TURBULENT AT HIGH AND LOW ROTATION RATES: EULERIAN AND LAGRANGIAN STATISTICS

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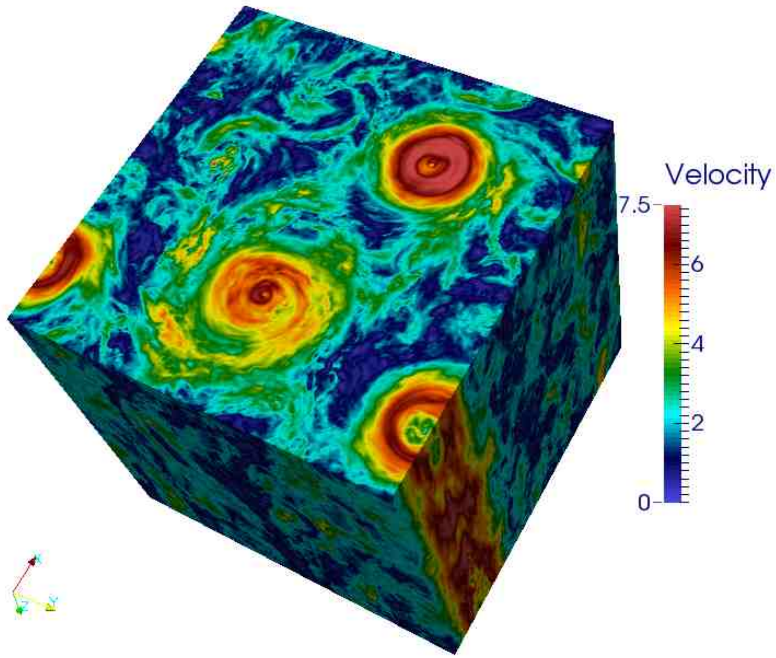
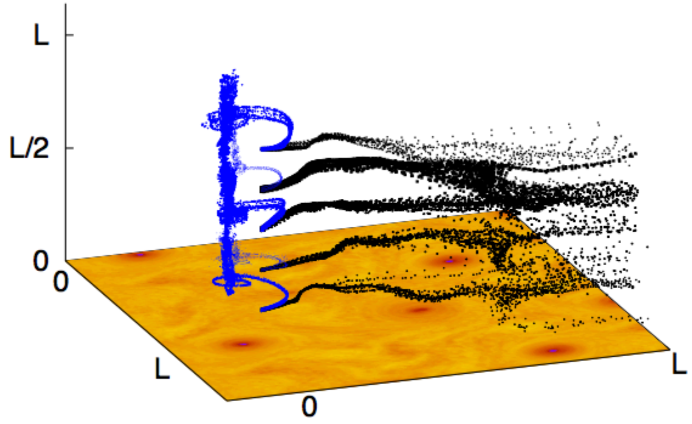
EJH2016





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ROTATING TURBULENCE
2015 – 55MH

THIS IS A STUDY ABOUT HIGH ORDER VELOCITY STRUCTURE FUNCTIONS AND VELOCITY PROBABILITY DENSITY FUNCTIONS IN TURBULENT ROTATING FLOW, TURBULENCE AND WAVES IN ROTATING THANK AND VORTICITY IN ROTATING FLUIDS:

High-order velocity structure functions in turbulent shear flows
F Anselmet, Y Gagne, EJ Hopfinger - J. Fluid Mech. 1984 - Cambridge Univ Press
Abstract: Measurements are presented of the velocity structure function on the axis of a turbulent jet at Reynolds numbers R, Q 852 and in a turbulent duct flow at R,= 515. Moments of the structure function up to the eighteenth order were calculated, primarily with a view to ...

Velocity probability density functions of high Reynolds number turbulence
B Castaing, Y Gagne, EJ Hopfinger - Physica D: Nonlinear Phenomena, 1990 - Elsevier
Abstract: This paper deals with the probability density function (PDF) of velocity differences between two points separated by distance r. Measurements of PDFs were made, for r lying in the inertial range, for two different flows: in a jet with R λ= 852 and in a wind tunnel with ...

Turbulence and waves in a rotating tank
EJ Hopfinger, FK Browand - Journal of Fluid Mech. 1982 - Cambridge Univ Press
Abstract A turbulent field is produced with an oscillating grid in a deep, rotating tank. Near the grid, the Rossby number is kept large, 0 (3-33), and the turbulence is locally unaffected by rotation. Away from the grid, the scale of the turbulence increases, the rms turbulent ...

Vortices in rotating fluids
EJ Hopfinger, G Heijst - Annual review of fluid mechanics, 1993
Abstract: The emergence of coherent vortex structures is a characteristic feature of quasi-geostrophic or two-dimensional turbulence and because of their relevance to large-scale geophysical flows, the dynamics of these structures has been studied increasingly over the past ...

NAVIER_STOKES EQS IN A ROTATING FRAME (NO BOUNDARIES)

DNS: A. Pouquet, P. Mininni, A. Alexakis, S. Chen, G. Eyink ....

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{\Omega} \times \mathbf{v} = \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{F} - \alpha \mathbf{v}$$

