Eulerian and Lagrangian turbulence on fractal Fourier set

EXPERIMENTS IN-SILICO:
CAN WE ASK QUESTIONS ABOUT THE ENERGY TRANSFER EVENTS
(BOTH TYPICAL AND EXTREME)
BY DECIMATING INTERACTIONS IN THE NON LINEAR TERM?

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Turbulence

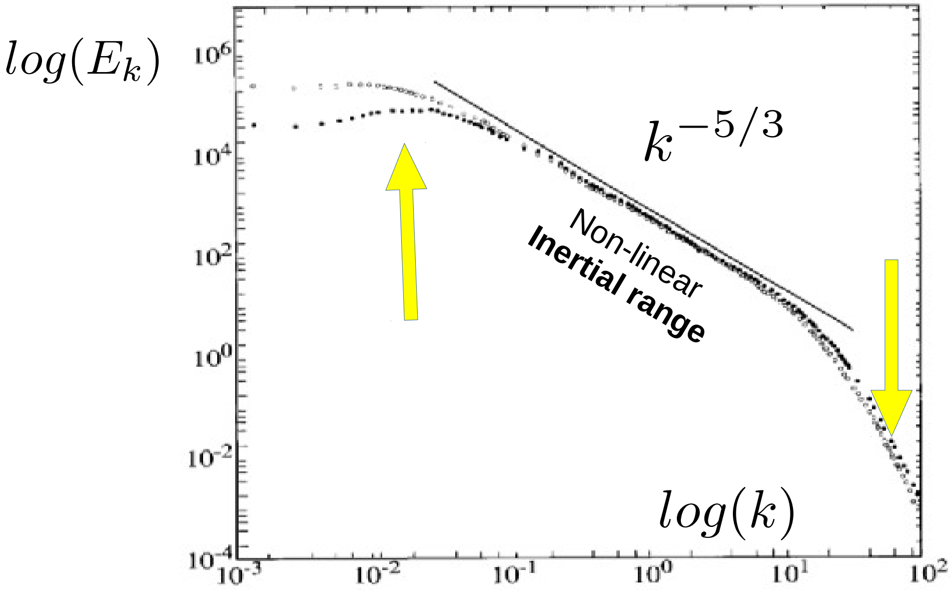
$$\begin{cases} \frac{\partial \bar{u}(\bar{x},t)}{\partial t} + \bar{u}(\bar{x},t)\cdot\nabla_{\bar{x}}\bar{u}(\bar{x},t) = -\nabla_{\bar{x}}p(\bar{x},t) + \nu\Delta_{\bar{x}}\bar{u}(\bar{x},t) + \bar{f}(\bar{x},t) \\ \nabla_{\bar{x}}\cdot\bar{u}(\bar{x},t) = 0 \\ +Boundary\ Conditions \end{cases}$$

$$\begin{cases} \hat{t} = t/t_0 \\ \hat{x} = x/l_0 \\ \hat{u} = u/u_0 \end{cases} \qquad \partial_t\hat{u} + \hat{u}\cdot\partial_{\hat{x}}\hat{u} = -\partial_{\hat{x}}\hat{P} + \frac{1}{Re}\partial^2_{\hat{x}}\hat{u} + \hat{f}(\hat{x},\hat{t})$$

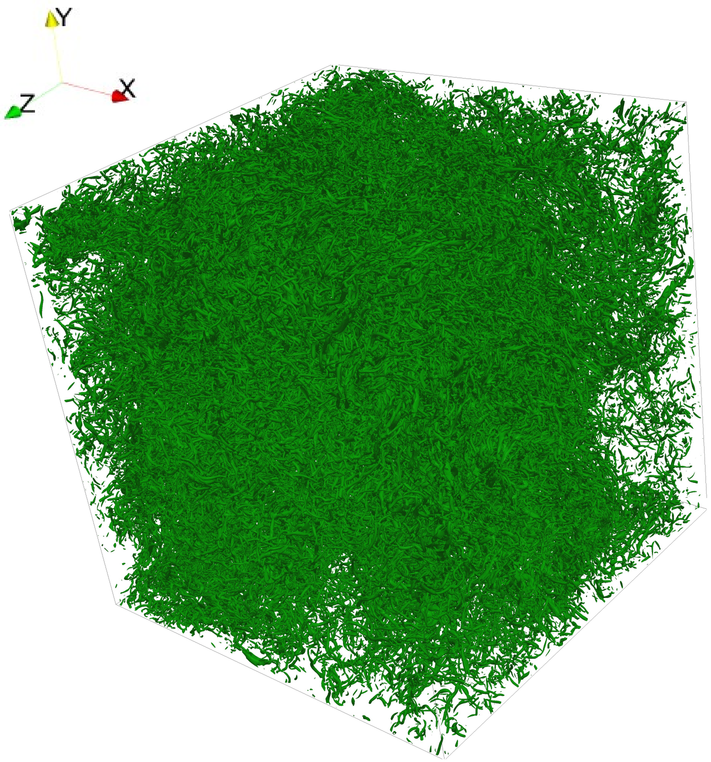
$$Re = \frac{l_0 u_0}{\nu} \qquad Re \sim \frac{\hat{u}\partial\hat{u}}{\nu\partial^2\hat{u}}$$

