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Turbulence under rotation
Eulerian and Lagrangian statistics

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JOINT WORK

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PRACE 2015
ROTATING TURBULENCE

FLUID DYNAMICS UNDER ROTATION

ROTATING PLASMA
accretion discs physics


PHYSICAL REVIEW LETTERS

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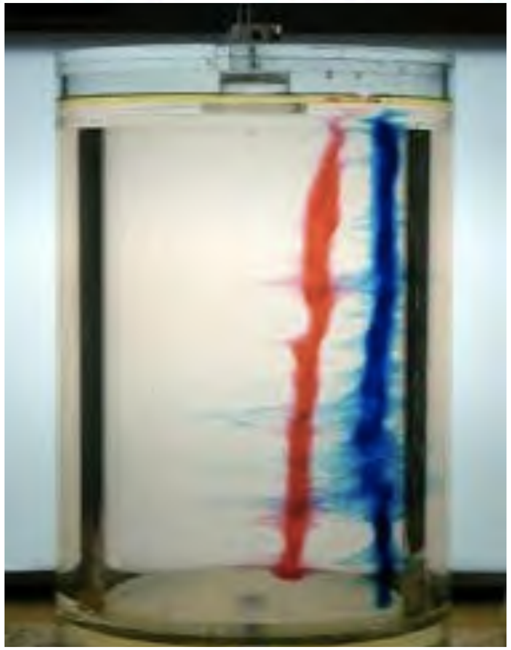
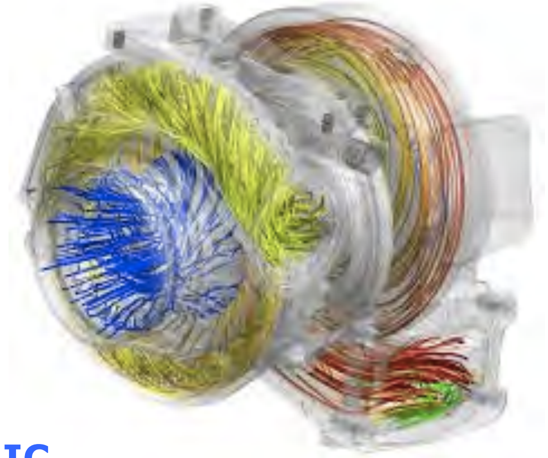
ROTATING CONVECTION



CYCLONIC-ANTICYCLONIC DYNAMICS

TURBOMACHINERY



INNER/OUTER PLANETARY DYNAMICS

GEOPHYSICAL ROTATING FLOWS

Earth rotation rate = $\boldsymbol{\Omega} = 7.2921 \times 10^{-5}$ rad/s
Coriolis parameter = $f = 2\,\Omega \sin(\varphi) = 10^{-4}$ rad/s at mid latitudes

Rossby number= Ro= V/(f L)
Ro small → *Coriolis forces dominate* *Ro ~ 0.1*
Ro large → *Inertial/centrifugal forces dominate* *Ro ~ 100-1000*

Reynolds number = Re = VL/ν
Re small → *viscous forces dominate* *Re ~ 1*
Re large → *Turbulence dominate* *Re ~* 10^{6} *-* 10^{10}

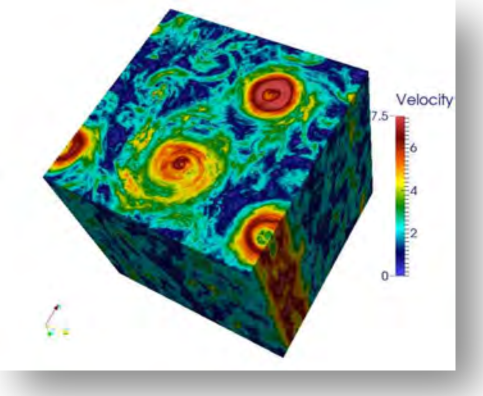
Tornado Ro ~ 1000
wind speeds are large, Re large

