# Extreme events in Turbulence and Burgerslence

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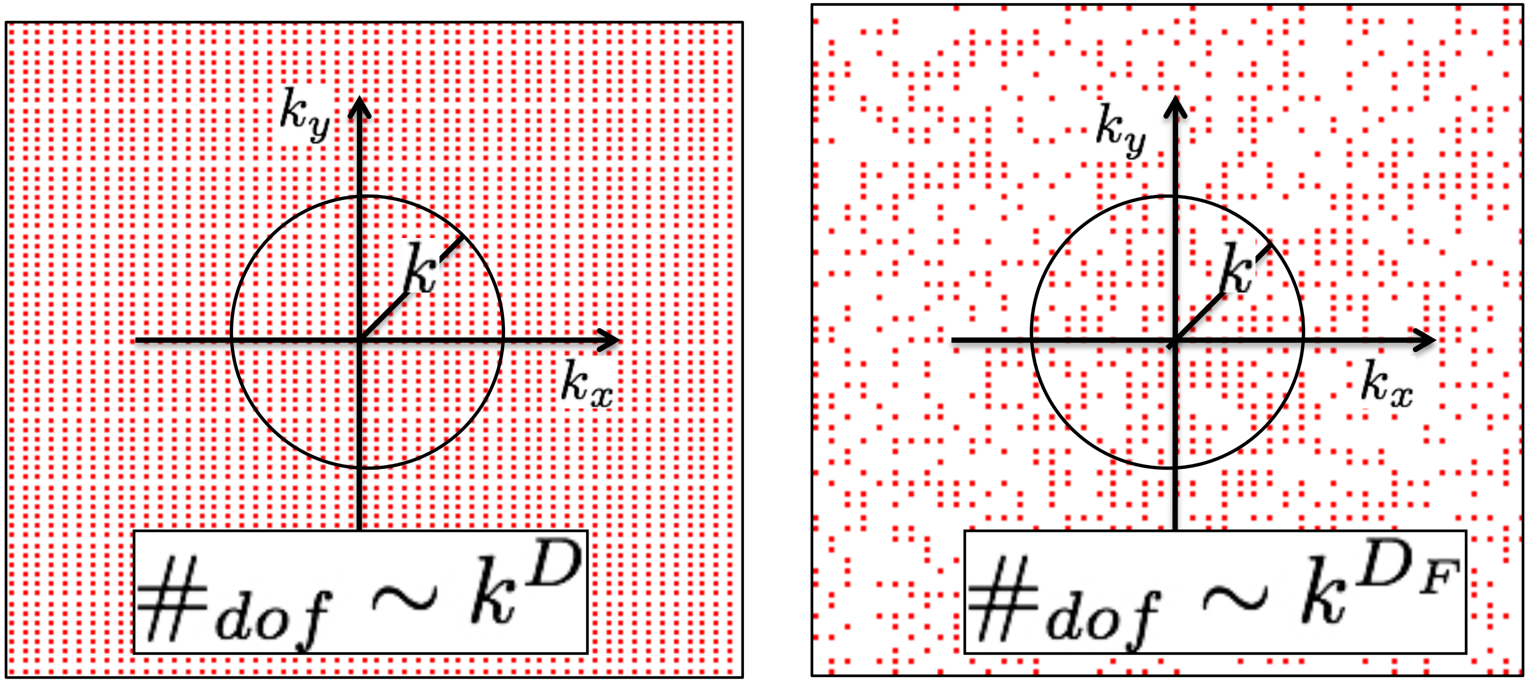
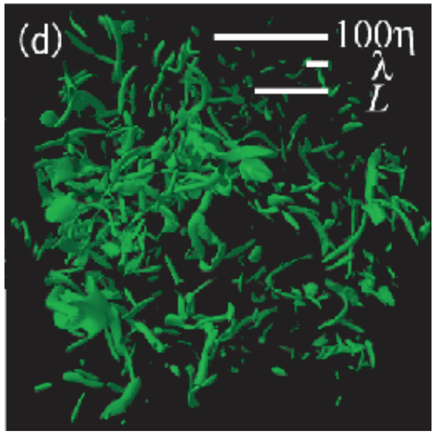


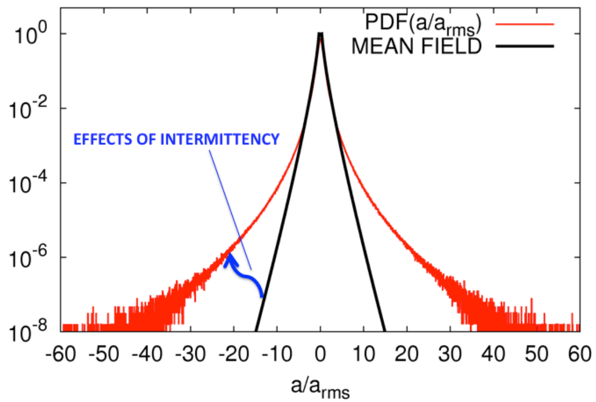

A.S. Lanotte (CNR, Italy)
S. Malapaka & R. Benzi (Tor Vergata Univ. Italy)
F. Toschi (TuE, The Netherlands)

## 3D HOMOGENEOUS AND ISOTROPIC TURBULENCE

$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v} \cdot \partial)\mathbf{v} = -\partial P + \nu \Delta \mathbf{v} + \mathbf{F} \\ \partial \cdot \mathbf{v} = 0 \\ + \ Boundary\ Conditions \end{cases}$$

**EXPERIMENTS IN-SILICO:**
**CAN WE ASK QUESTIONS ABOUT THE ENERGY TRANSFER EVENTS**
**(BOTH TYPICAL AND EXTREME)**
**BY DECIMATING INTERACTIONS IN THE NON LINEAR TERM?**




ACCELERATION PROBABILITY DISTRIBUTION FUNCTION (PDF) AT RE ~ $10^5$ [Bi04]
COMPARED WITH THE PREDICTION FROM MEAN FIELD (KOLMOGOROV THEORY)

L. B. et al *Phys. Rev. Lett.* **93**, 064502, 2004.

# Extreme events in computational turbulence

P. K. Yeung[a], X. M. Zhai[b], and Katepalli R. Sreenivasan[c,1]

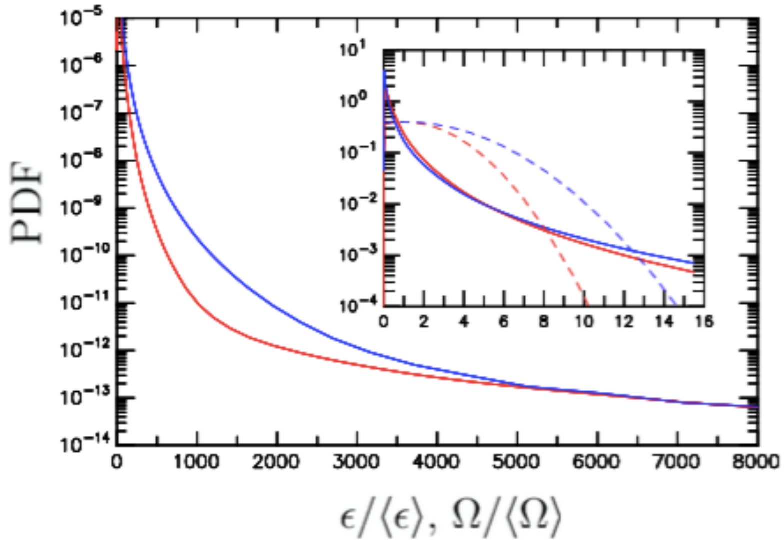



**Fig. 1.** Ensemble-averaged PDFs of normalized dissipation (red) and enstrophy (blue) from $8{,}192^3$ simulation at $R_\lambda \approx 1{,}300$, with $k_{max}\eta \approx 2$. *Inset* shows data for 0–16 mean values. Dashed curves in *Inset* show positive halves of Gaussian distributions with equal variances; they serve only a pedantic purpose because dissipation and enstrophy are both positive definite. Rare events occur enormously more frequently than can be anticipated by Gaussian distributions—by some 10 orders of magnitude when the abscissae values reach 50 or smaller, and by some 250 orders of magnitude for abscissae values of 1,000. Although the data shown are averaged over 14 instantaneous snapshots, the main features are robust: Every snapshot possesses similar features.

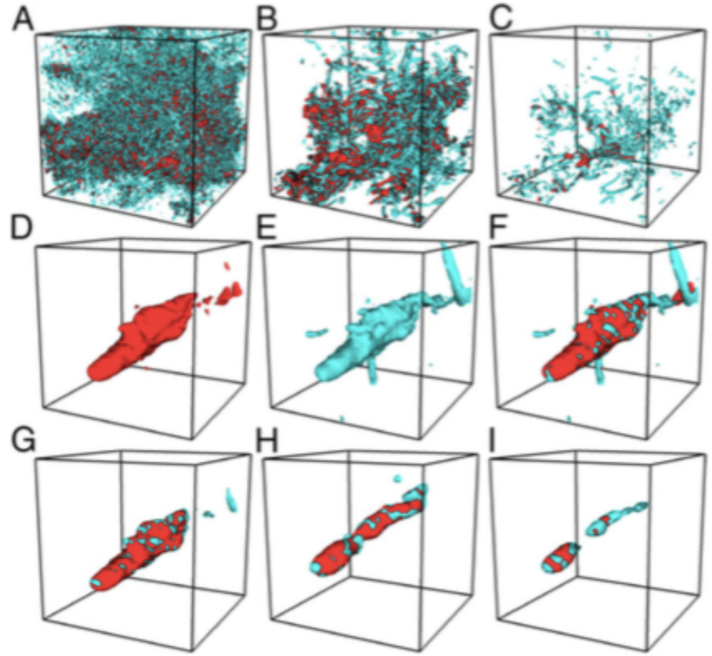



**Fig. 3.** Perspective views of 3D contour surfaces of dissipation (red) and enstrophy (cyan) extracted from a randomly chosen (but representative) $8{,}192^3$ instantaneous snapshot, at different thresholds (in multiples of mean values) and for different sized subcubes: (*A*) 10, $768^3$; (*B*) 30, $256^3$; (*C*) 100, $256^3$; (*D*–*F*): 300, $51^3$; (*G*) 600, $51^3$; (*H*) 4,800, $31^3$; and (*I*) 9,600, $31^3$. Both dissipation and enstrophy are shown in all frames but *D* and *E*.

# Extreme events in computational turbulence

P. K. Yeung[a], X. M. Zhai[b], and Katepalli R. Sreenivasan[c,1]

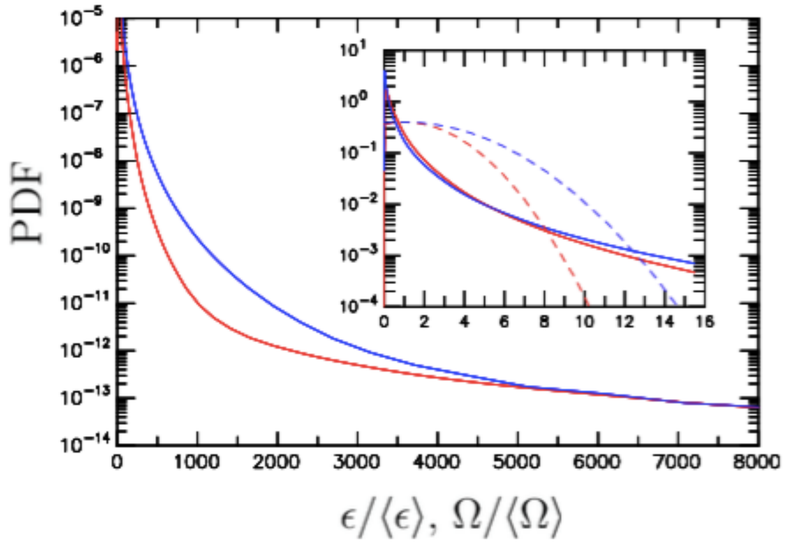
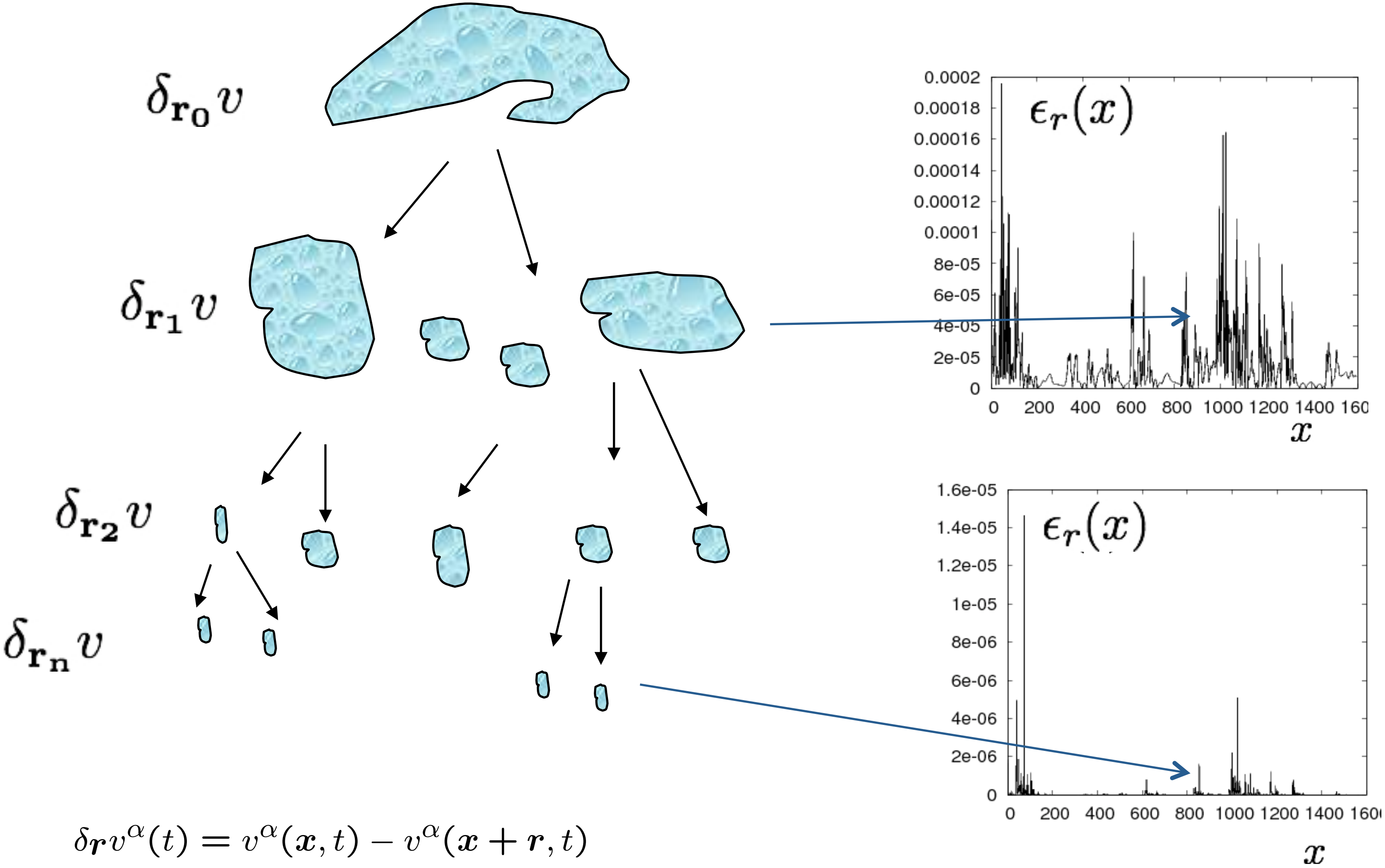



**Fig. 1.** Ensemble-averaged PDFs of normalized dissipation (red) and enstrophy (blue) from $8{,}192^3$ simulation at $R_\lambda \approx 1{,}300$, with $k_{max}\eta \approx 2$. *Inset* shows data for 0–16 mean values. Dashed curves in *Inset* show positive halves of Gaussian distributions with equal variances; they serve only a pedantic purpose because dissipation and enstrophy are both positive definite. Rare events occur enormously more frequently than can be anticipated by Gaussian distributions—by some 10 orders of magnitude when the abscissae values reach 50 or smaller, and by some 250 orders of magnitude for abscissae values of 1,000. Although the data shown are averaged over 14 instantaneous snapshots, the main features are robust: Every snapshot possesses similar features.