$\mathbf{\Omega}$ =rotation

$$P = P_0 + \frac{1}{2} |\mathbf{\Omega} \times \mathbf{r}|^2$$

$\mathbf{F}$ =large scale Forcing

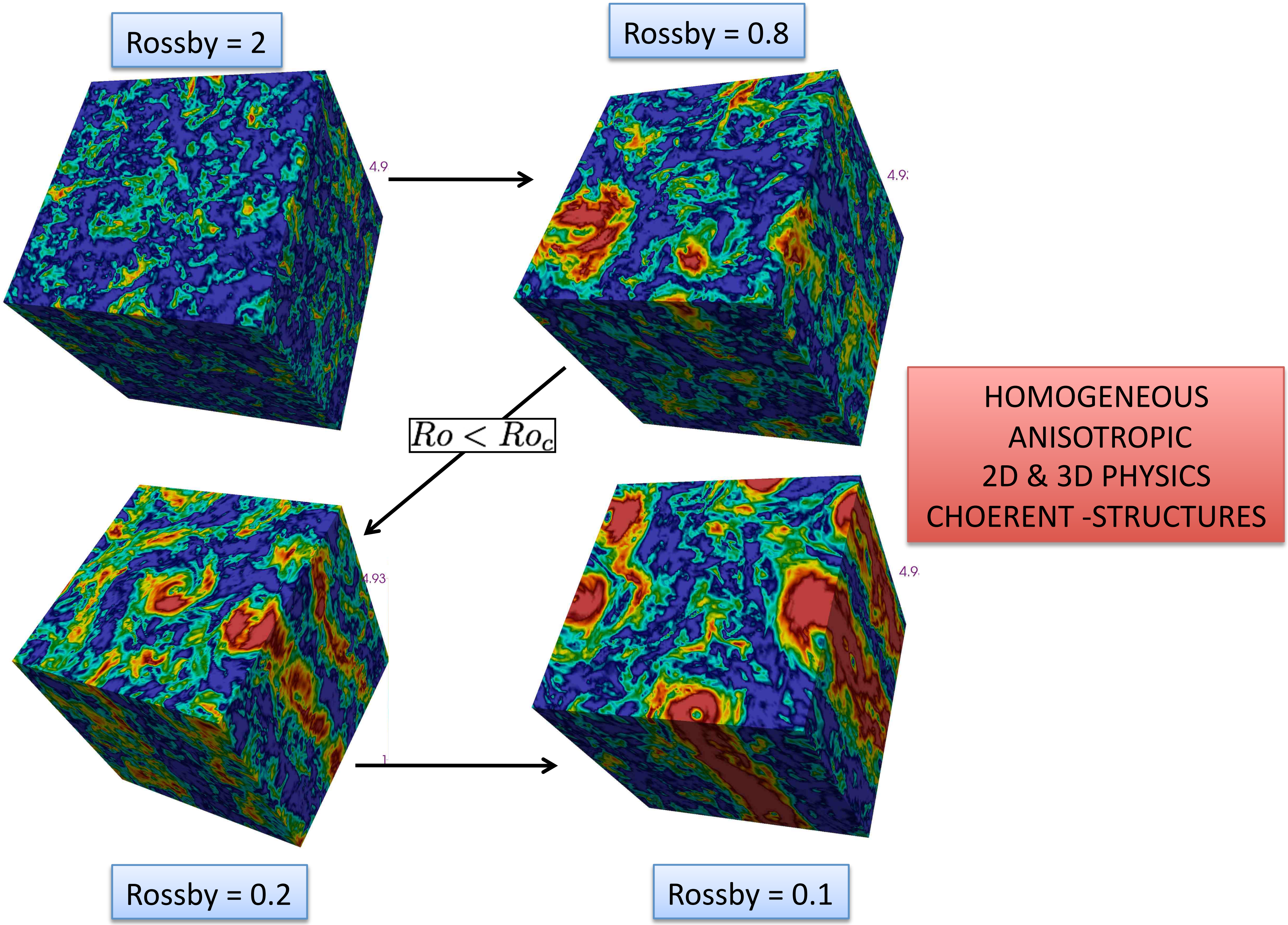
$\alpha =$ large scale energy sink

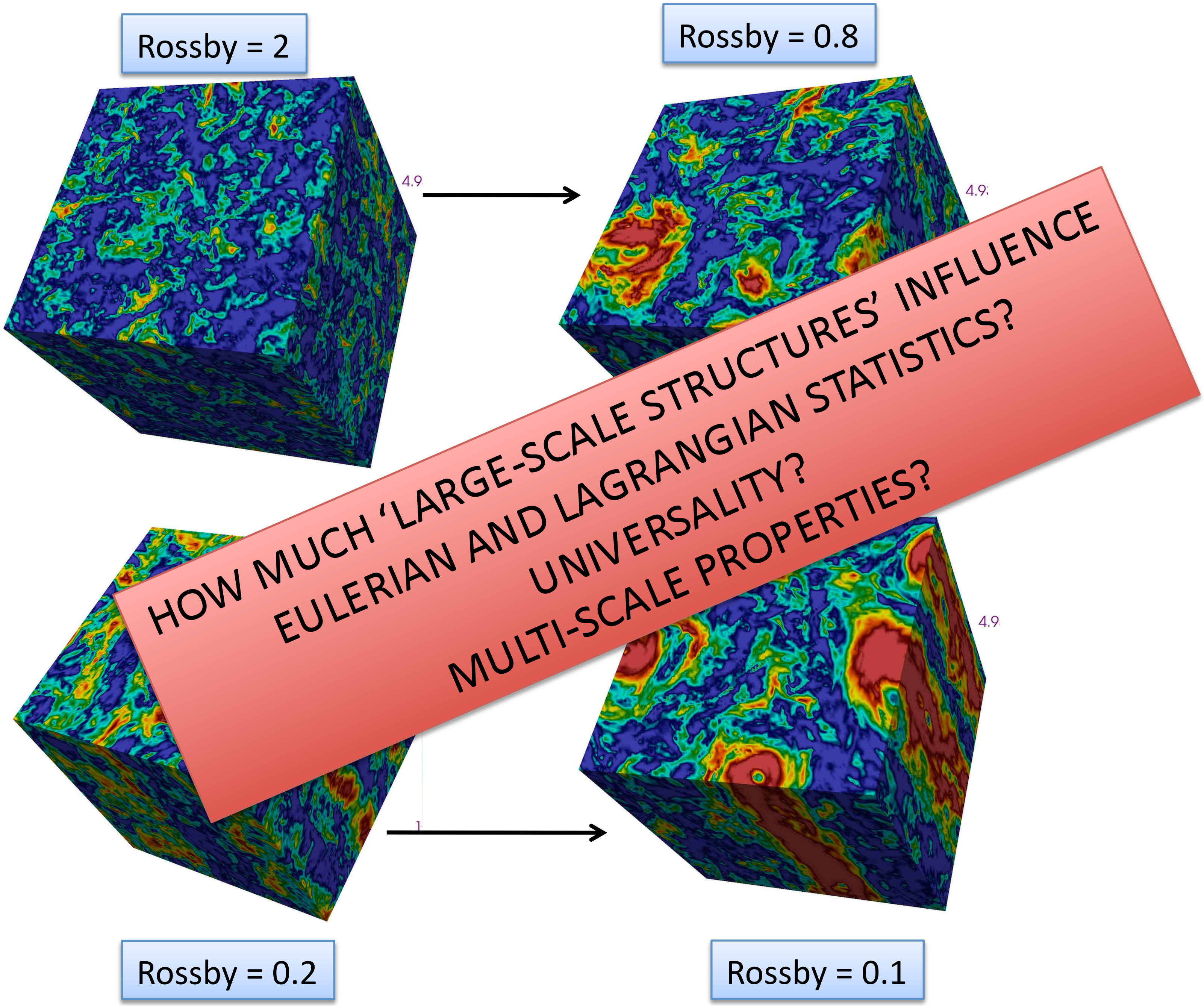
ROSSBY NUMBER ~ NON-LINEAR/ROTATION

$$\mathrm{Ro} \sim \frac{v_0}{\Omega L_0}$$

$\mathrm{Ro} \geq Ro_c \rightarrow$ FORWARD ENERGY TRANSFER

$\mathrm{Ro} \leq Ro_c \rightarrow$ FORWARD & BACKWARD ENERGY TRANSFER


Rossby = 2
Rossby = 0.8
$Ro < Ro_c$
Rossby = 0.2
Rossby = 0.1
HOMOGENEOUS
ANISOTROPIC
2D & 3D PHYSICS
CHOERENT -STRUCTURES