..fully developed

$$Re \to \infty$$



Inertial Range Intermittency

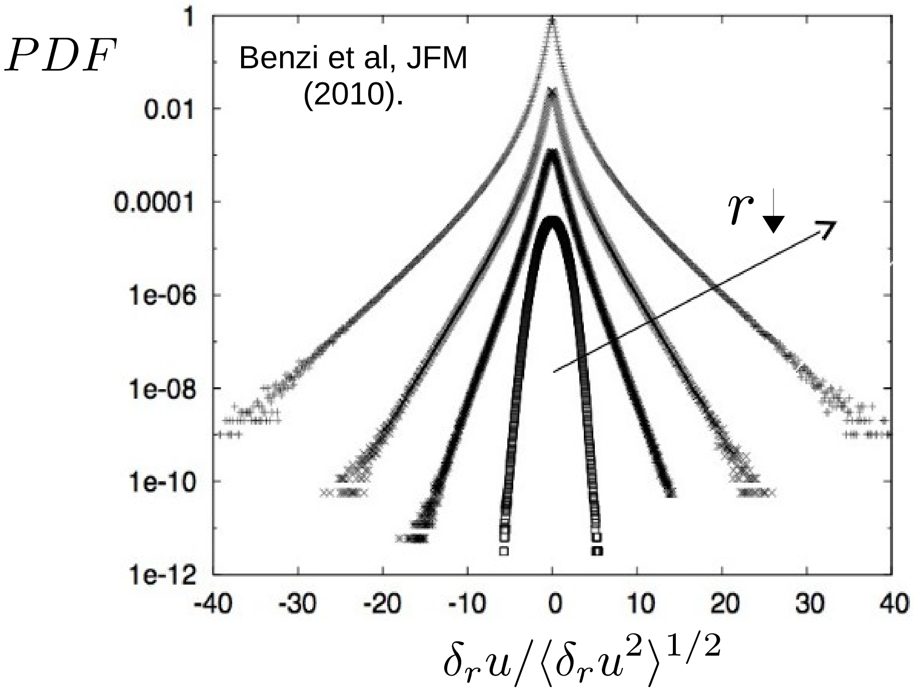


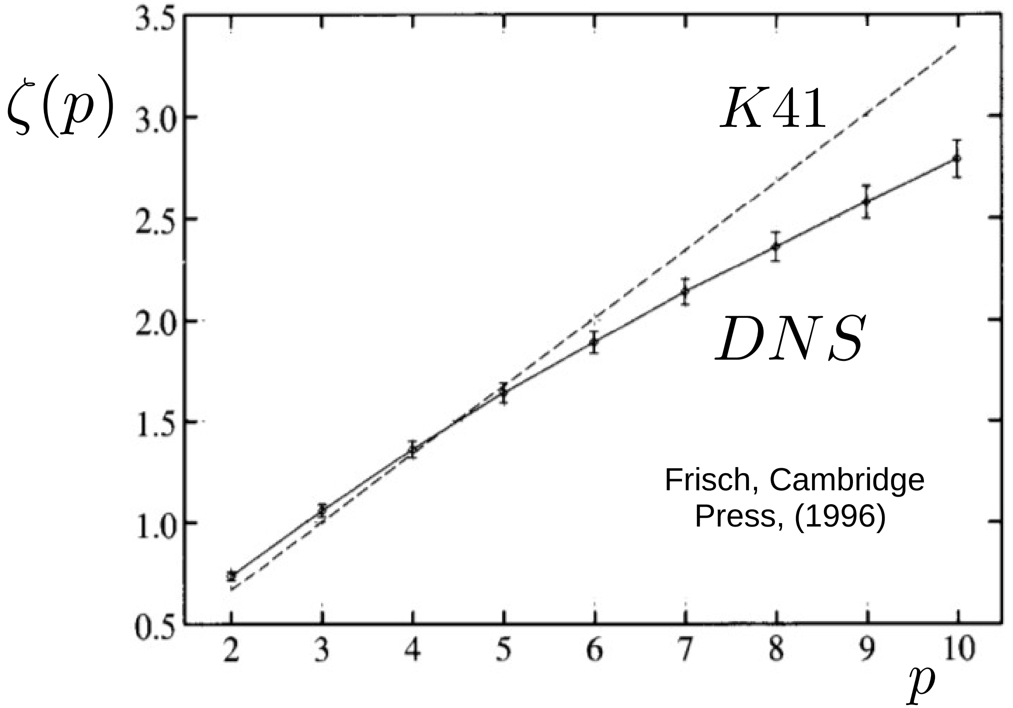
$$\delta_r u = (\mathbf{u}(\mathbf{x}+r) - \mathbf{u}(\mathbf{x}))\cdot\mathbf{r}$$

$$S_p(r) = \langle(\delta_r u)^p\rangle$$

$\sim r^{\frac{p}{3}}$ **K41**

$\sim r^{\zeta(p)}$ **numerics + experiments**





Extreme events in computational turbulence

P. K. Yeung[a], X. M. Zhai[b], and Katepalli R. Sreenivasan[c,1]

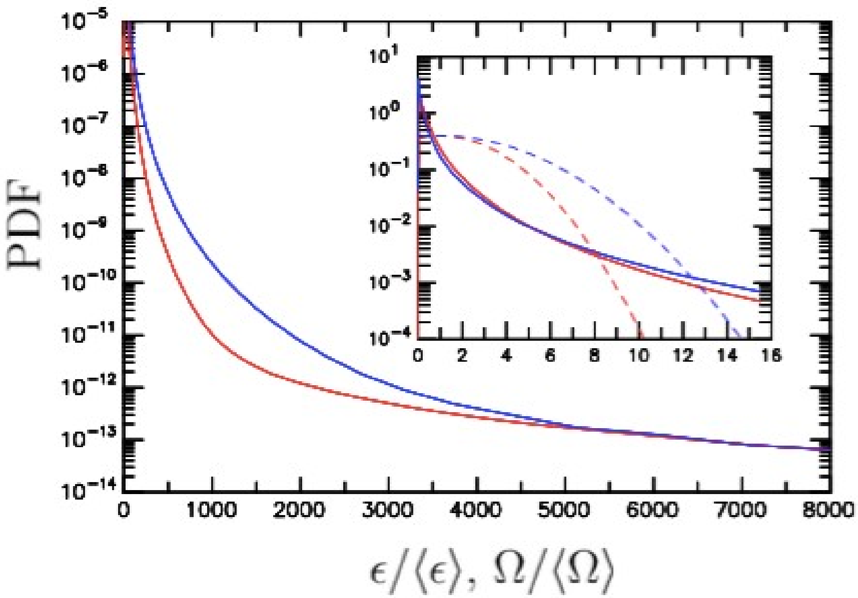


Fig. 1. Ensemble-averaged PDFs of normalized dissipation (red) and enstrophy (blue) from $8{,}192^3$ simulation at $R_\lambda \approx 1{,}300$, with $k_{max}\eta \approx 2$. *Inset* shows data for 0–16 mean values. Dashed curves in *Inset* show positive halves of Gaussian distributions with equal variances; they serve only a pedantic purpose because dissipation and enstrophy are both positive definite. Rare events occur enormously more frequently than can be anticipated by Gaussian distributions—by some 10 orders of magnitude when the abscissae values reach 50 or smaller, and by some 250 orders of magnitude for abscissae values of 1,000. Although the data shown are averaged over 14 instantaneous snapshots, the main features are robust: Every snapshot possesses similar features.

