

# Effects of thermal fluctuations on a nanoligament fragmentation

Xiao Xue<sup>1,2</sup>, Mauro Sbragaglia<sup>1</sup>, Luca Biferale<sup>1</sup>, Federico Toschi<sup>2</sup>

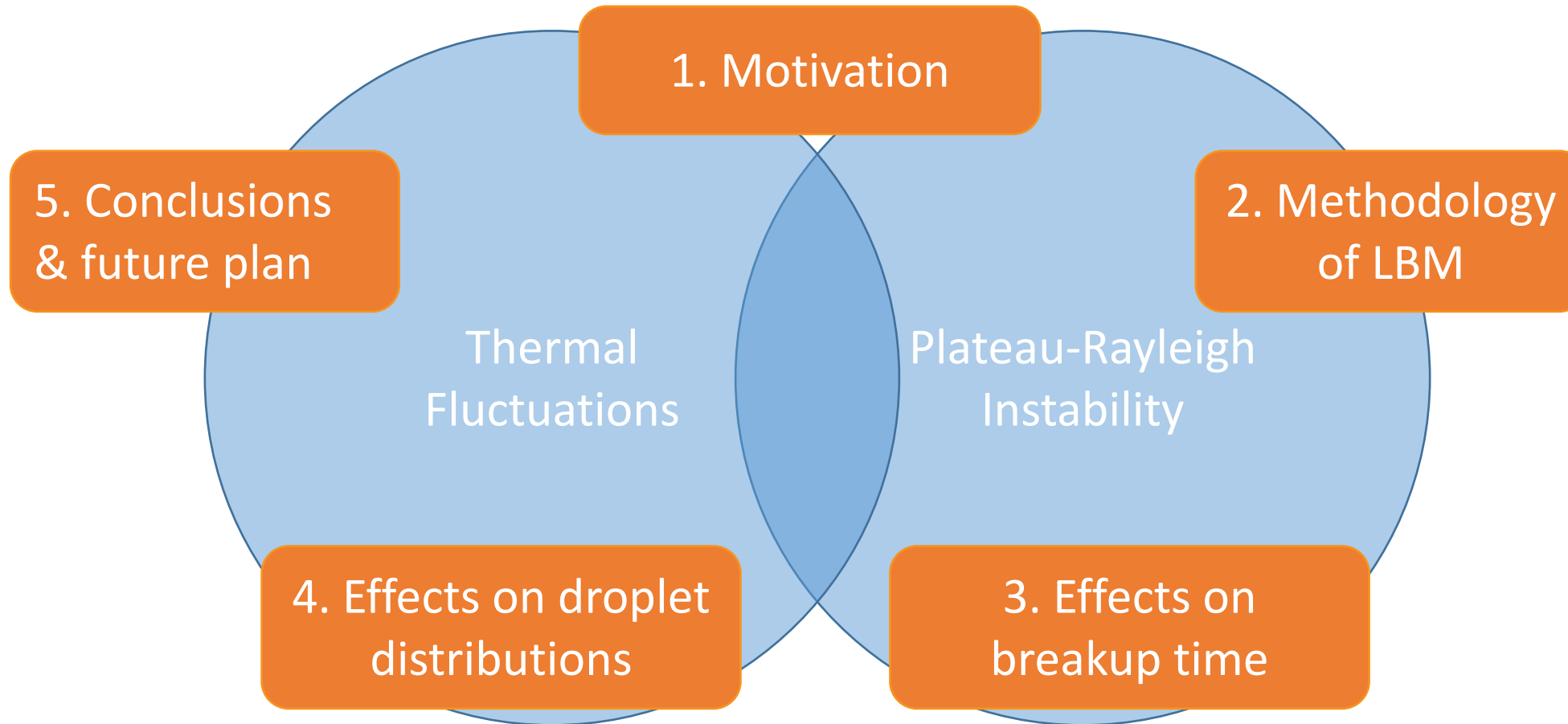
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Cambridge, U.K., 12.07.2018



# Content

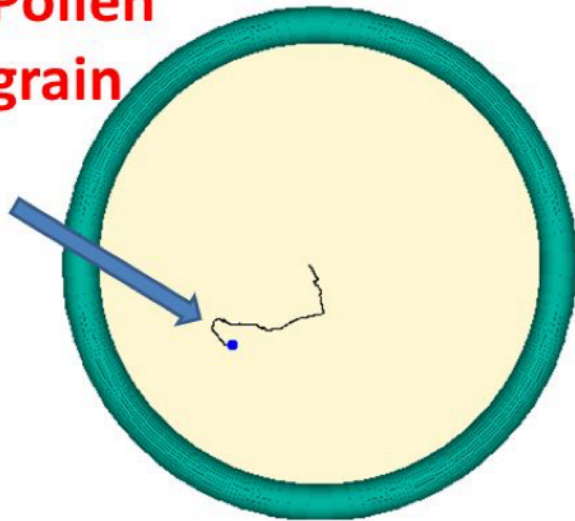


# Motivation



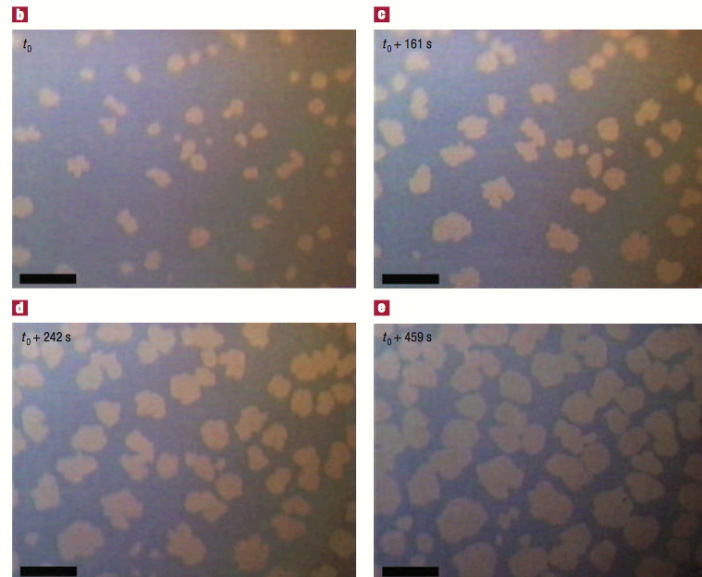
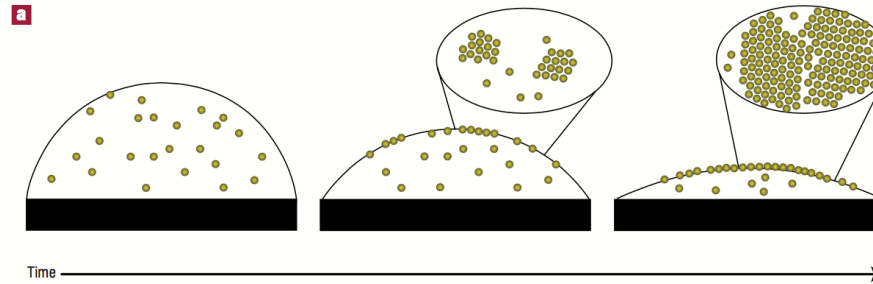
1827 Robert Brown

**Pollen grain**

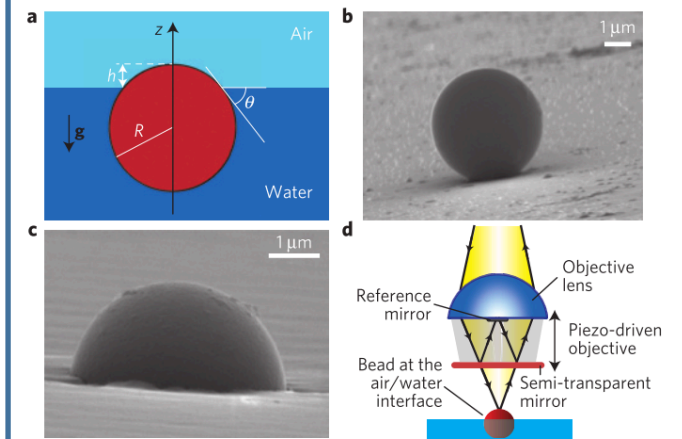


Pollen of the plant suspended in water under a microscope

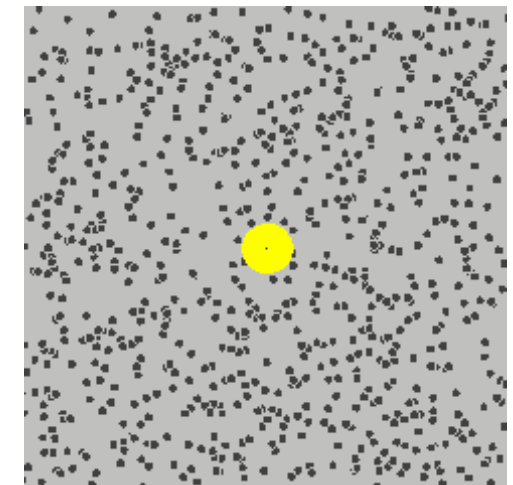
TP Bigioni et. al. Nature materials, 2006



Self assembly particle in industry application & medical application

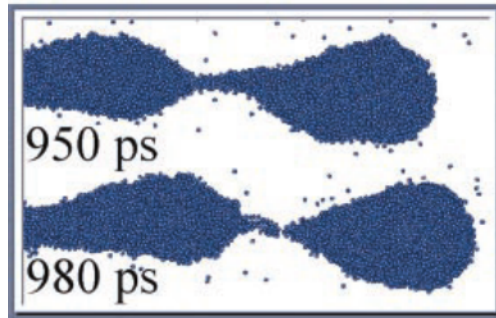


Giuseppe Boniello, et al. ,Nature material, 2015



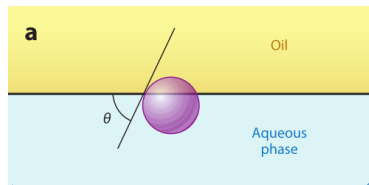
[https://en.wikipedia.org/wiki/Brownian\\_motion](https://en.wikipedia.org/wiki/Brownian_motion)