Ocean Ro ~ 0.01-0.1
current speeds are small, Re large

PLAN

DIRECT AND INVERSE KINETIC ENERGY TRANSFERS

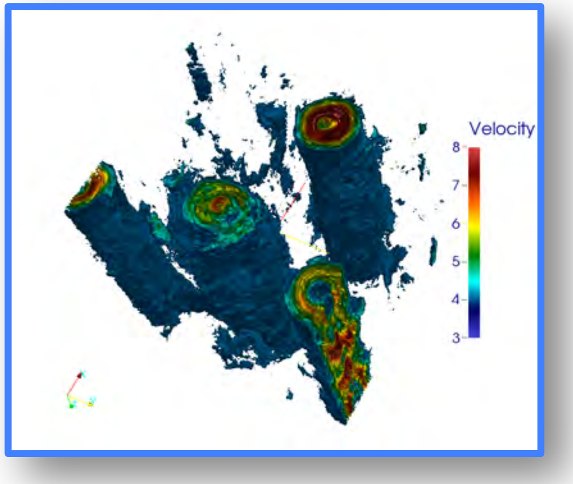
NUMERICAL EXPERIMENT : DNS@CINECA

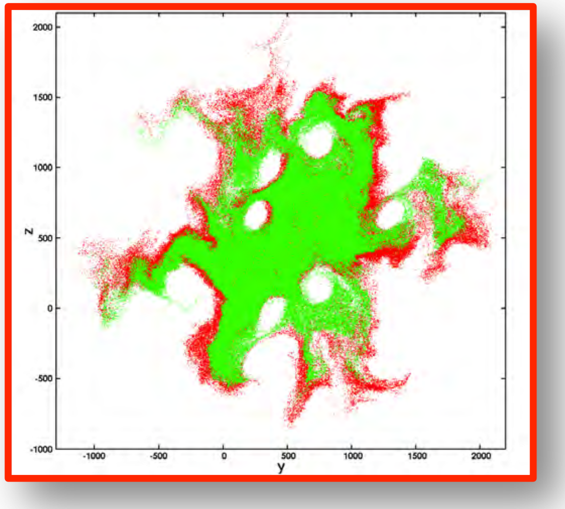


EULERIAN STATISTICS : i.e. measured at fixed spatial points

LAGRANGIAN STATISTICS : i.e. . measured along particle trajectories

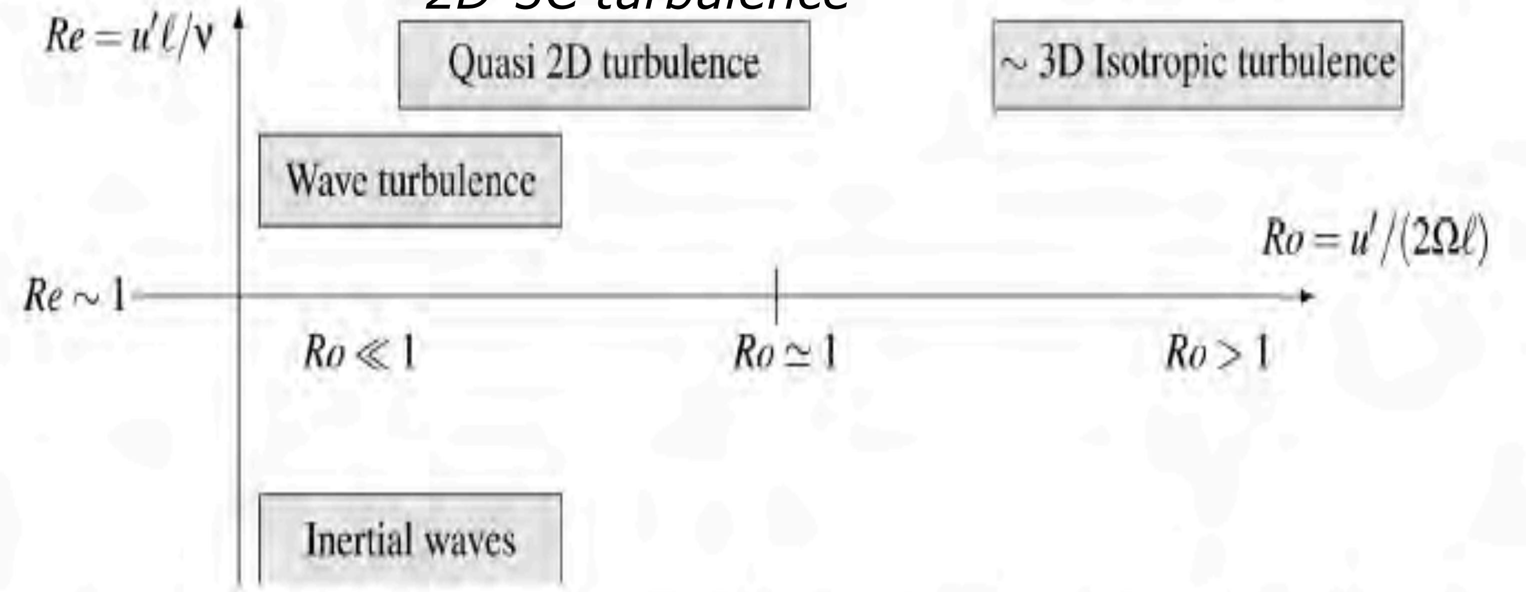
CONCLUSIONS





ROTATING TURBULENCE : *background*

Rossby-Reynolds parameter space



2D-3C turbulence

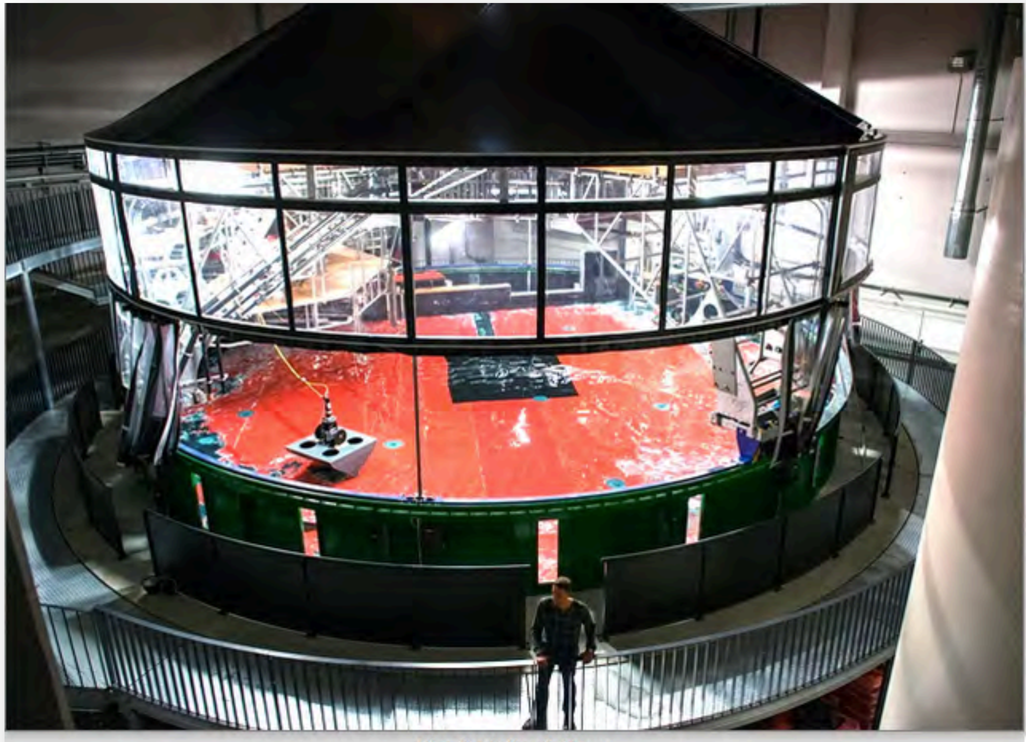
Fig. 1 Schematic of the different regimes of rotating turbulence in the Rossby–Reynolds parametric plane

(Godeferd & Moisy, Applied Mechanis Review 67, 2015)

ROTATING TURBULENCE : *lab experiment*

Coriolis Facility- LEGI, Grenoble

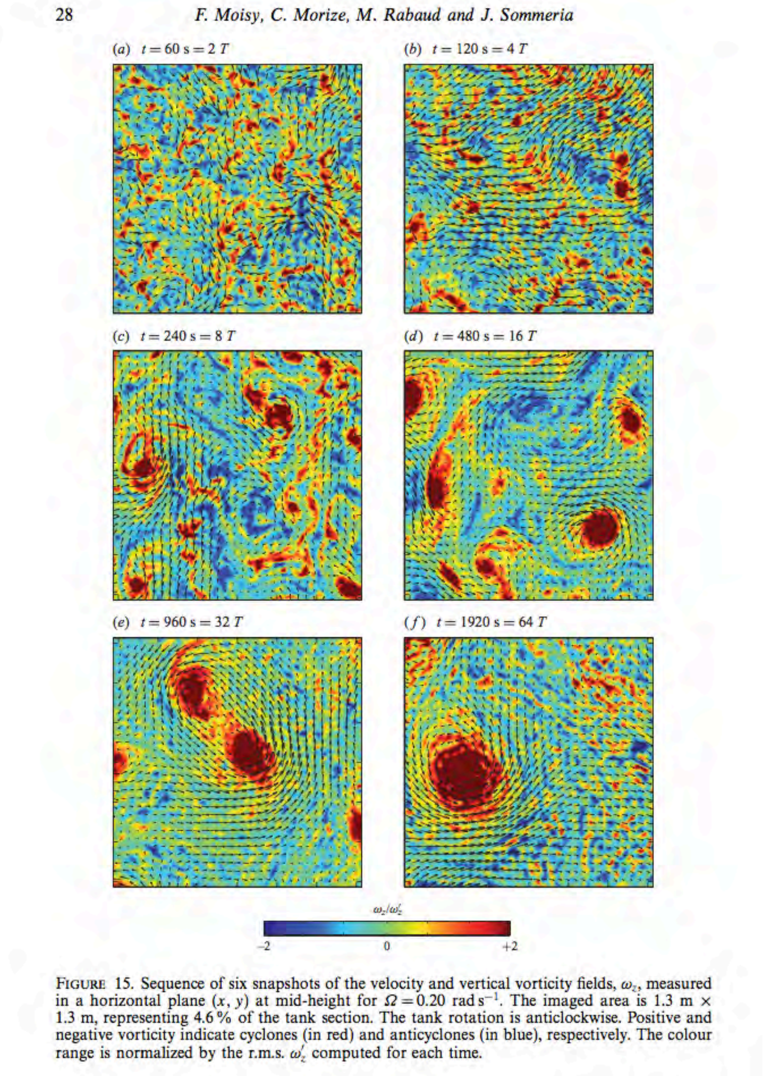
13 mt diameter – fluid dynamics largest rotating platform in the world



The Coriolis platform

Photothèque CNRS M. Fresillon

Decaying Experiment, J. Fluid Mech. 2011



28 F. Moisy, C. Morize, M. Rabaud and J. Sommeria

FIGURE 15. Sequence of six snapshots of the velocity and vertical vorticity fields, ω_z, measured in a horizontal plane (x, y) at mid-height for $\Omega = 0.20\ \mathrm{rad\,s^{-1}}$. The imaged area is 1.3 m × 1.3 m, representing 4.6 % of the tank section. The tank rotation is anticlockwise. Positive and negative vorticity indicate cyclones (in red) and anticyclones (in blue), respectively. The colour range is normalized by the r.m.s. ω_z' computed for each time.

