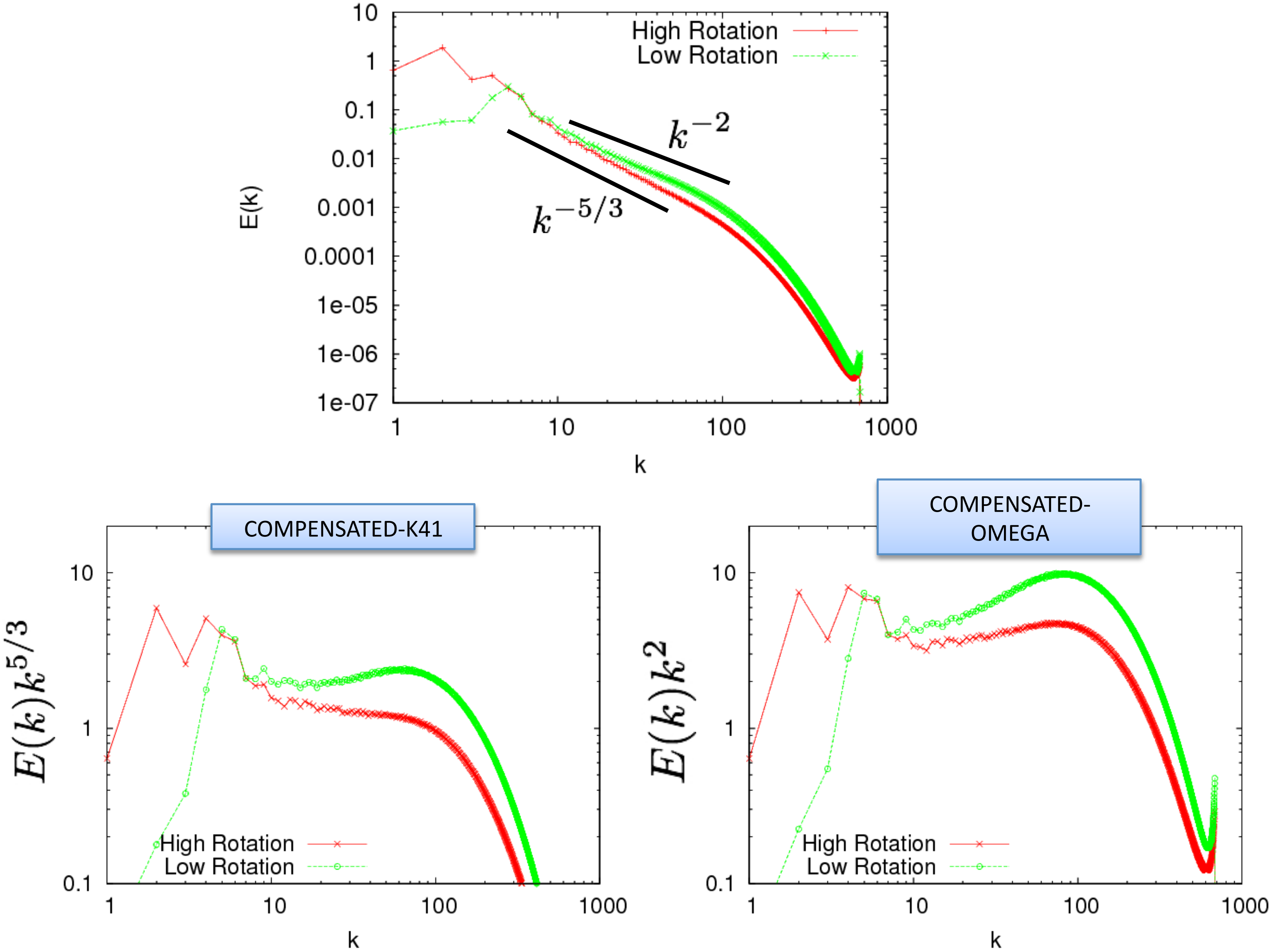
forcing $\mathbf{f}_L \to$ OU process peaked on $k_f = 5$ (Sawford, Phys. Fluids A 91)