# Motivation



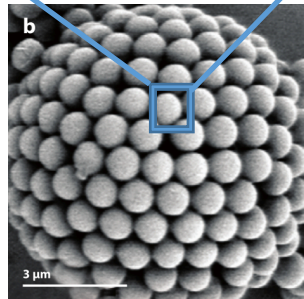
Breakup of nanojets<sup>1</sup>

M. Moseler and U. Landman, Science, 2000



Dinsmore et al. 2002

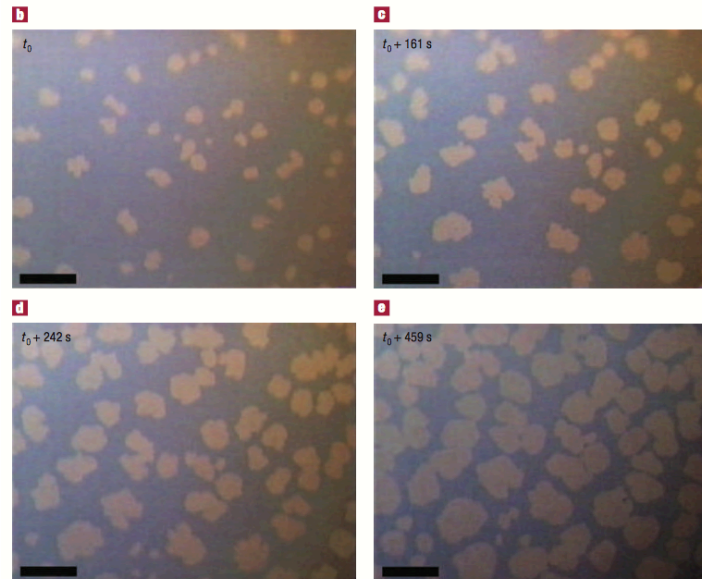
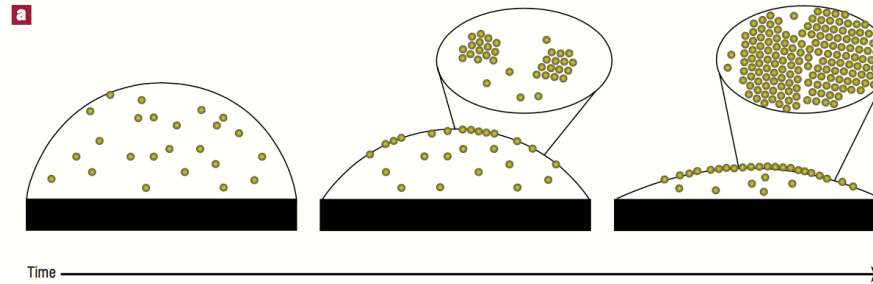
CC Berton-Carabin, et al., 2015



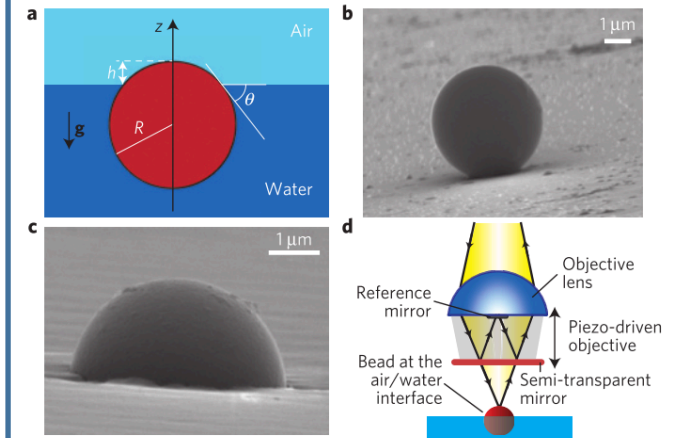
Aveyard et al. 2003

Pickering emulsions

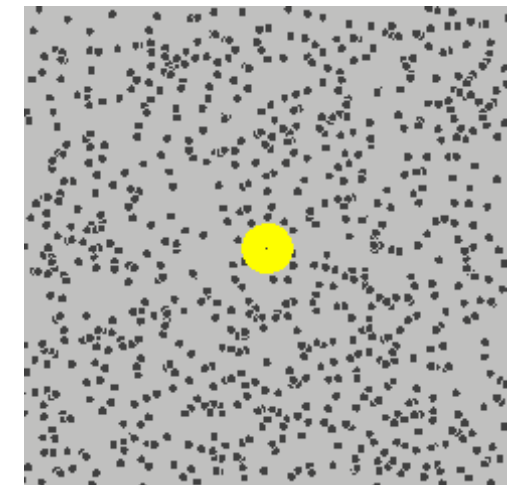
TP Bigioni et. al. Nature materials, 2006



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Giuseppe Boniello, et al. ,Nature material, 2015



[https://en.wikipedia.org/wiki/Brownian\\_motion](https://en.wikipedia.org/wiki/Brownian_motion)

# Plateau-Rayleigh Instability

Ohnesorge number

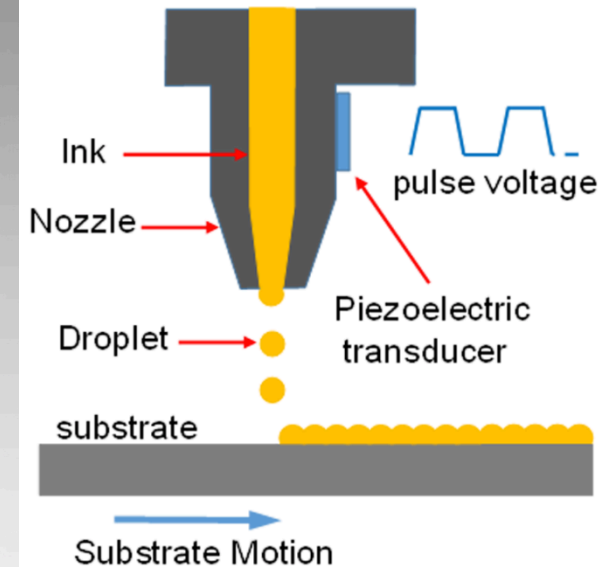
$$Oh = \mu_l \sqrt{\rho_l / (\sigma R_0)}$$

Capillary time

$$T_{cap} = \sqrt{\rho_l R_0^3 / \sigma}$$



Droplet formation at faucet



Inkjet printing



Microscopic scale

Fluctuating multicomponent lattice

Boltzmann model

Mesoscopic scale

$$\partial_t \rho_{tot} + \nabla \cdot (\rho_{tot} \mathbf{v}_{tot}) = 0$$

$$\partial_t \rho_{r,b} + \nabla \cdot (\rho_{r,b} \mathbf{v}_{tot}) = \nabla \cdot (D \nabla \mu + \Phi)$$

Noise term

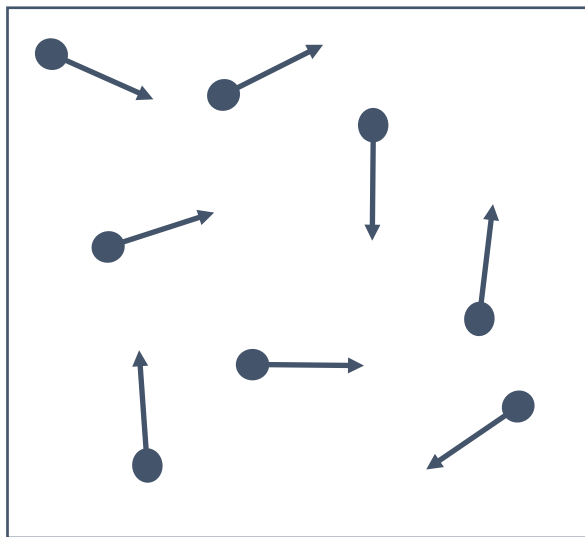
$$\partial_t (\rho_{tot} \mathbf{v}_{tot}) + \nabla \cdot (\rho_{tot} \mathbf{v}_{tot} \mathbf{v}_{tot}) = -\nabla \mathbf{P} + \nabla \cdot \left\{ \eta \left[ \nabla \mathbf{v}_{tot} + (\nabla \mathbf{v}_{tot})^T \right] + \Sigma \right\}$$

$$\Phi = \sqrt{2k_B T D} \hat{\mathbf{W}}, \Sigma = \sqrt{\eta k_B T} (\mathbf{W} + \mathbf{W}^T)$$

Gaussian noise

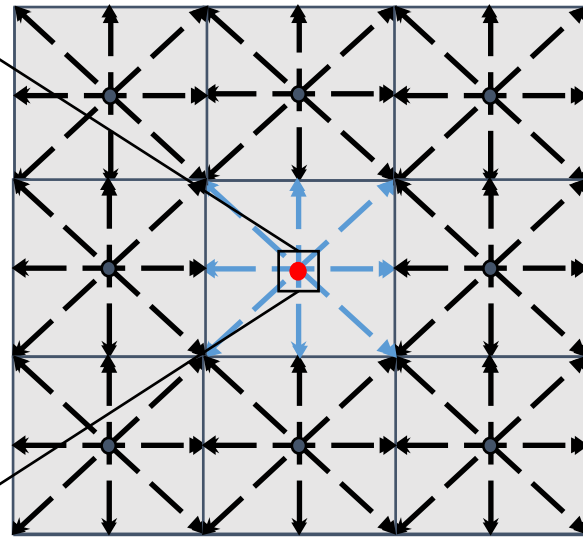
$$\rho_{tot} = \rho_r + \rho_b \quad \mathbf{v}_{tot} = \frac{\rho_r \mathbf{v}_r + \rho_b \mathbf{v}_b}{\rho_r + \rho_b}$$