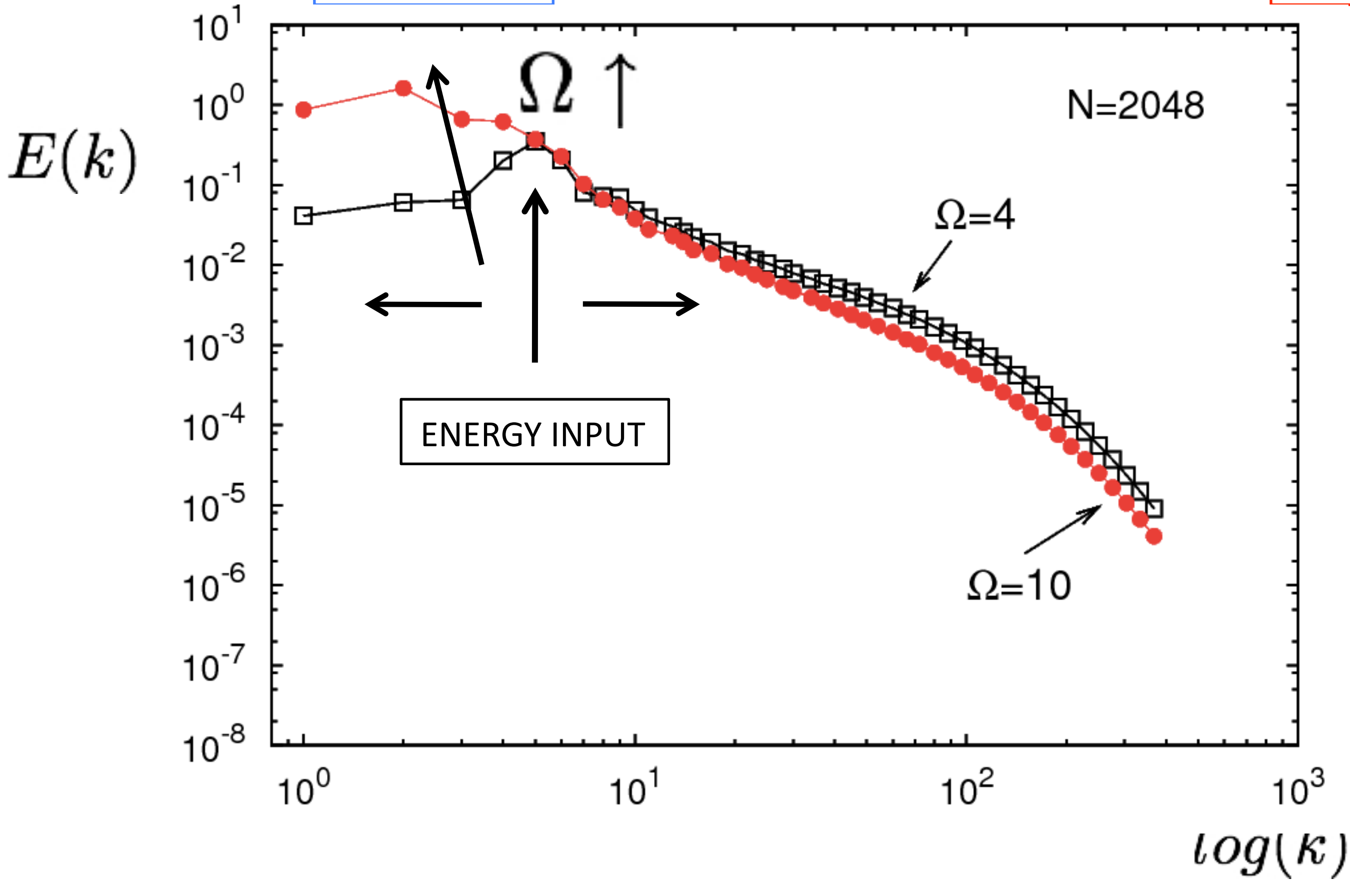

Rossby = 2
Rossby = 0.8
Rossby = 0.2
Rossby = 0.1
HOW MUCH 'LARGE-SCALE STRUCTURES' INFLUENCE
EULERIAN AND LAGRANGIAN STATISTICS?
UNIVERSALITY?
MULTI-SCALE PROPERTIES?

HOMOGENEOUS-ANISOTROPIC

$$\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} + \boldsymbol{\Omega}\times\mathbf{v} = \nabla P + \nu\nabla^2\mathbf{v} + \mathbf{F} - \alpha\mathbf{v}$$

ROTATION

DAMPING:



FORCING: 2°-order OU-PROCESSS: ISOTROPIC, HOMOGENEOUS **NOT** DELTA-CORRELATED

PHYSICS OF FLUIDS VOLUME 11, NUMBER 6 JUNE 1999

Transfer of energy to two-dimensional large scales in forced, rotating three-dimensional turbulence

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Fabian Waleffe
Departments of Mathematics & Engineering Physics, University of Wisconsin–Madison, Madison, Wisconsin 53706

$$\mathbf{u}(\mathbf{k}) = u^+(\mathbf{k})e^{+it\omega^+(\mathbf{k})}\mathbf{h}^+(\mathbf{k}) + u^-(\mathbf{k})e^{+it\omega^-(\mathbf{k})}\mathbf{h}^-(\mathbf{k})$$

$$i\boldsymbol{k}\times\boldsymbol{h}^\pm = \pm k\boldsymbol{h}^\pm \qquad \omega^\pm(\mathbf{k}) = \pm 2\Omega\frac{k_z}{k}$$

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0}\ \sum_{s_p,s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_p p - s_q q)$$

$$e^{i(\omega^{s_k}+\omega^{s_p}+\omega^{s_q})t/Ro} \times [u^{s_p}(\mathbf{p})u^{s_q}(\mathbf{q})]^*. \qquad (15)$$

TRIADIC WAVE-INTERACTIONS

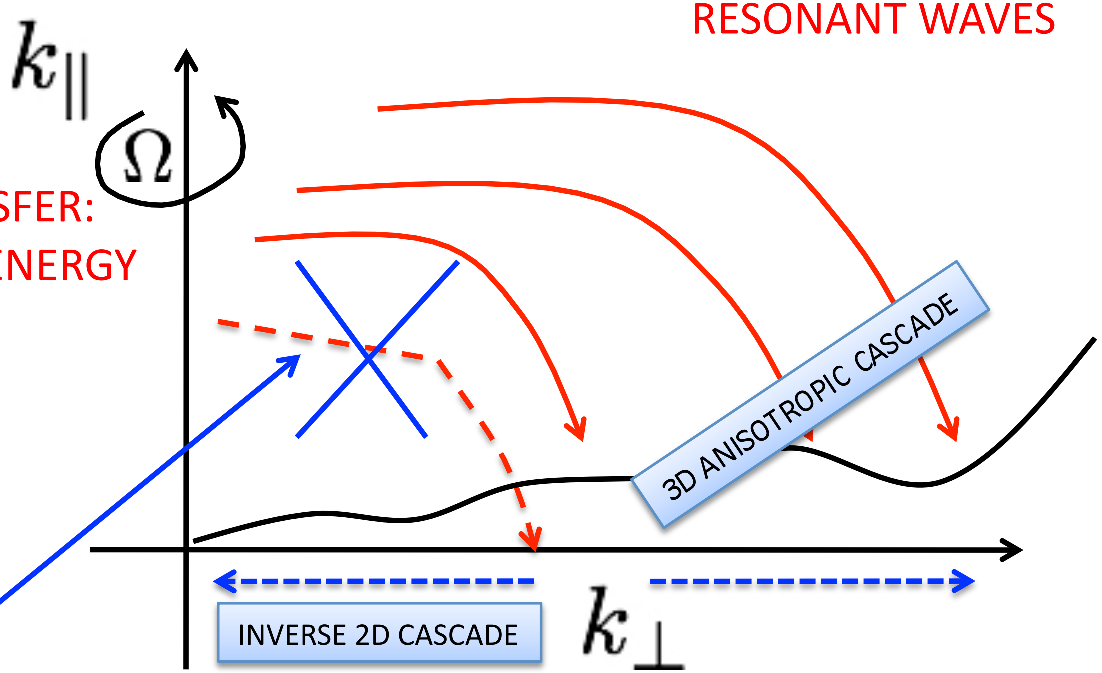
$$\omega^{s_k}+\omega^{s_p}+\omega^{s_q}=0$$

$$Ro \to 0$$

$$Ro \to 0 \qquad \omega^{s_k} + \omega^{s_p} + \omega^{s_q} = 0$$

RESONANT WAVES

ANISOTROPIC ENERGY TRANSFER:
WAVES TEND TO TRANSFER ENERGY
TOWARD THE $k_\perp$ BUT ...



FORBIDDEN !!!
NO DIRECT TRANSFER FROM 3D RESONANT WAVES TO 2D MODES

THERE EXISTS A BUFFER REGION IN THE K-SPACE CLOSE TO THE 2D MODES
WHERE TRIADIC RESONANT WAVES ARE LESS AND LESS EFFICIENT:

-) O(Ro) INTERACTIONS
-) QUARTET-INTERACTIONS
-) TURBULENCE

1 WHAT ARE THE INTERACTIONS/MECHANISMS RESPONSIBLE
FOR THE INVERSE ENERGY CASCADE, 2D-3D?

2. WHAT ABOUT THE SCALL-SCALES VELOCITY STATISTICS IN PRESENCE
OF A LARGE SCALE INVERSE ENERGY TRANSFER: EFFECTS OF CHOERENT
VORTEX STRUCTURES

OUR DNS DATA-BASE (EULERIAN + LAGRANGIAN)

NEW FEATURES:

1) IDEAL FORCING MECHANISM (AS NEUTRAL AS POSSIBLE: ISOTROPIC; NON HELICAL, TIME-COLORED) + LARGE SCALE FRICTION
2) UNPRECEDENTED NUMERICAL RESOLUTION/SCALE SEPARATION (**UP TO 4096^3**)
3) LAGRANGIAN STATISTICS (MILLIONS OF TRACERS AND INERTIAL PARTICLES)