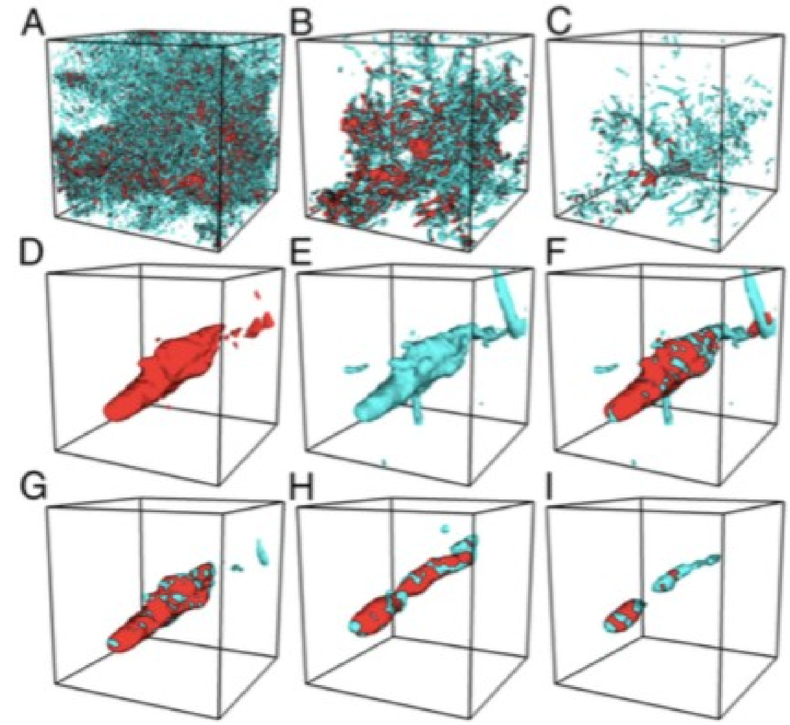
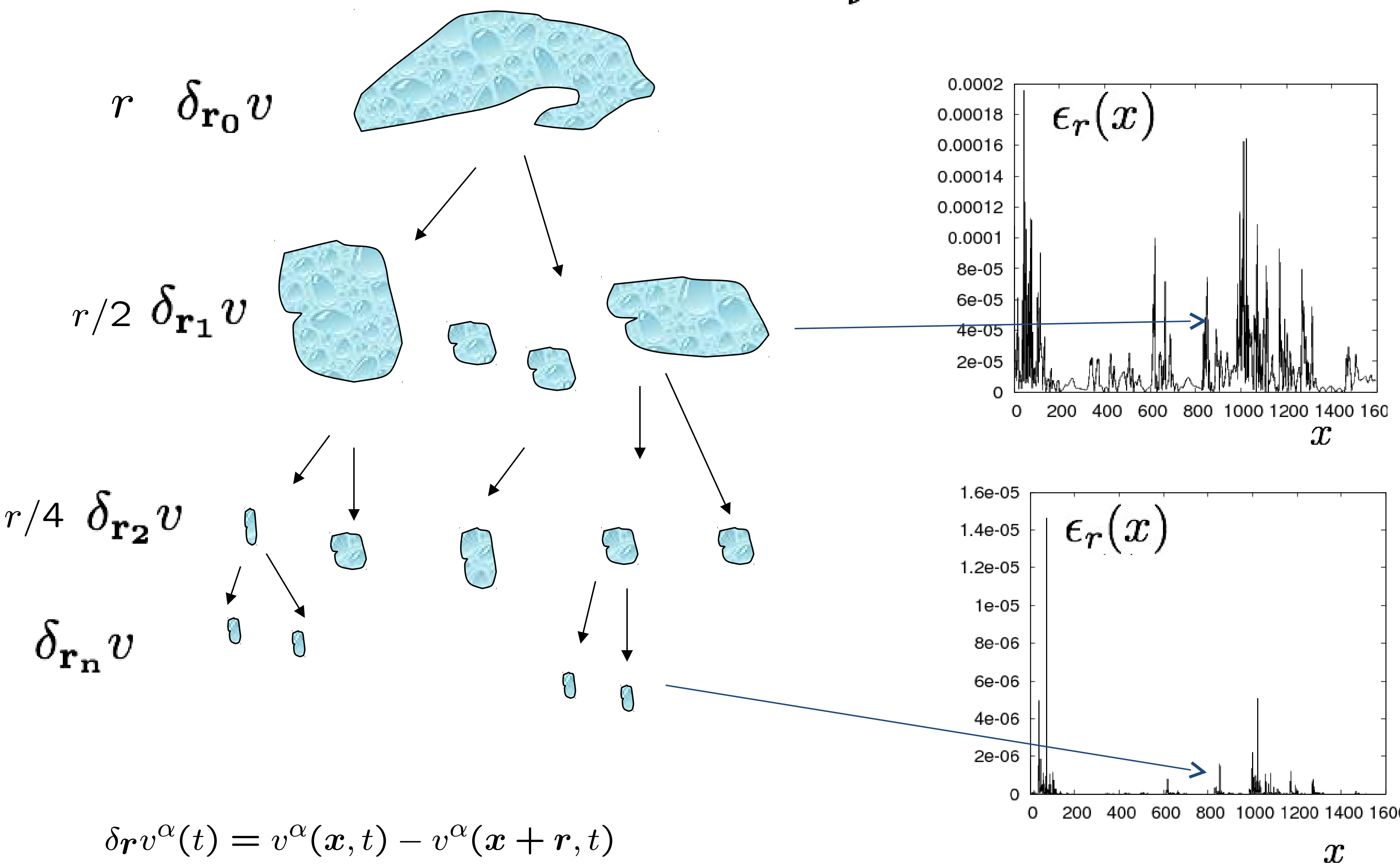


Fig. 3. Perspective views of 3D contour surfaces of dissipation (red) and enstrophy (cyan) extracted from a randomly chosen (but representative) $8{,}192^3$ instantaneous snapshot, at different thresholds (in multiples of mean values) and for different sized subcubes: (*A*) 10, 768^3; (*B*) 30, 256^3; (*C*) 100, 256^3; (*D–F*): 300, 51^3; (*G*) 600, 51^3; (*H*) 4,800, 31^3; and (*I*) 9,600, 31^3. Both dissipation and enstrophy are shown in all frames but *D* and *E*.

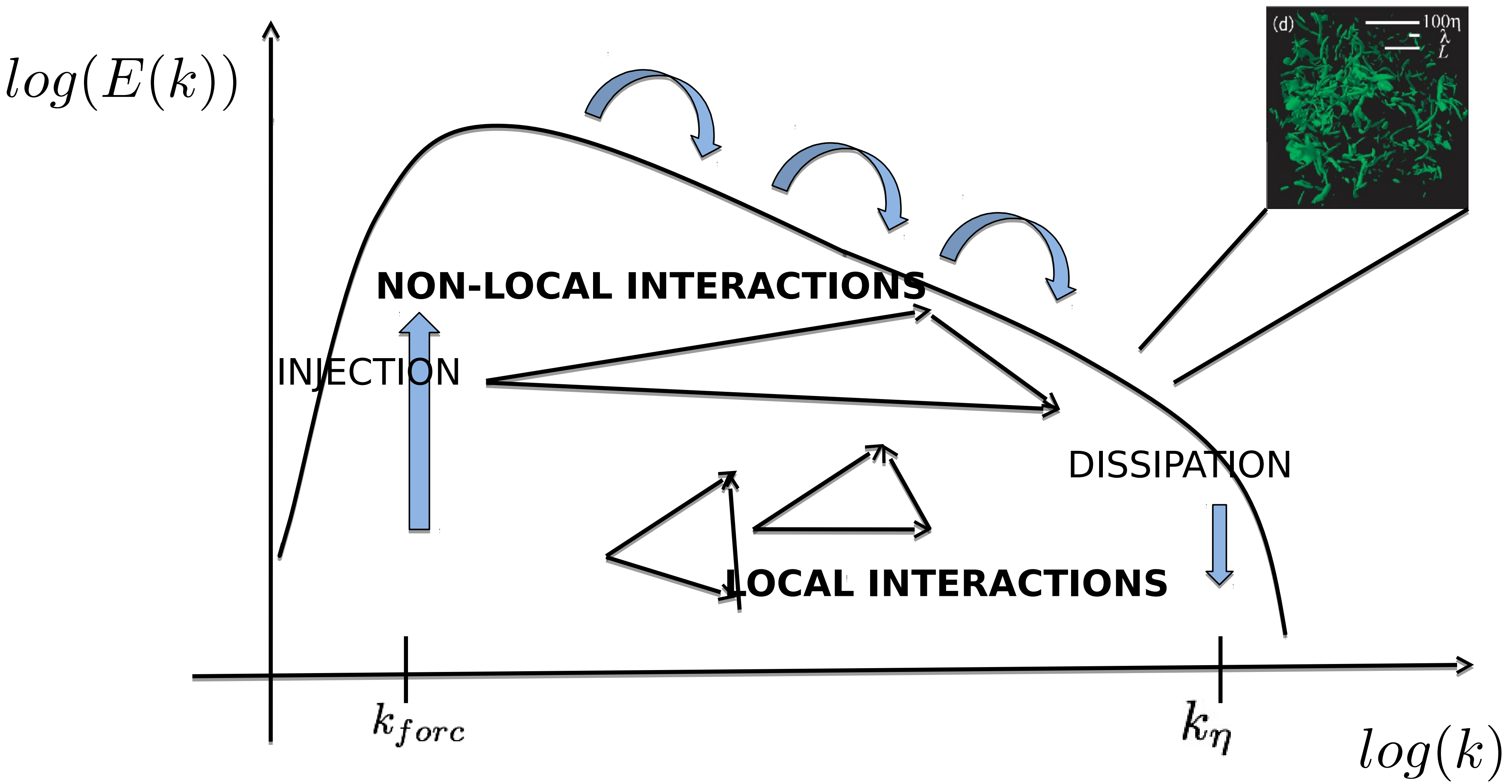
Phenomenological models: spatio-temporal Richardson cascade