Microscopic scale



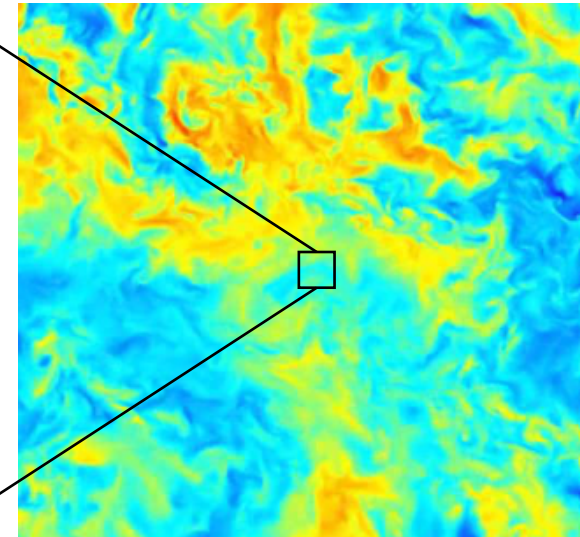
Particle methods

Mesoscopic scale



Lattice Boltzmann method

Macroscopic scale



Navier-Stokes

## Objective:

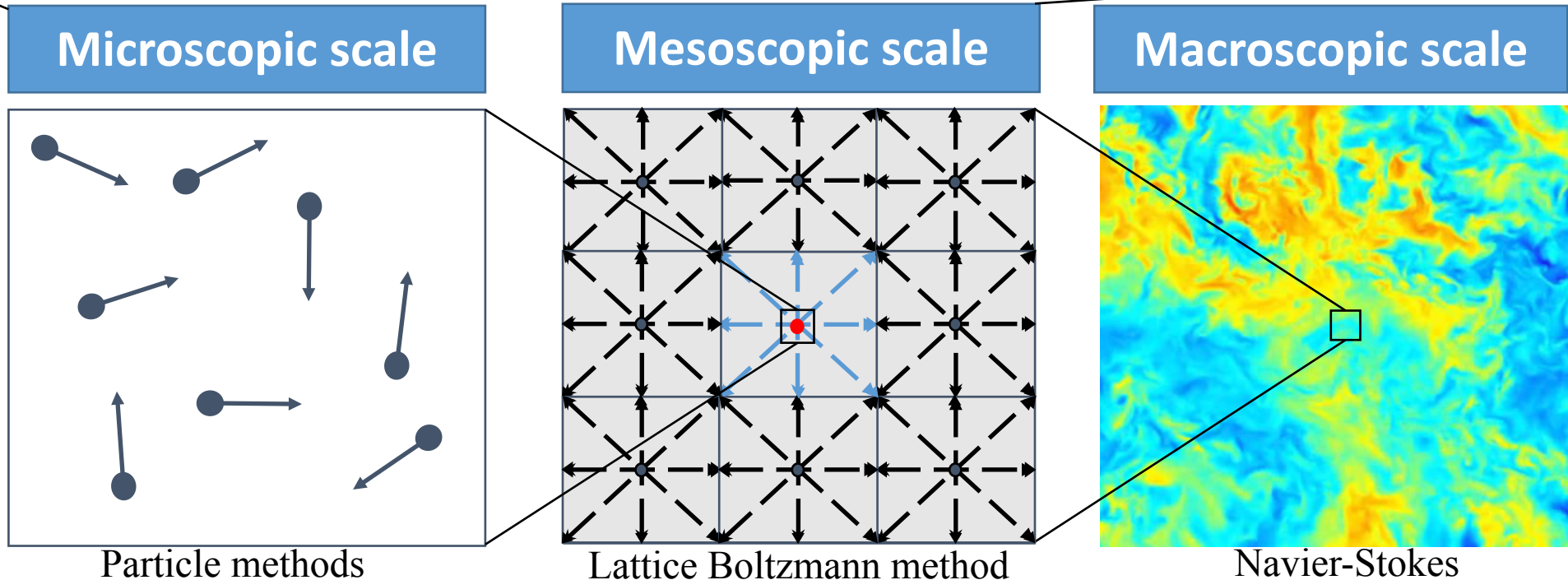
### Understanding thermal fluctuations on nano-ligaments break-up

$$\partial_t \rho_{tot} + \nabla \cdot (\rho_{tot} \mathbf{v}_{tot}) = 0 \quad \partial_t \rho_{r,b} + \nabla \cdot (\rho_{r,b} \mathbf{v}_{tot}) = \nabla \cdot (D \nabla \mu + \Phi)$$

$$\partial_t (\rho_{tot} \mathbf{v}_{tot}) + \nabla \cdot (\rho_{tot} \mathbf{v}_{tot} \mathbf{v}_{tot}) = -\nabla \mathbf{P} + \nabla \cdot \{ \eta [ \nabla \mathbf{v}_{tot} + (\nabla \mathbf{v}_{tot})^T ] + \Sigma \}$$

$$\Phi = \sqrt{2k_B T D \hat{\mathbf{W}}}, \Sigma = \sqrt{\eta k_B T} (\mathbf{W} + \mathbf{W}^T)$$

$$\rho_{tot} = \rho_r + \rho_b \quad \mathbf{v}_{tot} = \frac{\rho_r \mathbf{v}_r + \rho_b \mathbf{v}_b}{\rho_r + \rho_b}$$



Streaming

$$f_i^{r,b} = f_i^{r,b}(x - c_i \Delta t, t - \Delta t)$$

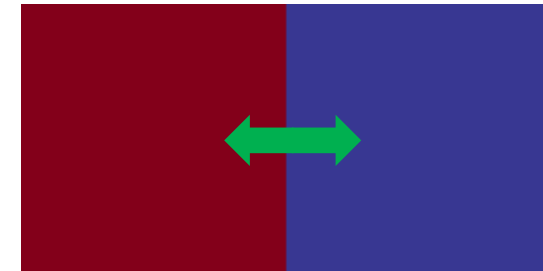
Collision

$$f_i^{r,b}(x + c_i \Delta t, t + \Delta t) = f_i^{r,b}(\mathbf{x}, t) + \mathcal{L}^{r,b}(f_i(\mathbf{x}, t)) + \underline{F_{sc}^{r,b}} + \xi_{noise}^{r,b}$$

Hydrodynamics  
quantities

$$\rho^{r,b} = \sum_i f_i^{r,b}$$

$$\rho^{r,b} \mathbf{u}^{r,b} = \sum_i f_i^{r,b} \mathbf{c}_i$$



Shan-Chen forcing

$$F_{sc}^{r,b}(\mathbf{x}, t) = - \sum_{\mu} G_{12} \sum_i \omega_i \varphi_{\mu}(\mathbf{x}, t) \varphi_{\mu}(\mathbf{x} + c_i \Delta t, t)$$



Streaming

$$f_i^{r,b} = f_i^{r,b}(x - c_i \Delta t, t - \Delta t)$$

Collision

$$f_i^{r,b}(x + c_i \Delta t, t + \Delta t) = f_i^{r,b}(\mathbf{x}, t) + \mathcal{L}^{r,b}(f_i(\mathbf{x}, t)) + F_{sc}^{r,b} + \xi_{noise}^{r,b}$$

Hydrodynamics  
quantities

$$\rho^{r,b} = \sum_i f_i^{r,b}$$

$$\rho^{r,b} \mathbf{u}^{r,b} = \sum_i f_i^{r,b} \mathbf{c}_i$$

Noise correlations

$$\langle \xi_\rho^b \xi_\rho^b \rangle = \langle \xi_\rho^b \xi_\rho^r \rangle = 0$$

$$\langle \xi_j^b \xi_j^b \rangle = -\langle \xi_j^b \xi_j^r \rangle = 2\lambda k_B T \frac{\rho^b \rho^r}{\rho^b + \rho^r} \mathbf{1}$$

Streaming

$$f_i^{r,b} = f_i^{r,b}(x - c_i \Delta t, t - \Delta t)$$

Collision

$$f_i^{r,b}(x + c_i \Delta t, t + \Delta t) = f_i^{r,b}(\mathbf{x}, t) + \mathcal{L}^{r,b}(f_i(\mathbf{x}, t)) + F_{sc}^{r,b} + \xi_{noise}^{r,b}$$