The present simulations at the highest Reynolds numbers to date—with the small scales well resolved—suggest something even more complex: With increasing Reynolds numbers, the extreme events assume a form that is not characteristic of similar events at low Reynolds numbers. Our results show that, for the Reynolds numbers of these simulations, events as large as $10^5$ times the mean value obtain, albeit rarely. They appear chunky in character, unlike elongated vortex tubes. We track the temporal evolution of these extreme events and find that they are generally short-lived. Extreme magnitudes of energy dissipation rate and enstrophy occur essentially simultaneously in space and remain nearly colocated during their evolution. This is the insight of the present work. The fact that extreme events do not preserve the same form at different Reynolds numbers strongly underlines that the large amplitude events (more broadly, the phenomenon of intermittency) at one Reynolds number cannot be understood by some simple transformation of those at another (lower) Reynolds number. This may well be the source of anomaly that turbulence is known to possess (e.g., ref. 6). We do not yet know how this insight is related to the source of anomaly so beautifully explored quantitatively for the Kraichnan model (30) of passive scalar turbulence (e.g., refs. 31–33).

# spatio-temporal Richardson cascade

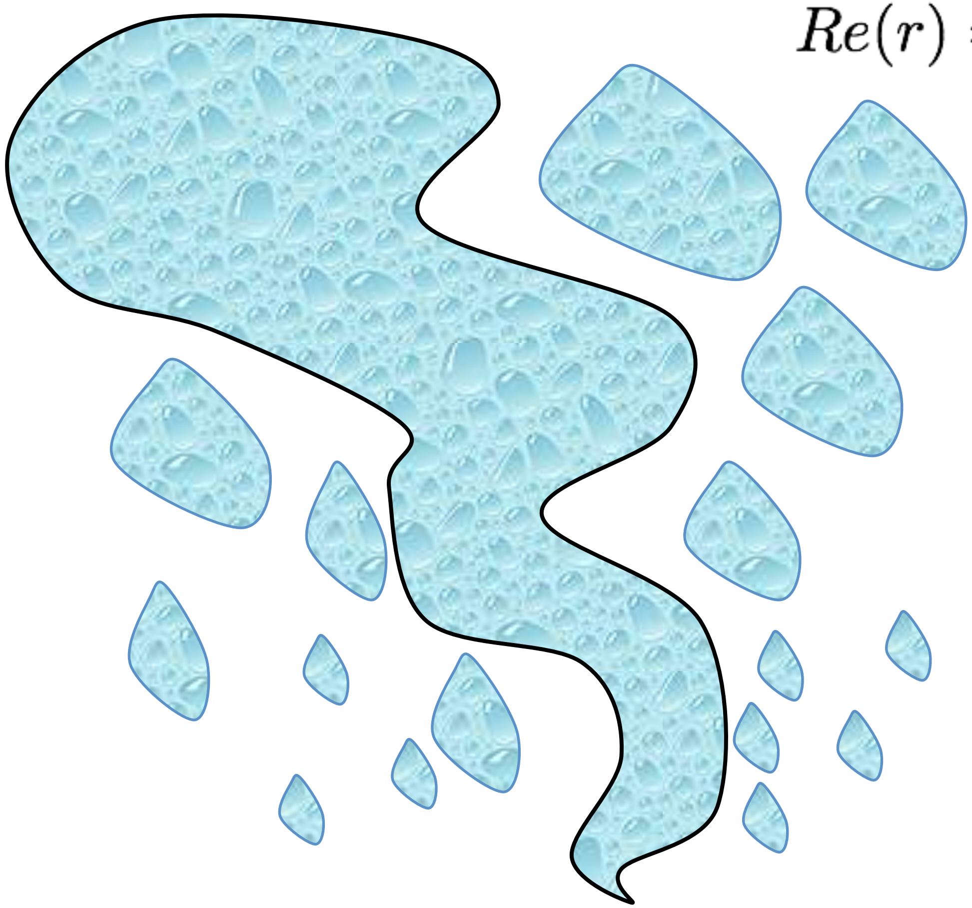
$$Re(r) = \frac{r\delta_r v}{\nu}$$

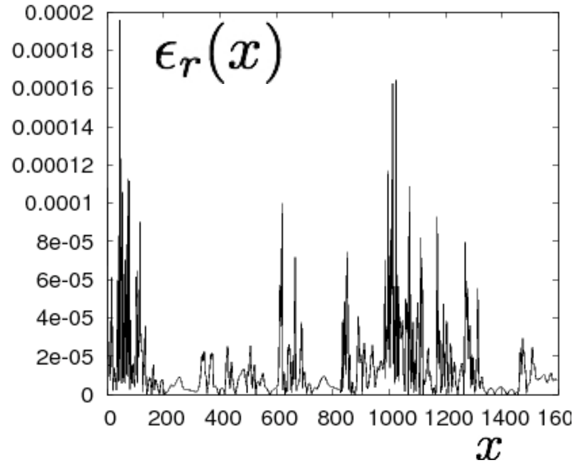


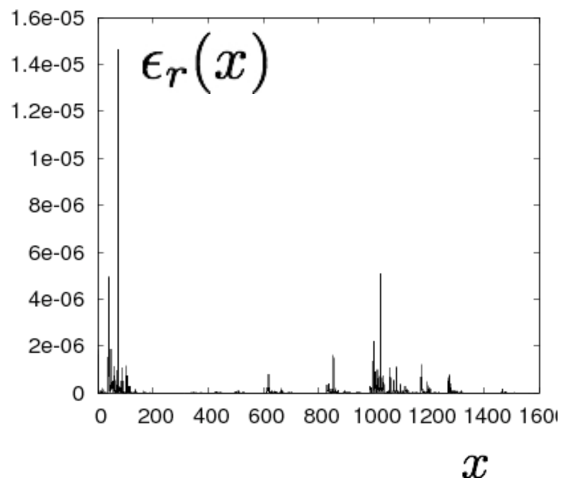

$$\delta_{\boldsymbol{r}} v^{\alpha}(t) = v^{\alpha}(\boldsymbol{x}, t) - v^{\alpha}(\boldsymbol{x} + \boldsymbol{r}, t)$$

$$S_n^{\bar{\alpha}}(\bar{\boldsymbol{r}}, \bar{t}) = \langle \delta_{\boldsymbol{r}_1} v^{\alpha_1}(t_1) \cdots \delta_{\boldsymbol{r}_n} v^{\alpha_n}(t_n) \rangle$$

$$Re(r) = \frac{r\delta_r v}{\nu}$$





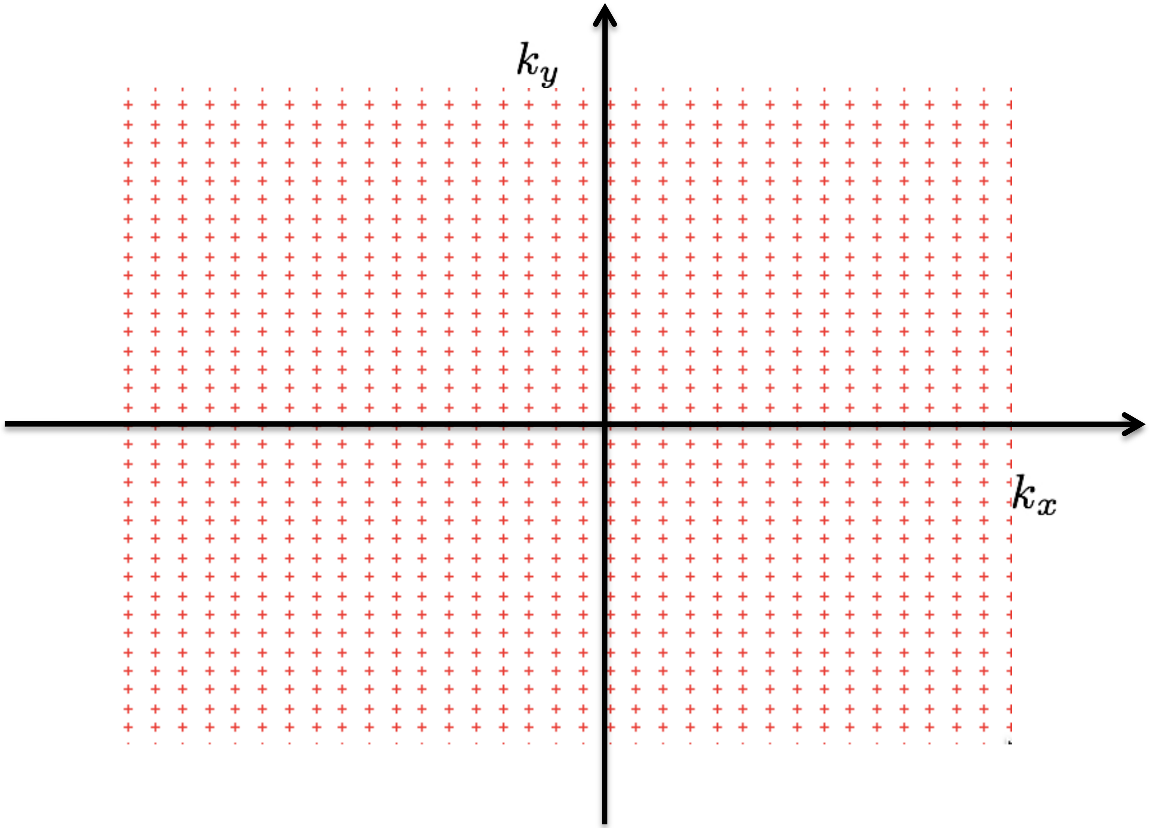




$$\delta_{\boldsymbol{r}} v^{\alpha}(t) = v^{\alpha}(\boldsymbol{x}, t) - v^{\alpha}(\boldsymbol{x} + \boldsymbol{r}, t)$$

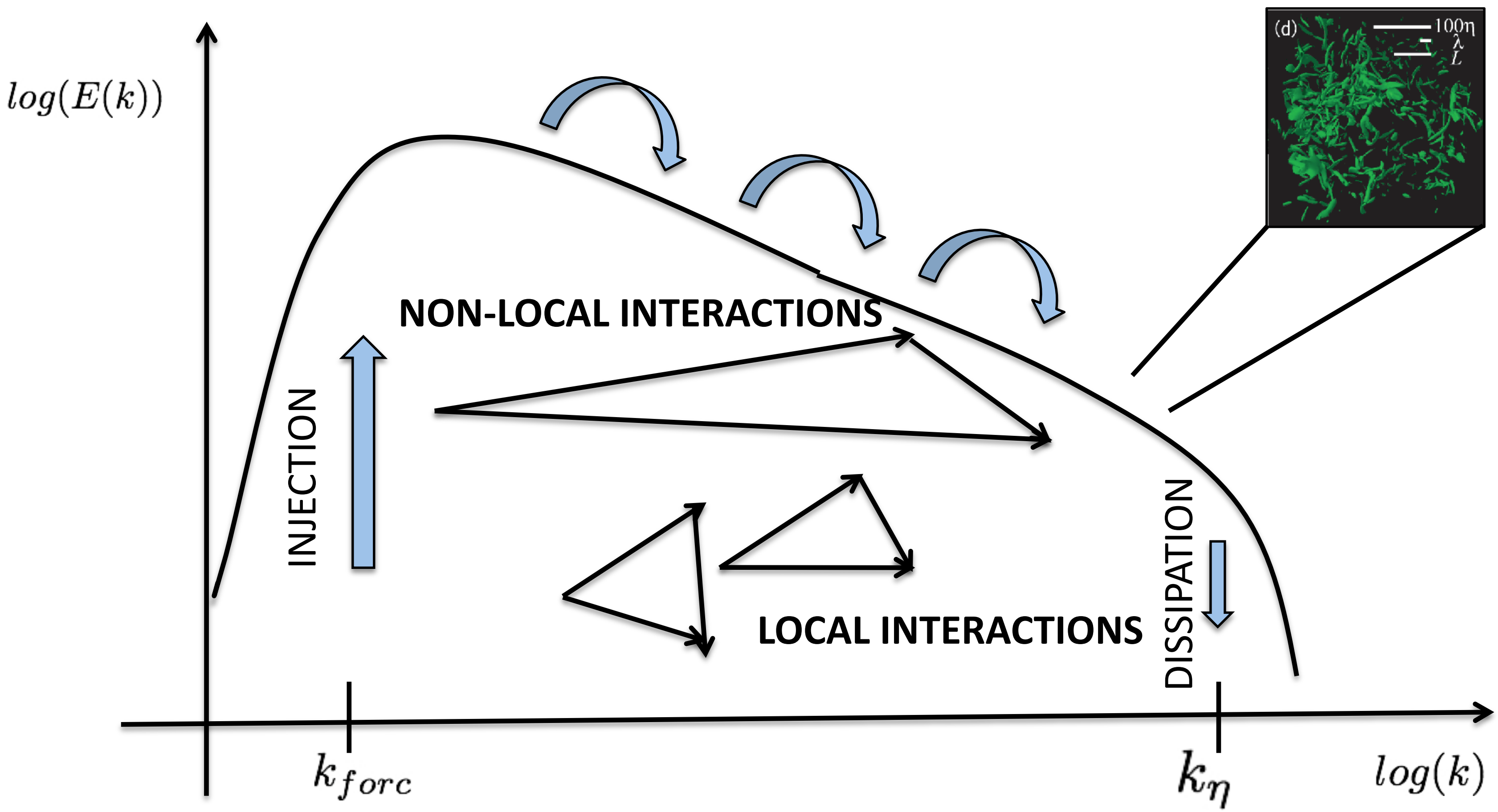
$$S_n^{\bar{\alpha}}(\bar{\boldsymbol{r}}, \bar{t}) = \langle \delta_{\boldsymbol{r}_1} v^{\alpha_1}(t_1) \cdots \delta_{\boldsymbol{r}_n} v^{\alpha_n}(t_n) \rangle$$

$$\frac{d\hat{u}_n(\boldsymbol{k},t)}{dt} + \left(\delta_{nm} - \frac{k_n k_m}{|k|^2}\right) \widehat{\boldsymbol{NL}}_m(\boldsymbol{k},t) = -\nu|\boldsymbol{k}|^2 \hat{u}_n(\boldsymbol{k},t) + \hat{f}_n(\boldsymbol{k},t)$$

$$\widehat{\boldsymbol{NL}}_m(\boldsymbol{k},t) = -i \sum_{\boldsymbol{k}'+\boldsymbol{k}''=\boldsymbol{k}} k''_j \, \hat{u}_m(\boldsymbol{k}',t)\hat{u}_j(\boldsymbol{k}'',t)$$