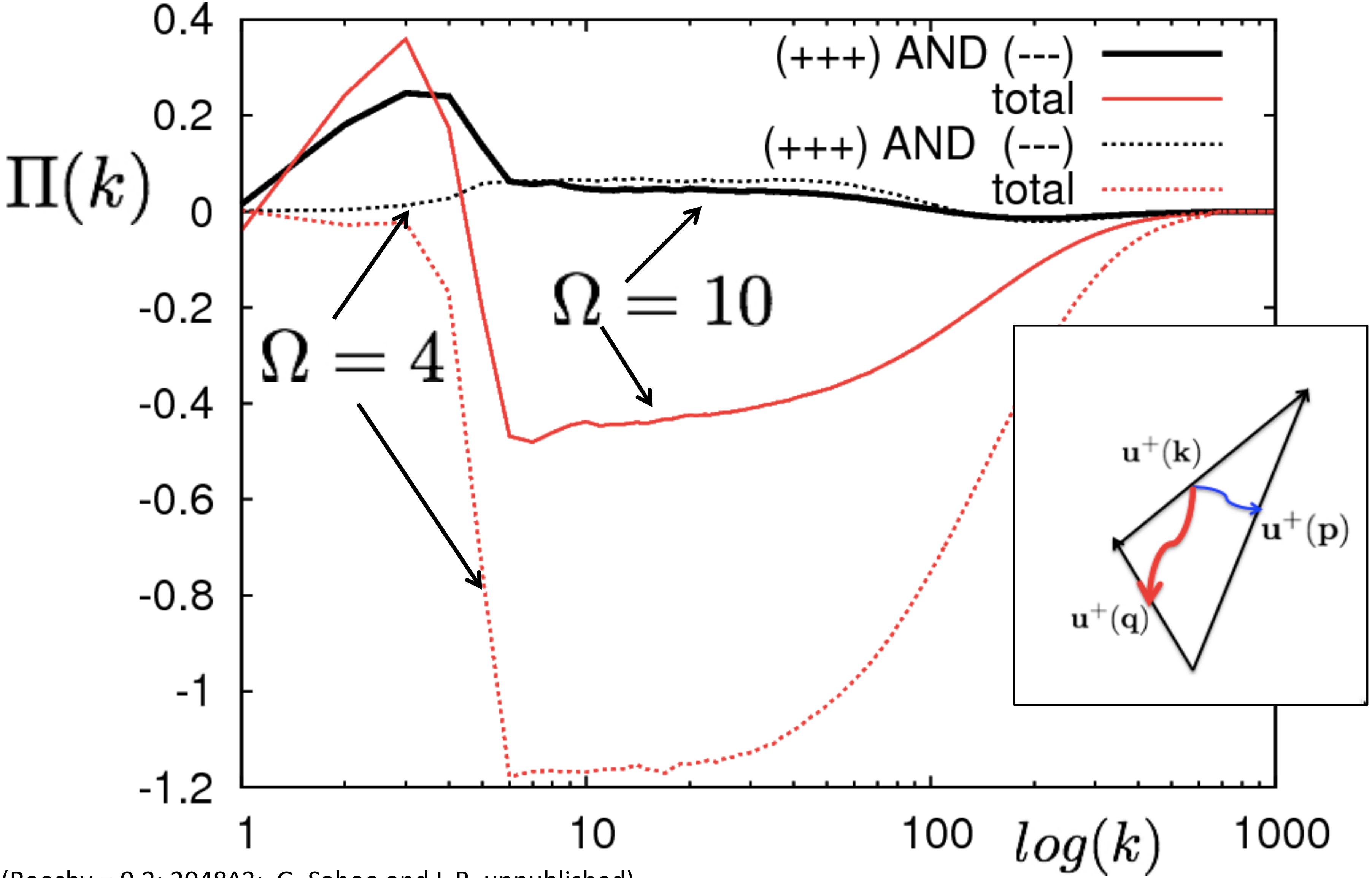
N	Ω	ν	ϵ	ϵ_f	u_0	η/dx	τ_η/dt	Re_λ	Ro	f_0	τ_f	T_0	α
1024	4	7×10^{-4}	1.2	1.2	1.05	0.67	120	150	0.78	0.02	0.023	0.17	0.0
1024	10	6×10^{-4}	0.46	0.59	1.6	0.76	294	580	0.24	0.02	0.023	0.25	0.1
2048	4	2.8×10^{-4}	1.2	1.2	1.05	0.67	380	230	0.76	0.02	0.023	0.17	0.0
2048	10	2.2×10^{-4}	0.45	0.64	1.7	0.72	550	1170	0.25	0.02	0.023	0.3	0.1
4096	10	1×10^{-4}	0.46	0.65	1.7	0.78	1010	1600	0.25	0.02	0.023	0.3	0.1

TABLE I: Eulerian dynamics parameters. N: number of collocation points per spatial direction; Ω: rotation rate; ν: kinematic viscosity; $\epsilon = \nu\int d^3x\sum_{ij}(\nabla_i u_j)^2$: viscous energy dissipation; $\epsilon_f = \int d^3x \sum_i f_i u_i$: energy injection; $u_0 = 1/3\int d^3x\sum_i u_i^2$: mean kinetic energy; $\eta = (\nu^3/\epsilon)^{1/4}$: Kolmogorov dissipative scale; $dx = L_0/N$: numerical grid spacing; $L_0 = 2\pi$: box size; $\tau_\eta = (\nu/\epsilon)^{1/2}$: Kolmogorov dissipative time; $Re_\lambda = (u_0\lambda)/\nu$: Reynolds number based on the Taylor micro-scale; $\lambda = (15\nu u_0^2/\epsilon)^{1/2}$: Taylor micro-scale; $Ro = (\epsilon_f k_f)^{1/3}/\Omega$: Rossby number defined in terms of the energy injection properties, where $k_f = 5$ is the wavenumber where the forcing is acting; f_0: intensity of the Ornstein-Uhlenbeck forcing; τ_f: decorrelation time of the forcing; $T_0 = u_0/L_0$: Eulerian large-scale eddy turn over time; α: coefficient of the damping term $\alpha\Delta^{-1}\boldsymbol{u}$.

MAX RESOLUTION

$$\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} + \boldsymbol{\Omega}\times\mathbf{v} = \nabla P + \nu\nabla^2\mathbf{v} + \mathbf{F} - \alpha\mathbf{v}$$

CONTRIBUTION TO THE INVERSE ENERGY FLUX MAINLY FROM TRIADS WITH SIGN-DEFINITE HELICITY



(Roosby = 0.2; 2048^3; G. Sahoo and L.B, unpublished)