Hydrodynamics  
quantities

$$\rho^{r,b} = \sum_i f_i^{r,b}$$

$$\rho^{r,b} \mathbf{u}^{r,b} = \sum_i f_i^{r,b} \mathbf{c}_i$$

Champman-Enskog expansion



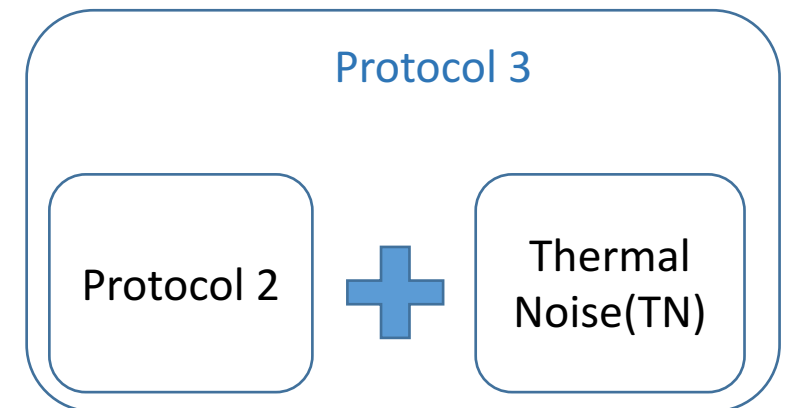
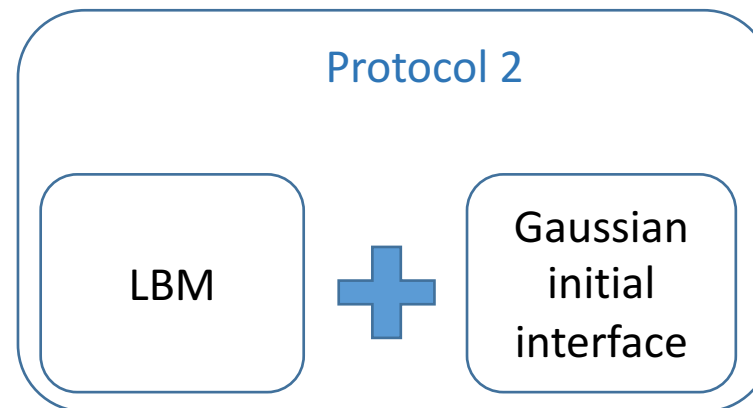
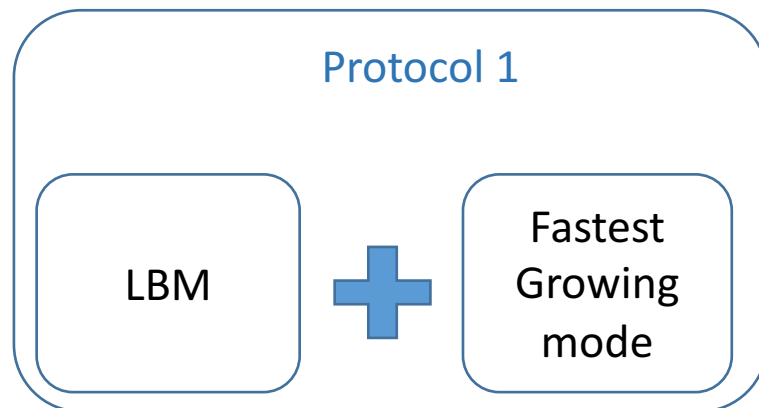
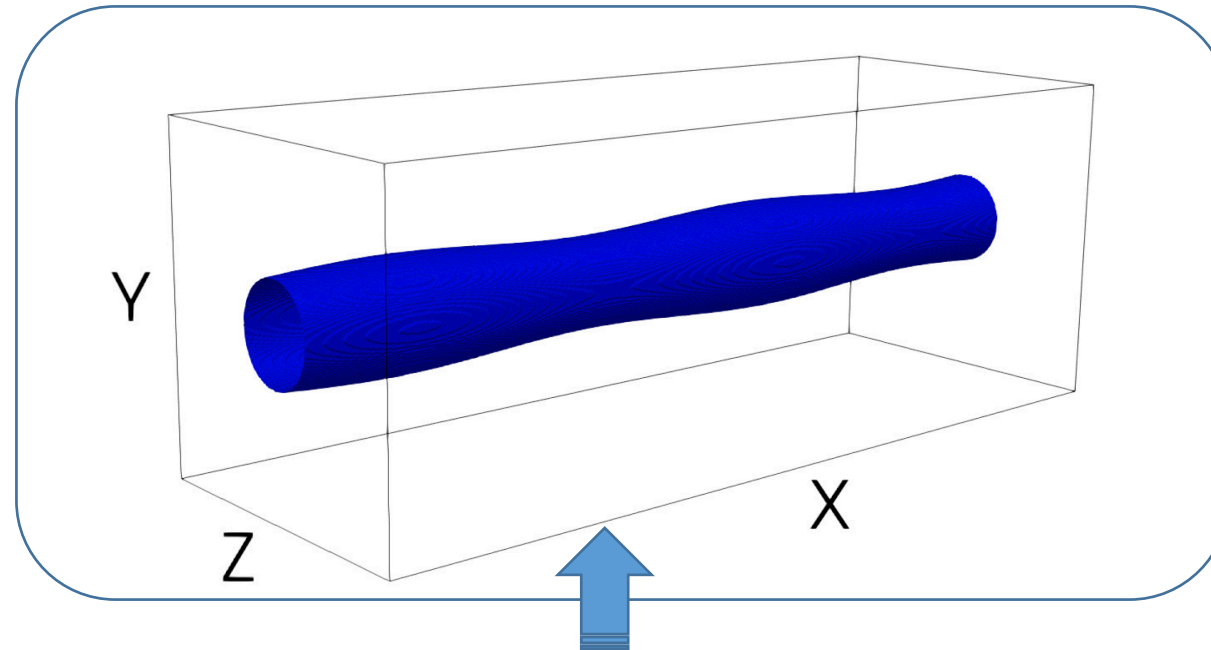
Navier-Stoke

$$\partial_t \rho_{tot} + \nabla \cdot (\rho_{tot} \mathbf{v}_{tot}) = 0$$

$$\partial_t \rho_{r,b} + \nabla \cdot (\rho_{r,b} \mathbf{v}_{tot}) = \nabla \cdot (D \nabla \mu + \Phi)$$

$$\partial_t (\rho_{tot} \mathbf{v}_{tot}) + \nabla \cdot (\rho_{tot} \mathbf{v}_{tot} \mathbf{v}_{tot}) = -\nabla \mathbf{P} + \nabla \cdot \{ \eta [ \nabla \mathbf{v}_{tot} + (\nabla \mathbf{v}_{tot})^T ] + \Sigma \}$$

