Optimal strategies to catch a drifting target in turbulence

ETC18

18th European Turbulence Conference
4-8 September 2023, Valencia, Spain

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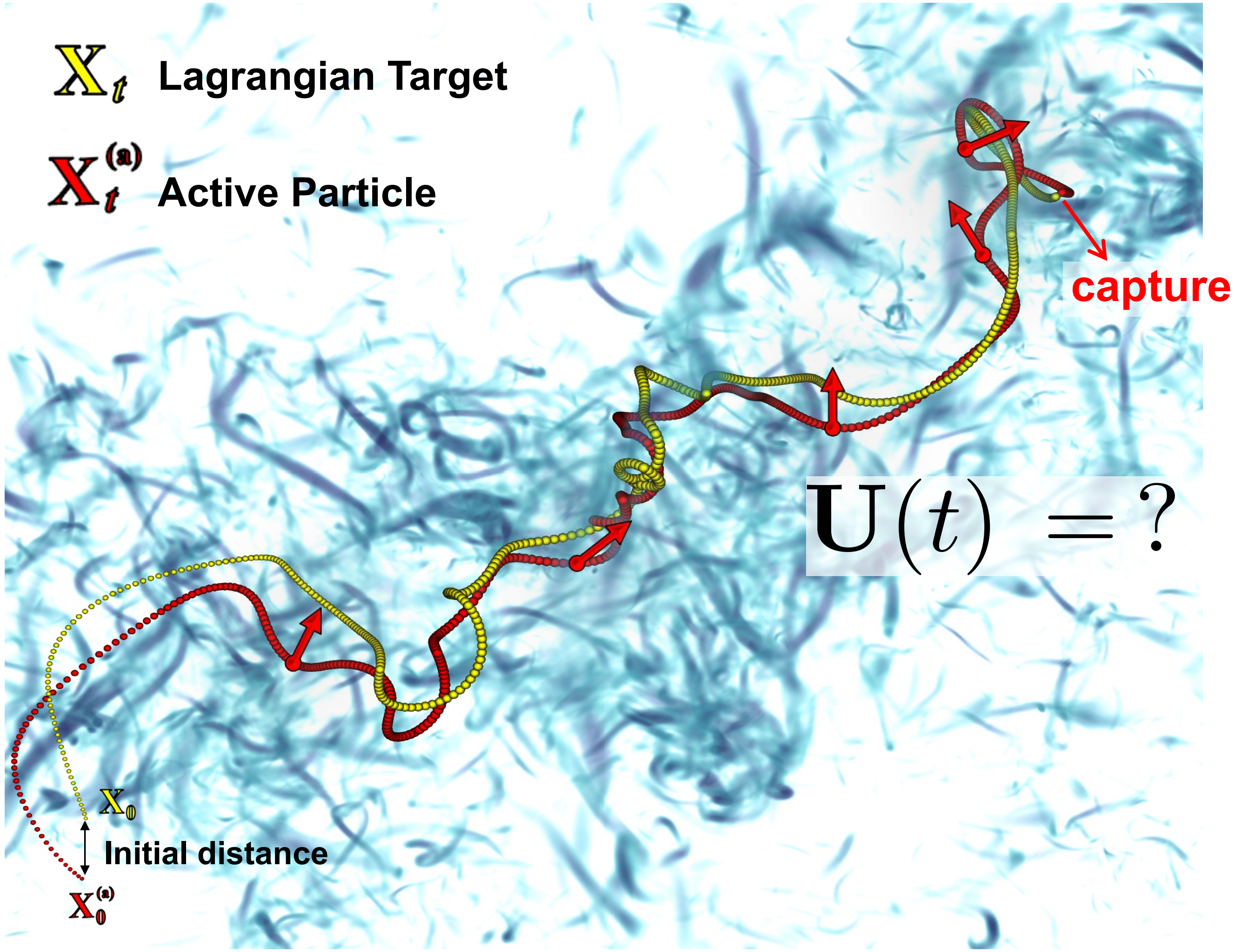
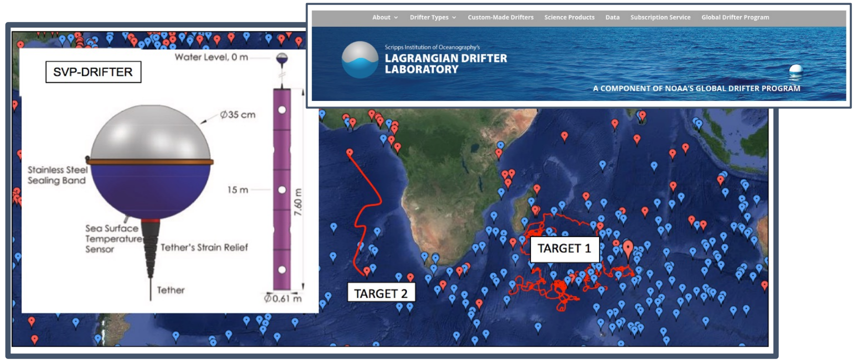