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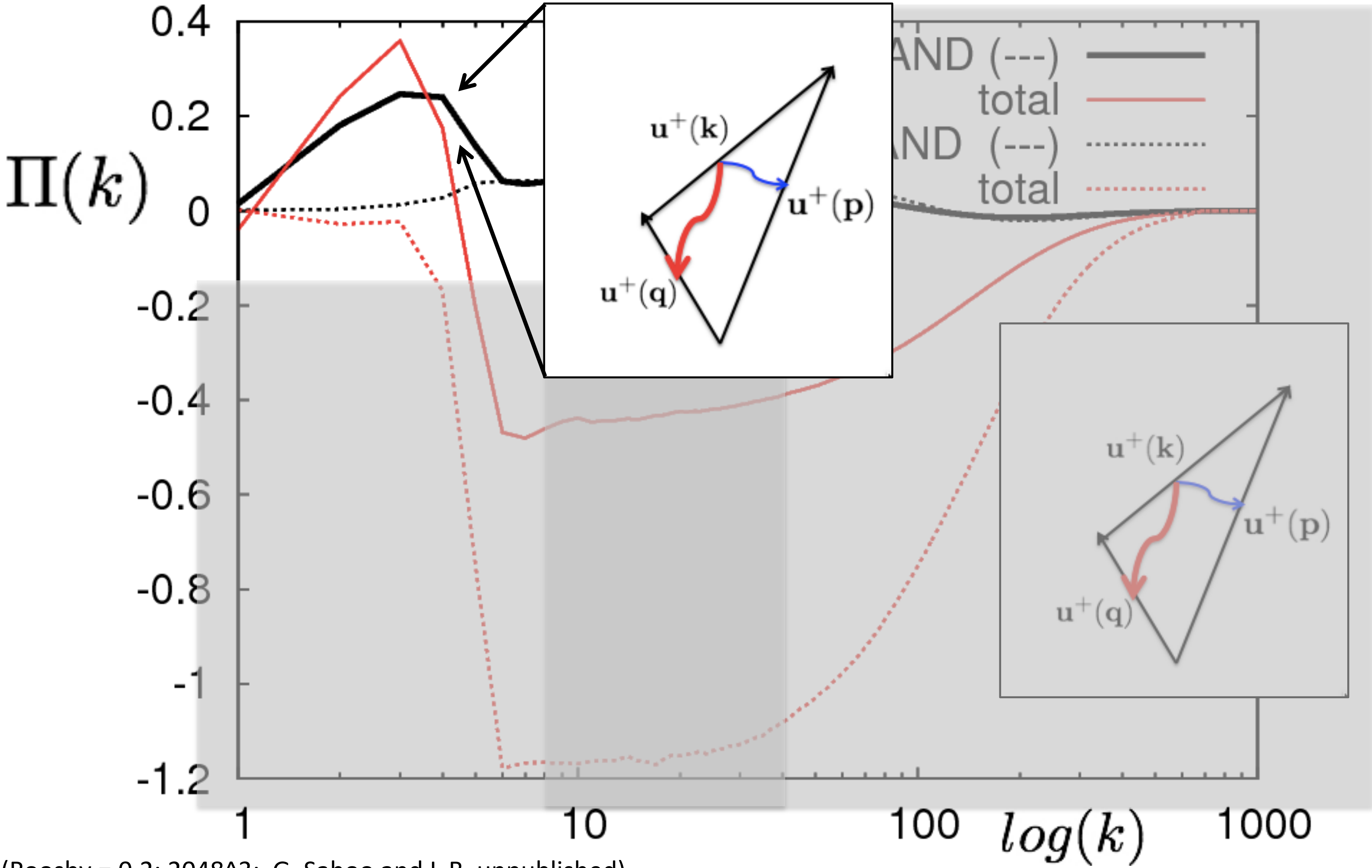
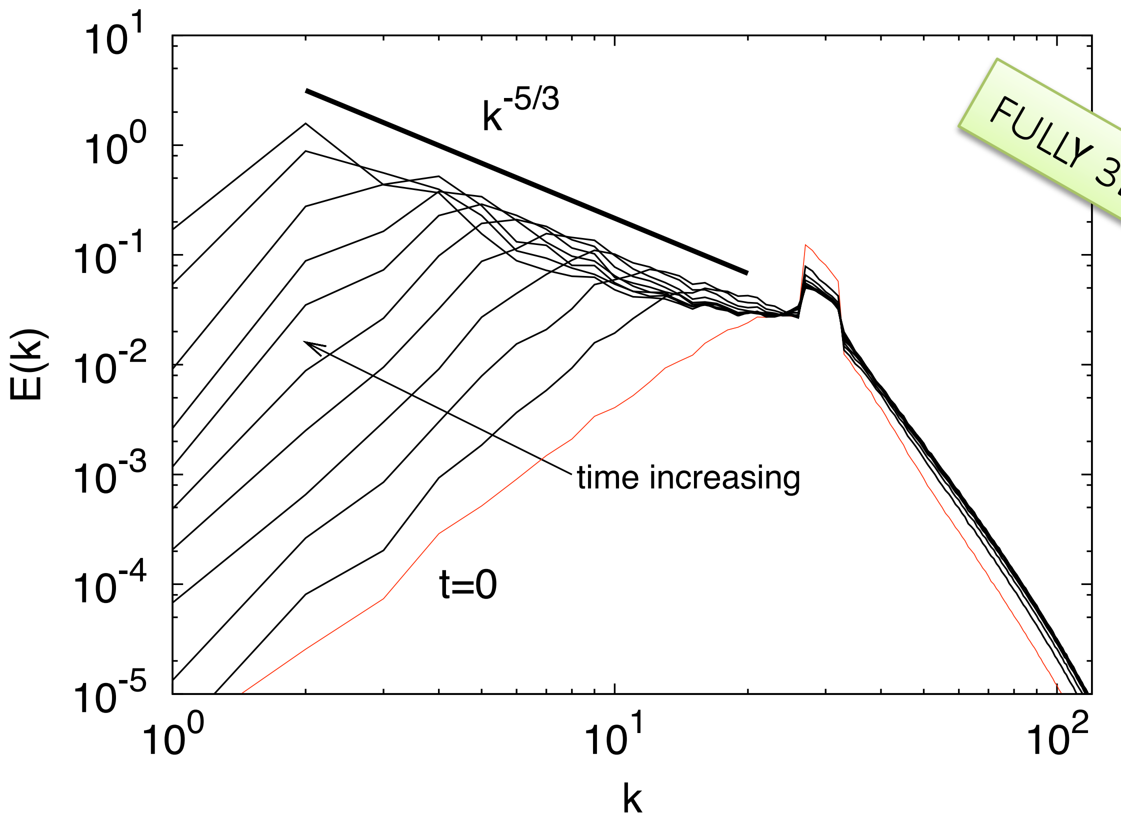
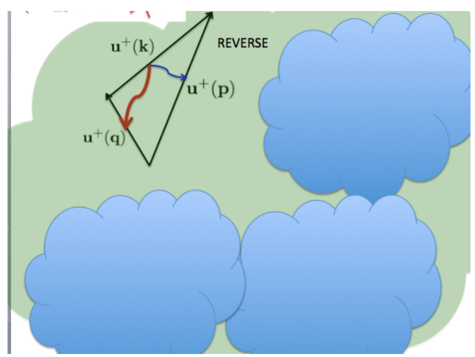


(Roosby = 0.2; 2048^3; G. Sahoo and L.B, unpublished)