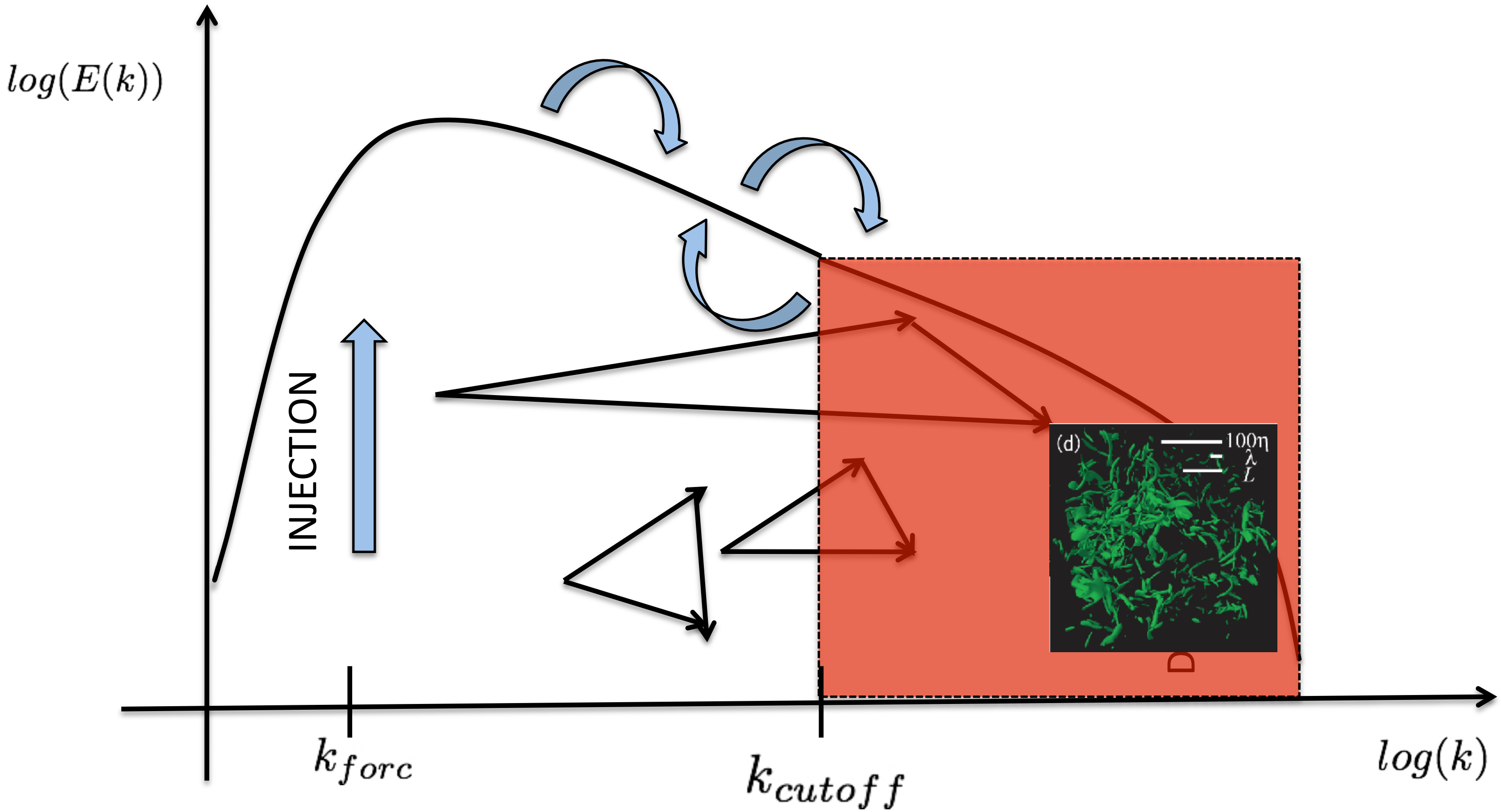



How many (and which) degrees of freedom do we need to preserve the main statistical properties of NS turbulence?

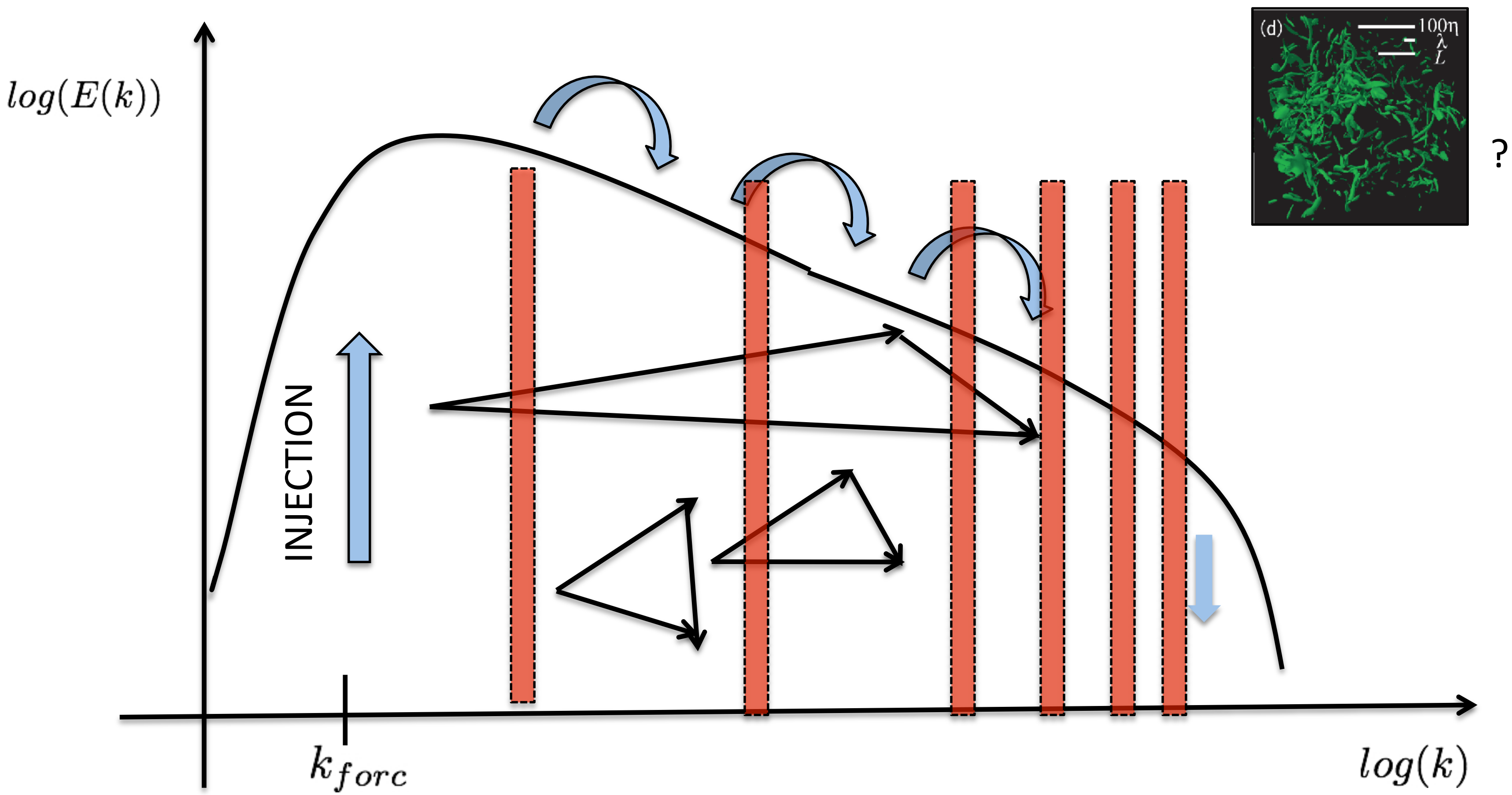


$log(E(k))$
NON-LOCAL INTERACTIONS
INJECTION
LOCAL INTERACTIONS
DISSIPATION
$k_{forc}$
$k_\eta$
$log(k)$
(d)
100η
λ
L

# LARGE EDDY SIMULATION




$$\partial_t \overline{v} = \overline{\overline{v}\partial_x \overline{v}} - \partial_x \overline{P} + \partial_x \Pi_{SG} + \nu \Delta \overline{v} + \overline{f}$$




$$\mathbf{v}^D(\mathbf{x},t) = \mathcal{P}^D \mathbf{v}(\mathbf{x},t) = \sum_{\mathbf{k}\in \mathcal{Z}^3} e^{i\mathbf{k}\cdot\mathbf{x}}\, \gamma_{\mathbf{k}} \mathbf{u}(\mathbf{k},t)\,.$$

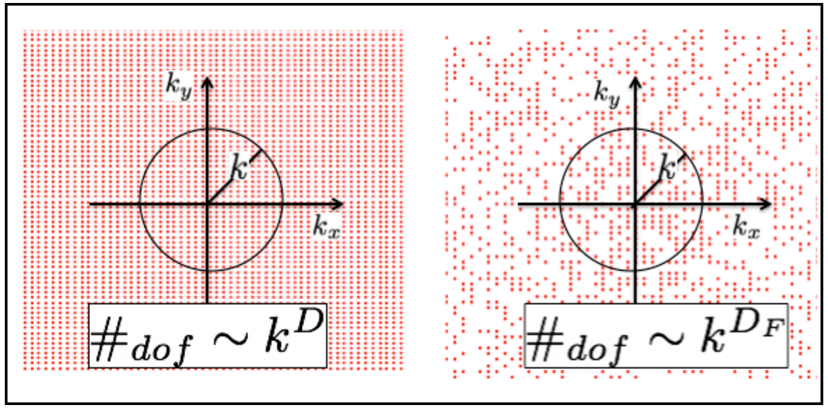
DECIMATED WITH PROBABILITY ~ $1 - k^{D_F - 3}$

# SELF-SIMILAR GALERKIN TRUNCATION

U. Frisch, A. Pomyalov, I. Procaccia and S. Ray PRL 2012
S. Grossmann, D. Lohse and A. Reeh, PRL 1996

$$\partial_t \overline{v} = \overline{\overline{v}\partial_x \overline{v}} - \partial_x \overline{P} + \xcancel{\partial_x \Pi_{SG}} + \nu \Delta \overline{v} + \overline{f}$$

$$\partial_x \Pi_{SG} = \overline{v \partial_x v} - \overline{\overline{v}\partial_x \overline{v}}$$




HOMOGENEOUS & ISOTROPIC & SELF-SIMILAR (NO EXTERNAL SCALES)
ENERGY & HELICITY INVISCID INVARIANTS
REAL PDE (INFINITE NUMBER OF DEGREES OF FREEDOM)

# INVERSE ENERGY CASCADES IN 3D (→ 2D) ?

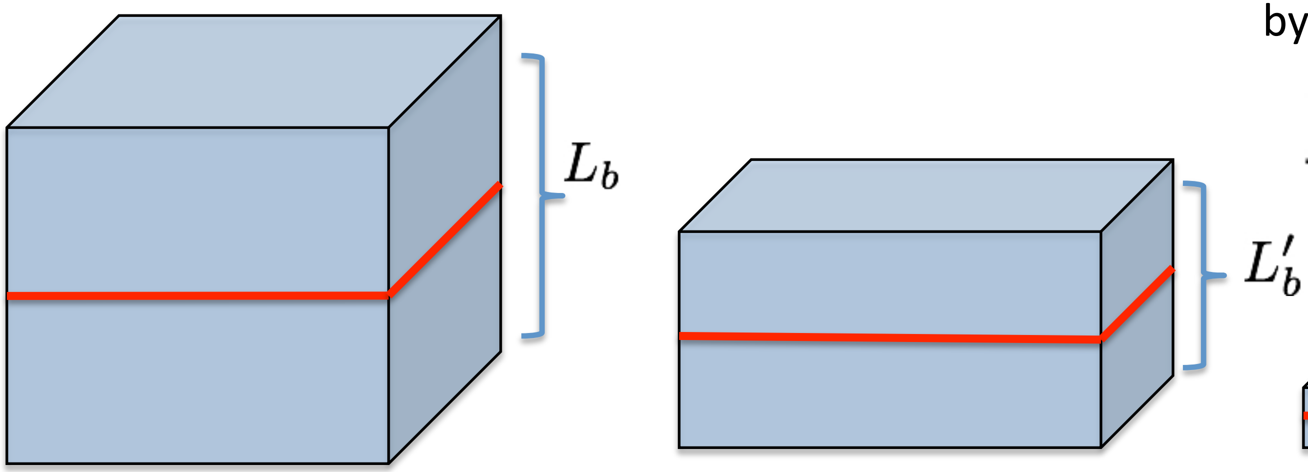
PRL 104, 184506 (2010) PHYSICAL REVIEW LETTERS week ending 7 MAY 2010

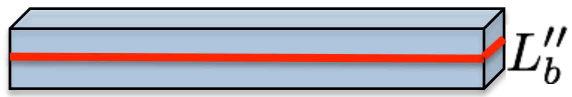
**Turbulence in More than Two and Less than Three Dimensions**