Effect of the Coriolis force

$$\mathbf{F}_C = 2(\mathbf{v} - \beta \mathbf{u}) \times \mathbf{\Omega}$$

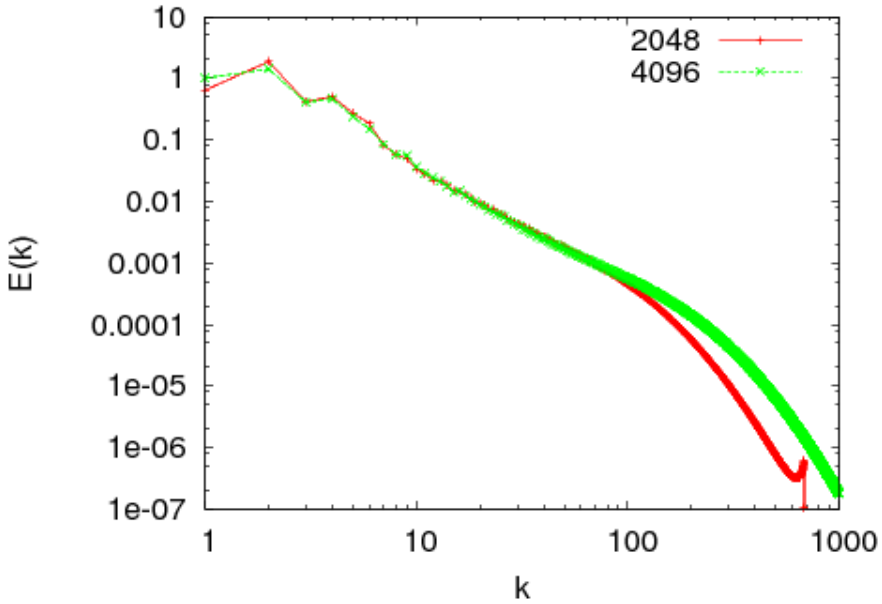
On cyclonic vortex (pressure minimum):



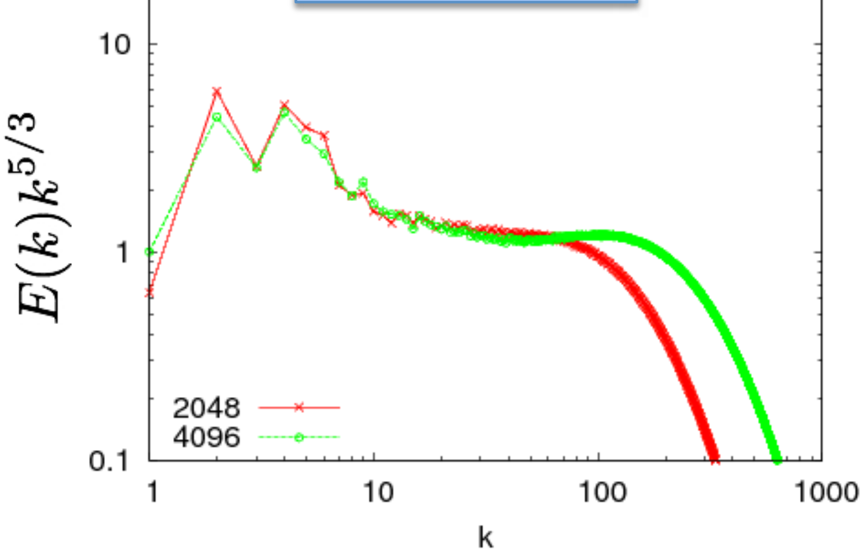


E(k)
k
High Rotation
Low Rotation
k^{-2}
$k^{-5/3}$
COMPENSATED-K41
$E(k)k^{5/3}$
High Rotation
Low Rotation
k
COMPENSATED-OMEGA
$E(k)k^{2}$
High Rotation
Low Rotation
k