$$Re(r) = \frac{r\delta_r v}{\nu}$$



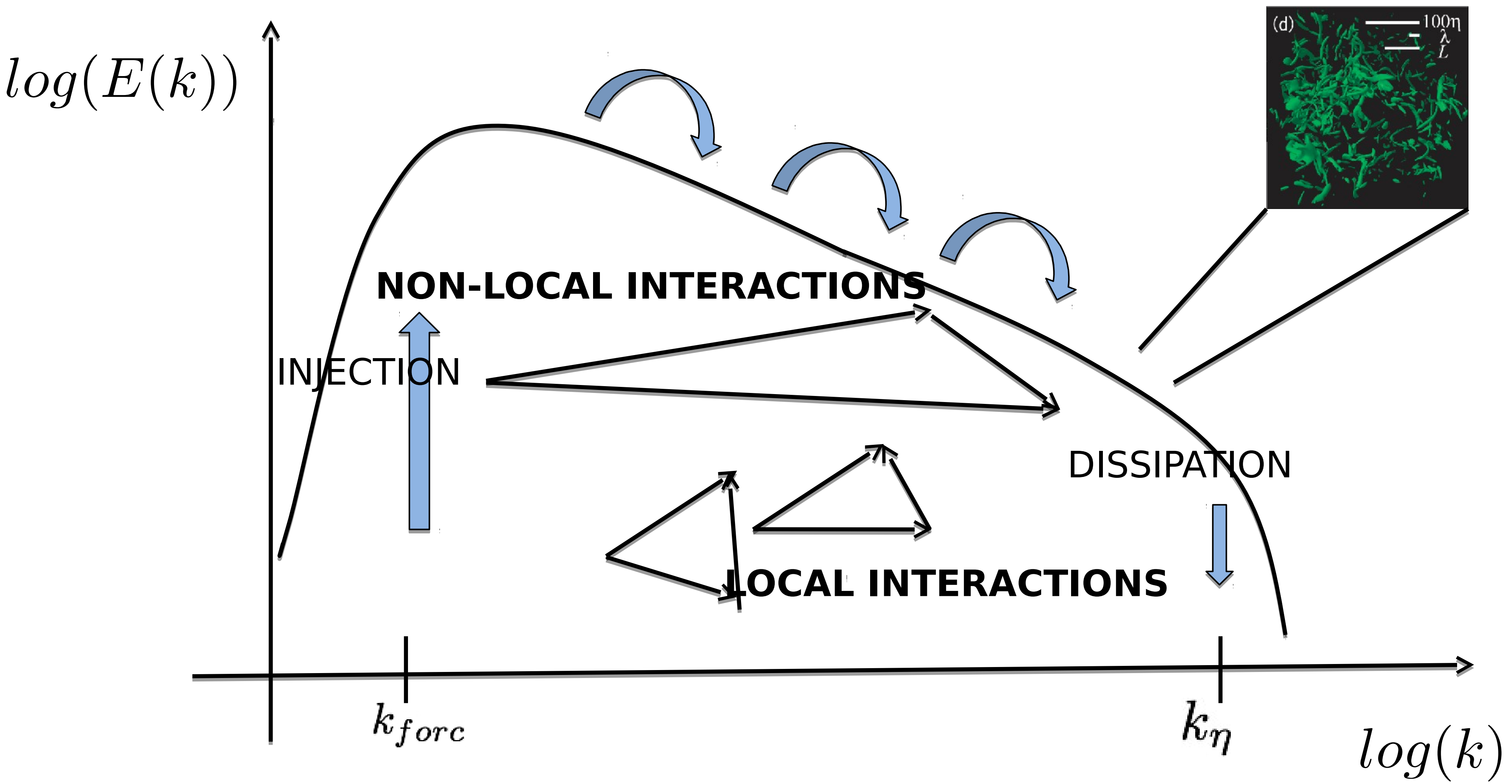
$$\delta_{\boldsymbol{r}} v^{\alpha}(t) = v^{\alpha}(\boldsymbol{x}, t) - v^{\alpha}(\boldsymbol{x} + \boldsymbol{r}, t)$$

$$S_n^{\bar{\alpha}}(\bar{\boldsymbol{r}}, \bar{t}) = \langle \delta_{\boldsymbol{r}_1} v^{\alpha_1}(t_1) \cdots \delta_{\boldsymbol{r}_n} v^{\alpha_n}(t_n) \rangle$$