Time-Dependent 3D Turbulent Flow

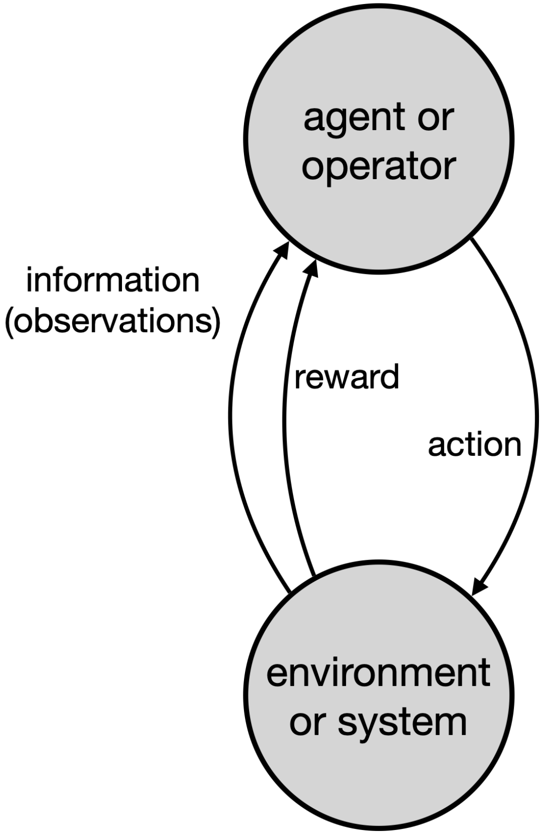
$$\begin{cases} \dot{\mathbf{X}}_t = \boldsymbol{v}(\mathbf{X}_t, t) \\ \dot{\mathbf{X}}_t^{(a)} = \boldsymbol{v}(\mathbf{X}_t^{(a)}, t) + \mathbf{U}(t) \\ \mathbf{U}(t) = V_s \hat{\boldsymbol{n}}(t) \end{cases}$$

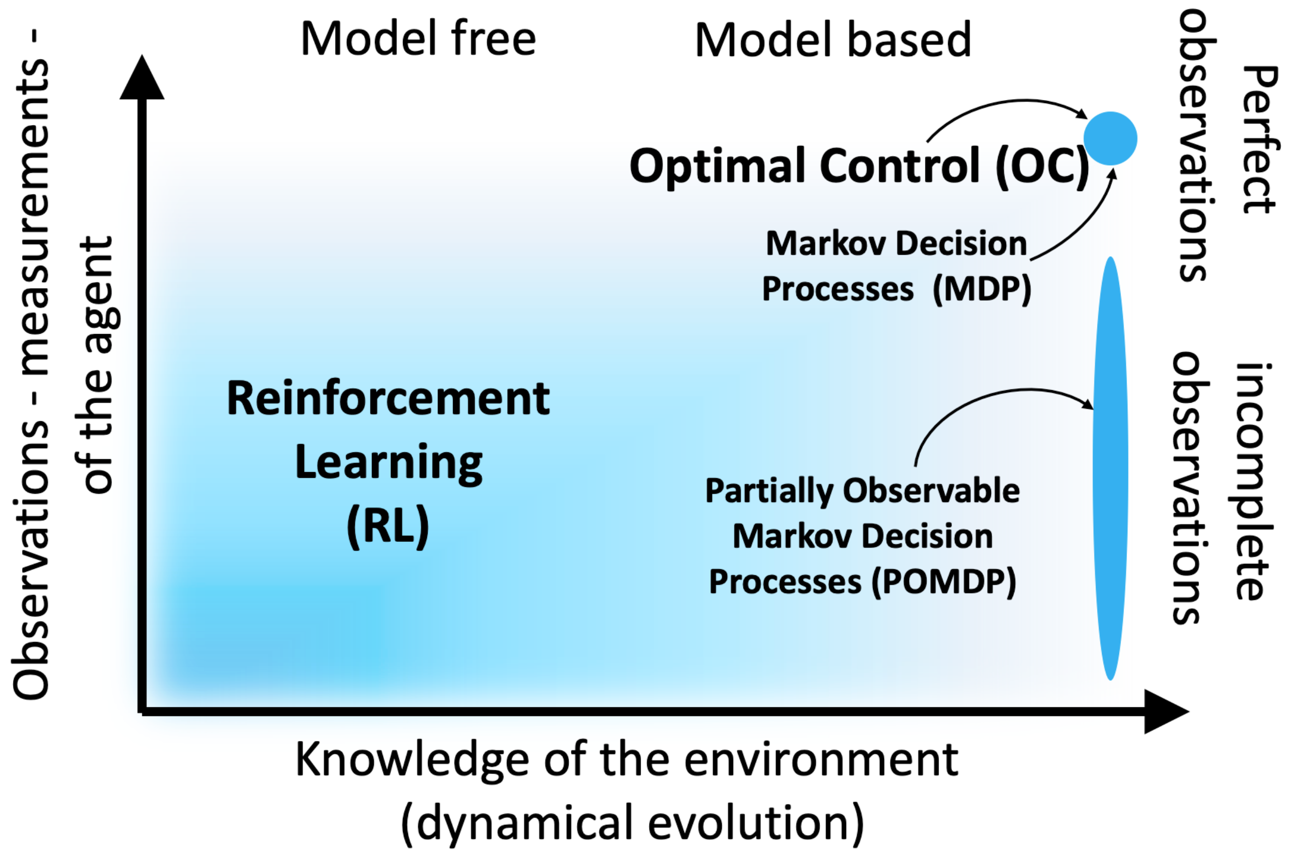
Small active control compared to the underling flow!