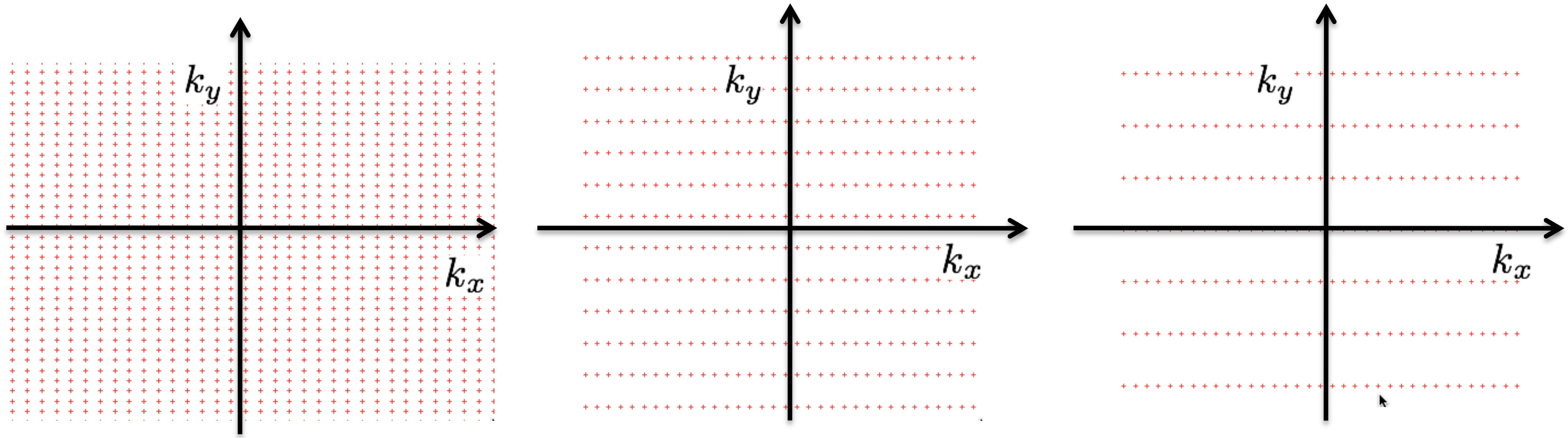
Antonio Celani,[1] Stefano Musacchio,[2,3] and Dario Vincenzi[3]

by geometrical constraints

$$L_b/L_f \to 0$$



# Turbulence in non-integer dimensions by fractal Fourier decimation

Uriel Frisch,[1] Anna Pomyalov,[2] Itamar Procaccia,[2] and Samriddhi Sankar Ray[1]
[1] *UNS, CNRS, OCA, Laboratoire Cassiopée, B.P. 4229, 06304 Nice Cedex 4, France*
[2] *Department of Chemical Physics, The Weizmann Institute of Science, Rehovot 76100, Israel*
(Dated: August 8, 2011)

Fractal decimation reduces the effective dimensionality of a flow by keeping only a (randomly chosen) set of Fourier modes whose number in a ball of radius $k$ is proportional to $k^D$ for large $k$. At the critical dimension $D = 4/3$ there is an equilibrium Gibbs state with a $k^{-5/3}$ spectrum, as in [V. L'vov *et al.*, Phys. Rev. Lett. **89**, 064501 (2002)]. Spectral simulations of fractally decimated two-dimensional turbulence show that the inverse cascade persists below $D = 2$ with a rapidly rising Kolmogorov constant, likely to diverge as $(D - 4/3)^{-2/3}$.

$$E(k) = \frac{k^{D-1}}{\alpha + \beta k^2}; \quad \beta > 0, \quad \alpha > -\beta,$$

$$D = 4/3$$

Enstrophy equipartition :
5/3 Kolmogorov spectrum

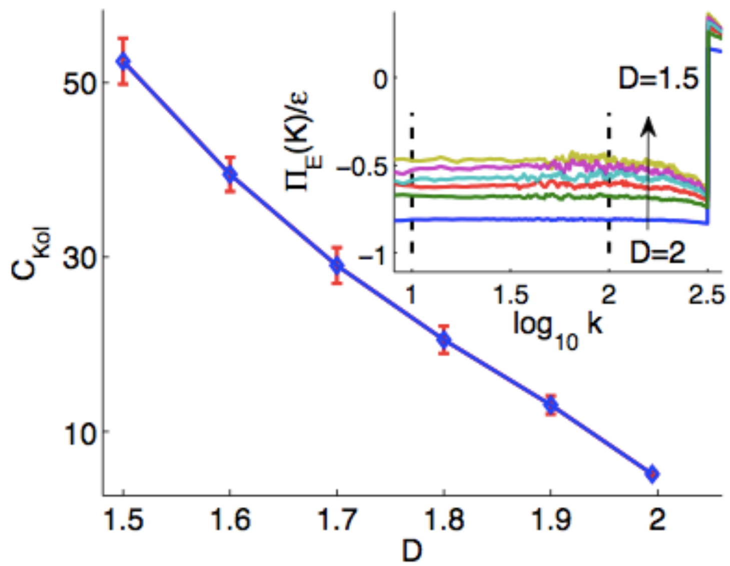



FIG. 3. (Color online) Dependence of the Kolmogorov constant on $D$. The lowest value, at $D = 2$, is about 5. The inset

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# Developed Turbulence: From Full Simulations to Full Mode Reductions

Siegfried Grossmann,* Detlef Lohse,† and Achim Reeh‡

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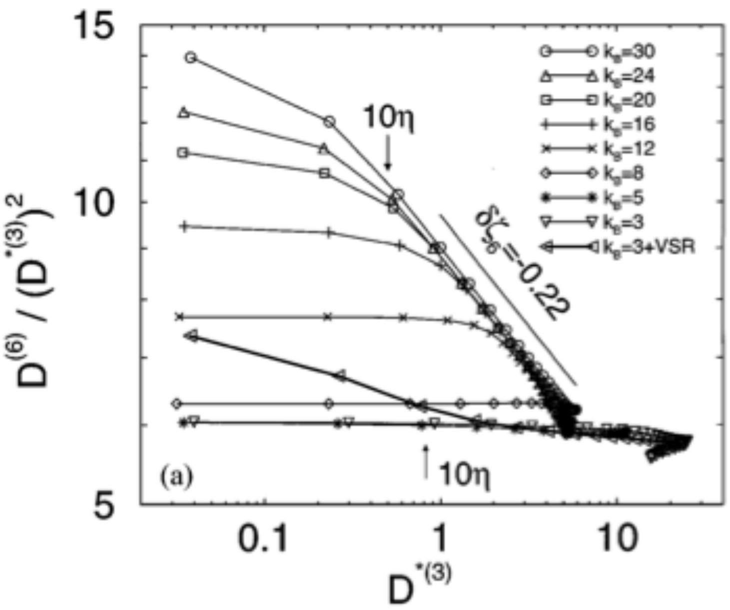
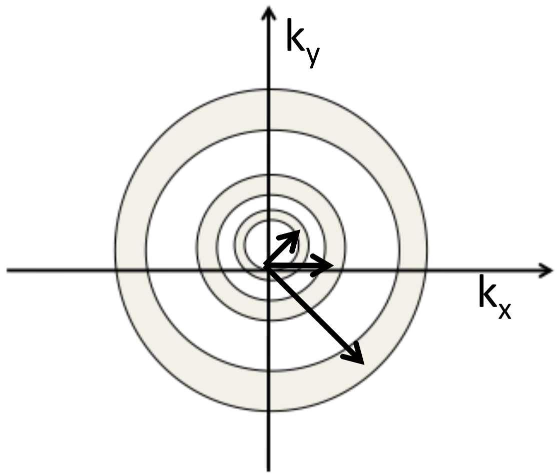
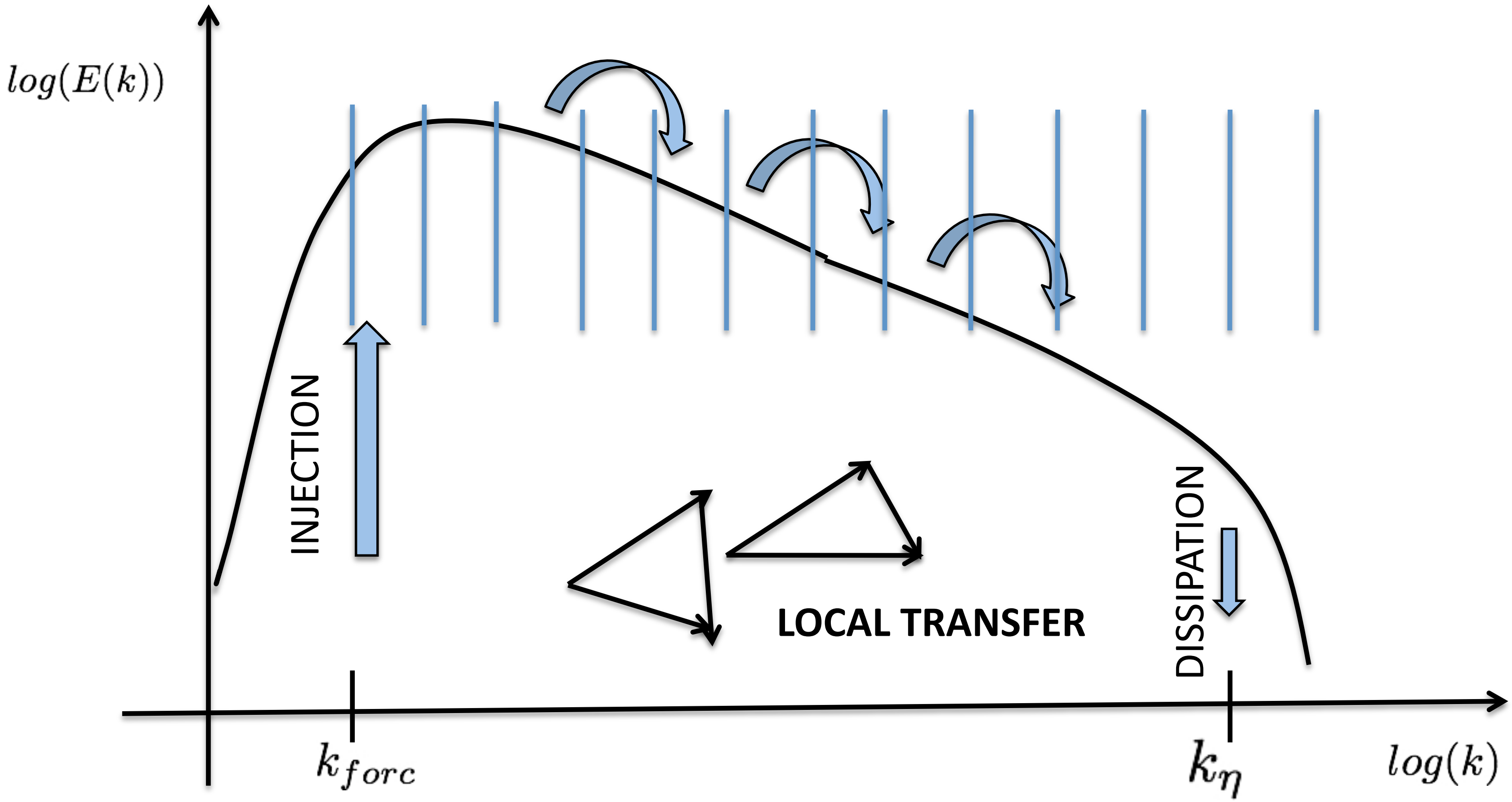
(Received 5 August 1996)

Developed Navier-Stokes turbulence is simulated with varying wave-vector mode reductions. The flatness and the skewness of the velocity derivative depend on the degree of mode reduction. They show a crossover towards the value of the full numerical simulation when the viscous subrange starts to be resolved. The intermittency corrections of the scaling exponents $\zeta_p$ of the $p$th order velocity structure functions seem to depend mainly on the proper resolution of the inertial subrange. *Universal* scaling properties (i.e., independent of the degree of mode reduction) are found for the relative scaling exponents $\rho_{p,q} = (\zeta_p - \zeta_3 p/3)/(\zeta_q - \zeta_3 q/3)$. [S0031-9007(96)01942-4]

PACS numbers: 47.27.Eq, 47.11.+j



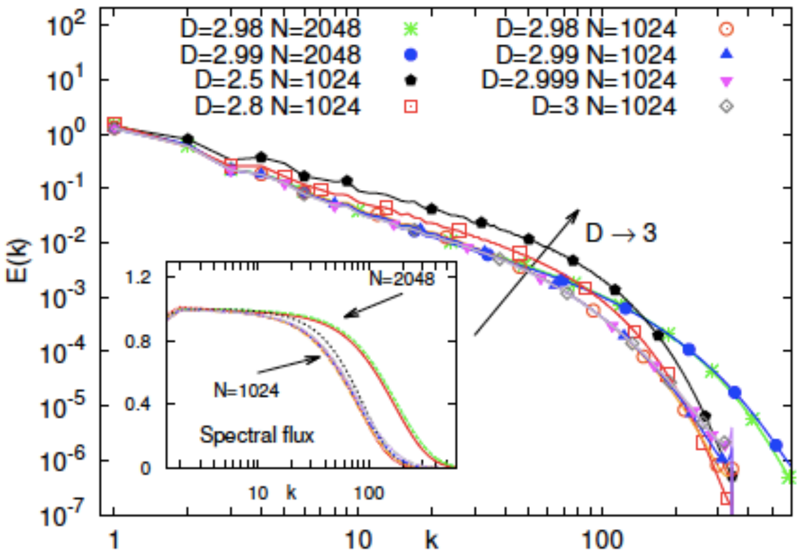



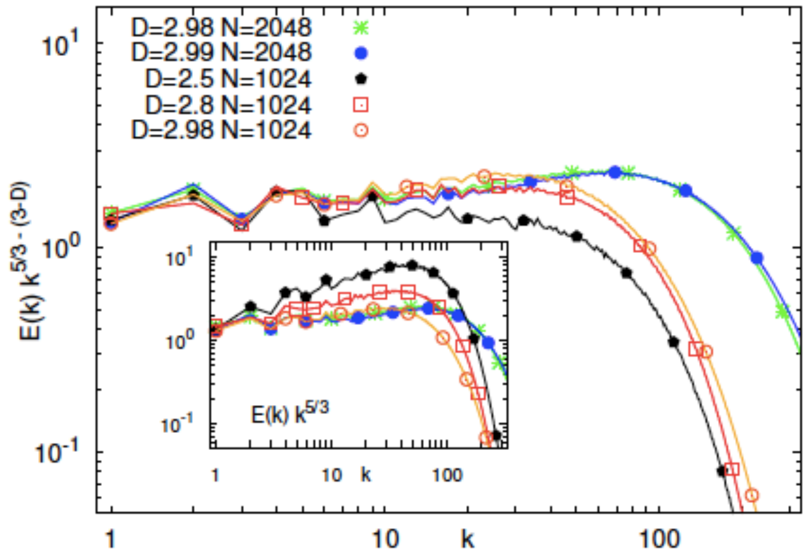



$$\frac{d}{dt}u(k_n) = k_n[a\,u(k_{n+2})u(k_{n+1}) + b\,u(k_{n+1})u(k_{n-1}) + c\,u(k_{n-2})u(k_{n-1})] - \nu k_n^2 u(k_n)$$