Cyclone-Anticyclone Asymmetry

- Cyclones merge into large ones
- Anticyclones are weaker and less compact
- Background of vorticity fluctuations of random sign advected by large-scale mostly cyclonic vortices

Homogeneous Navier-Stokes eqs. in a Rotating Frame

(no walls or other boundaries)

$$\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} + 2\boldsymbol{\Omega}\times\mathbf{v} = -\nabla P + \nu\nabla^2\mathbf{v} + \mathbf{F} - \alpha\mathbf{v}$$

Coriolis force

$\boldsymbol{\Omega}$ = rotation

$$P = P_0 + \frac{1}{2}|\boldsymbol{\Omega}\times\mathbf{r}|^2$$

$\mathbf{F}$ = Large-scale Forcing

$\alpha =$ Large-scale energy sink

ROSSBY NUMBER ~ NON-LINEAR/ROTATION

$$Ro = \frac{V_0}{\Omega L_0}$$

$\mathrm{Ro} \geq Ro_c \rightarrow$ ***FORWARD KINETIC ENERGY CASCADE***

$\mathrm{Ro} \leq Ro_c \rightarrow$ ***FORWARD & BACKWARD ENERGY CASCADE***

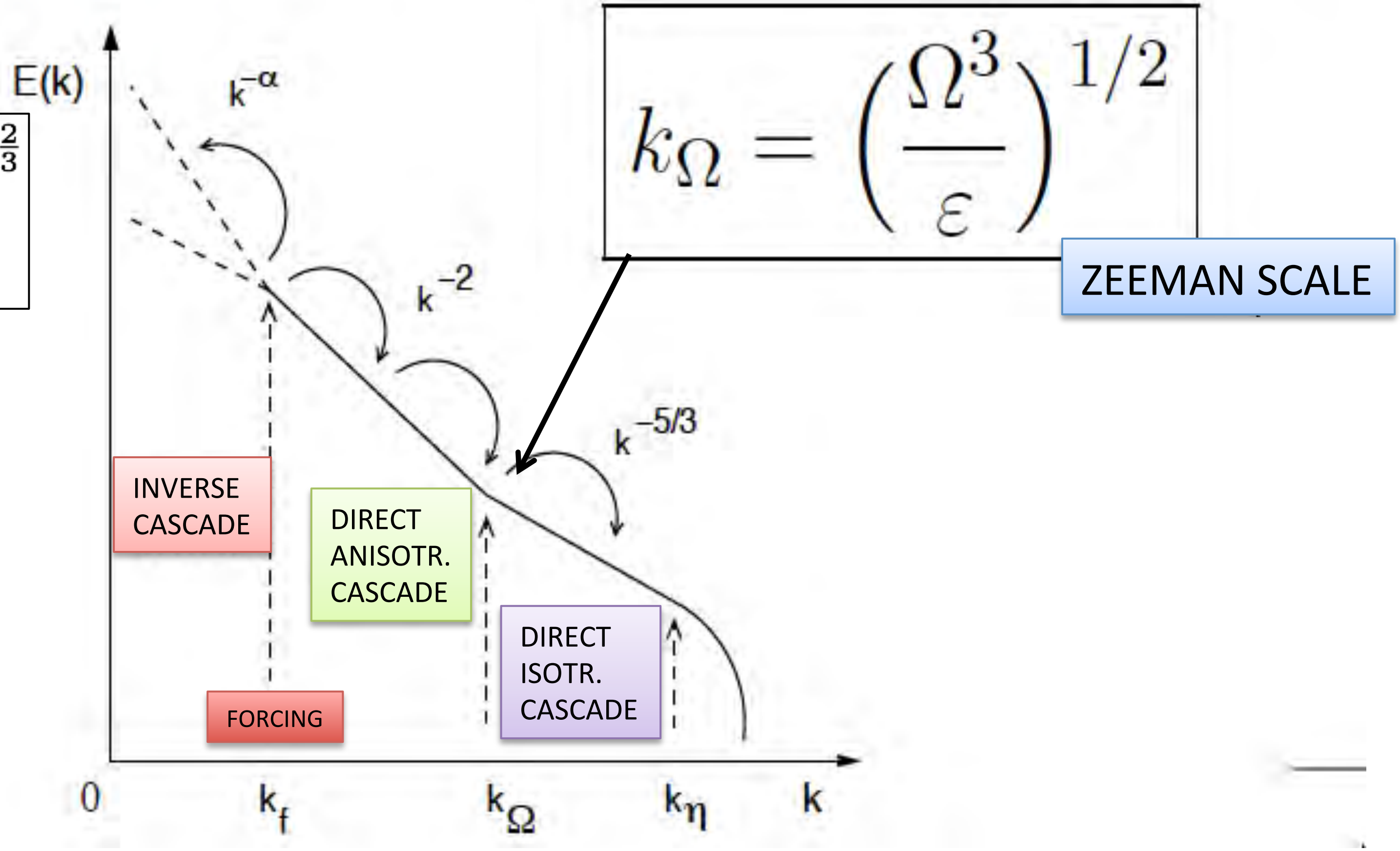
DIMENSIONAL ARGUMENT FOR THE ENERGY SPECTRUM BEHAVIOUR

- $k > k_\Omega \longrightarrow E(k) \sim \varepsilon^{2/3} k^{-5/3}$
- $k_f < k < k_\Omega \longrightarrow E(k) \sim (\varepsilon\Omega)^{1/2} k^{-2} \longleftrightarrow \boxed{\tau_{tr}(k) \sim \frac{\tau_{nl}(k)^2}{\tau_\Omega}}$
- $k < k_f \longrightarrow E(k) \sim \Omega^2 k^{-3}$? $E(k) \sim k^{-5/3}$

