Turbulence structure subjected to "precession-like" rotation

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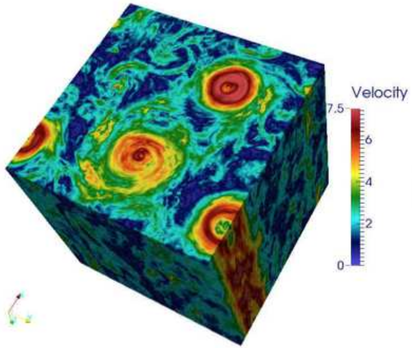

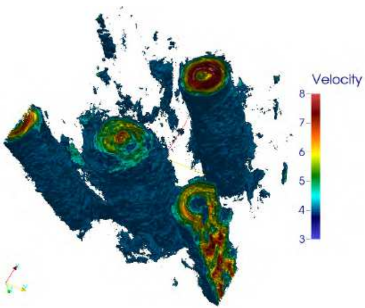

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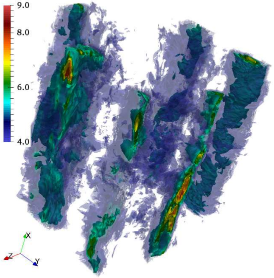
- Rotation effect on turbulence studied for fixed rotation axis
- Forward energy cascade weakened
  Inverse cascade amplified
- Quasi-2D-behavior: columnar structures along rotation axis



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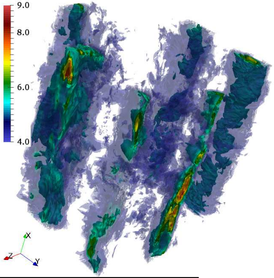

What happens if orientation of rotation axis is varied?

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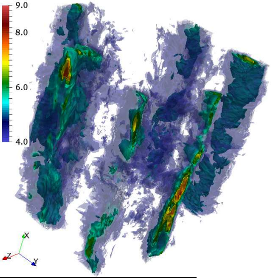
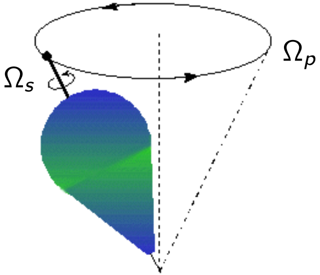


What happens if orientation of rotation axis is varied?

- Precession- rotational motion of spin axis of rotating body
- E.g. earth's precession period $= 26000$ years
- Weak Precession sustains turb. (*Malkus* '68, *Goto et. al.* '07):
  $\boxed{0 < \Gamma \ll 1}$ $\boxed{R_\lambda \equiv u' L/\nu \gg 1}$
- Geophysical applications: $\Gamma \sim O(10^{-7})$



Poincaré number $\boxed{\Gamma \equiv \Omega_p/\Omega_s}$

Equations

- Preliminary exploratory study!
- Co-ordinate system with angular velocity $\boldsymbol{\Omega} \equiv \boldsymbol{\Omega}(t)$:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} + \frac{d(\boldsymbol{\Omega} \times \mathbf{r})}{dt} = -\nabla p + \mathbf{f} + \nu \boldsymbol{\Delta} \mathbf{u} - \gamma \boldsymbol{\Delta}^{-1} \mathbf{u}$$

- γ: large scale damping constant; $\boldsymbol{\Delta}$: Laplacian operator
- Sub-volumes close to rotation axis: $\boxed{r \ll L}$
- $\left| d(\boldsymbol{\Omega} \times \mathbf{r})/dt \right| \ll |\boldsymbol{\Omega} \times \mathbf{u}|$ for $\boxed{0 < \Gamma \ll 1}$ and/or $\boxed{r \ll L}$.

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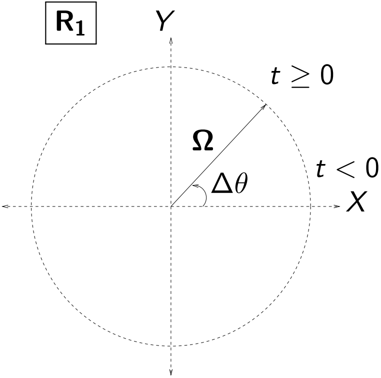
- Solve:

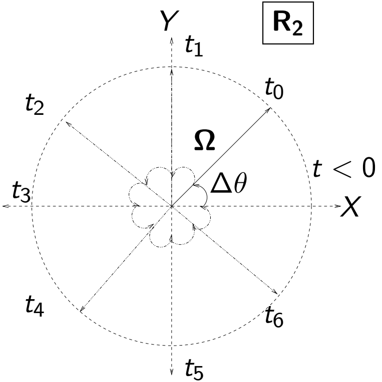
$$\boxed{\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -\nabla p + \mathbf{f} + \nu \mathbf{\Delta} \mathbf{u} - \gamma \mathbf{\Delta}^{-1} \mathbf{u}}$$