Bohr T., Jensen M. H., Paladin G. and Vulpiani A., *Dynamical Systems Approach to Turbulence*, Cambridge, in press (1998)

L.B. Annu. Rev. Fluid. Mech. 35, 441, 2003




$$E^D(k) = \int_{|\mathbf{k}_1|=k} d^3k_1\, \gamma_{\mathbf{k}_1} \int d^3k_2\, \gamma_{\mathbf{k}_2} \langle\, \mathbf{u}(\mathbf{k}_1)\, \mathbf{u}(\mathbf{k}_2)\, \rangle\, .$$

$$\Pi^D(k) = \int_{|\mathbf{k}_1|<k} d^3k_1 \gamma_{\mathbf{k}_1} \int d^3k_2\, d^3k_3\, \gamma_{\mathbf{k}_2}\gamma_{\mathbf{k}_3} S(\mathbf{k}_1|\mathbf{k}_2,\mathbf{k}_3)\, ,$$

$$\mathbf{u}(\mathbf{k}) \sim k^{-a}$$

$$\Pi^D(\lambda k) \;\sim\; \lambda^{3D+1-3a}\Pi^D(k).$$

$$a = D + 1/3 \rightarrow E^D(k) \sim E^{K41}(k) k^{3-D}$$

| DF | 2.5 | 2.8 | 2.98 | 2.99 | 2.999 | 3.0 |
|---|---|---|---|---|---|---|
| 1024^3 | X | X | X | X | X | X |
| 2048^3 | | | X | X | | |

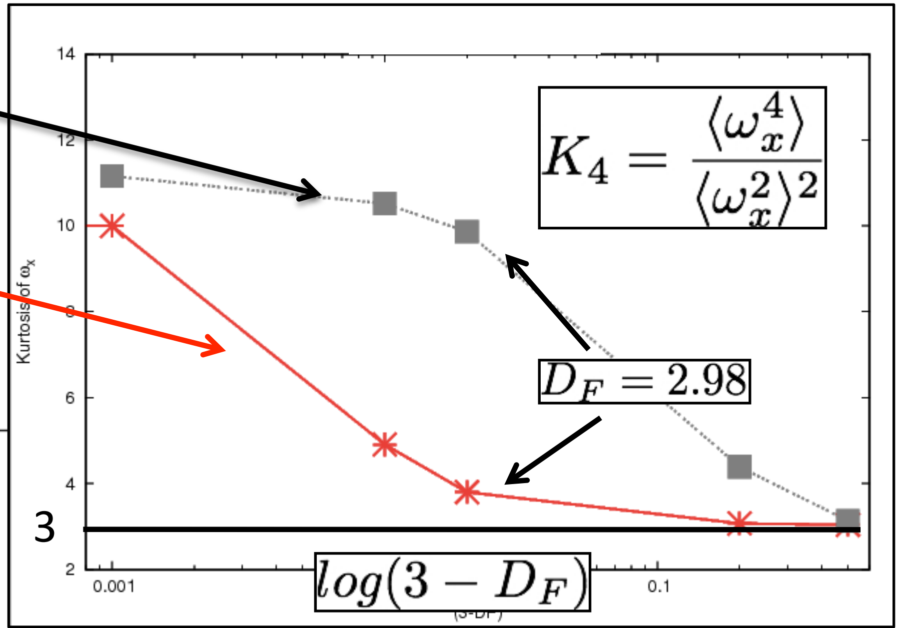
| DF | 2.5 | 2.8 | 2.98 | 2.99 | 2.999 | 3.0 |
|---|---|---|---|---|---|---|
| 1024^3 | 3% | 25% | 87% | 93% | 99% | 100% |

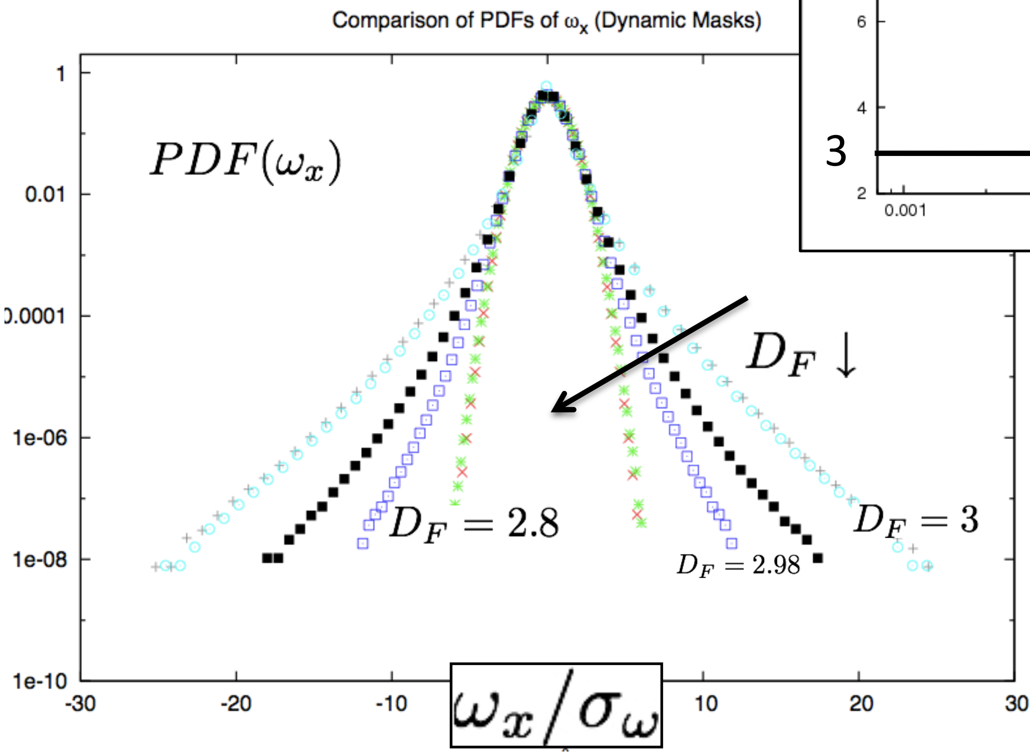
A.S. Lanotte, R. Benzi, L.B, S. Malapaka and F. Toschi PRL 2015

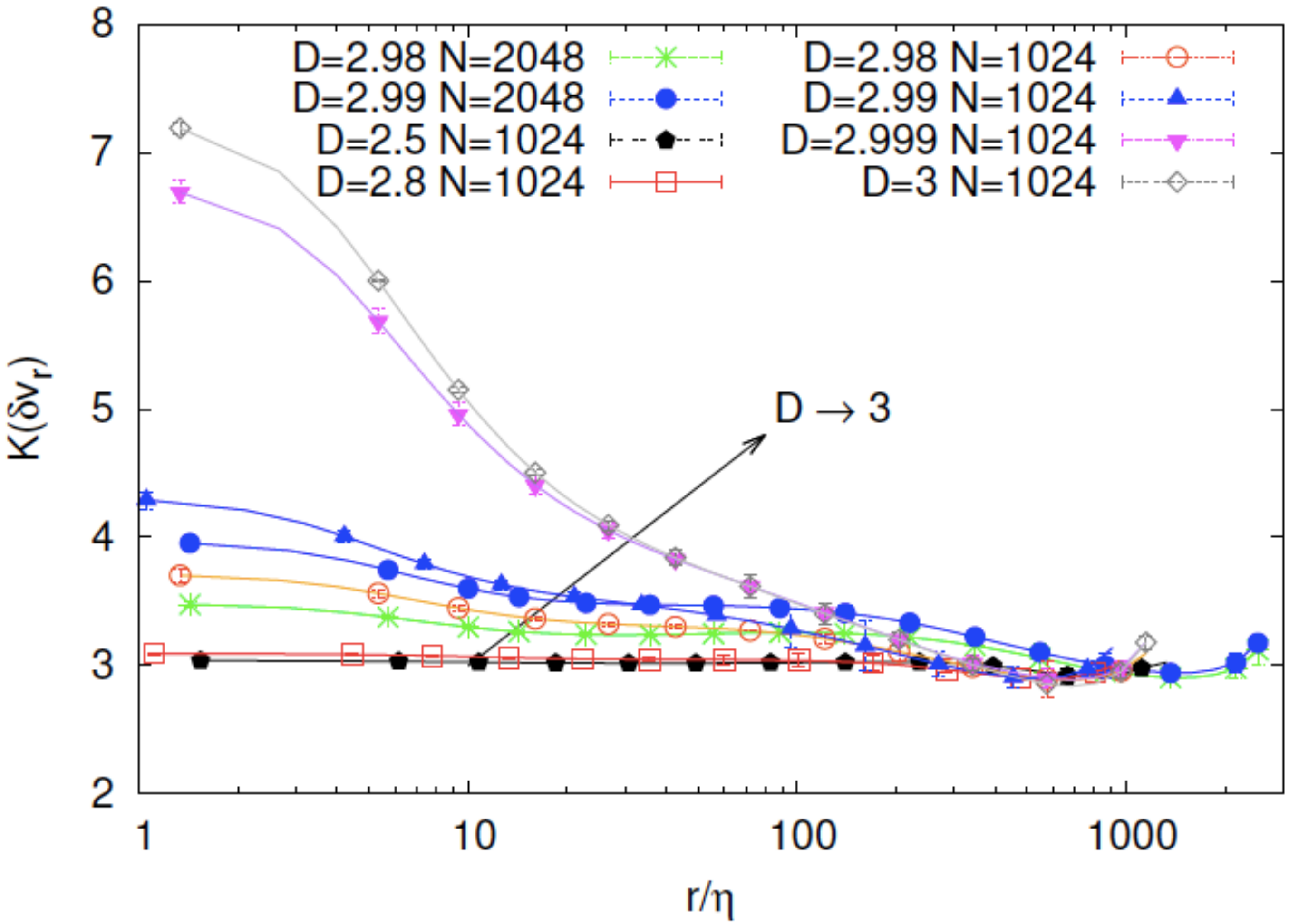
# PDF OF VORTICITY AT CHANGING FRACTAL DIMENSION

$$\begin{cases} \partial_t \mathbf{v} = B(\mathbf{v},\mathbf{v}) + \Delta \mathbf{v} + \mathbf{f} \\ \mathbf{v} \rightarrow P^{D_F} \mathbf{v} \end{cases}$$

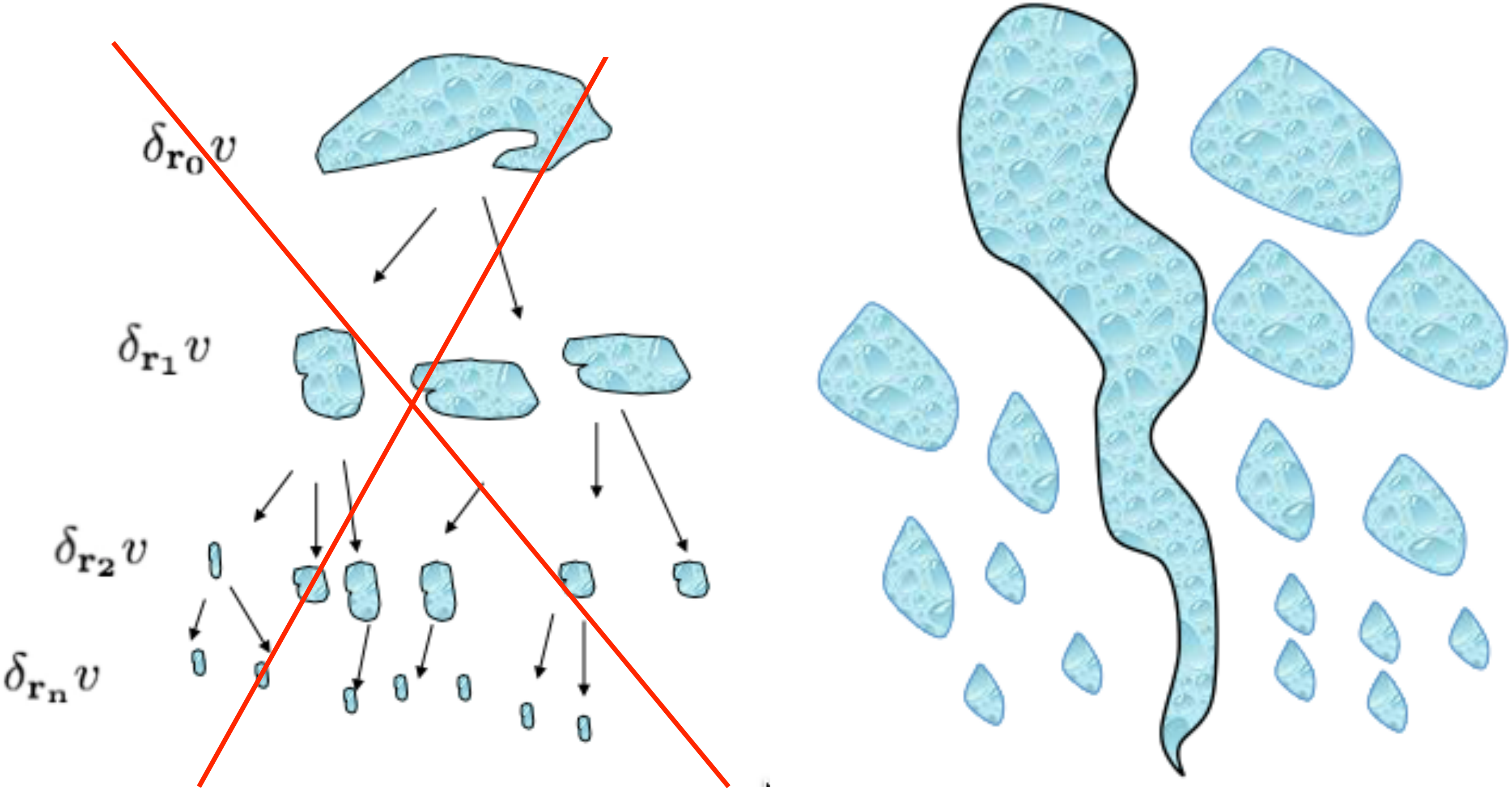
$$\partial_t \mathbf{v}^{D_F} = P^{D_F} B(\mathbf{v}^{D_F}, \mathbf{v}^{D_F}) + \Delta \mathbf{v}^{D_F} + \mathbf{f}^{D_F}$$





D=2.98 N=2048
D=2.99 N=2048
D=2.5 N=1024
D=2.8 N=1024
D=2.98 N=1024
D=2.99 N=1024
D=2.999 N=1024
D=3 N=1024
D → 3
K(δv_r)
r/η