# Simulation set up



# Thermal fluctuation impact on the break-up process

## 1. Ligament breaks up faster under the influence of thermal fluctuations?

## 2. What is the impact of thermal fluctuations on Droplet distributions?

**Thermal length**

$$\ell_T = \sqrt{k_B T / \sigma}$$

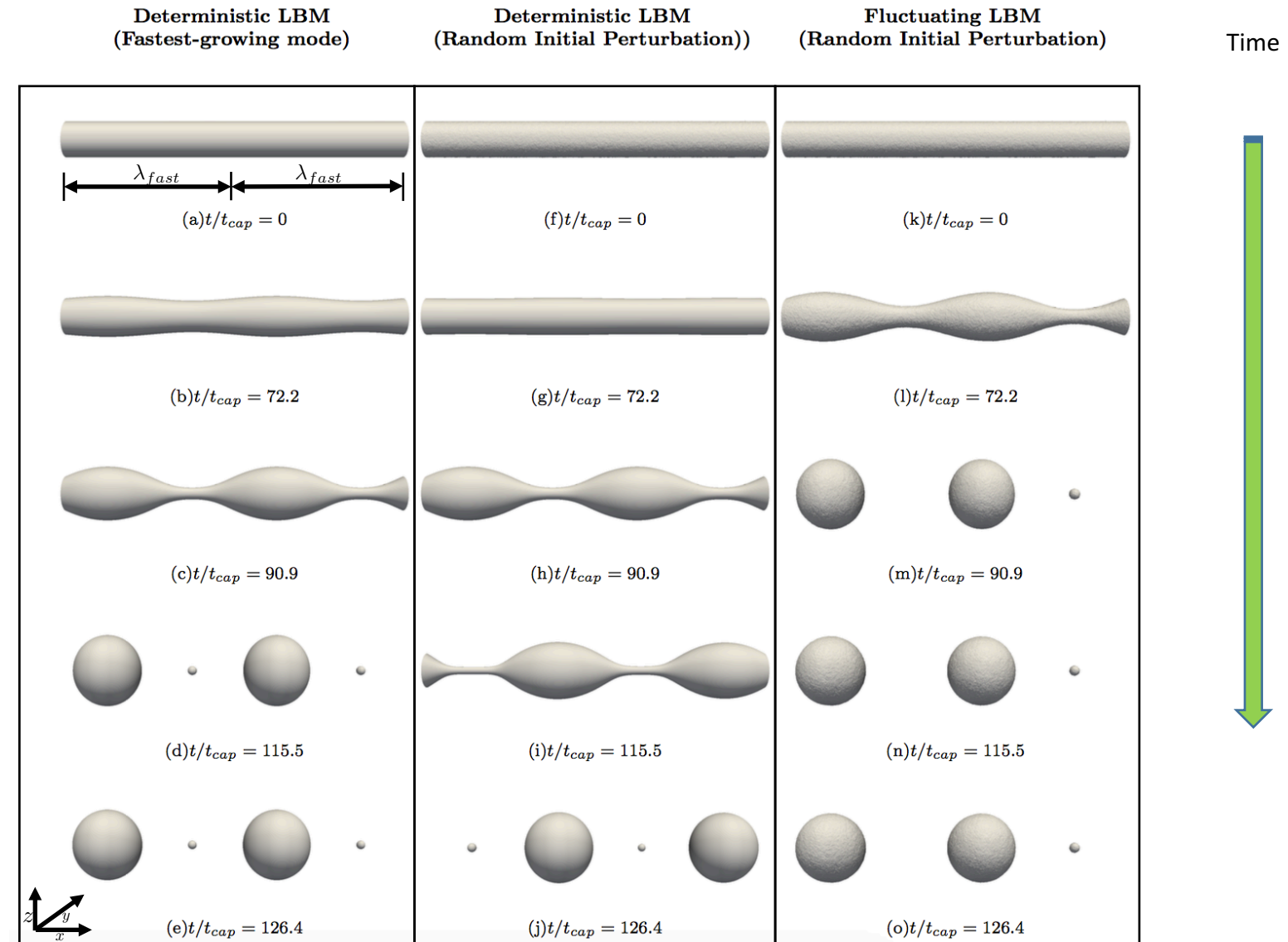
**Capillary time**

$$T_{cap} = \sqrt{\rho_l R_0^3 / \sigma}$$

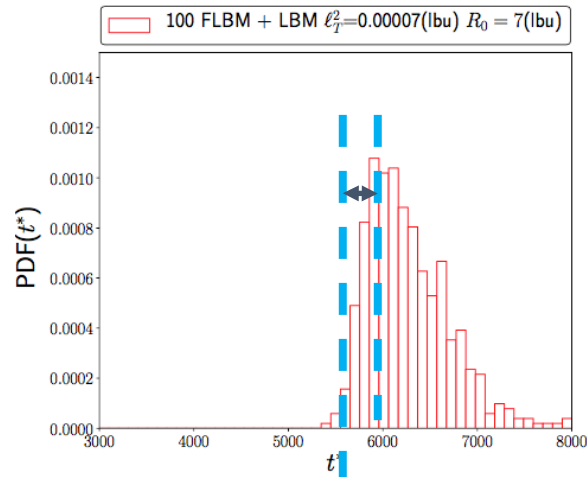
Domain size: 192X192X512

Thermal length:  $\ell_T = 0.1$

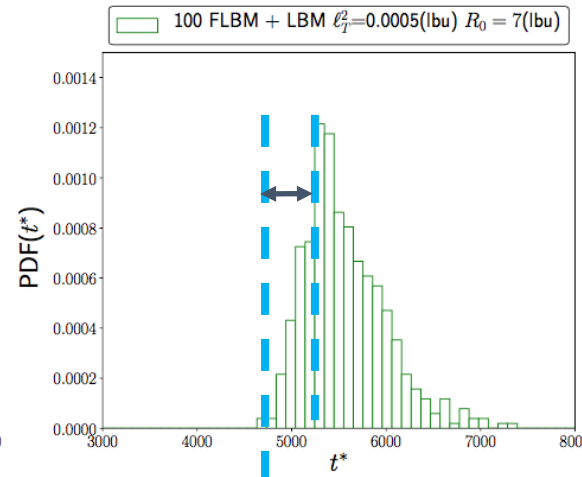
X.Xue et al. accepted at PRE, arXiv:1804.09520



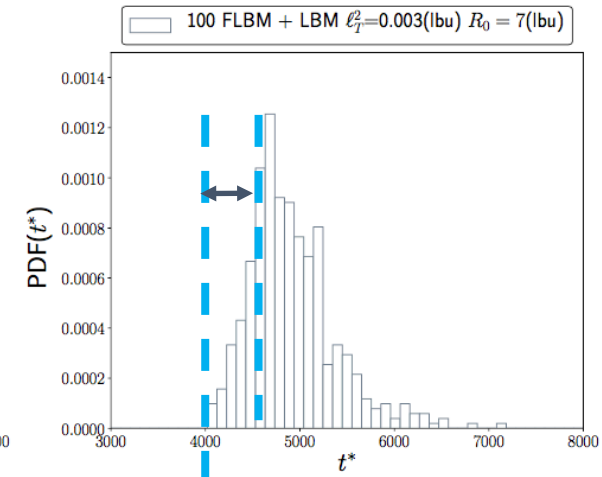
without TN



(a)  $\ell_T^2 = 0.00007$

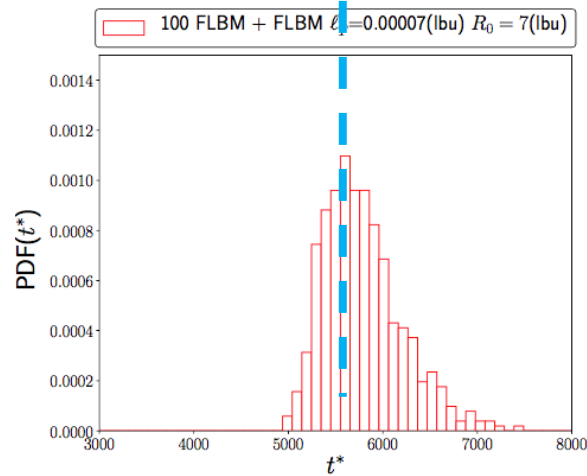


(b)  $\ell_T^2 = 0.0005$

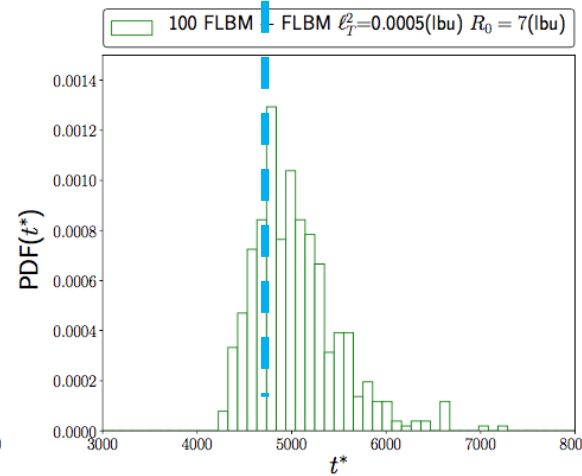


(c)  $\ell_T^2 = 0.003$

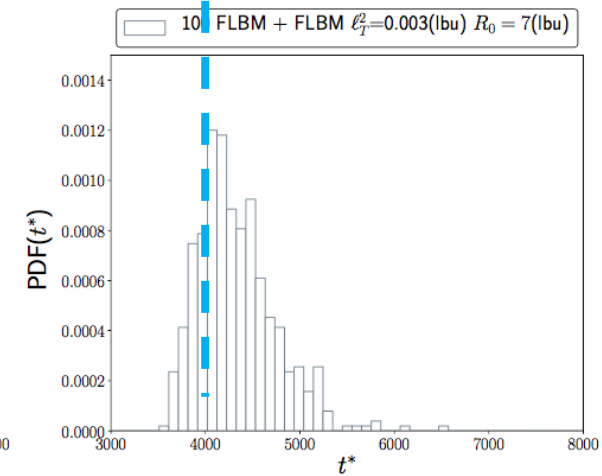
with TN



(d)  $\ell_T^2 = 0.00007$



(e)  $\ell_T^2 = 0.0005$

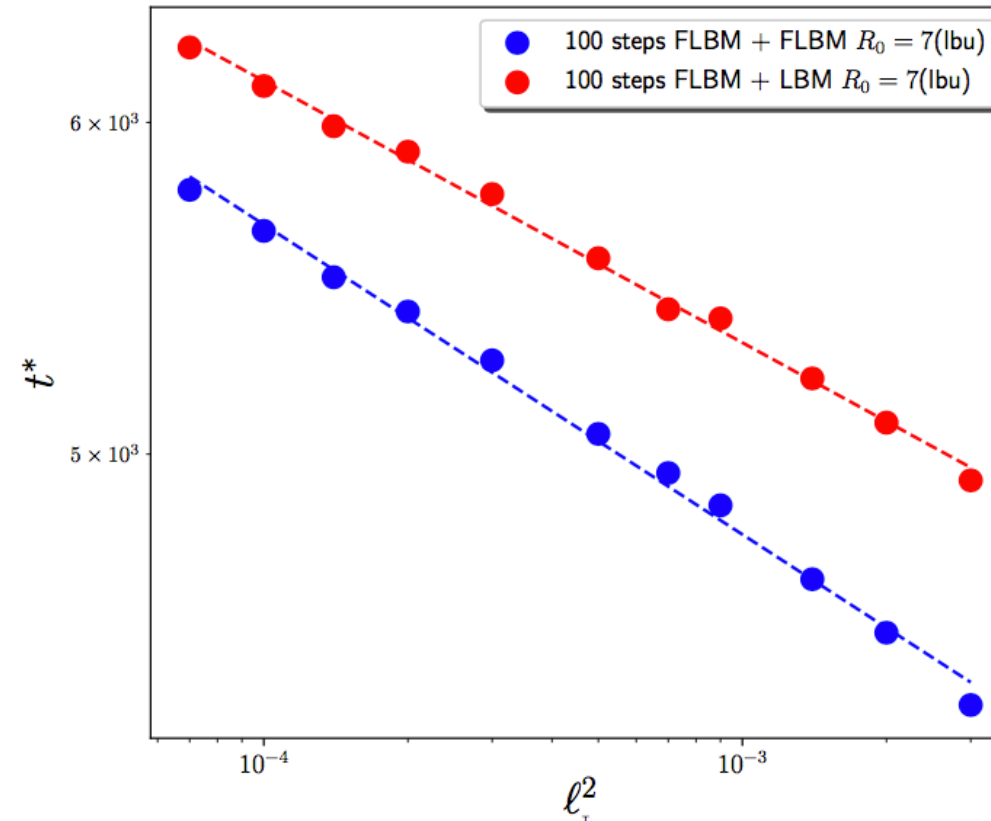


(f)  $\ell_T^2 = 0.003$

Break up time PDF for **LBM** and **FLBM** at fixed  $R_0 = 7$

# Thermal fluctuation impact on the break-up process

- Initial condition of the hydrodynamics can the **decrease** the breakup time
- Thermal fluctuations **enhance** the effect of acceleration