TWO TIME SCALES

$$\tau_{nl}(k) \sim \epsilon^{-\frac{1}{3}} k^{-\frac{2}{3}}$$

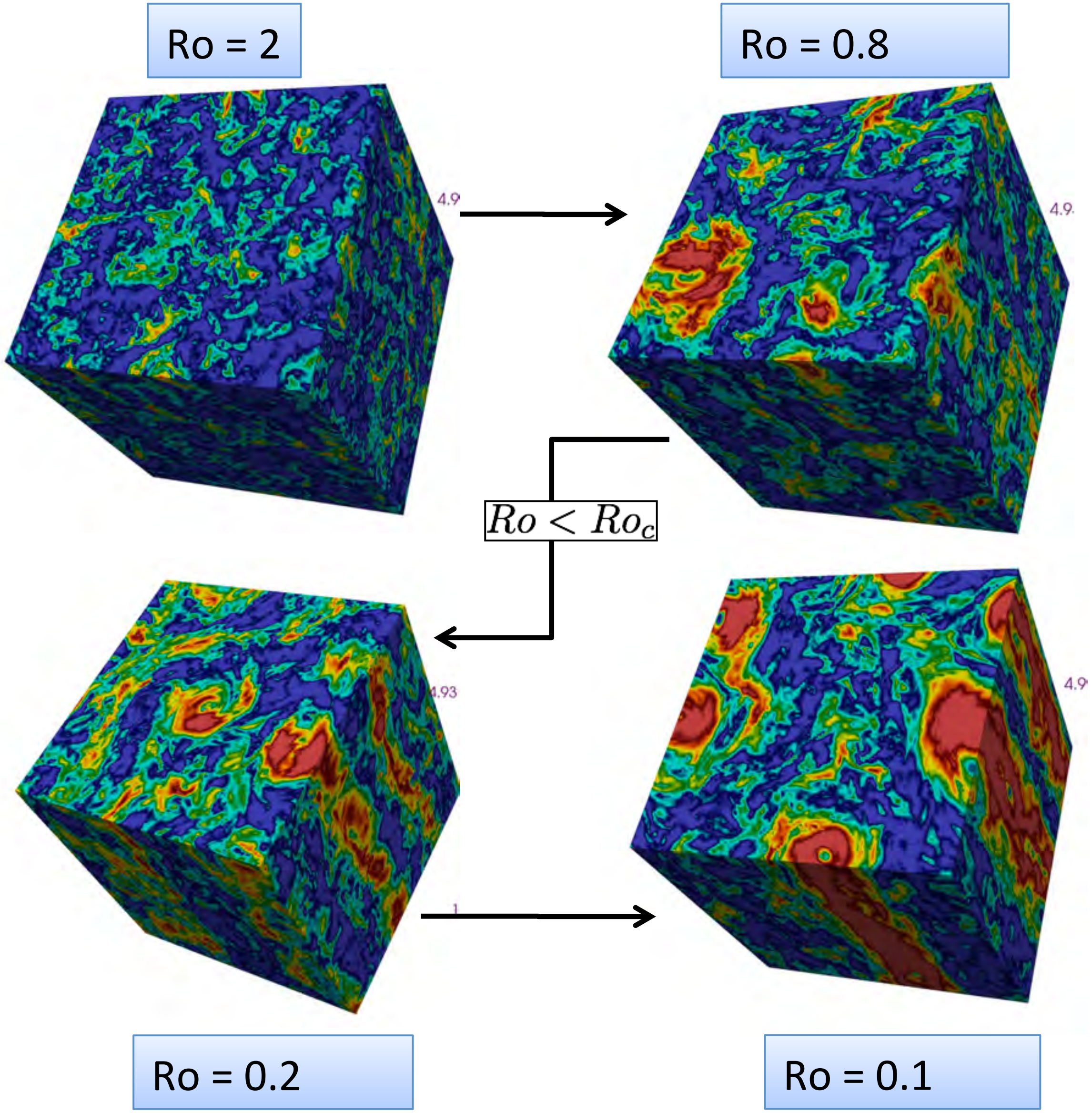
$$\tau_\Omega \sim 1/\Omega$$



$$k_\Omega = \left(\frac{\Omega^3}{\varepsilon}\right)^{1/2}$$

ZEEMAN SCALE

COEXISTENCE OF 2D AND 3D PHENOMENOLOGY IN HOMOGENEOUS ANISOTROPIC TURBULENCE



Ro = 2

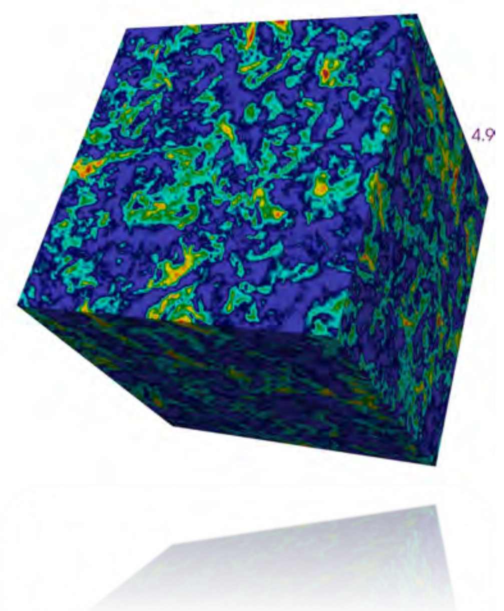
Ro = 0.8

$Ro < Ro_c$

Ro = 0.2

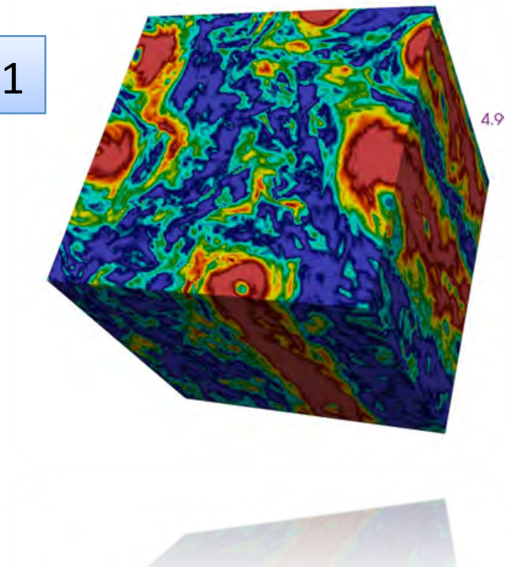
Ro = 0.1

MAIN QUESTIONS



Ro = 2

Ro = 0.1



How much **LARGE-SCALE STRUCTURES** influence
Eulerian and Lagrangian dynamics ?

Is their statistical signature **UNIVERSAL** ?

Can we disentangle the effects of
inertial waves - columnar structures - turbulent background ?

Does the flow become **less intermittent**
when it is fast rotating?

DNS@CINECA

KEY FEATURES:

1) **Ideal forcing mechanism**: Statistically Isotropic and Homogeneous (non helical), Gaussian, stochastic 2nd order Ornstein – Uhlebeck process (not white noise!)

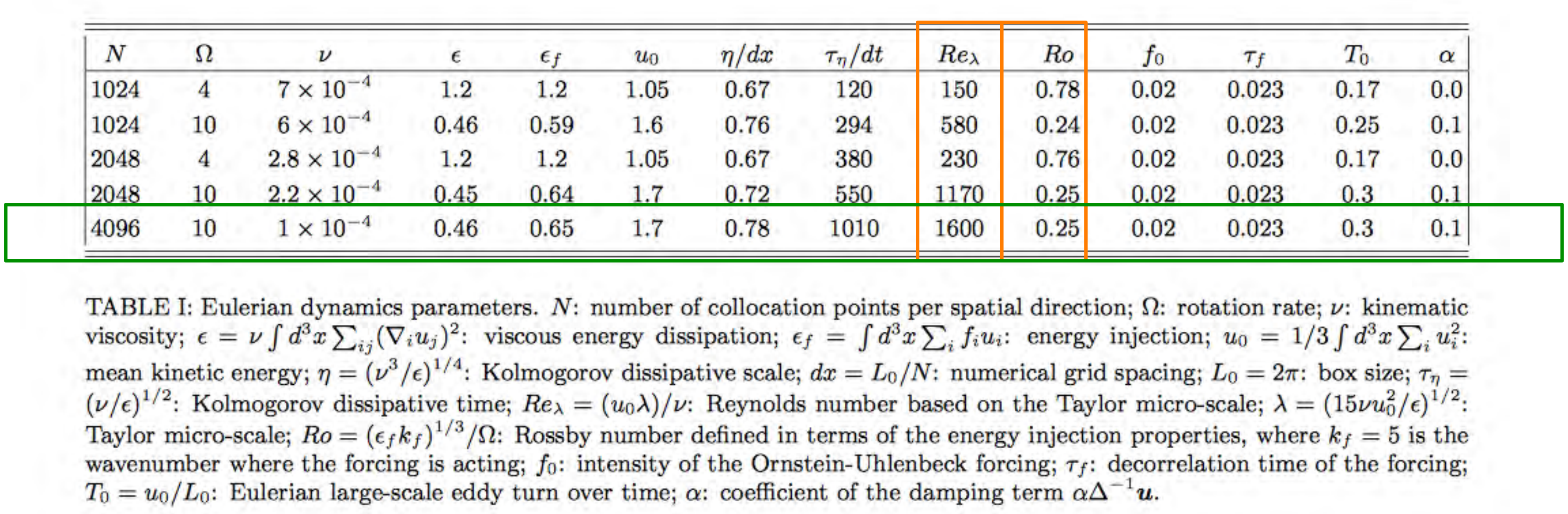
Code details

- Pseudo-Spectral
- Exact integration rotation
- Pencil FFT, HDF5 I/O
- Particles injected with different rotation axis →Memory/CPU balance

2) **Large-scale friction** to avoid pile-up of energy at large scales

3) **Unprecedented Numerical Resolution** : grid points up to $N^3 = 4096^3$

4) **Lagrangian Dynamics**: millions of tracer & light particles & heavy particles *(see it later)*