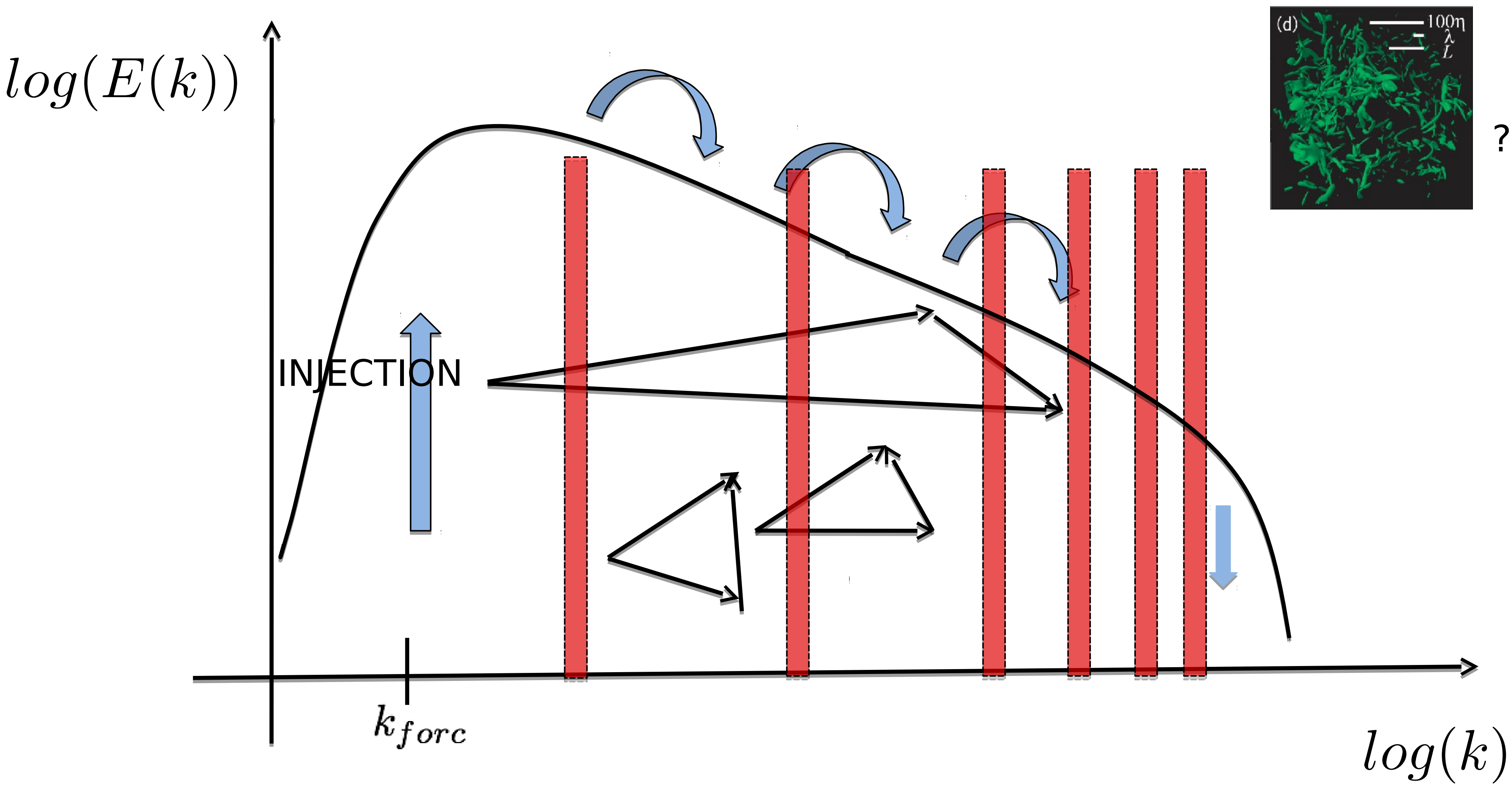


$$\partial_t \hat{u}_n(\mathbf{k},t) + \left(\delta_{nm} - \frac{k_n k_m}{|k|^2}\right) NL_m(\mathbf{k},t) = -\nu|\mathbf{k}|^2 \hat{u}_n(\mathbf{k},t) + \hat{f}_n(\mathbf{k},t)$$

$$NL_m(\mathbf{k},t) = -i \sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}} k''_j \hat{u}_m(\mathbf{k}',t)\hat{u}_j(\mathbf{k}'',t)$$



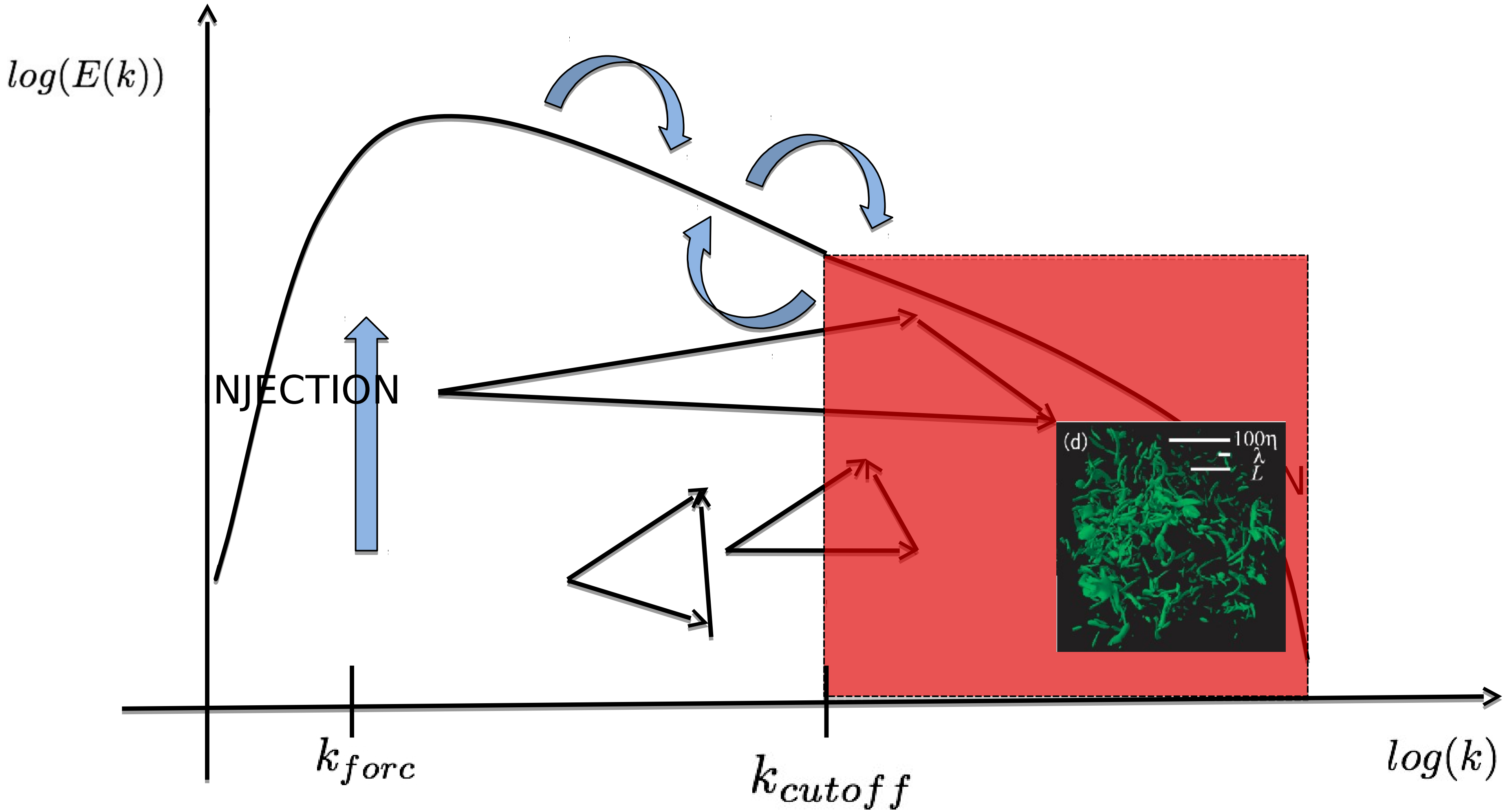
How many (and which) degrees of freedom do we need to preserve the main statistical properties of NS turbulence?



$$u^{D_F}(x,t) = P_{D_F} u(x,t) = \sum_{k \in Z^3} e^{ikx} \theta(k) \hat{u}(k,t)$$

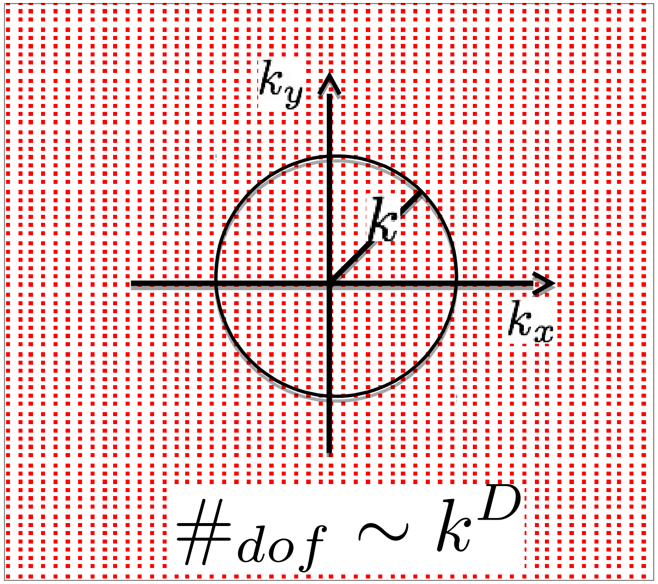
$$\theta(k) = \begin{cases} 1 \quad with \quad probability \quad \sim (k/k_0)^{D_F - 3} \\ 0 \quad with \quad probability \quad \sim 1 - (k/k_0)^{D_F - 3} \end{cases}$$