Break up time as function of **LBM and FLBM** at fixed  $R_0 = 7$



# Thermal fluctuation impact on the break-up process

1. Ligament breaks up faster under the influence of thermal fluctuations?

2. What is the impact of thermal fluctuations on Droplet distributions?

Thermal length

$$\ell_T = \sqrt{k_B T / \sigma}$$

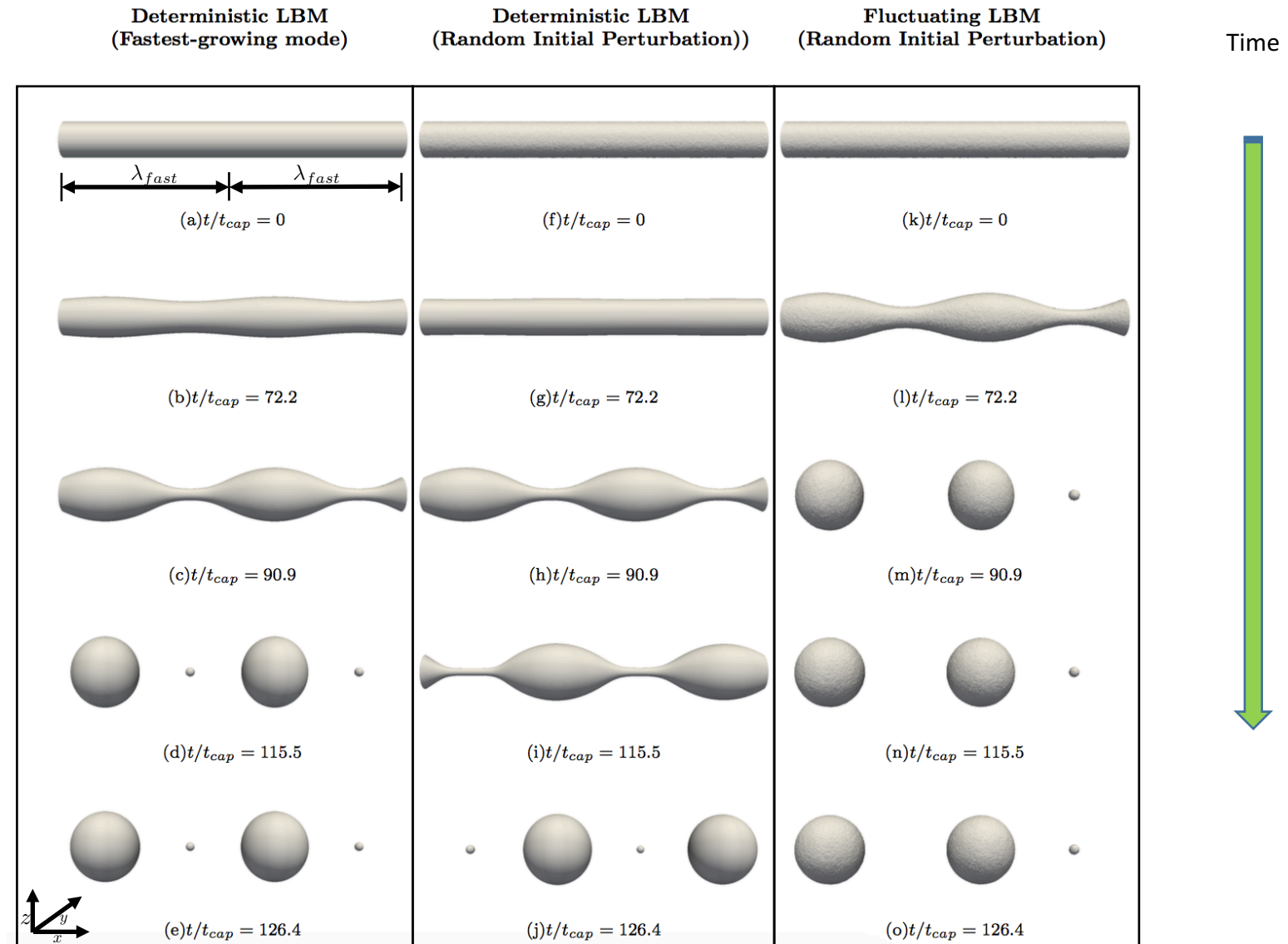
Capillary time

$$T_{cap} = \sqrt{\rho_l R_0^3 / \sigma}$$

Domain size: 192X192X512

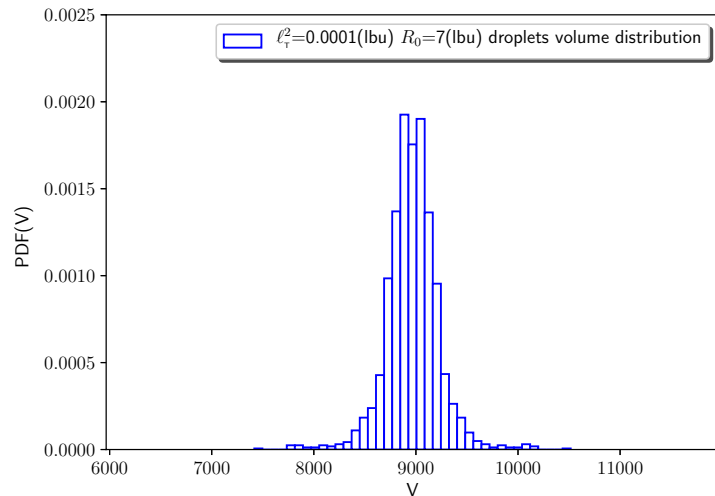
Thermal length:  $\ell_T = 0.1$

X.Xue et al. accepted at PRE, arXiv:1804.09520

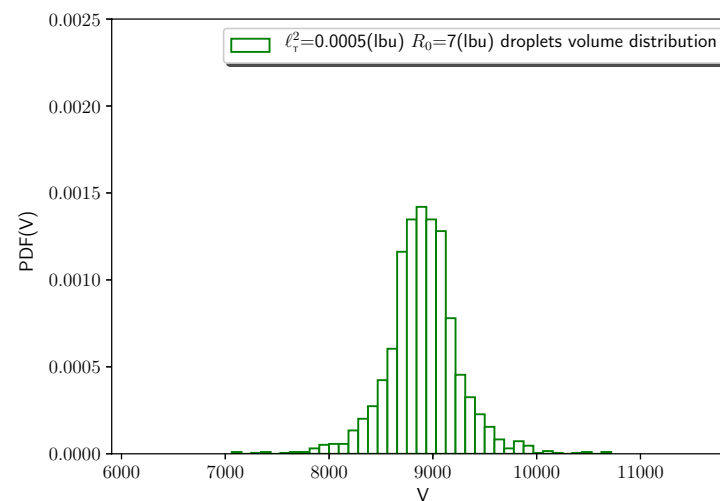


# Thermal fluctuations enhanced droplets' polydispersity

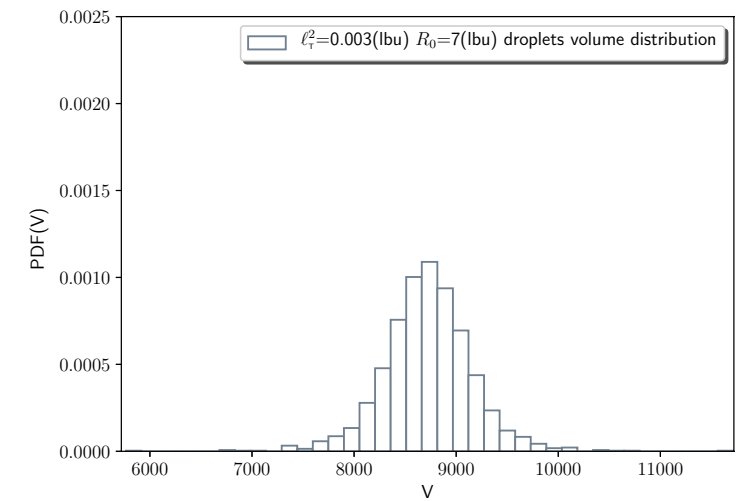
- Standard deviation of the droplet volumes are **increasing** with increasing of
- **What is shape of the distribution?**