N	Ω	ν	ϵ	ϵ_f	u_0	η/dx	τ_η/dt	Re_λ	Ro	f_0	τ_f	T_0	α
1024	4	7×10^{-4}	1.2	1.2	1.05	0.67	120	150	0.78	0.02	0.023	0.17	0.0
1024	10	6×10^{-4}	0.46	0.59	1.6	0.76	294	580	0.24	0.02	0.023	0.25	0.1
2048	4	2.8×10^{-4}	1.2	1.2	1.05	0.67	380	230	0.76	0.02	0.023	0.17	0.0
2048	10	2.2×10^{-4}	0.45	0.64	1.7	0.72	550	1170	0.25	0.02	0.023	0.3	0.1
4096	10	1×10^{-4}	0.46	0.65	1.7	0.78	1010	1600	0.25	0.02	0.023	0.3	0.1

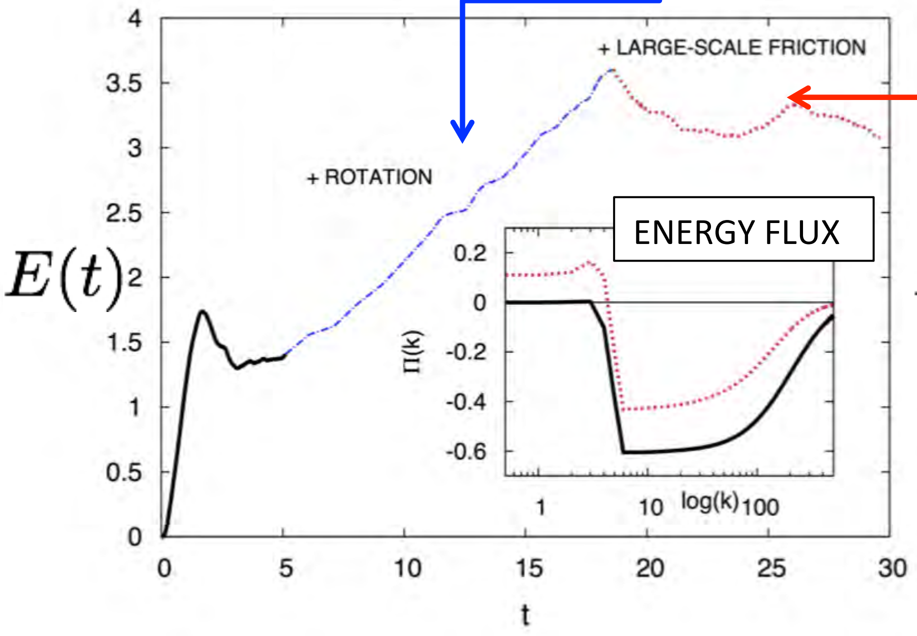
TABLE I: Eulerian dynamics parameters. N: number of collocation points per spatial direction; Ω: rotation rate; ν: kinematic viscosity; $\epsilon = \nu \int d^3x \sum_{ij} (\nabla_i u_j)^2$: viscous energy dissipation; $\epsilon_f = \int d^3x \sum_i f_i u_i$: energy injection; $u_0 = 1/3 \int d^3x \sum_i u_i^2$: mean kinetic energy; $\eta = (\nu^3/\epsilon)^{1/4}$: Kolmogorov dissipative scale; $dx = L_0/N$: numerical grid spacing; $L_0 = 2\pi$: box size; $\tau_\eta = (\nu/\epsilon)^{1/2}$: Kolmogorov dissipative time; $Re_\lambda = (u_0\lambda)/\nu$: Reynolds number based on the Taylor micro-scale; $\lambda = (15\nu u_0^2/\epsilon)^{1/2}$: Taylor micro-scale; $Ro = (\epsilon_f k_f)^{1/3}/\Omega$: Rossby number defined in terms of the energy injection properties, where $k_f = 5$ is the wavenumber where the forcing is acting; f_0: intensity of the Ornstein-Uhlenbeck forcing; τ_f: decorrelation time of the forcing; $T_0 = u_0/L_0$: Eulerian large-scale eddy turn over time; α: coefficient of the damping term $\alpha\Delta^{-1}\boldsymbol{u}$.

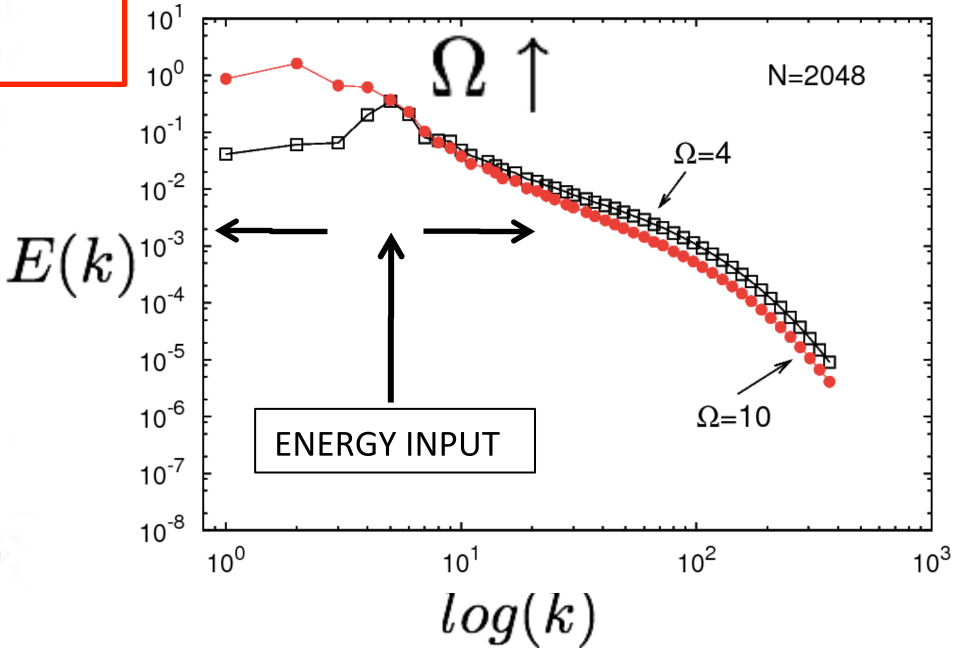
Steady State in Rotating Navier-Stokes

$$\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} + 2\boldsymbol{\Omega}\times\mathbf{v} = -\nabla P + \nu\nabla^2\mathbf{v} + \mathbf{F} - \alpha\mathbf{v}$$

ROTATION

DAMPING:
ECKMAN
FRICTION

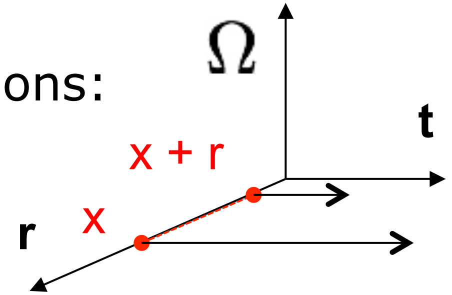




A WAY to DISENTANGLE TURBULENCE from VORTICES

Decompose the total VELOCITY field v into a two-dimensional with 3 components field + 3D fluctuations:

$$\mathbf{v}_{TOT}(x,y,z|t) = \bar{\mathbf{v}}_{2D}(x,y|t) + \mathbf{v}'(x,y,z|t)$$