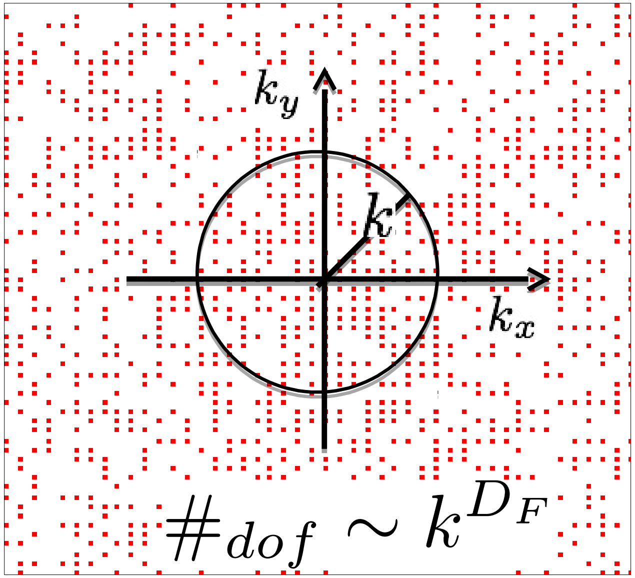
LARGE EDDY SIMULATION



Cutoff + Model for small scales

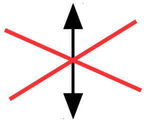
SELF-SIMILAR GALERKIN TRUNCATION





HOMOGENEOUS & ISOTROPIC & SELF-SIMILAR (NO EXTERNAL SCALES)
ENERGY & HELICITY INVISCID INVARIANTS
REAL PDE (INFINITE NUMBER OF DEGREES OF FREEDOM)

$$\partial_t \hat{u}_n(\mathbf{k},t) + \left(\delta_{nm} - \frac{k_n k_m}{|k|^2}\right) NL_m(\mathbf{k},t) = -\nu|\mathbf{k}|^2 \hat{u}_n(\mathbf{k},t) + \hat{f}_n(\mathbf{k},t)\,; \quad \hat{u}(\mathbf{k},t) \to P_{D_f}\hat{u}(\mathbf{k},t)$$



$$\partial_t \hat{u}_n^{D_f}(\mathbf{k},t) + \left(\delta_{nm} - \frac{k_n k_m}{|k|^2}\right) P_{D_f} NL_m^{D_f}(\mathbf{k},t) = -\nu|\mathbf{k}|^2 \hat{u}_n^{D_f}(\mathbf{k},t) + \hat{f}_n^{D_f}(\mathbf{k},t)$$

Turbulence in non-integer dimensions by fractal Fourier decimation

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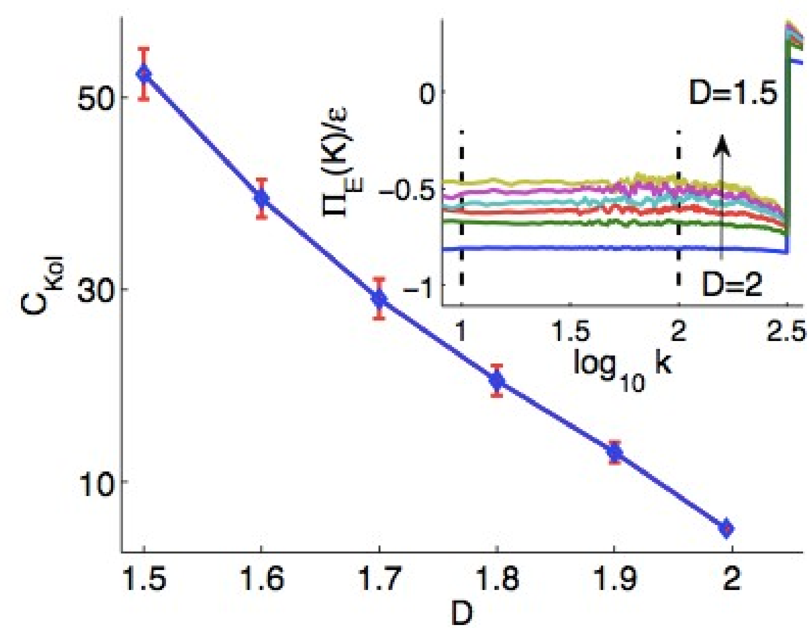
(Dated: August 8, 2011)

Fractal decimation reduces the effective dimensionality of a flow by keeping only a (randomly chosen) set of Fourier modes whose number in a ball of radius k is proportional to k^D for large k. At the critical dimension $D = 4/3$ there is an equilibrium Gibbs state with a $k^{-5/3}$ spectrum, as in [V. L'vov *et al.*, Phys. Rev. Lett. **89**, 064501 (2002)]. Spectral simulations of fractally decimated two-dimensional turbulence show that the inverse cascade persists below $D = 2$ with a rapidly rising Kolmogorov constant, likely to diverge as $(D - 4/3)^{-2/3}$.