Goal: **minimize** the **separation/capture** in a finite time horizon



Control Theory





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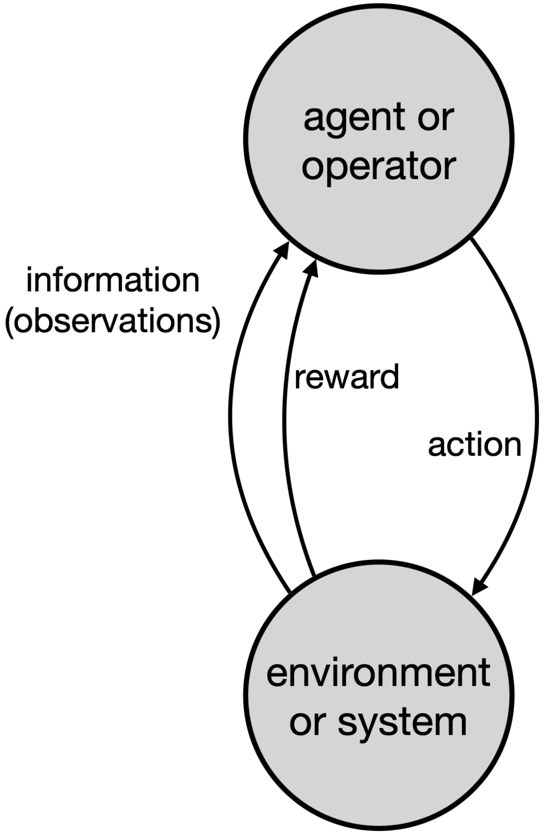
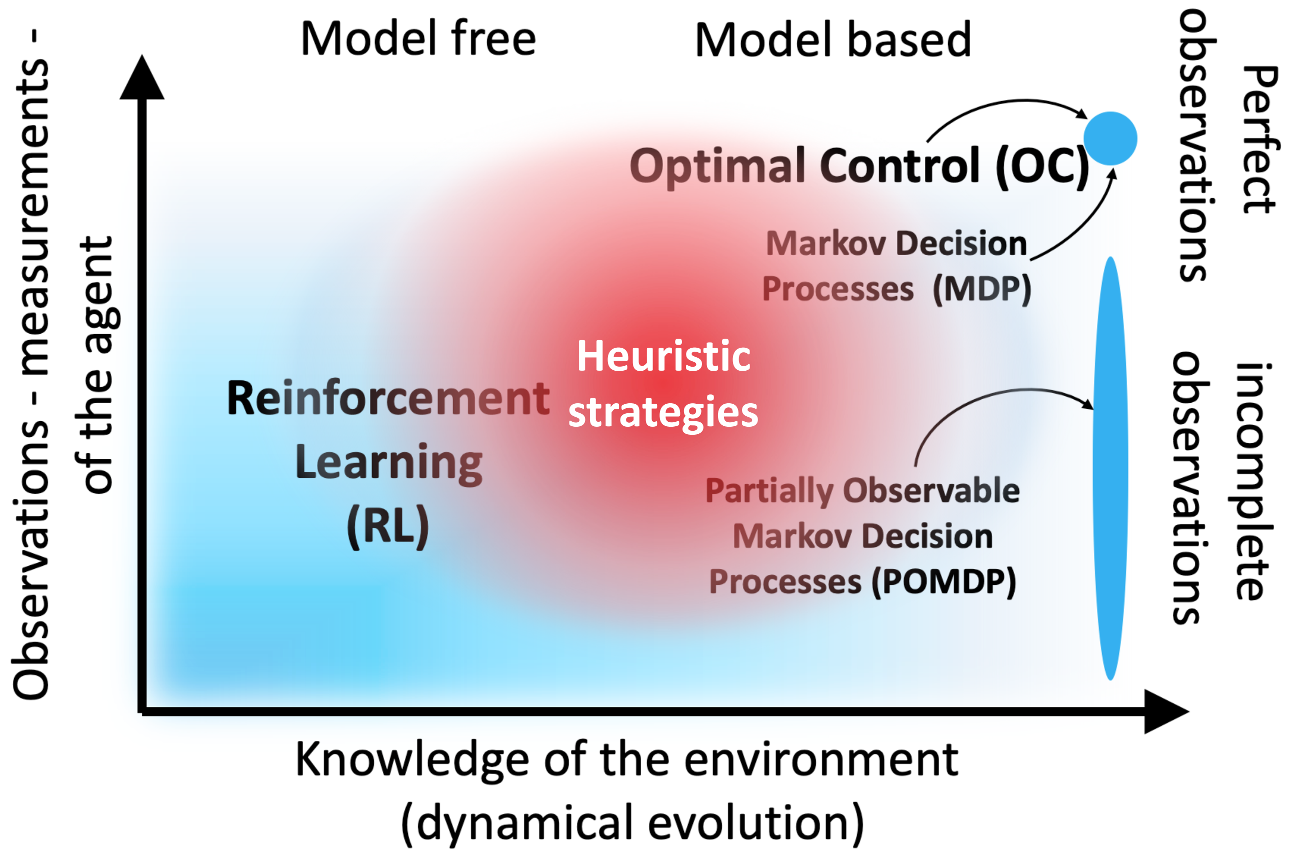
Control Theory