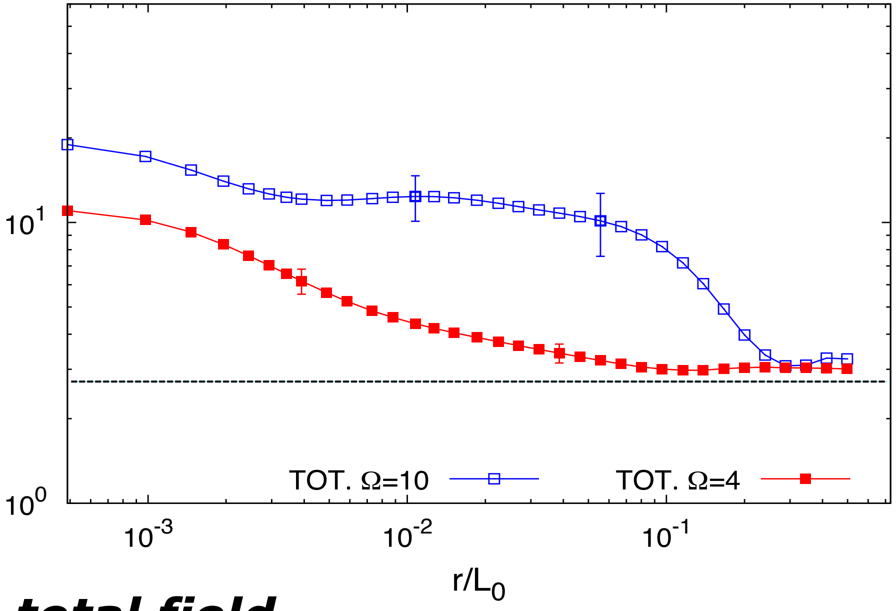


And measure transverse, perpendicular moments:

$$S_{\perp}^{(n)} = \langle [(\mathbf{v}(\mathbf{x}+\mathbf{r}) - \mathbf{v}(\mathbf{x})) \cdot \hat{\mathbf{t}}]^n \rangle$$

FLATNESS

$$\boxed{K_{\perp}^{(4)}(r) \equiv \frac{S_{\perp}^{(4)}(r)}{(S_{\perp}^{(2)}(r))^2}}$$



Much above Gaussian value,
but No scaling if we consider the total field

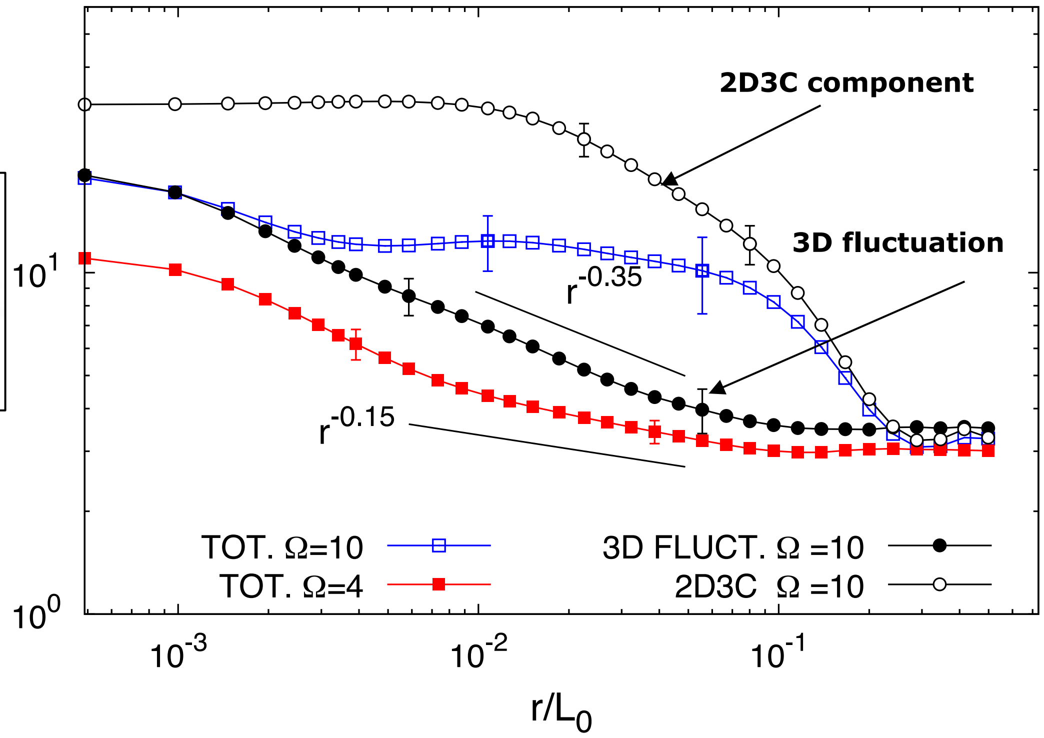
INTERMITTENT BEHAVIOUR

2D3C mean field 3D fluctuating field

$$\mathbf{v}(x,y,z|t) = \bar{\mathbf{v}}_{2D}(x,y|t) + \mathbf{v}'(x,y,z|t)$$

$$S_{\perp}^{(n)} = \langle[(\mathbf{v}(\mathbf{x}+\mathbf{r}) - \mathbf{v}(\mathbf{x}))\cdot\hat{\mathbf{t}}]^n\rangle$$

$$\boxed{K_{\perp}^{(4)}(r) \equiv \frac{S_{\perp}^{(4)}(r)}{(S_{\perp}^{(2)}(r))^2}}$$



- NON-GAUSSIAN properties need to be assessed with proper decomposition
- After filtering the 2D3C component, SCALE INTERMITTENT DEPENDANCE is back and differs from non rotating turbulence

NEWTON LAW FOR INERTIAL PARTICLES – *1 way coupling*

A small spherical partical of radius R and density ρ_p
moving in a fluid of velocty u & density ρ_f

NEW

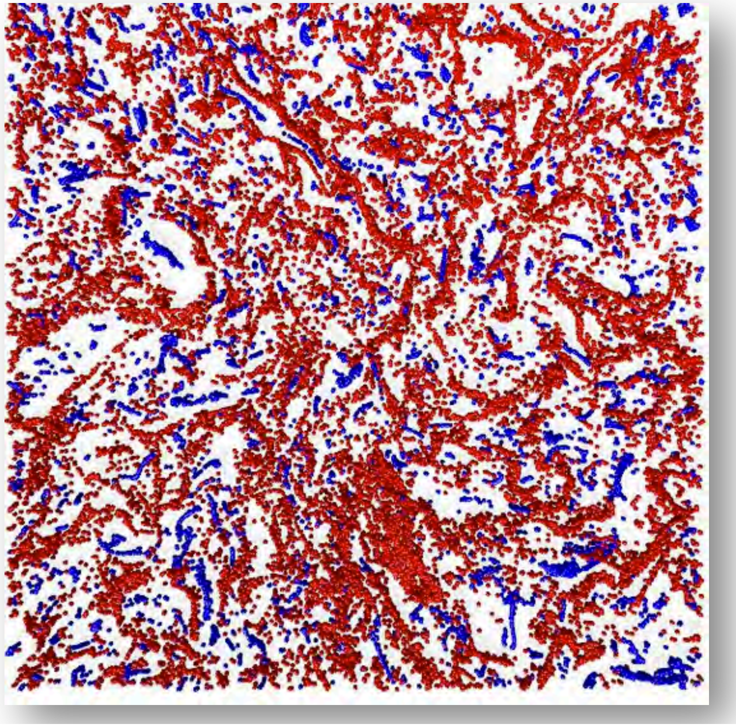
$$\frac{d\mathbf{v}}{dt} = \underbrace{\beta \frac{D\mathbf{u}}{Dt}}_{\text{added mass}} \underbrace{- \frac{1}{\tau_p}(\mathbf{v} - \mathbf{u})}_{\text{Stokes drag}} + \underbrace{2(\mathbf{v} - \beta \mathbf{u}) \times \mathbf{\Omega}}_{\text{Coriolis force}} \underbrace{- (1 - \beta)\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})}_{\text{centripetal force}}$$

$$\boxed{\beta = \frac{3\rho_f}{2\rho_p + \rho_f}}$$

$\beta < 1$ **heavy particles**
$\beta > 1$ **light particles**

$$\boxed{St = \frac{\tau_p}{\tau_\eta}} \quad \tau_p = \frac{R^2}{3\nu\beta}$$

Clustering of inertial particles in
Homog. Isotr. Turbulence (***Ω = 0***)



HOW MUCH ROTATION MODIFIES CLUSTERING ?