1024^3 DNS

- Pseudo-spectral algorithm, $(2\pi)^3$ box, Periodic B.C



$\Delta\theta = \pi/4$



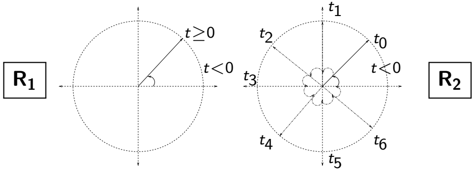
$t_i/\tau_\Omega = 10i \ , i = 0, 1, 2, \ldots$

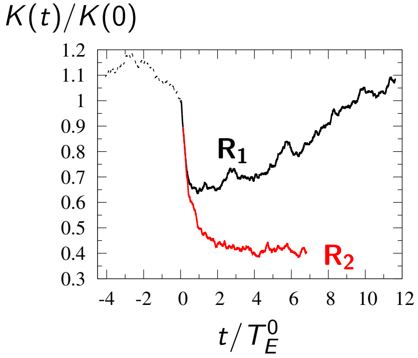
- I.C $(t < 0)$: $\mathbf{\Omega} = (10, 0, 0)$, $Ro \equiv \epsilon/(2K\Omega) = 0.0063$

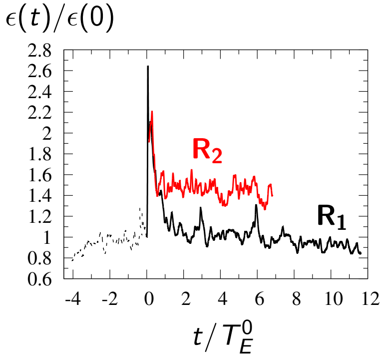
Mean kinetic energy (K), mean dissipation (ϵ)

$$K = \frac{1}{2}\langle u_i^2 \rangle$$

$$\epsilon = 2\nu \left\langle \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} \right\rangle$$

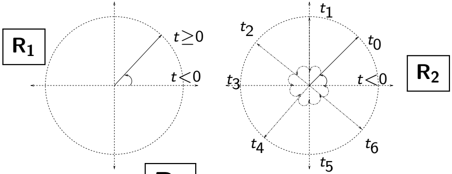






- Inverse cascade in $\mathbf{R_1}$ at late-time
- Larger ϵ in $\mathbf{R_2}$ $\Longrightarrow$ stronger spectral transfer down-scale

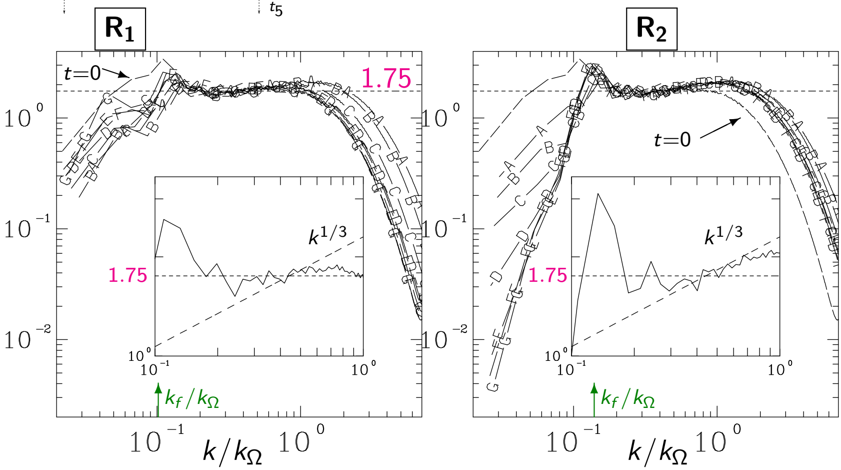
Energy spectrum $E(k,t) = C_\Omega(\epsilon(t)\Omega)^{1/2}k^{-2}$, $k_f \ll k \ll k_\Omega$