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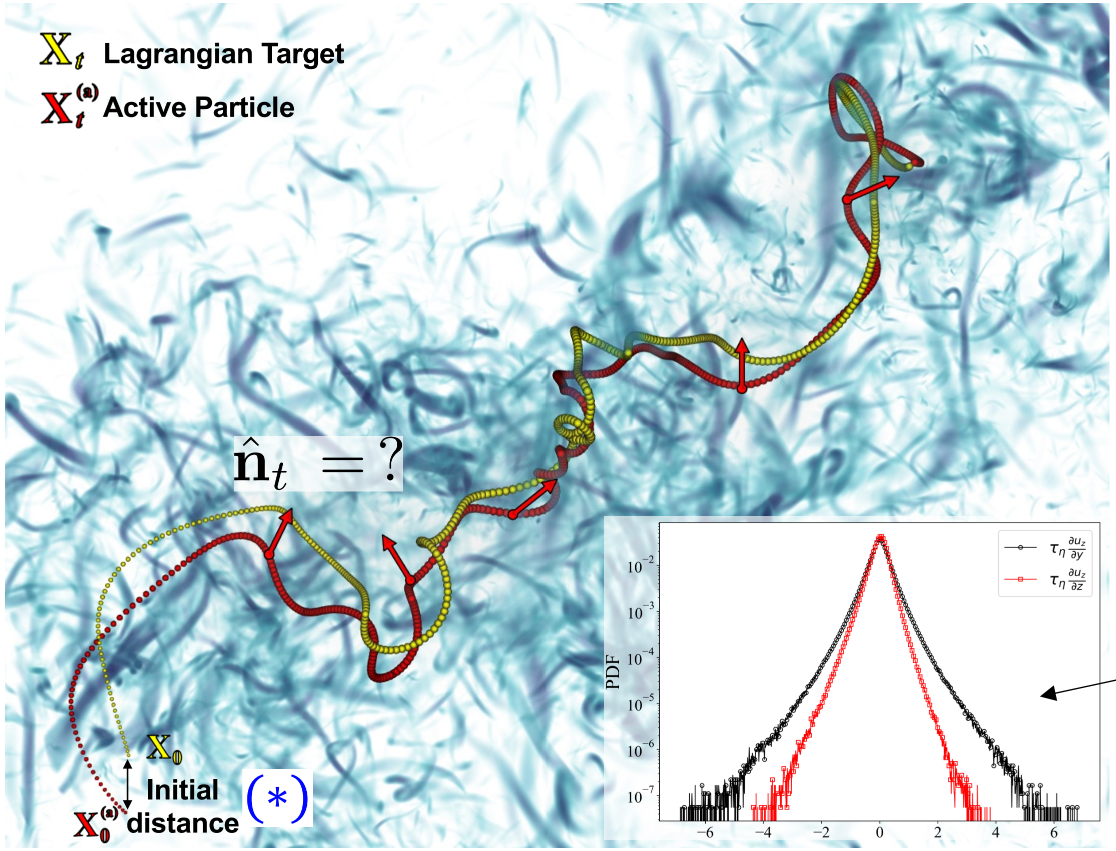
$$\boldsymbol{R}_t = \mathbf{X}_t^{(a)} - \mathbf{X}_t$$



(*) Small scales $\|\boldsymbol{R}_{t_0}\| < \eta$

$$\begin{cases} \dot{\mathbf{X}}_t = \boldsymbol{v}(\mathbf{X}_t, t) \\ \dot{\mathbf{X}}_t^{(a)} = \boldsymbol{v}(\mathbf{X}_t^{(a)}, t) + \mathbf{U}(t) \\ \mathbf{U}(t) = V_s \hat{\boldsymbol{n}}(t) \end{cases}$$

$$\boldsymbol{v}(\mathbf{X}_t^{(a)}, t) \simeq \boldsymbol{v}(\mathbf{X}_t, t) + \nabla \boldsymbol{v}_t \boldsymbol{R}_t$$

$$\dot{\boldsymbol{R}}_t = \nabla \boldsymbol{v}_t \boldsymbol{R}_t + \mathbf{U}(t)$$

Tools:		
(1) Optimal Control theory	**(2) Heuristic control strategies**	**(3) Reinforcement Learning**
Model based problem (full knowledge required).	Reactive controls based on local-only in time cues.	Model free tool. Some past memory can be considered.

Biferale, L., Bonaccorso, F., Buzzicotti, M., & **Calascibetta, C**. (2023). **TURB-Lagr. A database of 3d Lagrangian trajectories in homogeneous and isotropic turbulence**. (arXiv:2303.08662)

(1) **Optimal Control (OC) theory** – Pontryagin minimum principle

- **Model based** and analytical tool (Perfect knowledge required)
- **Constrained minimization problem**

Minimize $J = \|\boldsymbol{R}_{t_f}\|^2 + c\int_{t_0}^{t_f} dt\, \theta(\|\boldsymbol{R}_t\|^2 - \|\boldsymbol{R}_c\|^2)$

capture's distance

$$\|\boldsymbol{R}_c\| = \frac{\|\boldsymbol{R}_{t_0}\|}{100}$$

Imposing:

$$\begin{cases} \dot{\boldsymbol{R}}_t = \nabla \boldsymbol{v}_t \boldsymbol{R}_t + \mathbf{U}(t)\,, \\ \boldsymbol{R}_{t_0} = \text{given}\,, \\ \mathbf{U}(t) = V_s \hat{\boldsymbol{n}}(t) \\ \|\hat{\boldsymbol{n}}(t)\| = 1\,. \end{cases}$$

Euler-Lagrange equations

$$\|\boldsymbol{R}_{t_0}\| \sim \frac{V_s}{\lambda_{lyapunov}}$$ border of controllability

Minimize trajectories' separation

Minimize time of arrival at the desired distance

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(2) **Heuristic strategies**

Pure Pursuit (PP): constantly chooses the direction that points towards the moving target, $\hat{\boldsymbol{n}}(t) = -\hat{\boldsymbol{R}}_t$

Surfing Control (SC)*:
- constant gradients for a time τ_s (free parameter);
- maximization of the searcher displacement along the $\boldsymbol{R}_t$ direction;
- good for slowly varying $\boldsymbol{R}_t$ (i.e., at large scales)

$$\hat{\boldsymbol{n}}(t) = -\frac{\left[e^{(\tau_s - t)\nabla \boldsymbol{v}_{t_0}}\right]^{\mathrm{T}} \cdot \hat{\boldsymbol{R}}_{t_0}}{\left\| \left[e^{(\tau_s - t)\nabla \boldsymbol{v}_{t_0}}\right]^{\mathrm{T}} \cdot \hat{\boldsymbol{R}}_{t_0} \right\|}$$