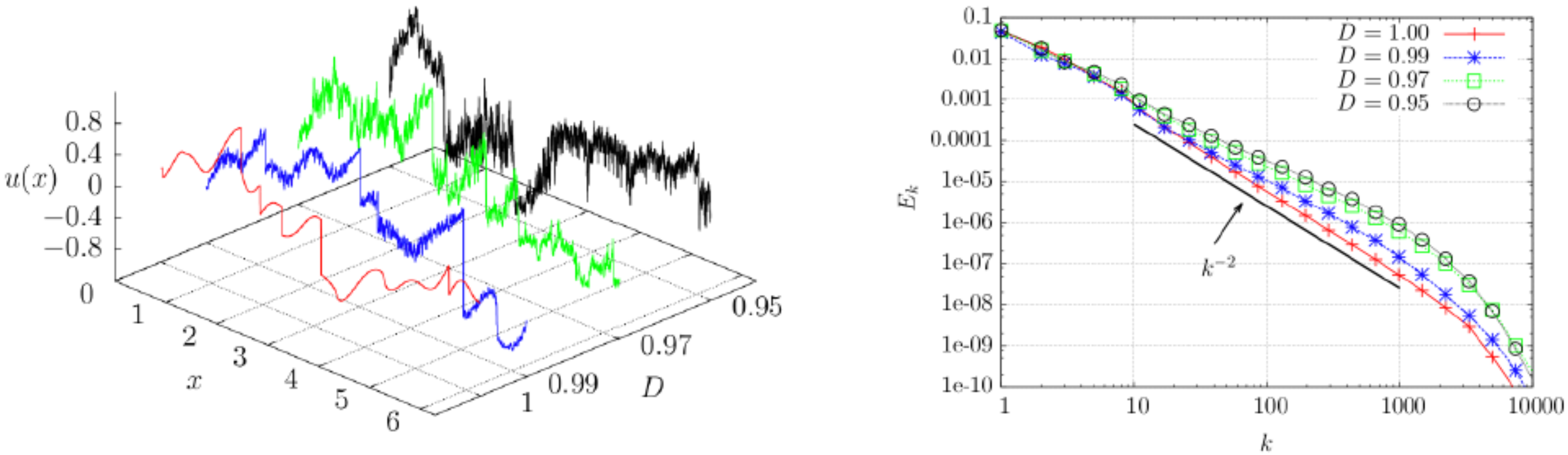
$\delta_{\mathbf{r_0}} v$
$\delta_{\mathbf{r_1}} v$
$\delta_{\mathbf{r_2}} v$
$\delta_{\mathbf{r_n}} v$

## THE SIMPLEST CASE

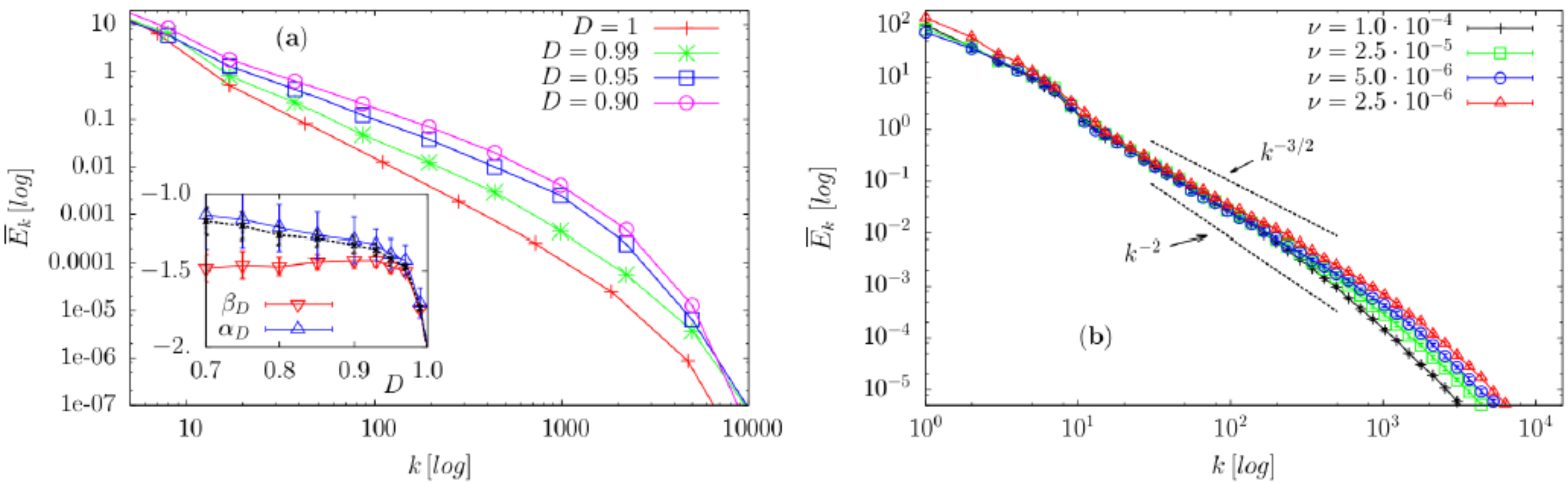
$$\frac{\partial v}{\partial t} + \frac{1}{2} P_D \frac{\partial v^2}{\partial x} = \nu \frac{\partial^2 v}{\partial x^2} + f,$$

TABLE I. $D$: system dimension; $D = 1$ denotes the ordinary non-decimated Burgers equation (Eq. 1), while $D < 1$ represents the decimated system as described in Eq. 3. $N$: number of collocation points. $\%(D)$: percentage of decimated wave numbers, where the first value is related to the lower resolution used while the second value is related to the higher one. $\nu$: value of the kinematic viscosity. $k_f$: the range of forced wavenumbers. $C_f$: the mean energy injection, $\langle uf \rangle$. $N_{mask}$: number of different random quenched masks. $dt$: time step used in the temporal evolution.

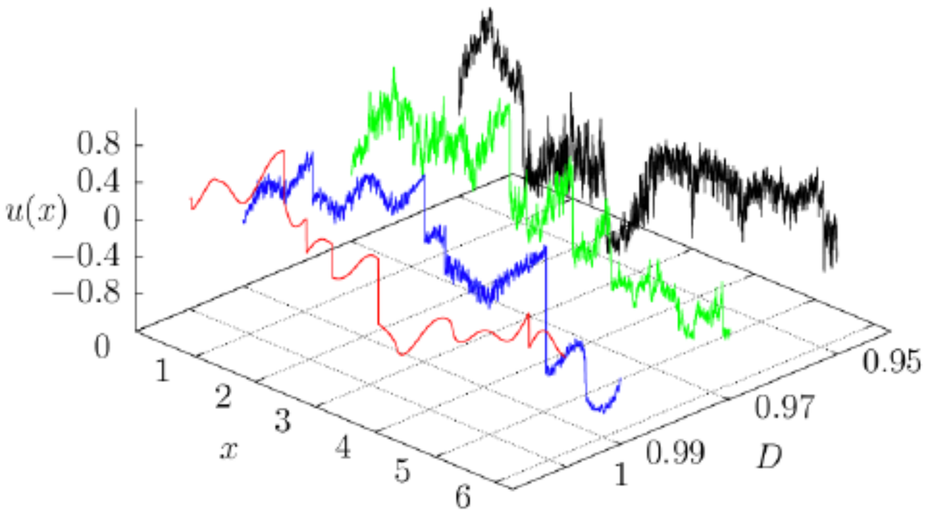
| $D$ | N | $\%(D)$ | $\nu$ | $k_f$ | $C_f$ | $N_{mask}$ | $dt$ |
|---|---|---|---|---|---|---|---|
| 1 | $2^{16} \div 2^{18}$ | 0 | $8 \times 10^{-5}$ | $[1 : 5 \div 10]$ | $0.01 \div 0.05$ | 0 | $5.5 \times 10^{-5}$ |
| 0.99 | $2^{16} \div 2^{19}$ | $8 \div 10$ | $2.5 \times 10^{-5}$ | $[1 : 5 \div 10]$ | $0.01 \div 0.05$ | 32 | $2.3 \times 10^{-5}$ |
| 0.97 | $2^{16} \div 2^{18}$ | $23 \div 27$ | $9 \times 10^{-6}$ | $[1 : 5 \div 10]$ | $0.01 \div 0.05$ | 64 | $2.0 \times 10^{-5}$ |
| 0.95 | $2^{16} \div 2^{18}$ | $36 \div 40$ | $5 \times 10^{-6}$ | $[1 : 5 \div 10]$ | $0.01 \div 0.05$ | 64 | $1.7 \times 10^{-5}$ |
| 0.90 | $2^{16} \div 2^{18}$ | $59 \div 64$ | $2 \times 10^{-6}$ | $[1 : 5 \div 10]$ | $0.01 \div 0.05$ | 64 | $1.6 \times 10^{-5}$ |
| 0.80 | $2^{16} \div 2^{18}$ | $83 \div 87$ | $8 \times 10^{-7}$ | $[1 : 5 \div 10]$ | $0.01 \div 0.05$ | 96 | $1.5 \times 10^{-5}$ |
| 0.70 | $2^{16} \div 2^{18}$ | $93 \div 95$ | $6.5 \times 10^{-7}$ | $[1 : 5 \div 10]$ | $0.01 \div 0.05$ | 96 | $1.5 \times 10^{-5}$ |

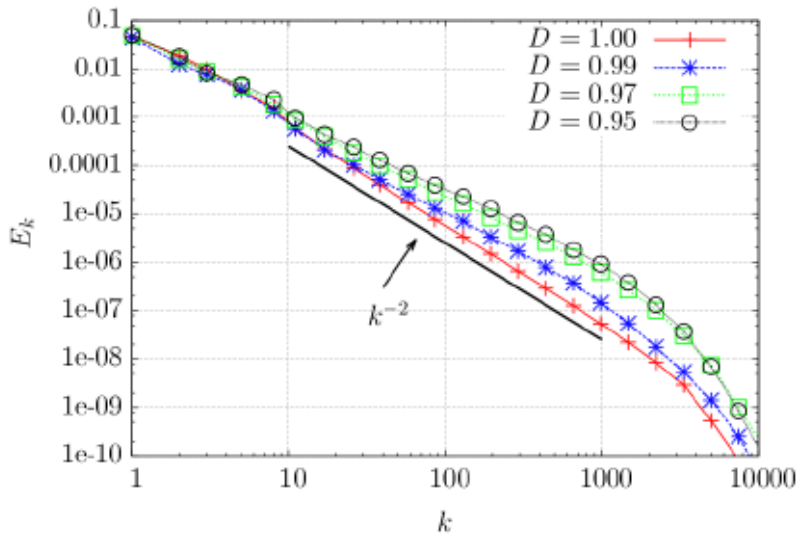


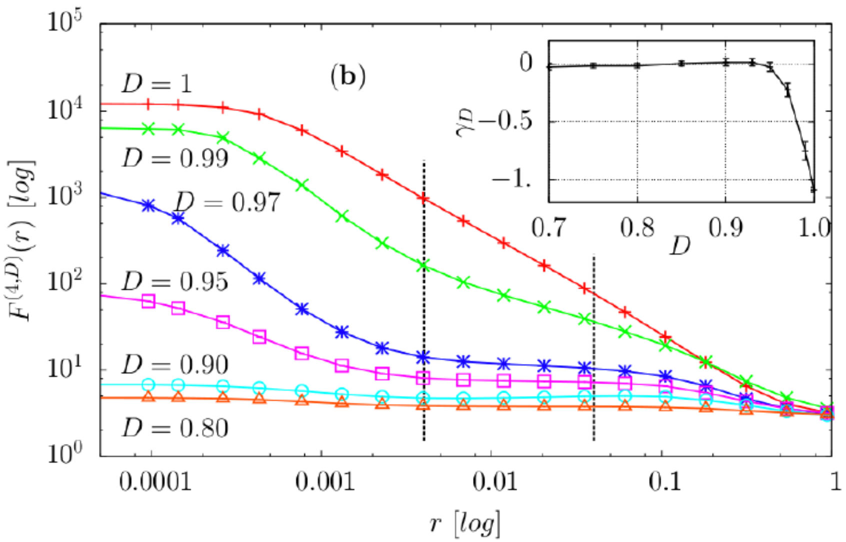
u(x)
x
D
0.95
0.97
0.99
$D = 1.00$
$D = 0.99$
$D = 0.97$
$D = 0.95$
$E_k$
$k$
$k^{-2}$

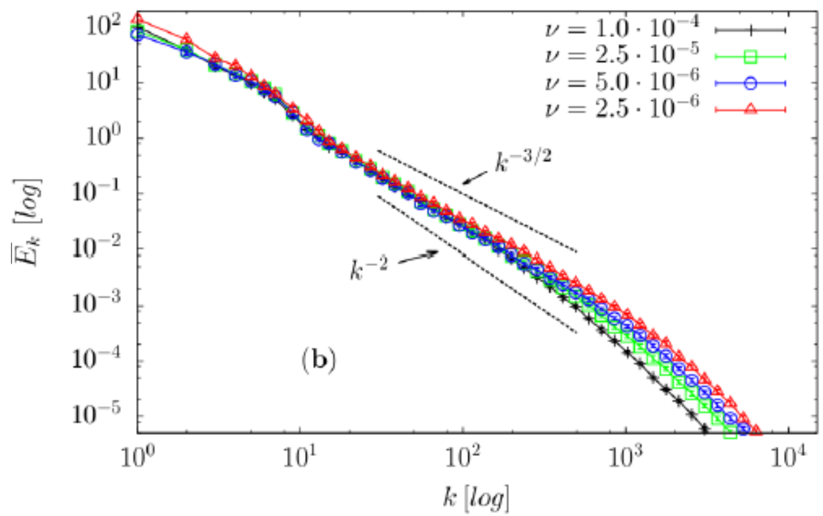



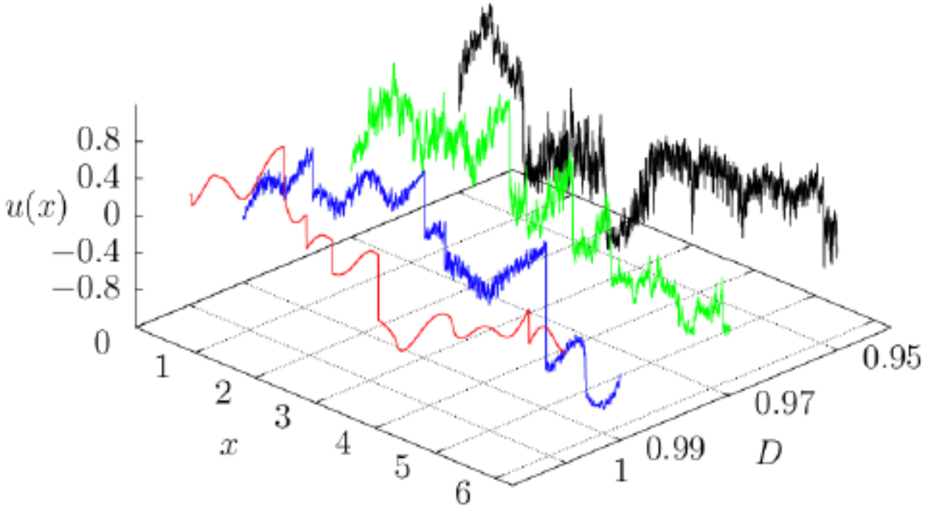
(a)
$D = 1$
$D = 0.99$
$D = 0.95$
$D = 0.90$
$\beta_D$
$\alpha_D$
$\overline{E}_k$ [log]
$k$ [log]
(b)
$\nu = 1.0 \cdot 10^{-4}$
$\nu = 2.5 \cdot 10^{-5}$
$\nu = 5.0 \cdot 10^{-6}$
$\nu = 2.5 \cdot 10^{-6}$
$k^{-3/2}$
$k^{-2}$