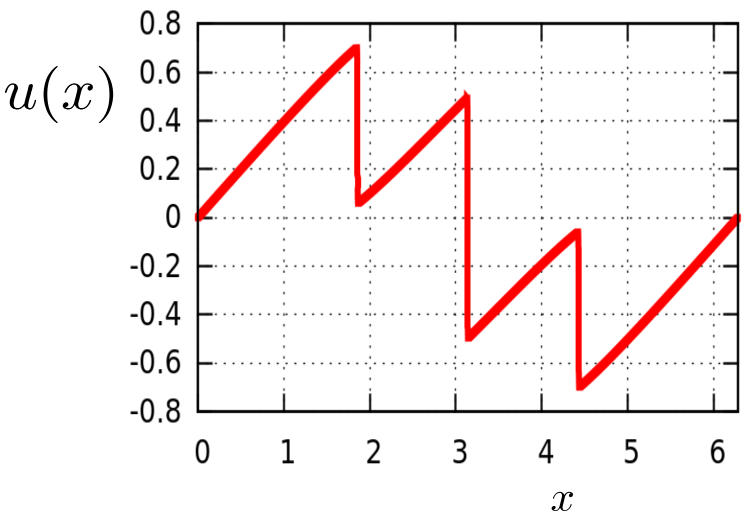
$$D = 4/3$$

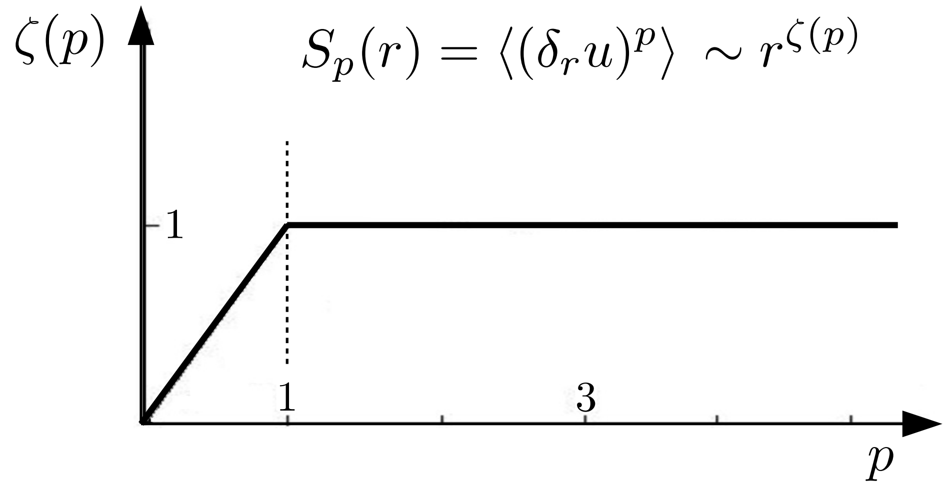
Enstrophy equipartition:
5/3 Kolmogorov spectrum



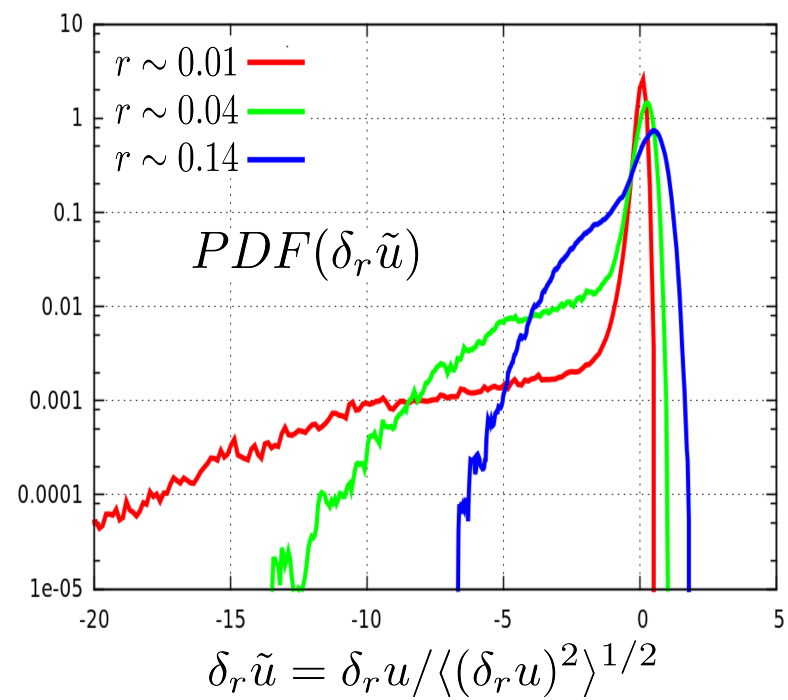
The simplest case:

$$\frac{\partial u(x,t)}{\partial t} + u\frac{\partial u(x,t)}{\partial x} = \nu\frac{\partial^2 u(x,t)}{\partial x^2} + f(x,t)$$

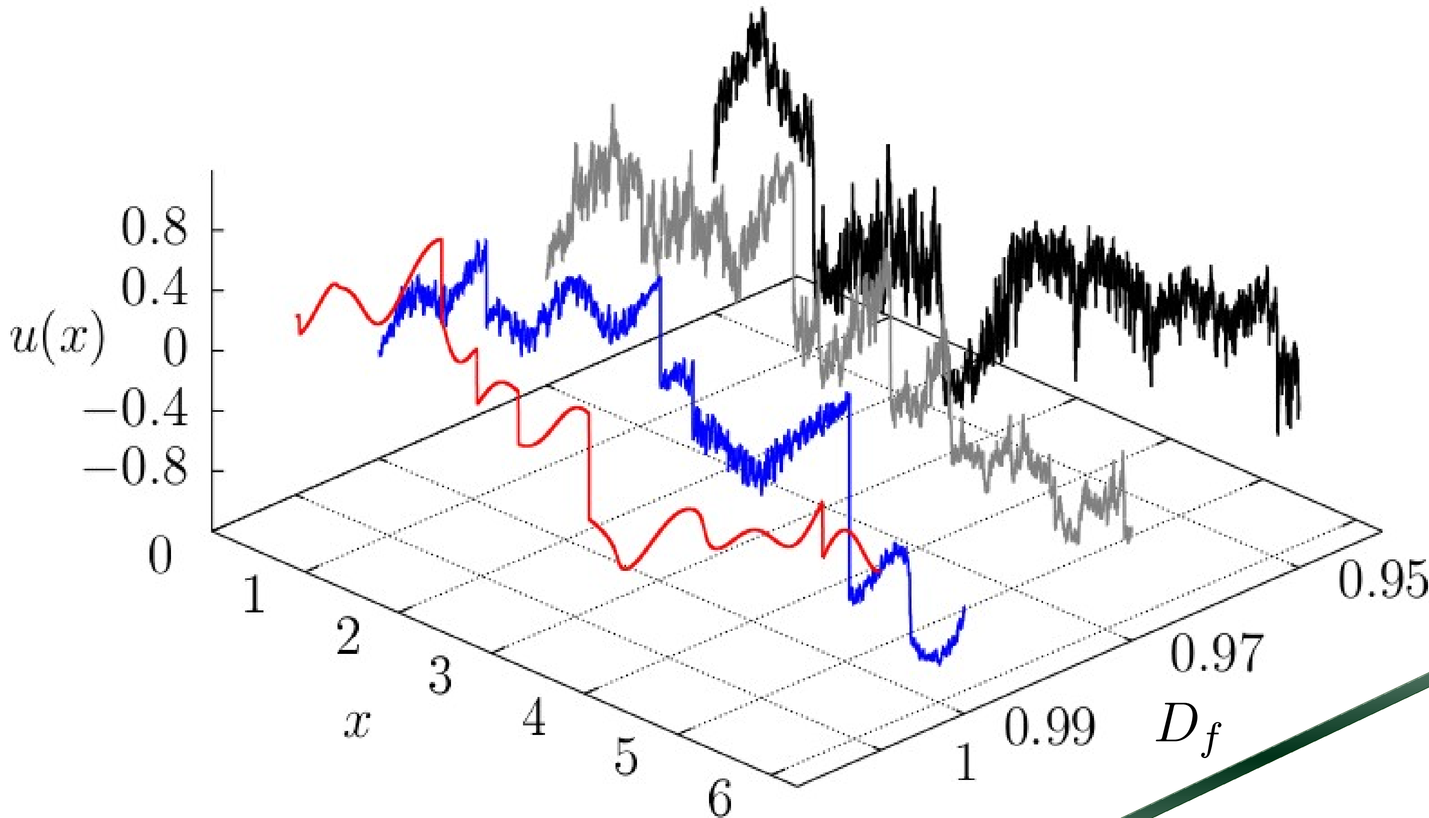




As vortices in real 3D turbulence, **shock produces a non-trivial statistics in the Burgers' velocity field**.