Perturbative Optimal Control (PO):
- 0^{th} order OC with constant gradients for a time τ_p (free parameter) ;
- valid at small scales

$$\hat{\boldsymbol{n}}(t) = -\frac{\left[e^{(\tau_p - t)\nabla \boldsymbol{v}_{t_0}}\right]^{\mathrm{T}} \cdot e^{(\nabla \boldsymbol{v})_{t_0} \tau_p} \cdot \hat{\boldsymbol{R}}_{t_0}}{\left\| \left[e^{(\tau_p - t)\nabla \boldsymbol{v}_{t_0}}\right]^{\mathrm{T}} \cdot e^{(\nabla \boldsymbol{v})_{t_0} \tau_p} \cdot \hat{\boldsymbol{R}}_{t_0} \right\|}$$

*Monthiller, Rémi, et al. **Surfing on Turbulence: A Strategy for Planktonic Navigation**. *Phys. Rev. Lett.* ***129****, 064502* (2022)

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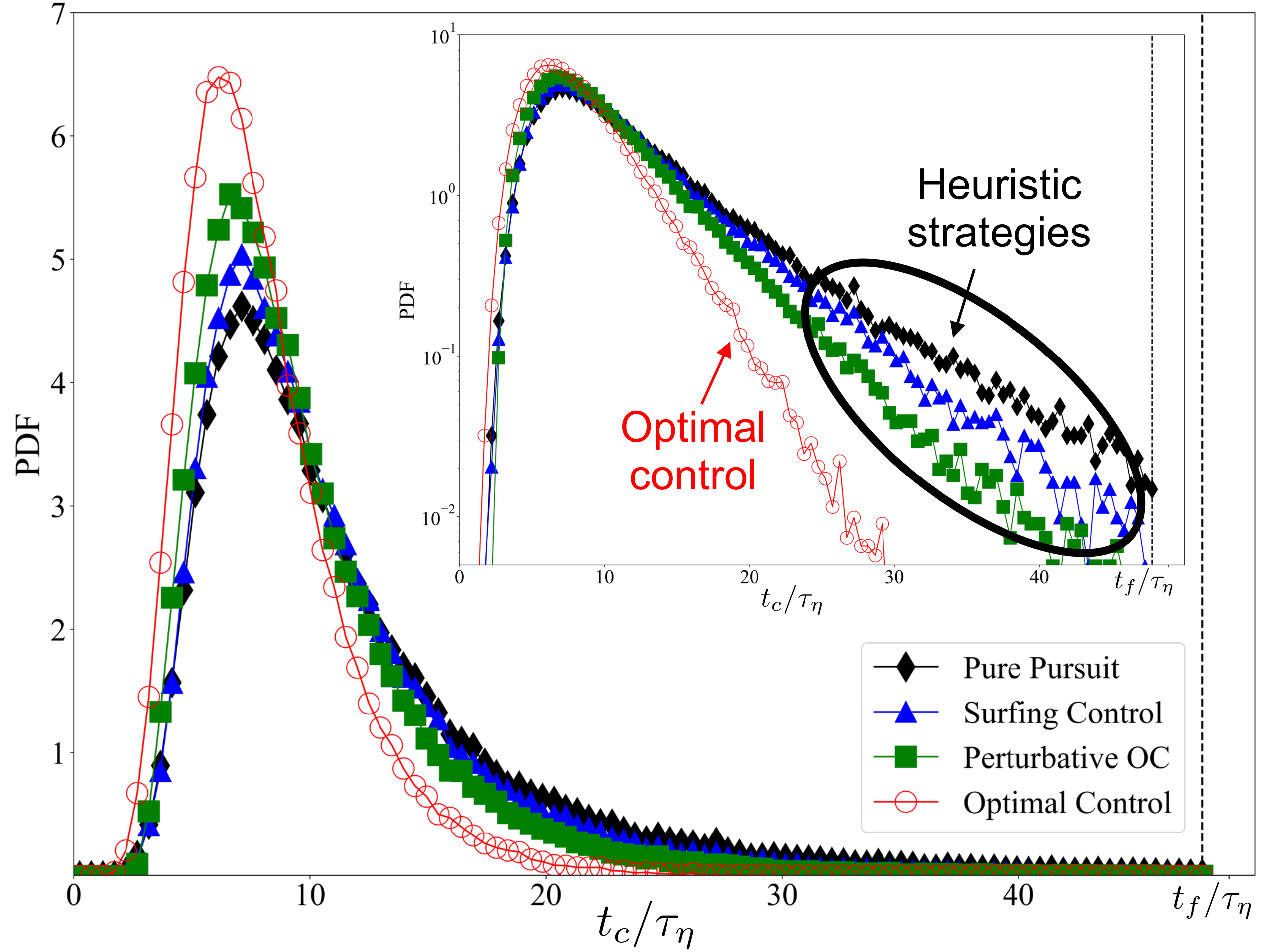
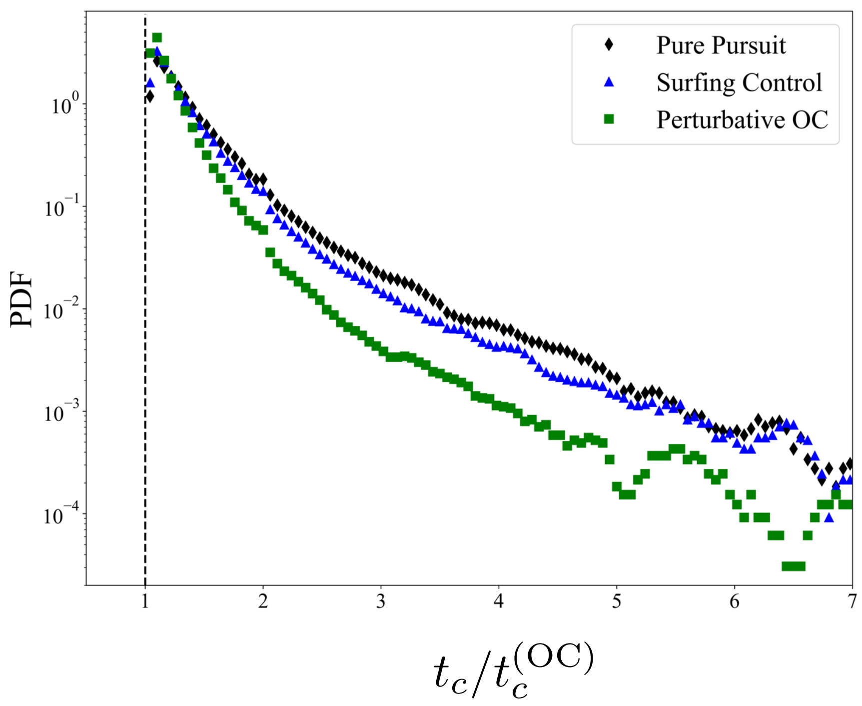
Optimal Control vs heuristic strategies: **capture time statistics**