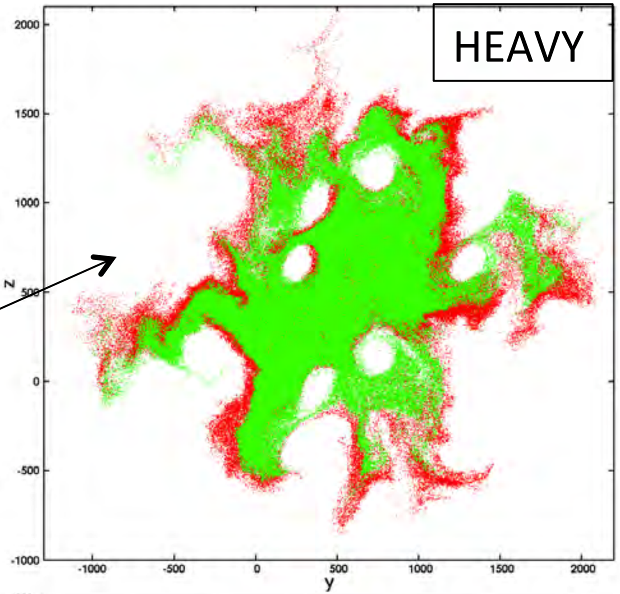
STRATEGY

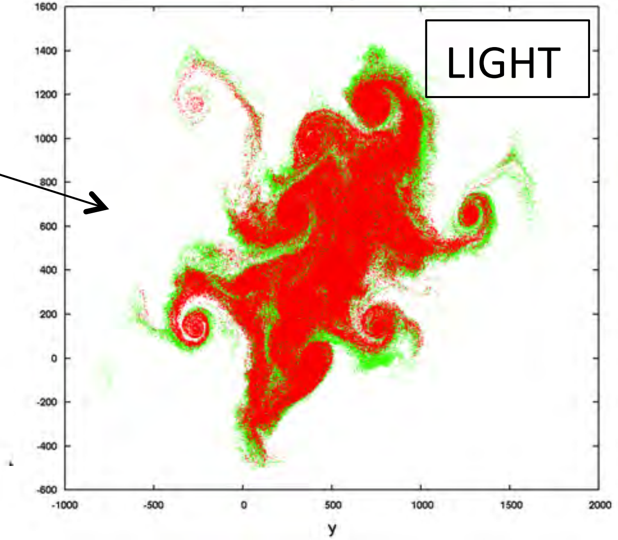
128 INJECTION AXIS at random in the flow

10^5 PARTICLES PER INJECTION, PER FAMILY

+ 10^6 PARTICLES PER FAMILY at random pos.

family	β	St	type
1	0.4	0.3	Heavy
2	0.4	0.7	
3	0.8	0.3	
4	0.8	0.7	
5	1.2	0.3	Light
6	1.2	0.7	
7	1.6	0.3	
8	1.6	0.7	
9	1.6	1	
10	1.6	5	





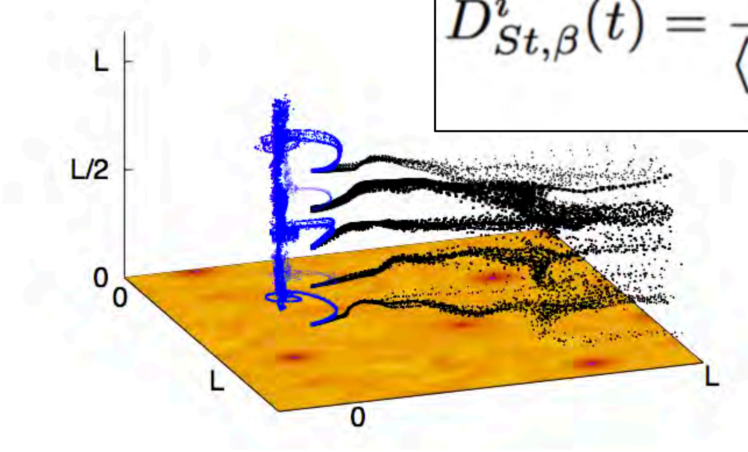
$$\frac{d\mathbf{v}}{dt} = \beta\frac{D\mathbf{u}}{Dt} - \frac{1}{\tau_p}(\mathbf{v}-\mathbf{u}) + 2(\mathbf{v}-\beta\mathbf{u})\times\boldsymbol{\Omega} - (1-\beta)\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})$$

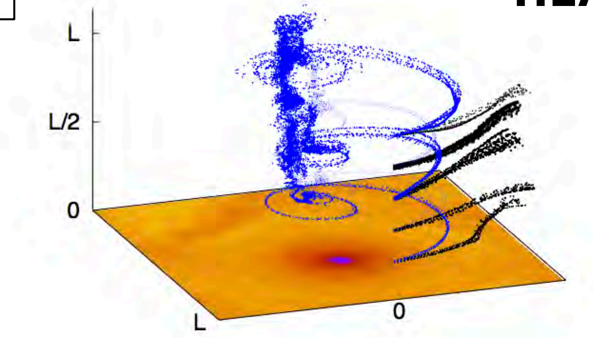
Centrifugal force effect depends on (β -1)

Centrifugal for heavy particles, centripetal for light particles

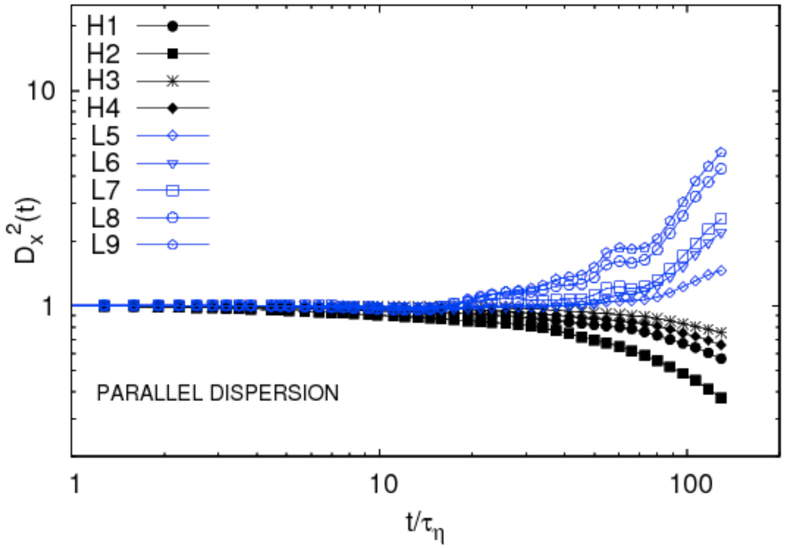
ROTATION SINGULAR EFFECT ON PARTICLES DIFFUSION: lift/splash effect

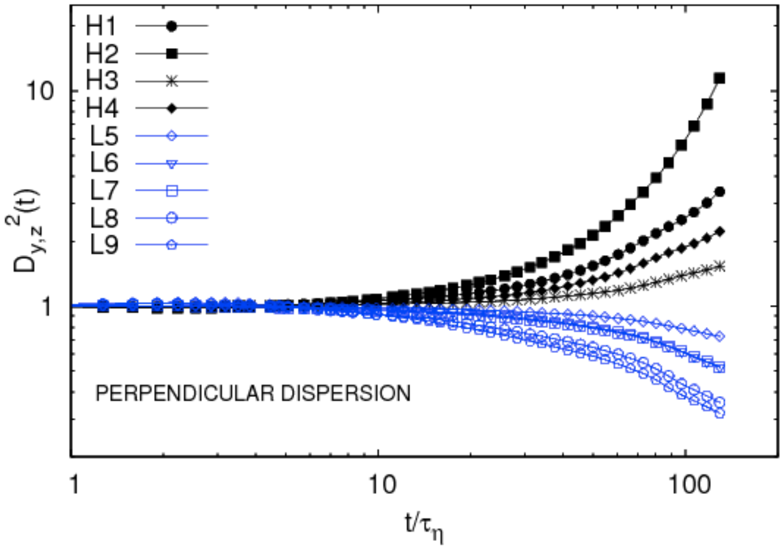
$$D^{i}_{St,\beta}(t) = \frac{\langle (r^{i}_{t} - r^{i}_{0})^{2} \rangle_{St,\beta}}{\langle (r^{i}_{t} - r^{i}_{0})^{2} \rangle_{tracer}}$$