$$\ell_T^2 = 0.0001$$



$$\ell_T^2 = 0.0005$$

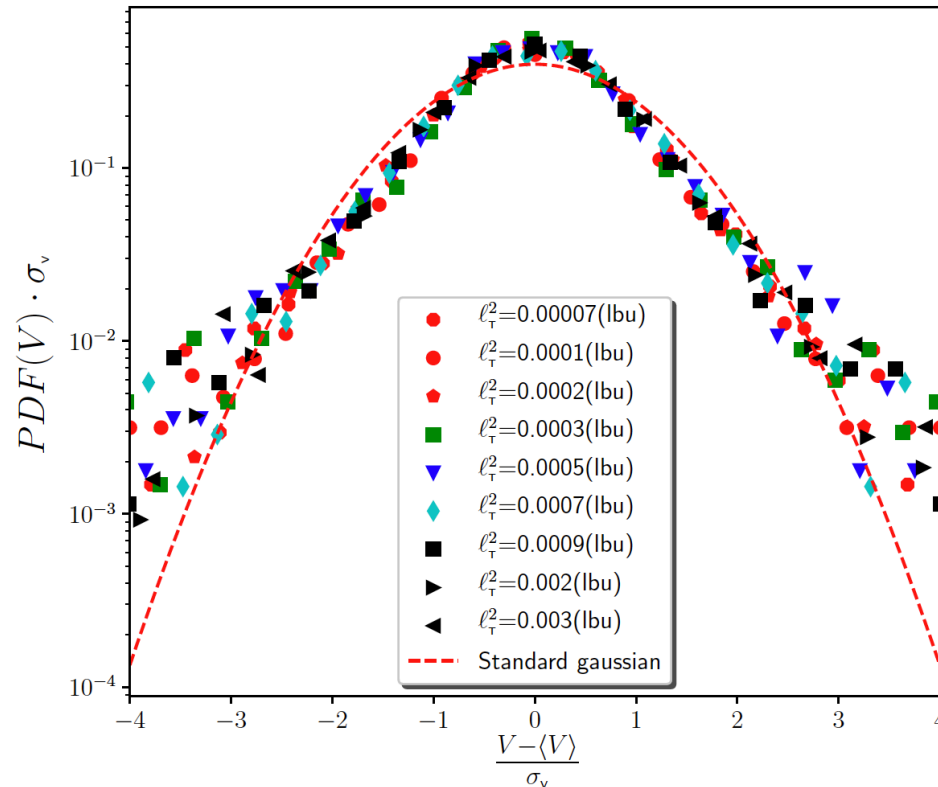


$$\ell_T^2 = 0.003$$

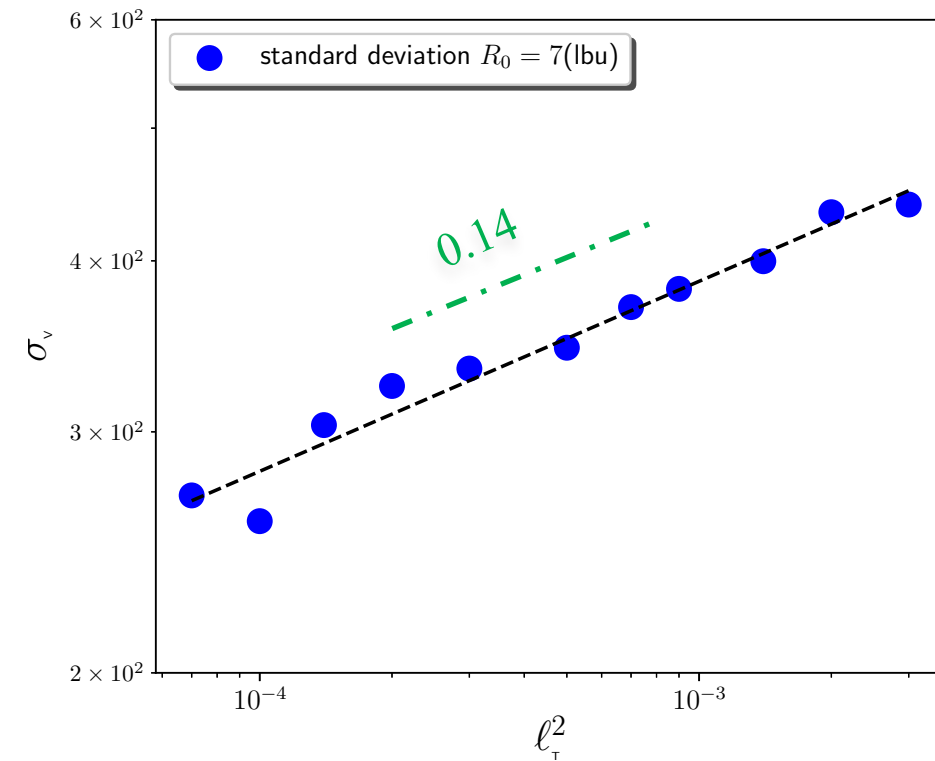


Distributions of droplets volumes at different values of  $\ell_T^2$  at fixed  $R_0 = 7$

# Droplet volumes distributions have small deviation from Gaussian distribution

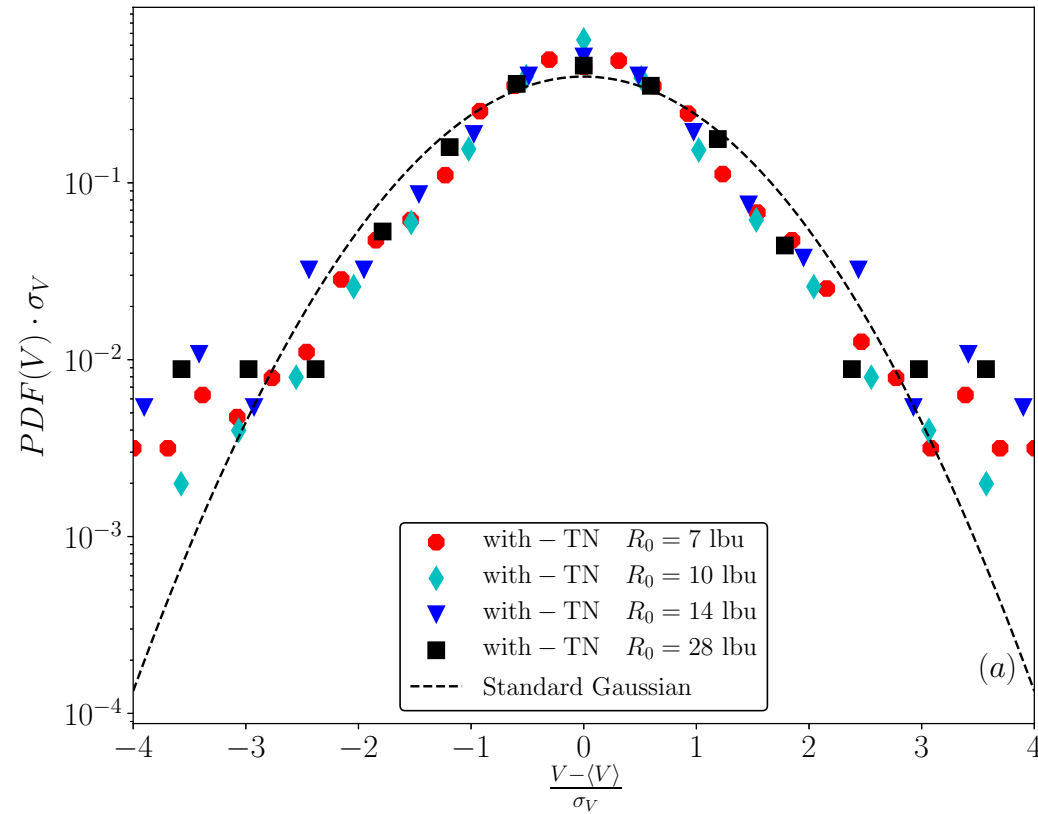


Normalized PDF for **fixed  $R_0 = 7$**   
vs Gaussian distributions

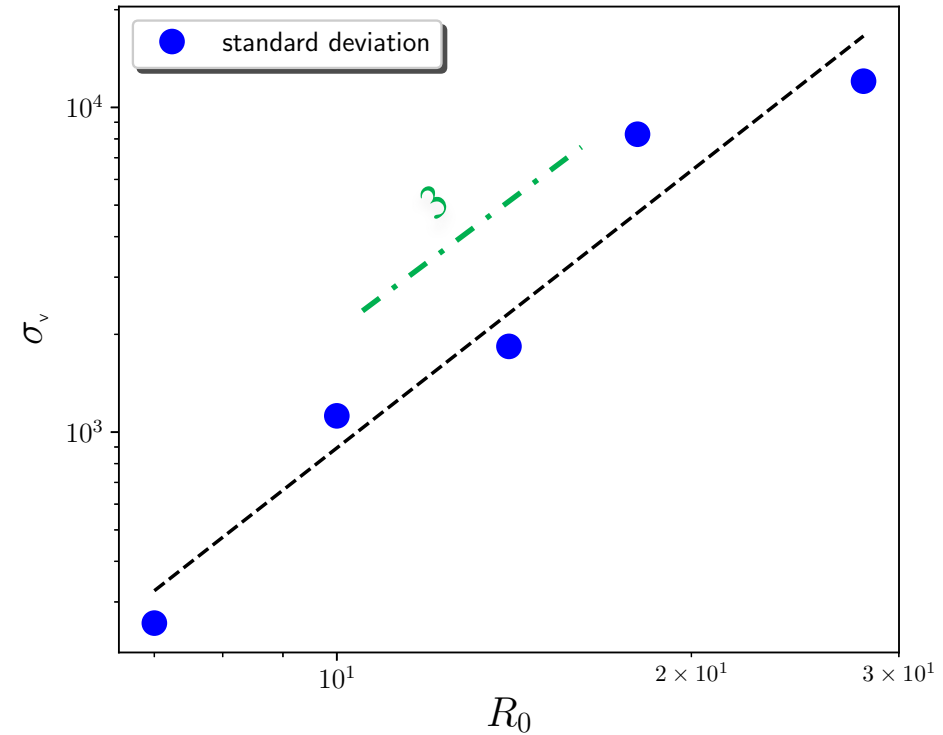


Standard deviation as function of  $l_\tau^2$  for  
**fixed  $R_0 = 7$**

# What about different resolutions?

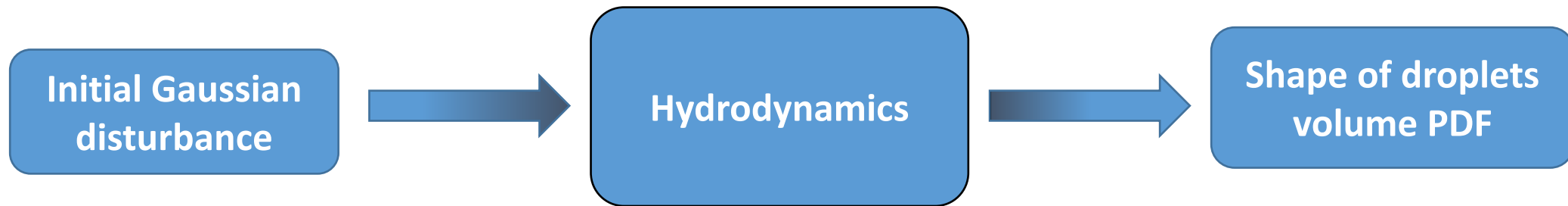


Normalized PDF for **fixed**  $l_T^2 = 0.0001$   
at **different**  $R_0$  vs Gaussian distributions



Standard deviation of initial radius as  
a function of  $R_0$  for **fixed**  $l_T^2 = 0.0001$

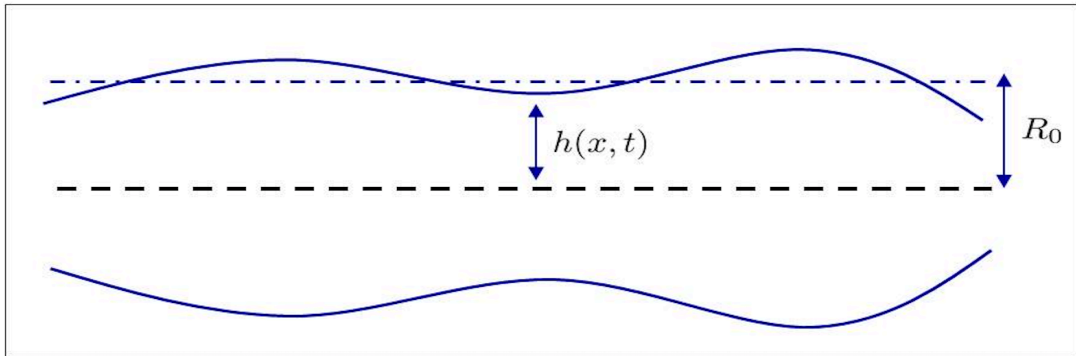
# Comparison with lubrication theory?