$$\dot{\boldsymbol{R}}_t = \nabla \boldsymbol{v}_t \boldsymbol{R}_t + \mathbf{U}(t)$$

t_c = **Capture time**: (time of arrival at the desired distance)



PDF of normalized capture time



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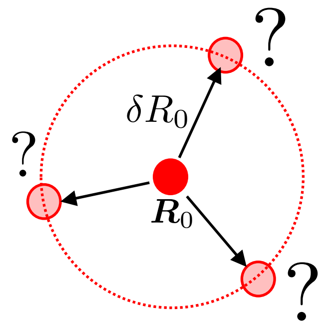
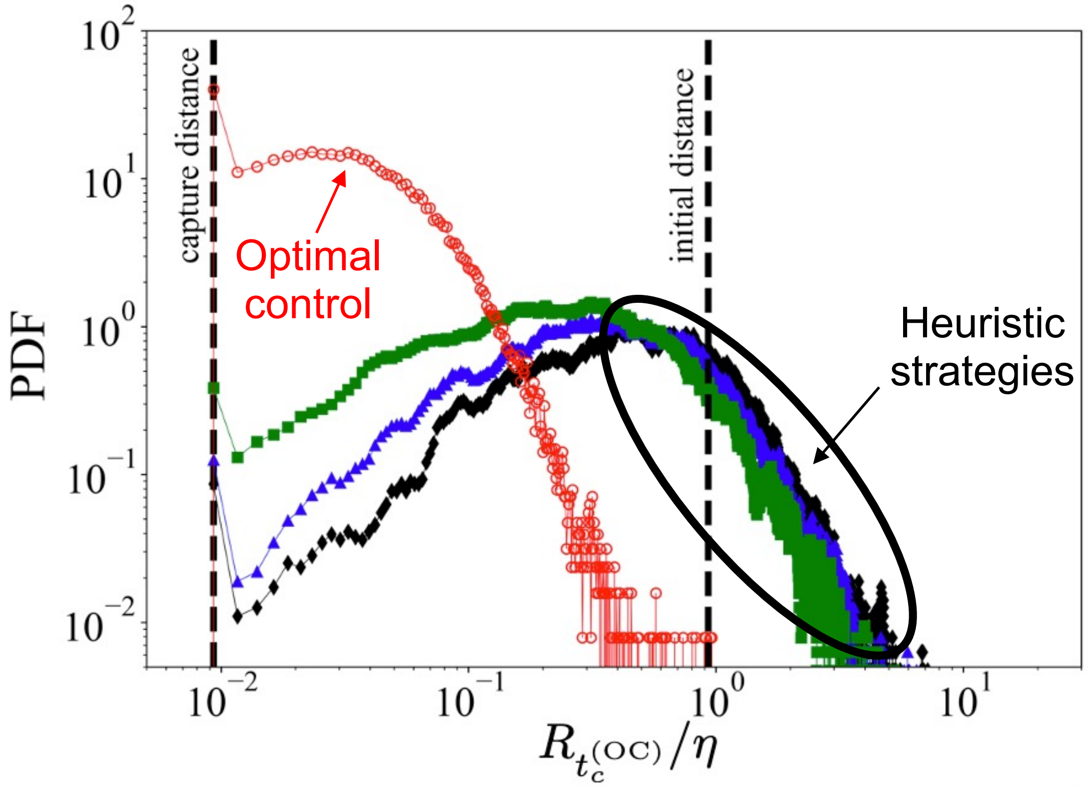
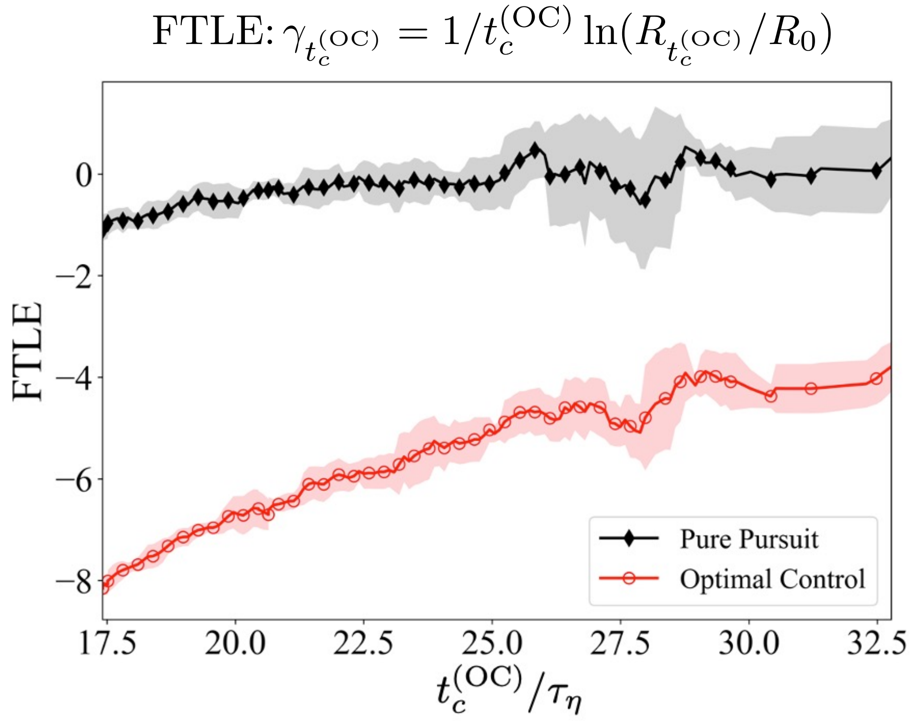
$t_c^{(\mathrm{OC})}$ = Capture time for the unperturbed OC trajectory $R_{t_c^{(\mathrm{OC})}}$ = Distance of perturbed trajectories