LIGHT
HEAVY

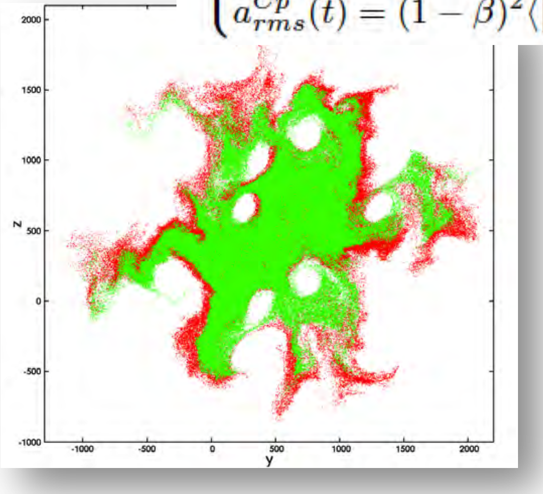




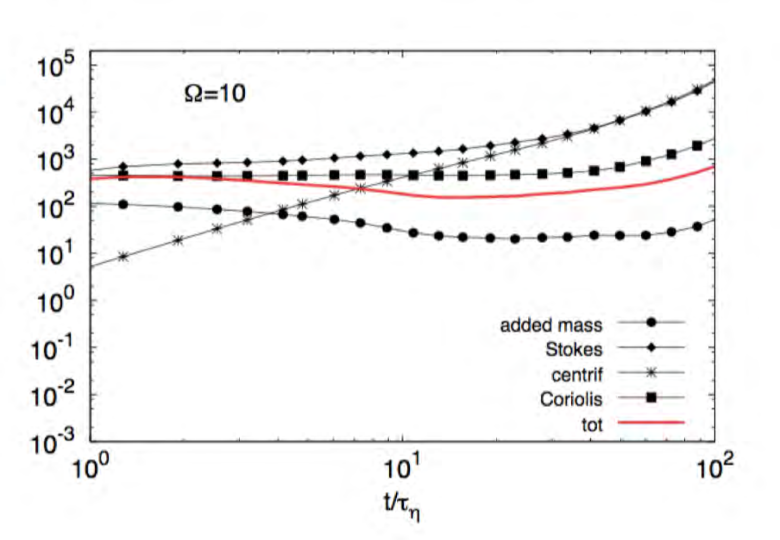
Light particles are ***lifted*** up and never explore the flow
Heavy particles are ***splashed*** into planes perpendicular to the rotation

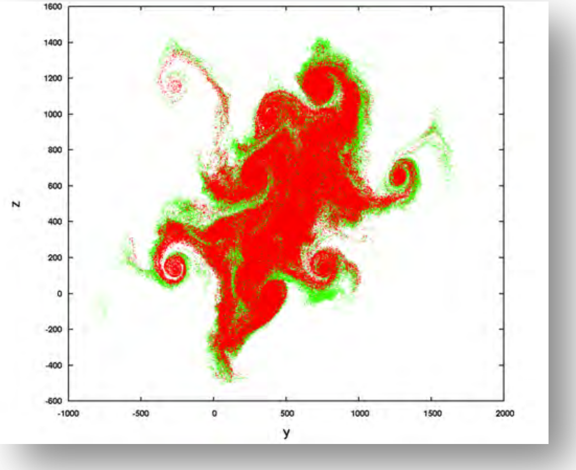
DETAILED DYNAMICAL ANALYSIS

$$\begin{cases} a^{tot}_{rms}(t) = \langle \dot{v}^2 \rangle; & \text{total} \\ a^{am}_{rms}(t) = \beta^2 \langle (D_t \mathbf{u})^2 \rangle; & \text{added mass} \\ a^{St}_{rms}(t) = 1/\tau_p^2 \langle (\mathbf{v} - \mathbf{u})^2 \rangle; & \text{Stokes drag} \\ a^{Co}_{rms}(t) = 4 \langle [\mathbf{\Omega} \times (\mathbf{v} - \beta \mathbf{u})]^2 \rangle; & \text{Coriolis} \\ a^{Cp}_{rms}(t) = (1-\beta)^2 \langle [\mathbf{\Omega} \times (\mathbf{\Omega} \times (\mathbf{r}_t - \mathbf{r}_0)]^2 \rangle; & \text{centripetal.} \end{cases}$$

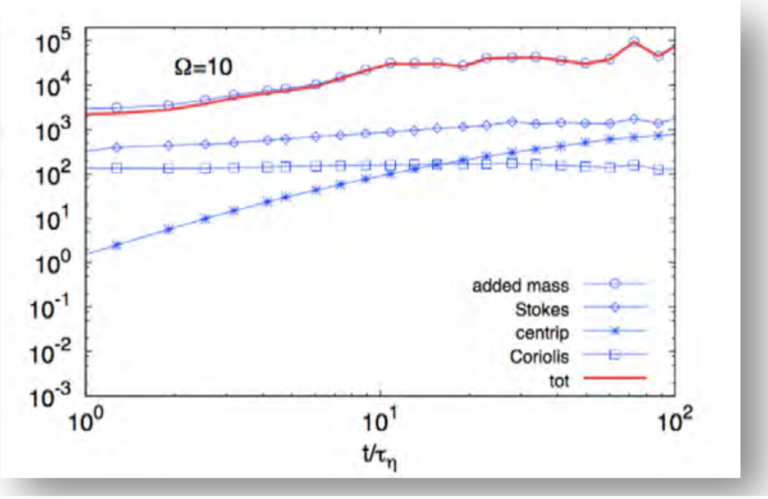


HEAVY





LIGHT



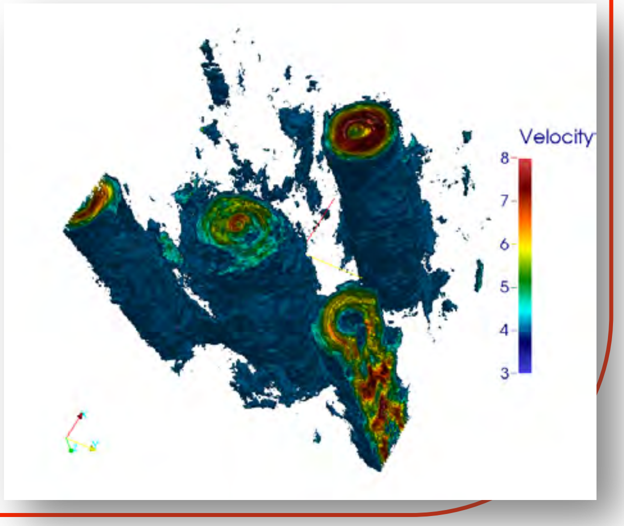
SUMMARY

Within PRACE (55M hours), we performed a
HIGH-RESOLUTION ROTATING TURBULENCE STUDY

First DNS able to SIMULTANEOUSLY control
both **EULERIAN & LAGRANGIAN STATISTICS**

Accurate, Ideal SET-UP for Stationary, Rotating Turbulence
Homogeneous & Isotropic, time-colored forcing
SCALE-SEPARATION achieved by forcing intermediate modes

At lo Ro, we generally observe
breaking of cyclone/antycyclone symmetry
with few (three) equal sign vortices.
Merging is not observed, but we can not exclude



CONCLUSIONS

Rotation affects Vortical Structures & background turbulence:
Universality of the statistics is not yet clear

Eulerian statistics show deviations from Gaussianity:
crucial the way Intermittency is measured (2D3C vs. 3D stat)

Vortical structures leave their signature on Lagrangian dynamics:
inertial particles are natural probes for rotating flows

Coherent structures and extreme events in rotating multiphase turbulent flows

L. Biferale, F. Bonaccorso, I. Mazzitelli, A. S. Lanotte, S. Musacchio, P. Perlekar, F. Toschi, M. van Hinsberg,

Phys. Rev. X submitted 2016