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$

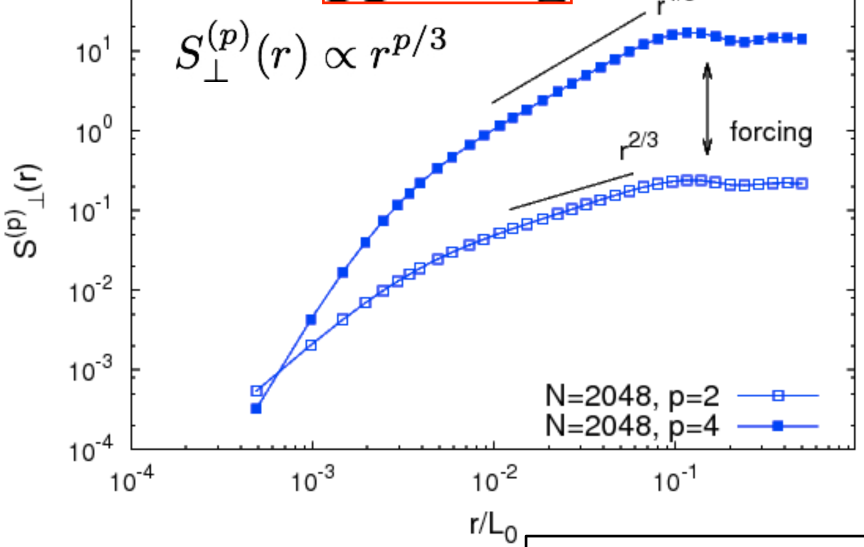




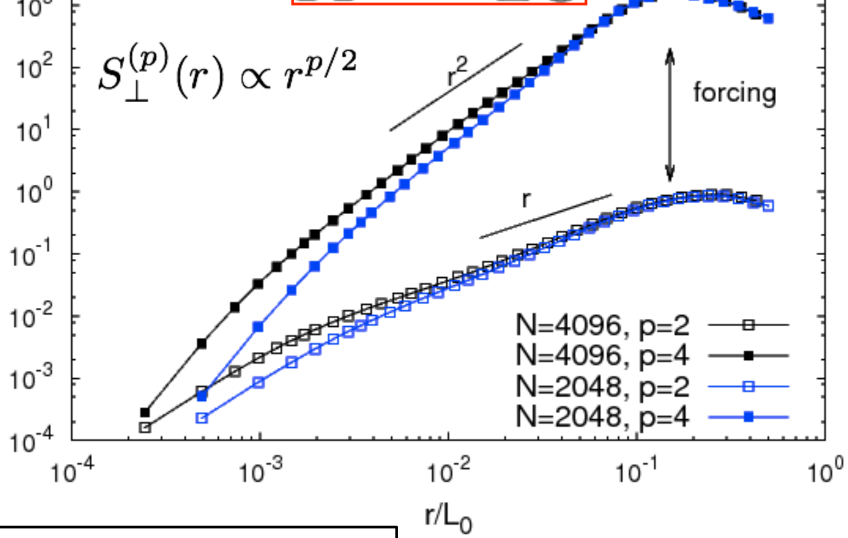
L.B., S. MUSACCHIO & F. TOSCHI Phys. Rev. Lett. 108 164501, 2012.

$$S_{\perp}^{(p)}(r) = \langle (\delta u(r)_{\perp})^p \rangle$$

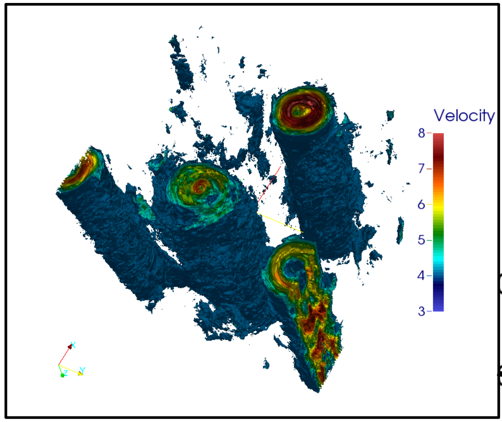
$\Omega = 4$

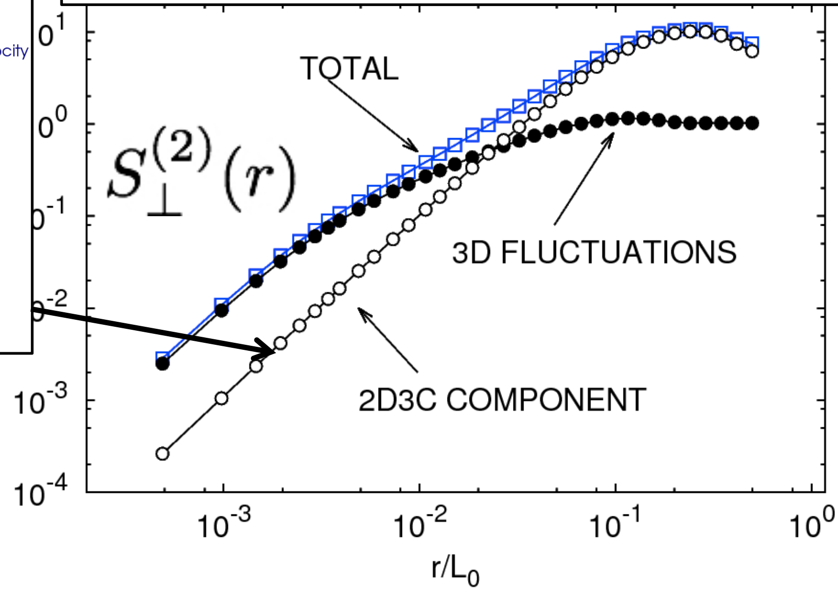


$\Omega = 10$



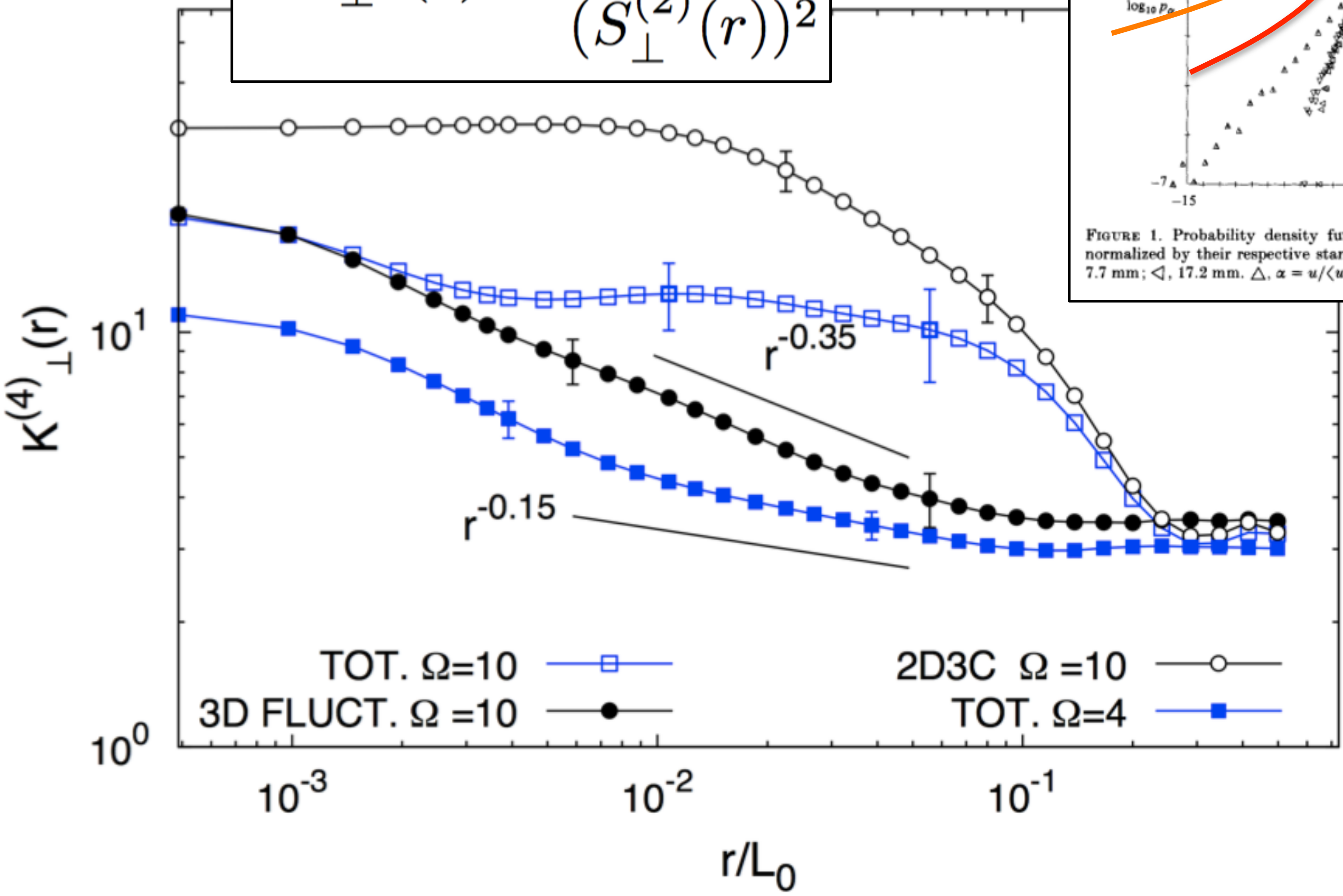
$$\boldsymbol{u}(x,y,z|t) = \boldsymbol{u}_{2D}(y,z|t) + \boldsymbol{u}'(x,y,z|t)$$