u(x)
0.8
0.4
0
−0.4
−0.8
0
1
2
3
4
5
6
x
1
0.99
0.97
0.95
D

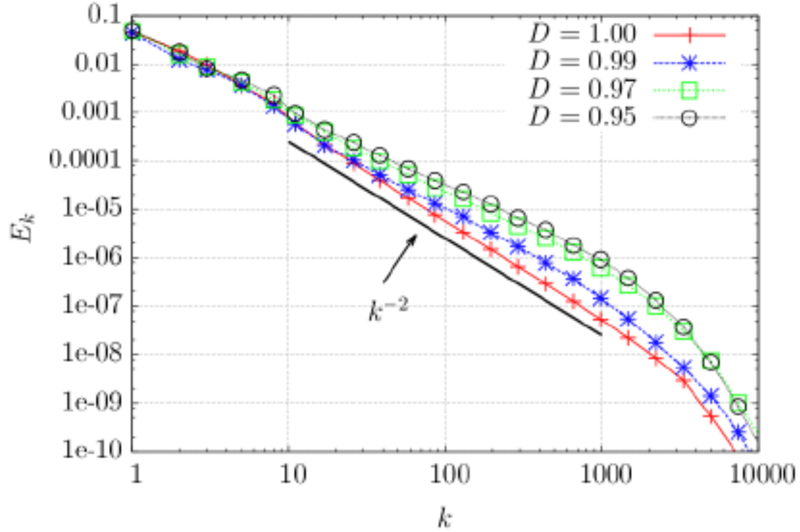

D = 1.00
D = 0.99
D = 0.97
D = 0.95
E_k
k^{-2}
k
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
1
10
100
1000
10000


(b)
D = 1
D = 0.99
D = 0.97
D = 0.95
D = 0.90
D = 0.80
F^{(4,D)}(r) [log]
r [log]
0.0001
0.001
0.01
0.1
1
γ_D
D
0.7
0.8
0.9
1.0

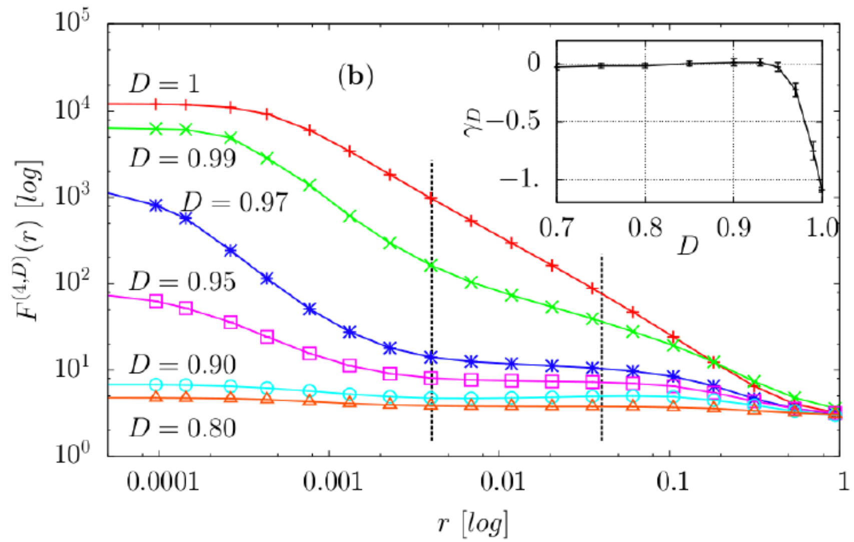

ν = 1.0 · 10^{-4}
ν = 2.5 · 10^{-5}
ν = 5.0 · 10^{-6}
ν = 2.5 · 10^{-6}
k^{-3/2}
k^{-2}
(b)
Ē_k [log]
k [log]



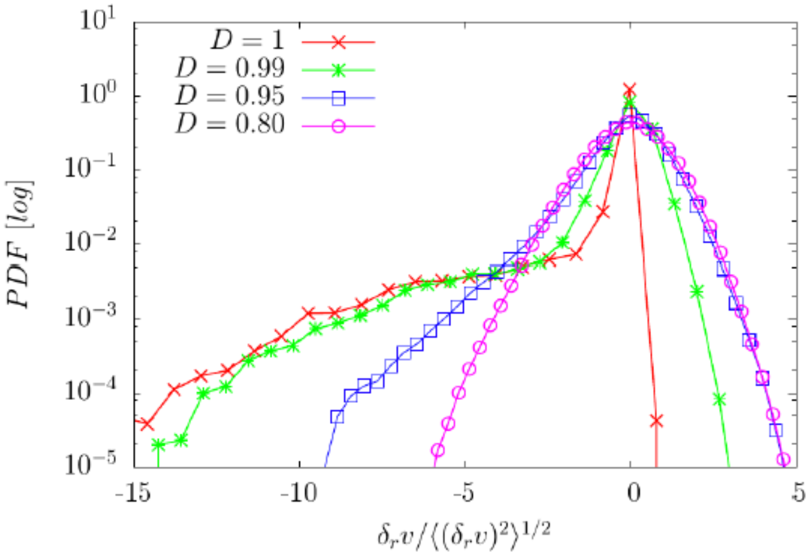
$u(x)$
0.8
0.4
0
−0.4
−0.8
0
1
2
3
4
5
6
$x$
1
0.99
0.97
0.95
$D$

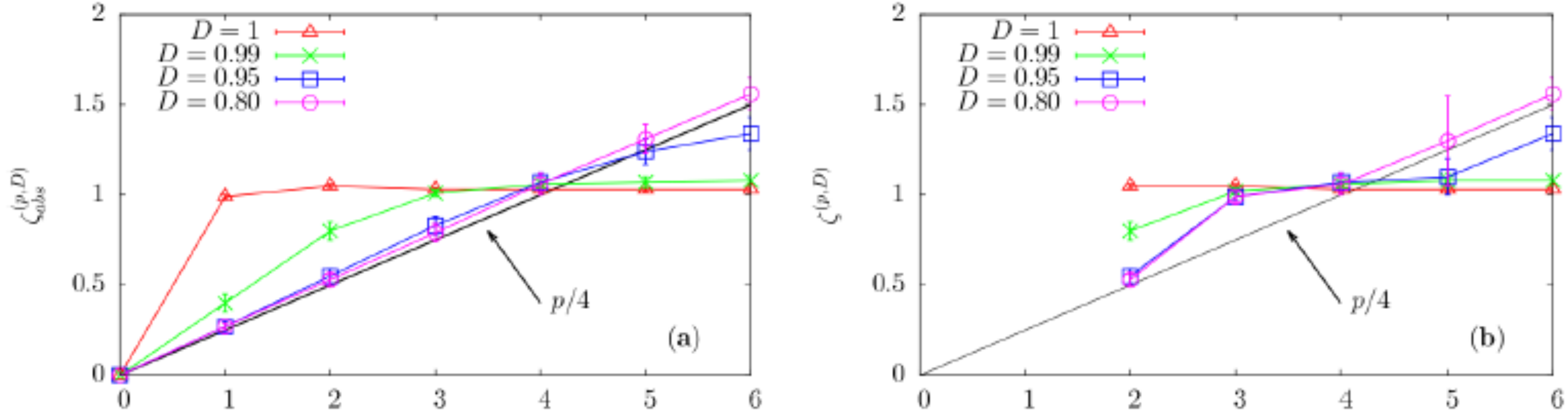
$D = 1.00$
$D = 0.99$
$D = 0.97$
$D = 0.95$
$E_k$
$k^{-2}$
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
1
10
100
1000
10000
$k$

(b)
$D = 1$
$D = 0.99$
$D = 0.97$
$D = 0.95$
$D = 0.90$
$D = 0.80$
$F^{(4,D)}(r)$ [log]
$r$ [log]
$10^5$
$10^4$
$10^3$
$10^2$
$10^1$
$10^0$
0.0001
0.001
0.01
0.1
1
$\gamma_D$
0
−0.5
−1.
0.7
0.8
0.9
1.0
$D$

$D = 1$
$D = 0.99$
$D = 0.95$
$D = 0.80$
$PDF$ [log]
$10^1$
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
-15
-10
-5
0
5
$\delta_r v / \langle (\delta_r v)^2 \rangle^{1/2}$




$$\begin{cases} S_{+}^{(p,D)}(r) = r^{p/4} + \text{smooth}; \\ S_{-}^{(p,D)}(r) = r^{p/4} + r + \text{smooth}, \end{cases}$$

$$\epsilon = kE_k/\tau_{tr}$$

$$\tau_{dec} \propto 1/k$$

$$\tau_{tr} = \tau_{nlt}^2/\tau_{dec}$$

$$\tau_{nlt} \propto (k^3 E_k)^{-1/2}$$

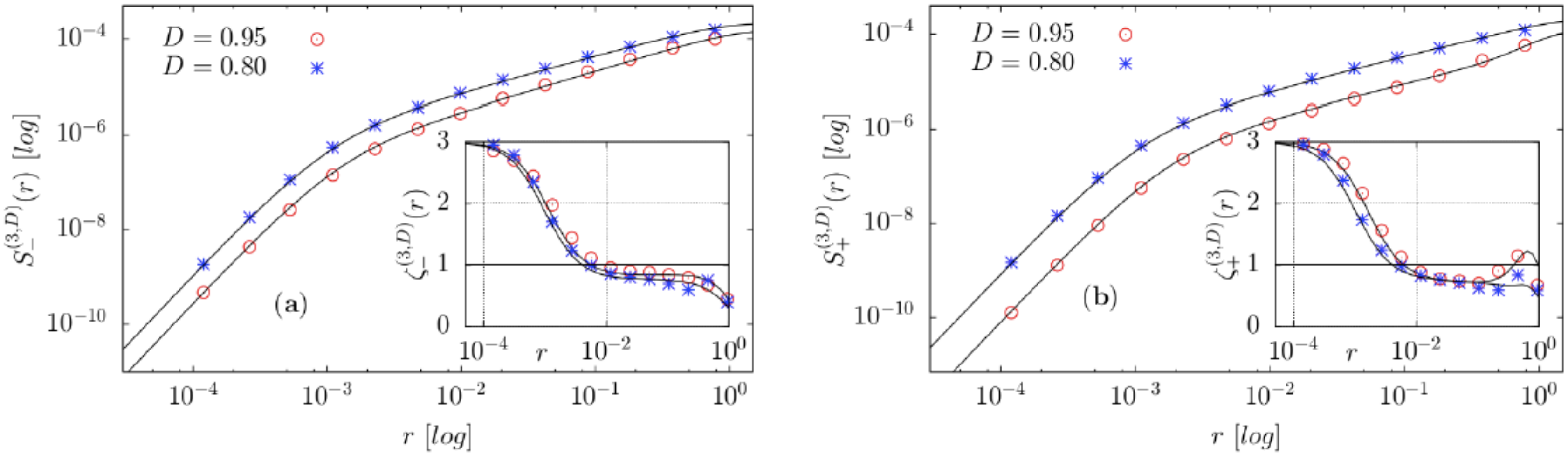
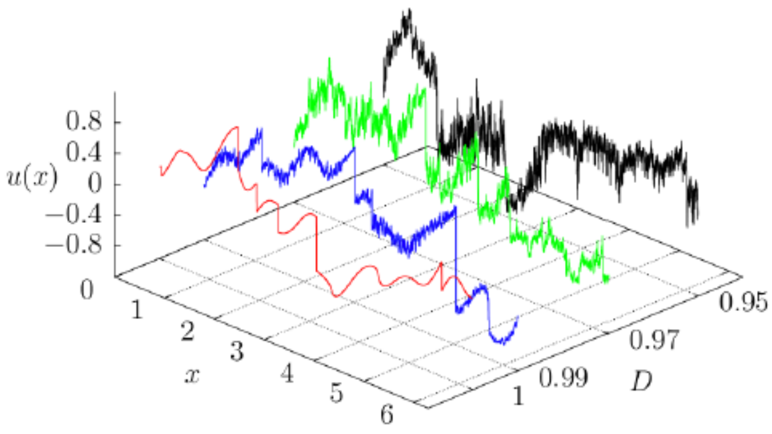
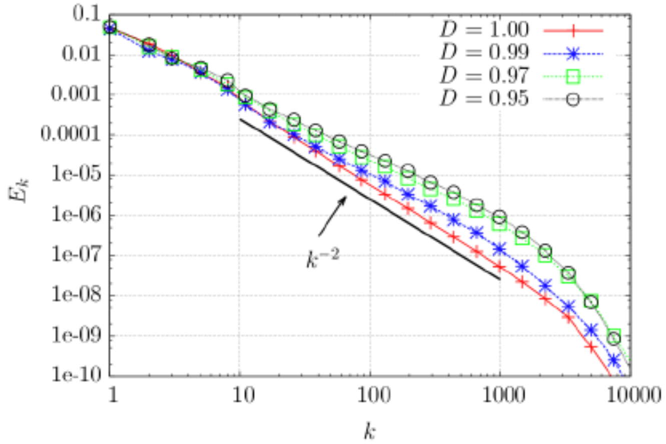



FIG. 5. (Color online) (a) Structure functions for $D = 0.80$ (blue stars) and $D = 0.95$ (red circles) for (a) the negative increments $S_-^{(3,D)}(r)$ and (b) positive increments structure functions $S_+^{(3,D)}(r)$; the black solid lines are the respective fitting functions (10). In the insets, with the same legend, we show the associated local slopes.