OC is stable under perturbations of the initial conditions!!!

1. The target trajectory becomes an attractor for the controlled dynamics.
2. Larger capture times are connected to more chaotic trajectories.





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pros & cons

Optimal Control (*)

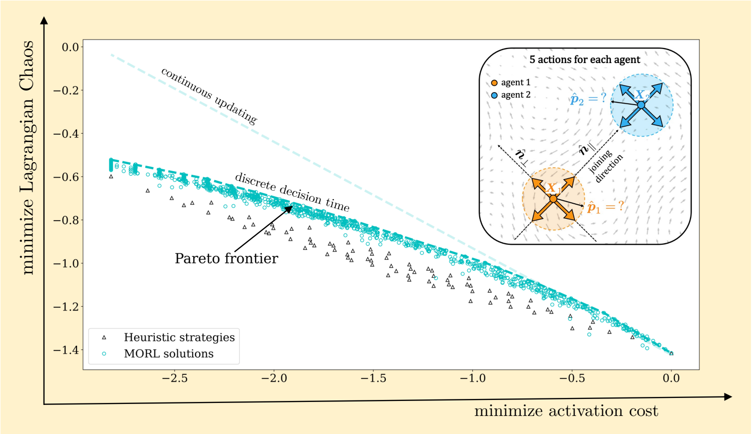
- \+ It is optimized
- \- It is model based and needs perfect information (fully history) from the environment
- \+ It is resilient to disturbances on the agent starting conditions
- \- It is difficult to consider a decision time in the control variable

Heuristic strategies (*)

- \- They are not optimized and display worse performances wrt OC
- \+ They need only partial information (reactive strategies)
- \+ They can be applied also with a discrete decision time

Reinforcement Learning (**) (not shown) →

- \+ It is optimized
- \+ It is model free
- \+ It needs partial information (past memory can be included)
- \- It is data-hungry



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(**) **Calascibetta, C.**, Biferale, L., Borra, F., Celani, A. and Cencini, M. **Taming Lagrangian Chaos with multi-objective reinforcement learning.** *Eur. Phys. J. E* ***46**, 9* (2023).