k_f forcing scale

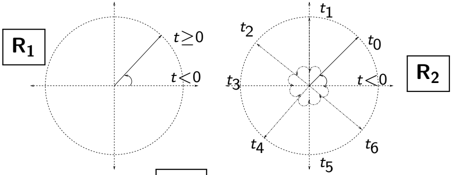
$k_\Omega = (\Omega^3/\epsilon)^{1/2}$

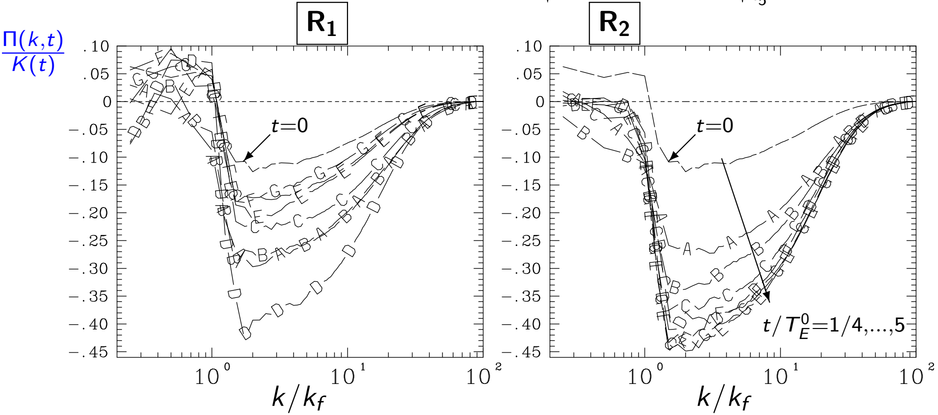
$A - G$: $t/T_E^0 = 1/4, 1/2, 1, 2, 3, 4, 5$



Spectral Flux

$$\Pi(k,t) \equiv \frac{d}{dt}\int_0^k E_c(p,t)\,dp$$

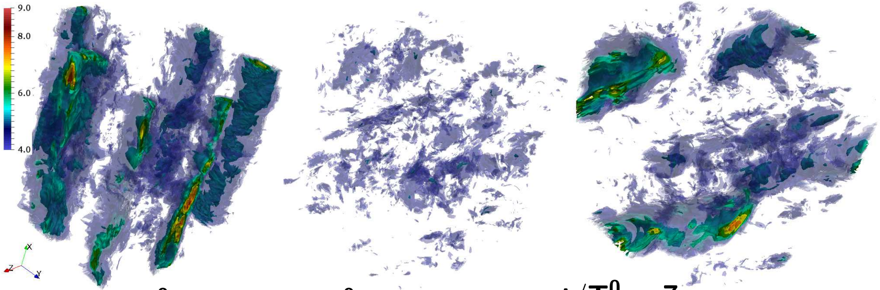




- $\mathbf{R_1}$: Positive flux $\Longrightarrow$ Inverse cascade
- $\mathbf{R_2}$: Negative flux $\Longrightarrow$ Forward cascade

Large scale structure (iso-contours of $\sqrt{K(t)}$)

R_1



$\mathbf{t/T_E^0 = 0}$ $\mathbf{t/T_E^0 = 1}$ $\mathbf{t/T_E^0 = 7}$

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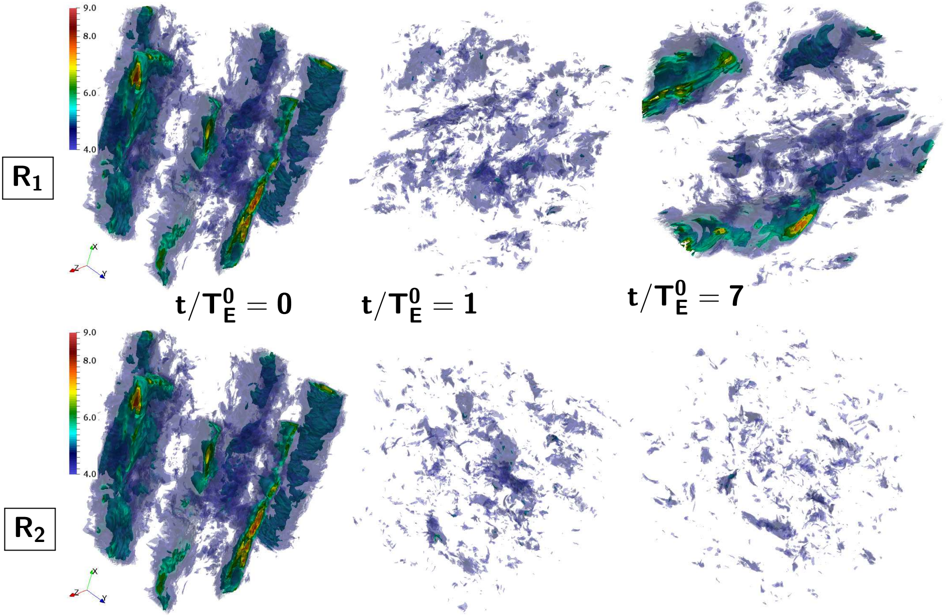

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- Neglected non-homogeneous term $d(\boldsymbol{\Omega} \times \mathbf{r})/dt$ in NSE
- Periodic B.C can cause large scale structures to wrap around lattice
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- For more details see *Iyer et. al. EPJE 2015* (available on `http://arxiv.org/abs/1511.06159`)

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