FLUCTUATIONS: FLATNESS

$$K^{(4)}_{\perp}(r) \equiv \frac{S^{(4)}_{\perp}(r)}{(S^{(2)}_{\perp}(r))^2}$$



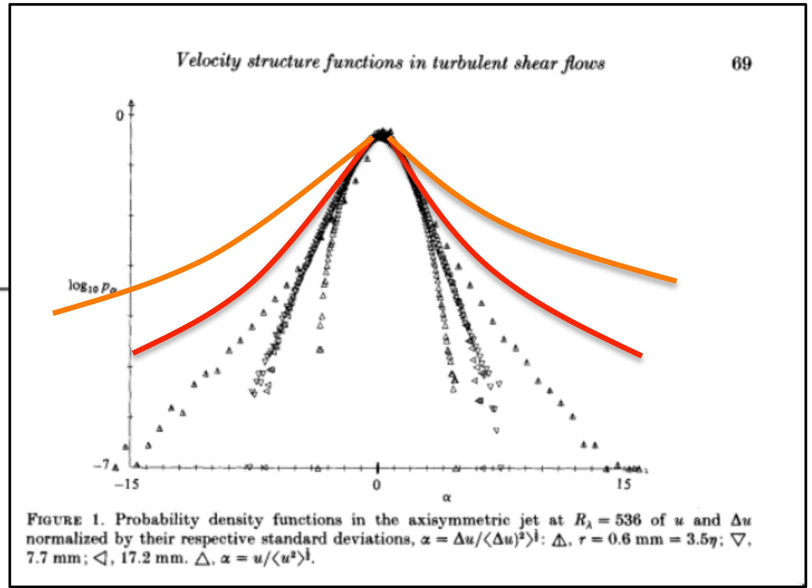

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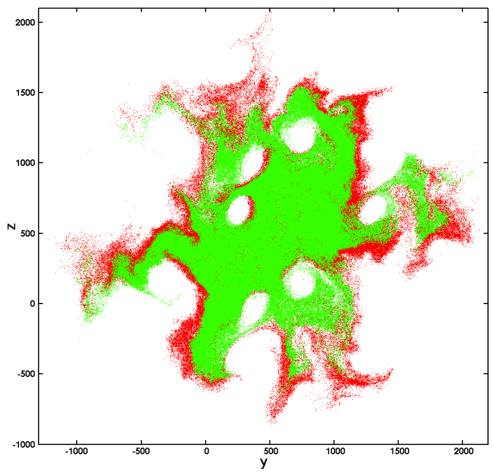
FIGURE 1. Probability density functions in the axisymmetric jet at $R_\lambda = 536$ of u and Δu normalized by their respective standard deviations, $\alpha = \Delta u/\langle(\Delta u)^2\rangle^{\frac{1}{2}}$: ▲, $r = 0.6$ mm $= 3.5\eta$; ▽, 7.7 mm; ◁, 17.2 mm. △, $\alpha = u/\langle u^2\rangle^{\frac{1}{2}}$.

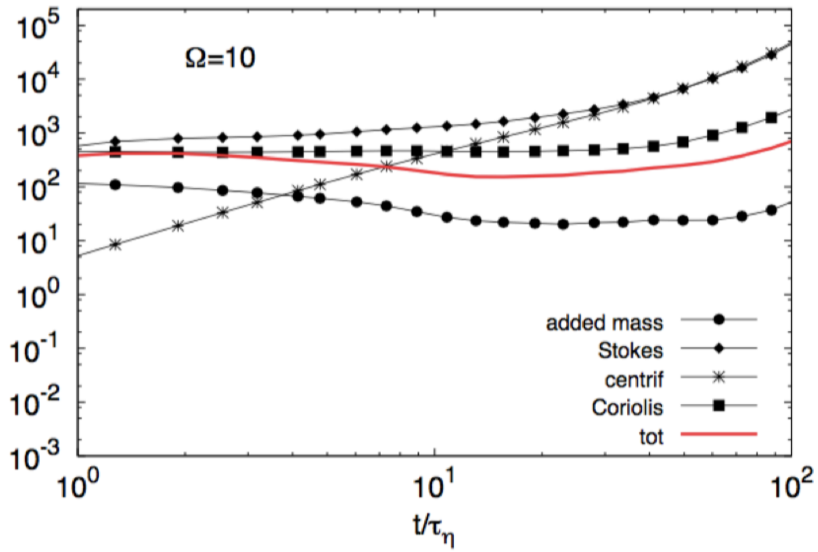
- NON-GAUSSIAN PROPERTIES DEPEND ON THE WAY YOU DECOMPOSE THE FIELD
- AFTER FILTERING THE 2D3C COMPONENT: SCALING PROPERTIES ARE BACK (BUT NOT HIT!)

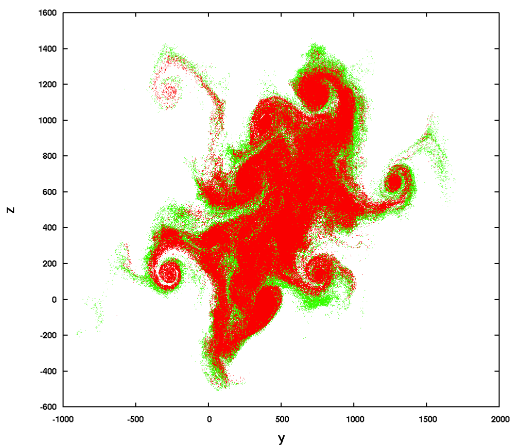
RMS FORCES ALONG TRAJECTORIES

$$\frac{d\mathbf{v}}{dt} = \beta\frac{D\mathbf{u}}{Dt} - \frac{1}{\tau_p}(\mathbf{v}-\mathbf{u}) + 2(\mathbf{v}-\beta\mathbf{u})\times\mathbf{\Omega} - (1-\beta)\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})$$