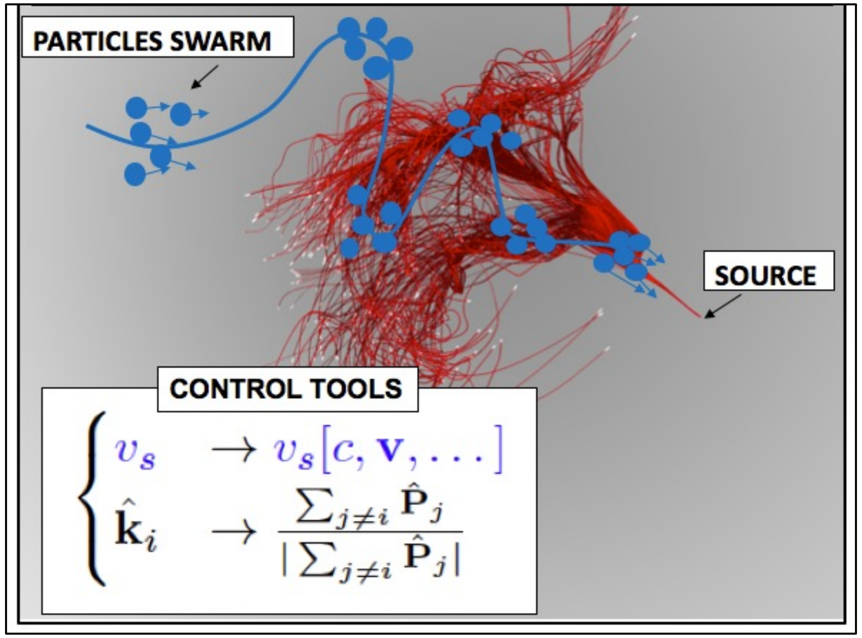
Conclusions

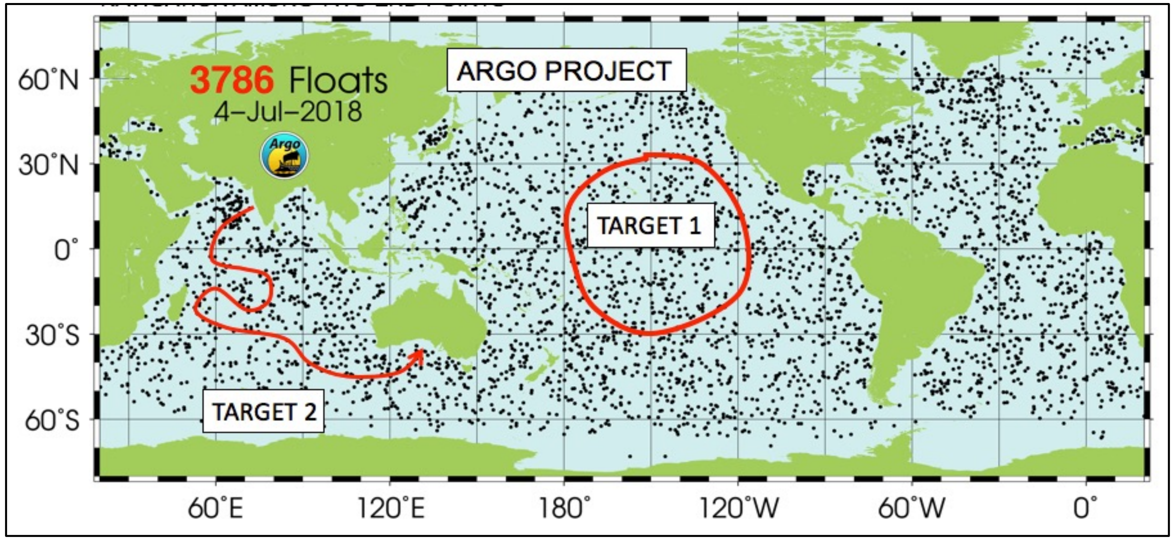
Open questions:

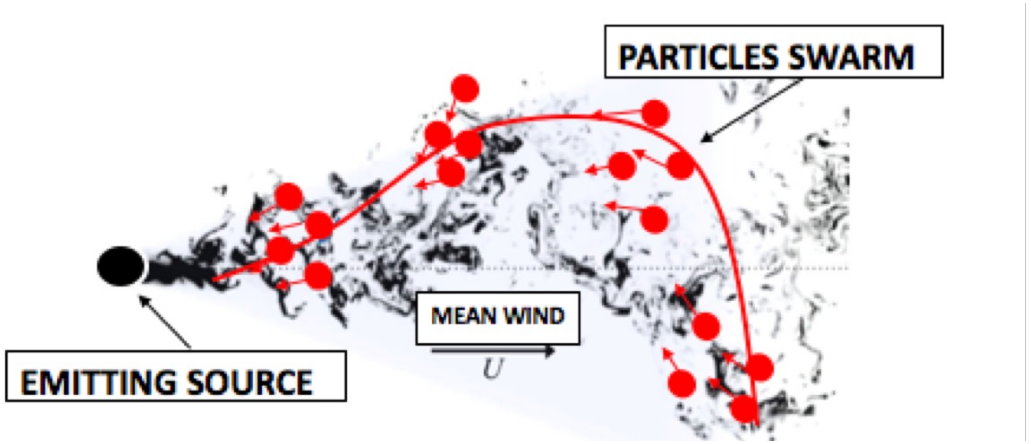
1. How to fill the gap between heuristic strategies and optimal control performances?
2. How to control a multi-agent system to minimize turbulent dispersion at larger range of scales (i.e., beyond the linear regime)?
3. Are the agents able to collaborate with each-other during the navigation?

Tools:

We can develop new heuristic controls with short-term memory
We can use RL to control autonomous swimmer to reach complex goals
We can apply OC theory as a benchmark to test the heuristic/data-driven solutions.







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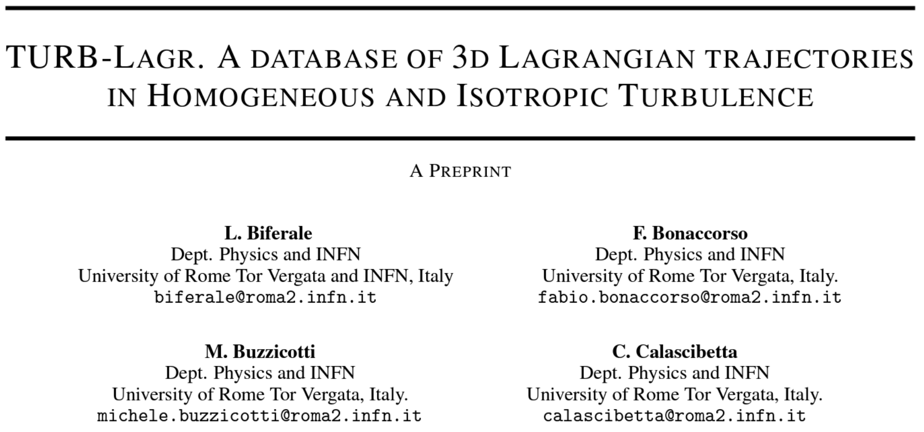
TURB-LAGR. A DATABASE OF 3D LAGRANGIAN TRAJECTORIES IN HOMOGENEOUS AND ISOTROPIC TURBULENCE

A PREPRINT

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