**Axisymmetric Lubrication theory (high viscosity ratio)**

$$\partial_t h + v h' + \frac{1}{2} v' h = 0$$

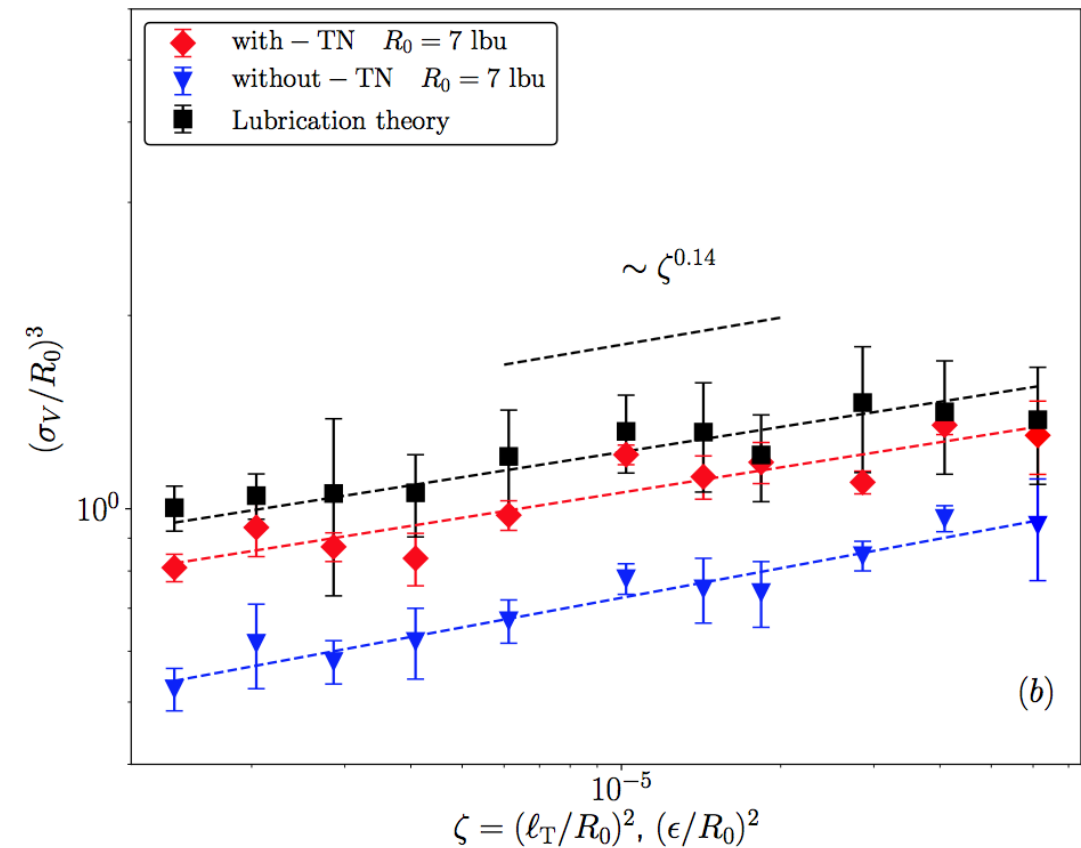
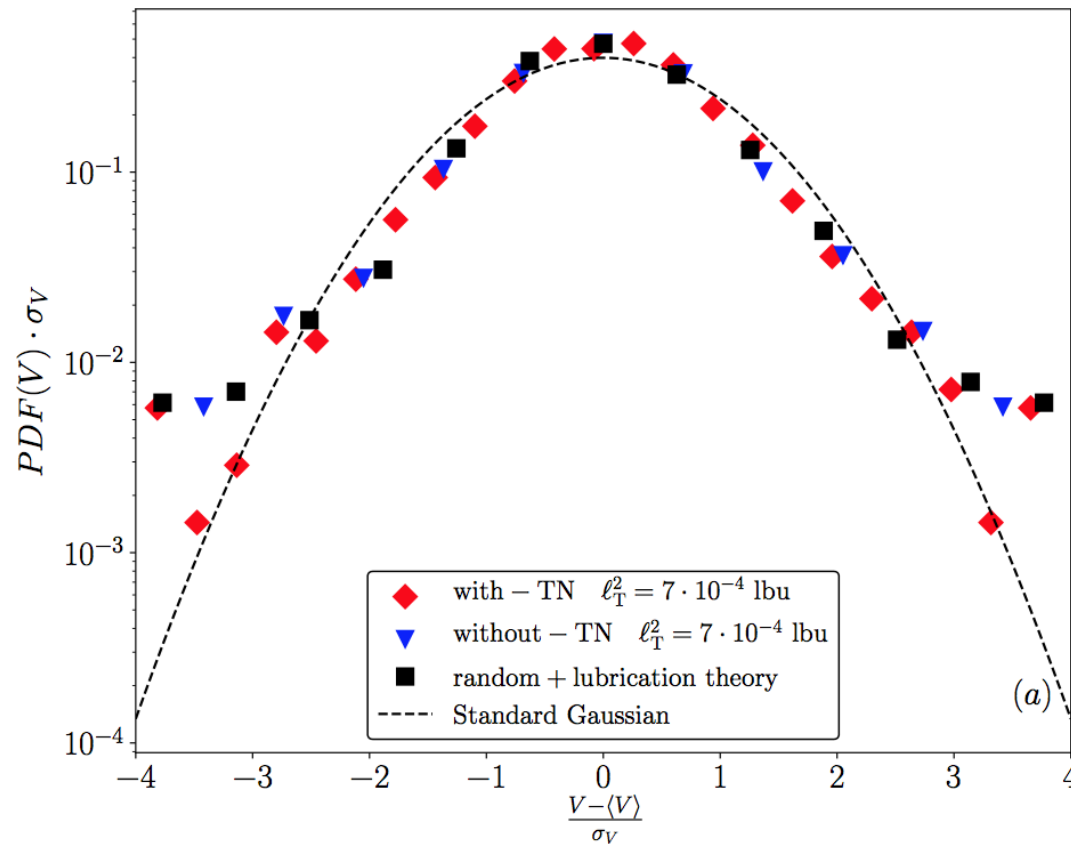
$$\partial_t v + v v' = -P' / \rho_l + 3\mu_l / \rho_l (h^2 v')' / h^2$$

$$P = \sigma \left[ \frac{1}{h(1 + (h')^2)^{\frac{1}{2}}} - \frac{h''}{(1 + (h')^2)^{\frac{3}{2}}} \right]$$


1. T. Driessen, R. Jeurissen, International Journal of Computational Fluid Dynamics, 2011

2. J Eggers, TF Dupont - Journal of fluid mechanics, 1994

# Thermal fluctuations amplified the droplet polydispersity



Normalized PDF for **lubrication theory, LBM, FLBM**

Comparison of **with-TN** and **without-TN** at different  $\ell_T^2$



# Summary and future plan

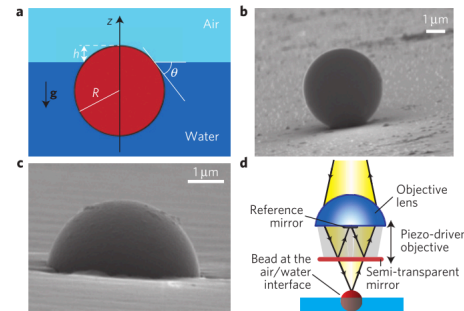
## Summary:

- ✓ We investigated the nano-ligament the by using fluctuating lattice Boltzmann method
- ✓ Thermal fluctuations can speed up the ligament break-up process
- ✓ Thermal fluctuations can amplify the droplets polydispersity



## Future work:

- Exploring nano-scale simulation with fluid-particle interaction



Giuseppe Boniello, et al. ,Nature material, 2015

*Thank you for your attention. Questions?*



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 European Union



# References

- [1] D Belardinelli, M Sbragaglia, L Biferale, M Gross, and F Varnik. Fluctuating multicomponent lattice boltzmann model. *Physical Review E*, 91(2):023313, 2015.
- [2] Sudhir Srivastava, JHM ten Thije Boonkkamp, and Federico Toschi. The lattice boltzmann method for contact line dynamics. 2011.
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- [4] S Van der Graaf, T Nisisako, C Schroen, RGM Van Der Sman, RM Boom. Lattice Boltzmann simulations of droplet formation in a T-shaped microchannel, *Langmuir* 22 (9), 4144-4152, 2006
- [5] K van Dijke, G Veldhuis, K Schroën, R Boom, Parallelized edge-based droplet generation (EDGE) devices, *Lab on a Chip* 9 (19), 2824-2830, 2009