$$u(k) = a(k) exp(i\phi_k)$$

$$\varphi^{k_3}_{k_1,k_2}(t) = \phi_{k_1}(t) + \phi_{k_2}(t) - \phi_{k_3}(t),$$

$$\dot{a}_k = k\,\theta_k \sum_{k_1} a_{k_1} a_{k_2}\,\theta_{k_1}\theta_{k_2}\sin(\varphi^{k}_{k_1,k_2})\,\delta_{k_1+k_2,k}\,,$$

$$\dot{\phi}_k = -k\,\theta_k \sum_{k_1} \frac{a_{k_1} a_{k_2}}{a_k}\,\theta_{k_1}\theta_{k_2}\cos(\varphi^{k}_{k_1,k_2})\,\delta_{k_1+k_2,k}\,,$$

$$\varphi_{k_1,k_2}^{k_3}(t) = \phi_{k_1}(t) + \phi_{k_2}(t) - \phi_{k_3}(t),$$

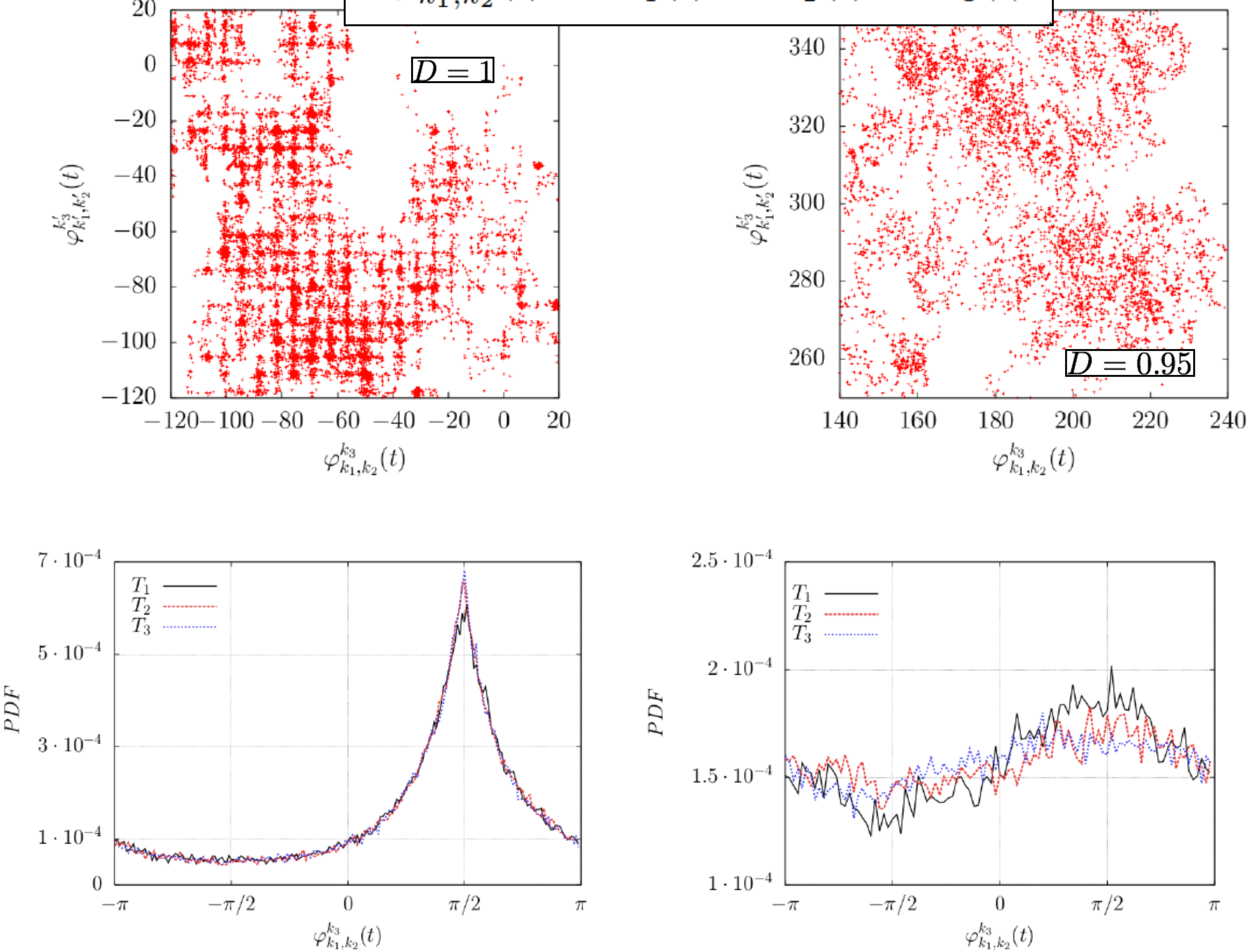
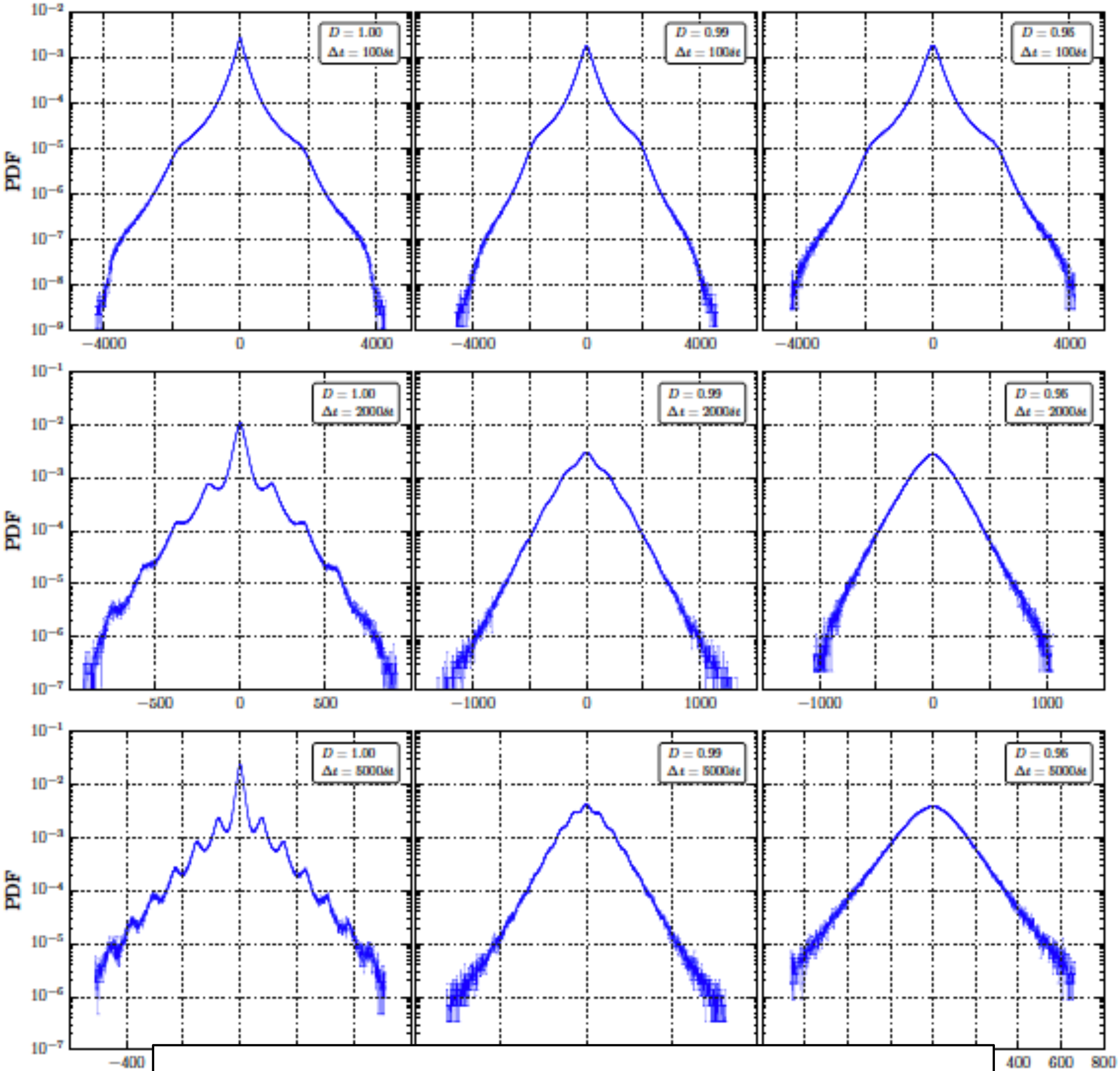
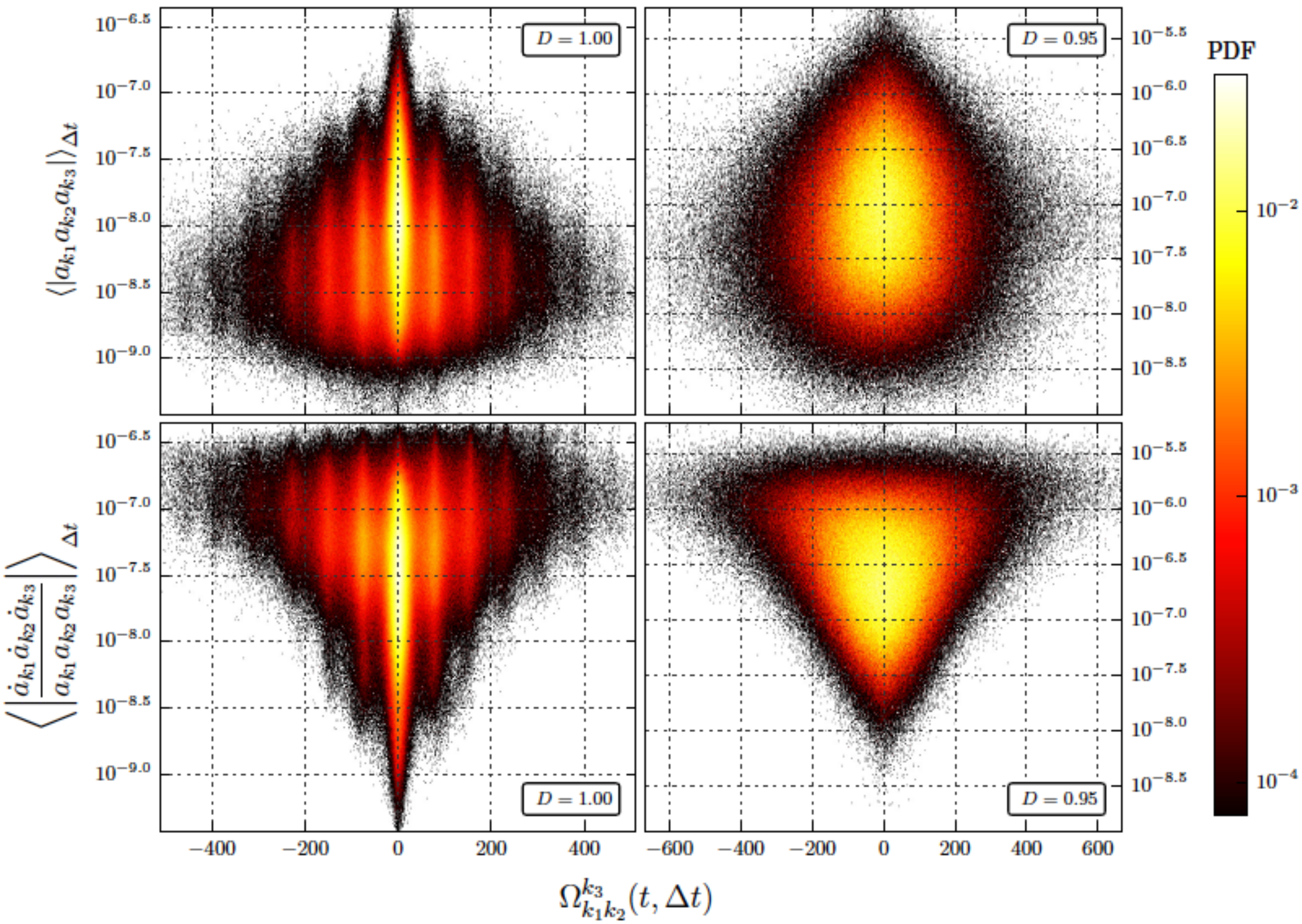



**Fig. 2.** Triad phase histograms computed during the temporal evolution of different triads in the inertial range. $T_1 : [k_1; k_2; k_3] =$




$$\Omega^{k_3}_{k_1 k_2}(t, \Delta t) \equiv \frac{1}{N} \sum_{t_i = t}^{t + \Delta t} \dot{\varphi}^{k_3}_{k_1 k_2}(t_i).$$



$D = 1.00$
$D = 0.95$
$D = 1.00$
$D = 0.95$
PDF
$10^{-2}$
$10^{-3}$
$10^{-4}$
$\langle |a_{k_1} a_{k_2} a_{k_3}| \rangle_{\Delta t}$
$\left\langle \left| \frac{\dot{a}_{k_1} \dot{a}_{k_2} \dot{a}_{k_3}}{a_{k_1} a_{k_2} a_{k_3}} \right| \right\rangle_{\Delta t}$
$10^{-6.5}$
$10^{-7.0}$
$10^{-7.5}$
$10^{-8.0}$
$10^{-8.5}$
$10^{-9.0}$
$10^{-5.5}$
$10^{-6.0}$
−600
−400
−200
0
200
400
600
$\Omega^{k_3}_{k_1 k_2}(t, \Delta t)$

# CONCLUSIONS

## FRACTAL DECIMATION: MILDEST REMOVAL OF DEGREE OF FREEDOM

### HOMOGENEOUS & ISOTROPIC & SELF SIMILAR

+ QUANTIFY IMPORTANCE OF LOCAL VS NON-LOCAL TRIADIC INTERACTIONS

+/- QUANTIFY IMPORTANCE OF $\#_{DOF}$ FOR VORTEX STRETCHING

+ CORRECTION IN THE MEAN RESPONSE (SPECTRUM) PROPORTIONAL TO 3-$D_F$: YOU CAN HAVE A LITTLE CHANGE IN THE SPECTRAL PROPERTIES AND STILL GAINING IN THE $\#_{DOF}$

+ CORRECTION TO FLUCTUATIONS: HUGE. SMALL SCALE VORTICITY IS STRONGLY SENSITIVE TO DECIMATION. "CHOERENT" SMALL-SCALE STRUCTURES FEEL GLOBAL CORRELATIONS ACROSS SCALES IN FOURIER: BAD NEWS FOR MODELING PEOPLE

+ HOW TO BRING INTERMITTENCY BACK TO NS EQUATIONS?

$$\partial_t \overline{v} = \overline{\overline{v}\partial_x \overline{v}} - \partial_x \overline{P} + \partial_x \Pi_{SG} + \nu \Delta \overline{v} + \overline{f}$$

+ CONCEPTUALLY DIFFERENT FROM KINEMATIC SIMULATIONS (FLUX)

+ WHAT ABOUT LAGRANGIAN DYNAMICS?

- WE STILL MISS A CLEAR DEFINITION OF INTERMITTENCY IN FOURIER SPACE -> BACK TO "CHOERENT STRUCTURES"