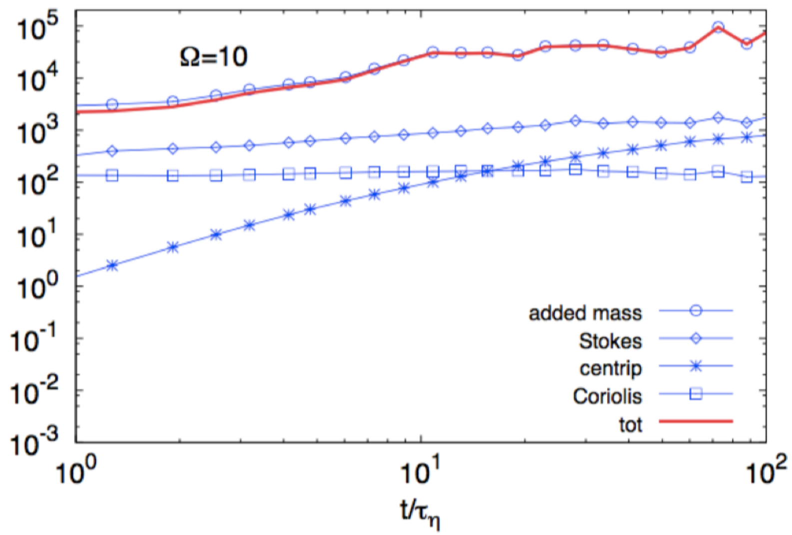


HEAVY



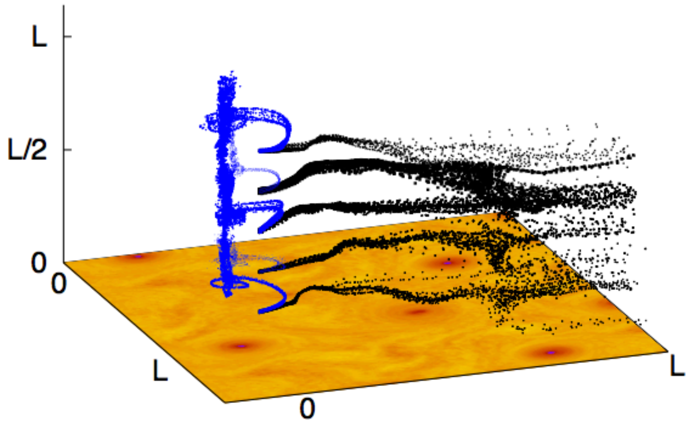


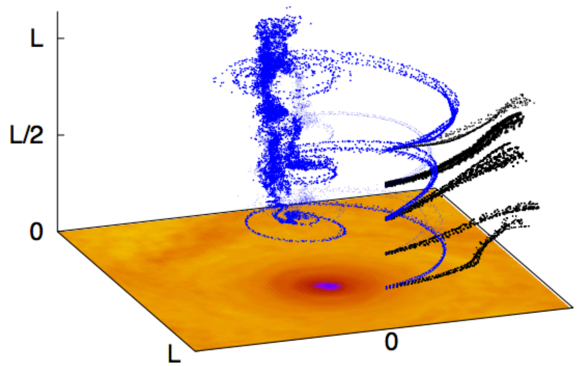
LIGHT

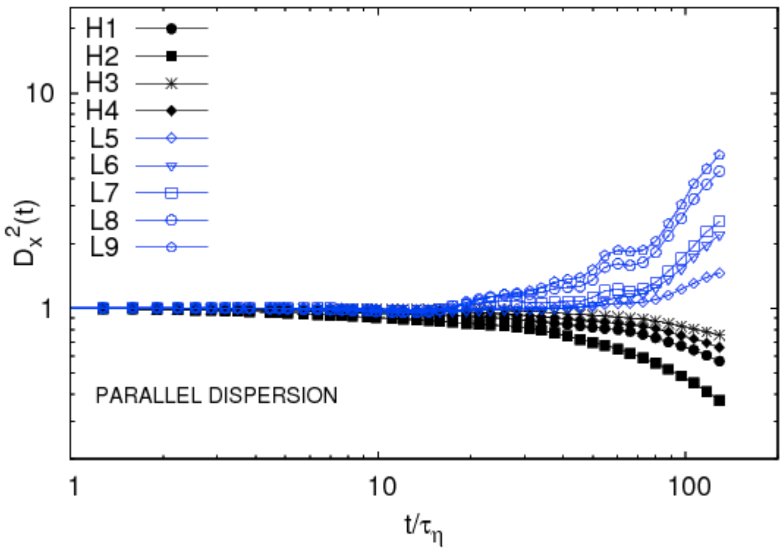


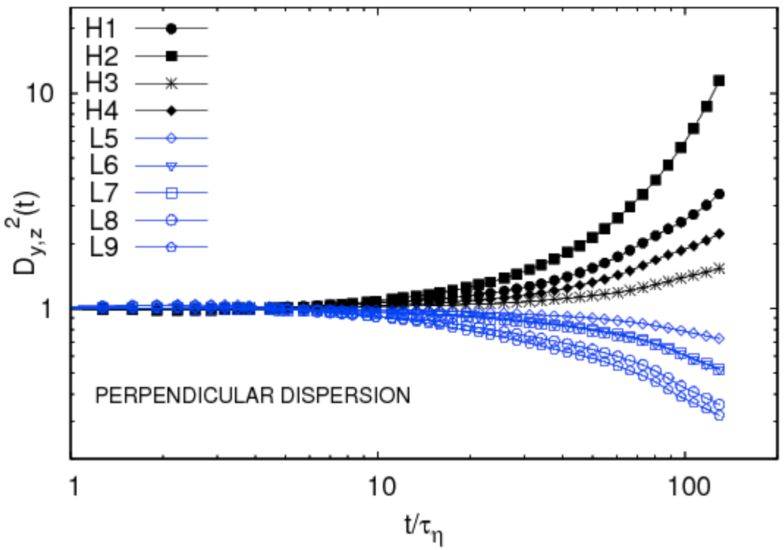
INERTIA: SINGULAR EFFECT ON SINGLE PARTICLE DISPERSION

$$D^{i}_{St,\beta}(t) = \frac{\langle (r^i_t - r^i_0)^2 \rangle_{St,\beta}}{\langle (r^i_t - r^i_0)^2 \rangle_{tracer}}$$









CONCLUSIONS

-HIGH RESOLUTION ROTATING TURBULENCE: FIRST ATTEMPT TO CONTROL SIMULTANEOUSLY EULERIAN & LAGRANGIAN STATISTICS

-IDEAL SET-UP (1): HOMOGENEOUS AND ISOTROPIC TIME-COLORED FORCING

-IDEAL SET-UP (2): SCALE-SEPARATION

-STRONG INFLUENCE OF LARGE-SCALE (NON-UNIVERSAL?) VORTICAL STRUCTURES

-DEPARTURE FROM GAUSSIANITY (DEPENDING ON HOW YOU MEASURE IT: 2D3C-3D3D)

-EFFECTS OF LARGE-SCALE STRUCTURES ON PARTICLES' DISPERSION

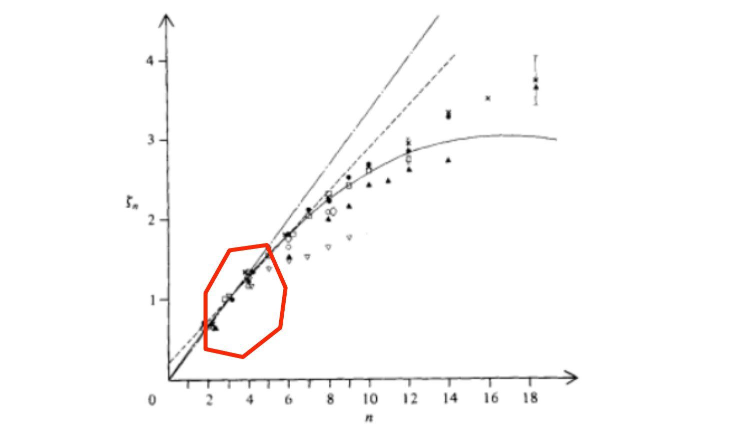


FIGURE 14. Variation of exponent ζ_n as a function of the order n. ●, $R_\lambda = 515$ (duct); □, 536; ×, 852. Symbols ○, ▲, ▽, ◇ are respectively the exponents given by Mestayer (1980); Vasilenko *et al.* (1975); Van Atta & Park (1972); and Antonia *et al.* (1982*a*). The solid curve is LN with $\mu = 0.2$, the dotted curve the β-model and the chain-dotted line Kolmogorov's (1941) model.

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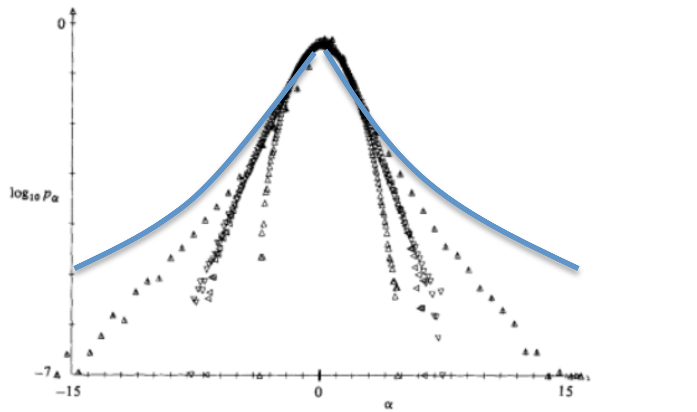

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L.B., F. Bonaccorso, I. Mazzitelli, A. Lanotte, S. Musacchio, P. Perlekar, F. Toschi and M. Hinsberg, PRX submitted 2016