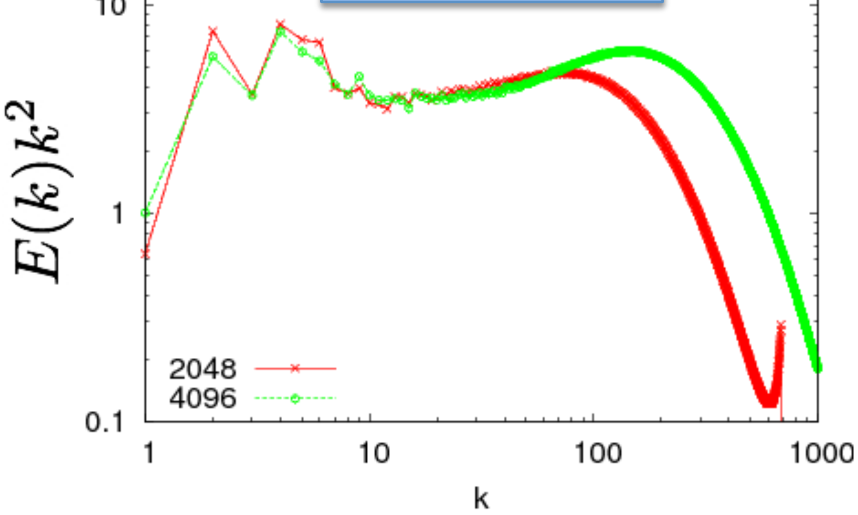
HIGH ROTATION



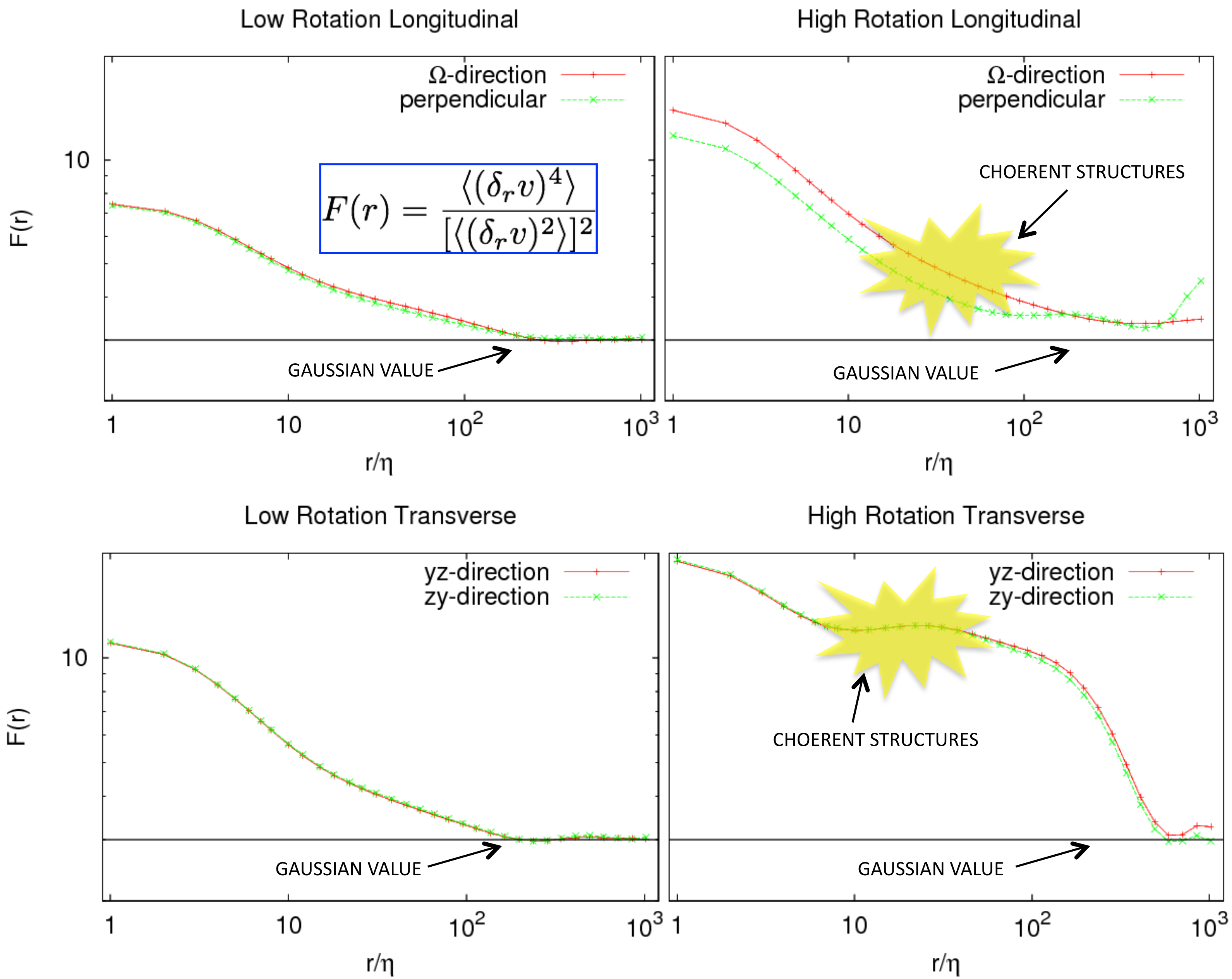
COMPENSATED-K41



COMPENSATED-OMEGA

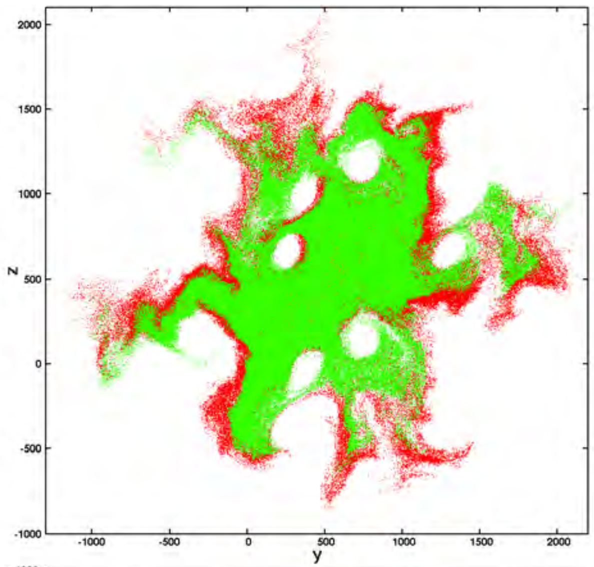


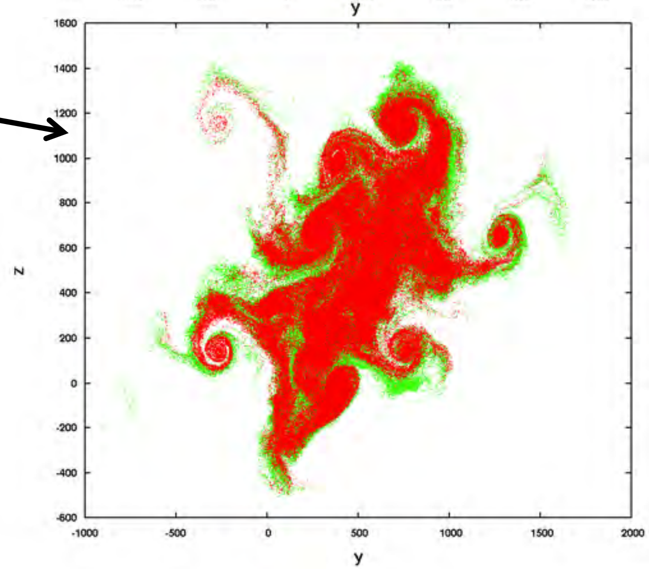
INTERMITTENCY -> EULERIAN FLATNESS



PREFERENTIAL SAMPLING-> WHERE DO PARTICLES GO?

family	β	St	type
1	0.4	0.3	Heavy
2	0.4	0.7	
3	0.8	0.3	
4	0.8	0.7	
5	1.2	0.3	Light
6	1.2	0.7	
7	1.6	0.3	
8	1.6	0.7	
9	1.6	1	
10	1.6	5	





CONCLUSIONS:

-HIGH RESOLUTION ROTATING TURBULENCE: FIRST ATTEMPT TO CONTROL SIMULTANEOUSLY EULERIAN & LAGRANGIAN STATISTICS

-IDEAL SET-UP (1): HOMOGENEOUS AND ISOTROPIC FORCING

-IDEAL SET-UP (2): SCALE-SEPARATION

-STRONG INFLUENCE OF LARGE-SCALE (NON-UNIVERSAL?) VORTICAL STRUCTURES

-DEPARTURE FROM GAUSSIANITY

-ENHANCED PREFERENTIAL SAMPLING