Real space evolution at changing of fractal dimension:



$D_f = 0.95$

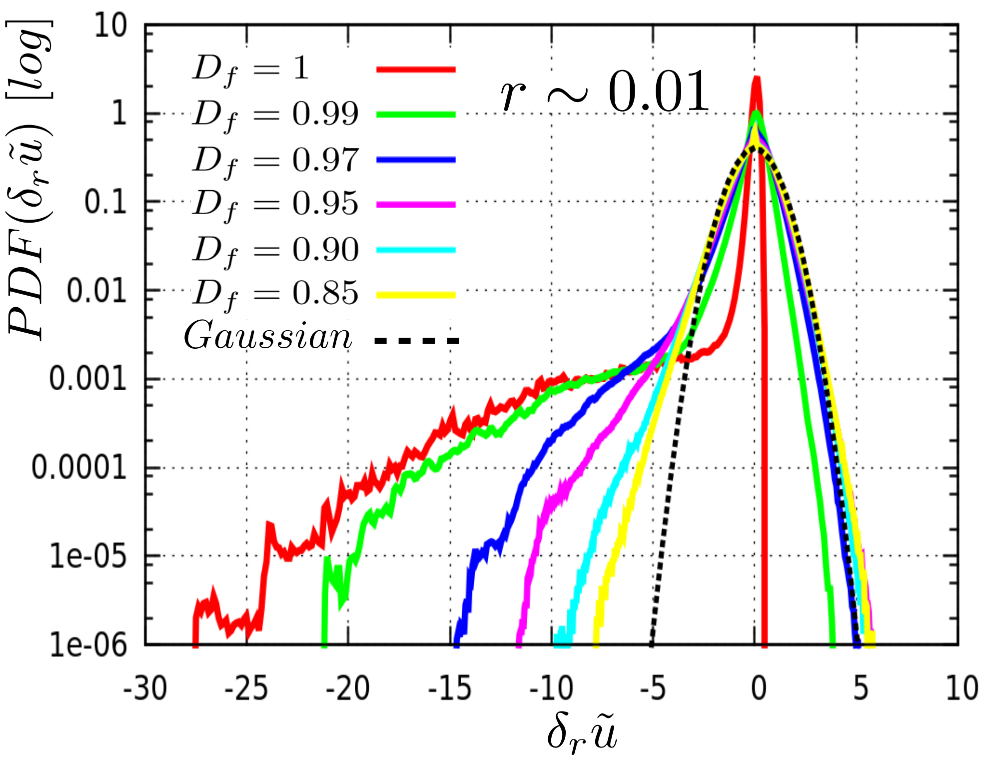
Random small scales structures

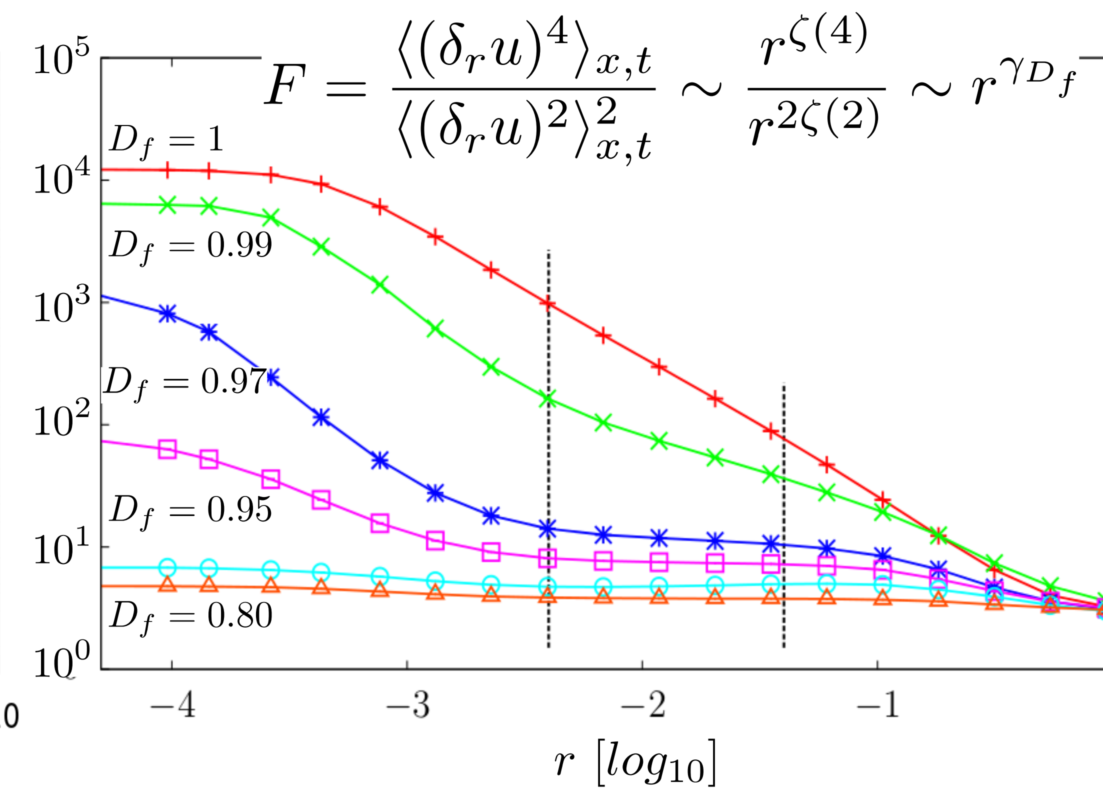
$D_f = 1$

Shock evolution in energy decaying

$$\partial_t u^{D_F}(x,t) + P_{D_f}\left[u^{D_F}\partial_x u^{D_F}(x,t)\right] = \nu \partial^2_{xx} u^{D_F}(x,t) + F^{D_f}$$

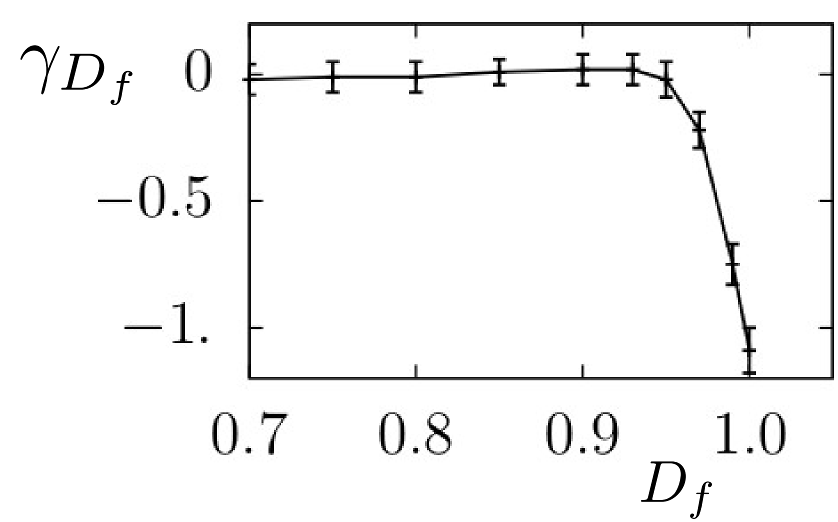
Statistical properties at different Fractal dimensions:





Self-similar fluctuations are introduced by decimation

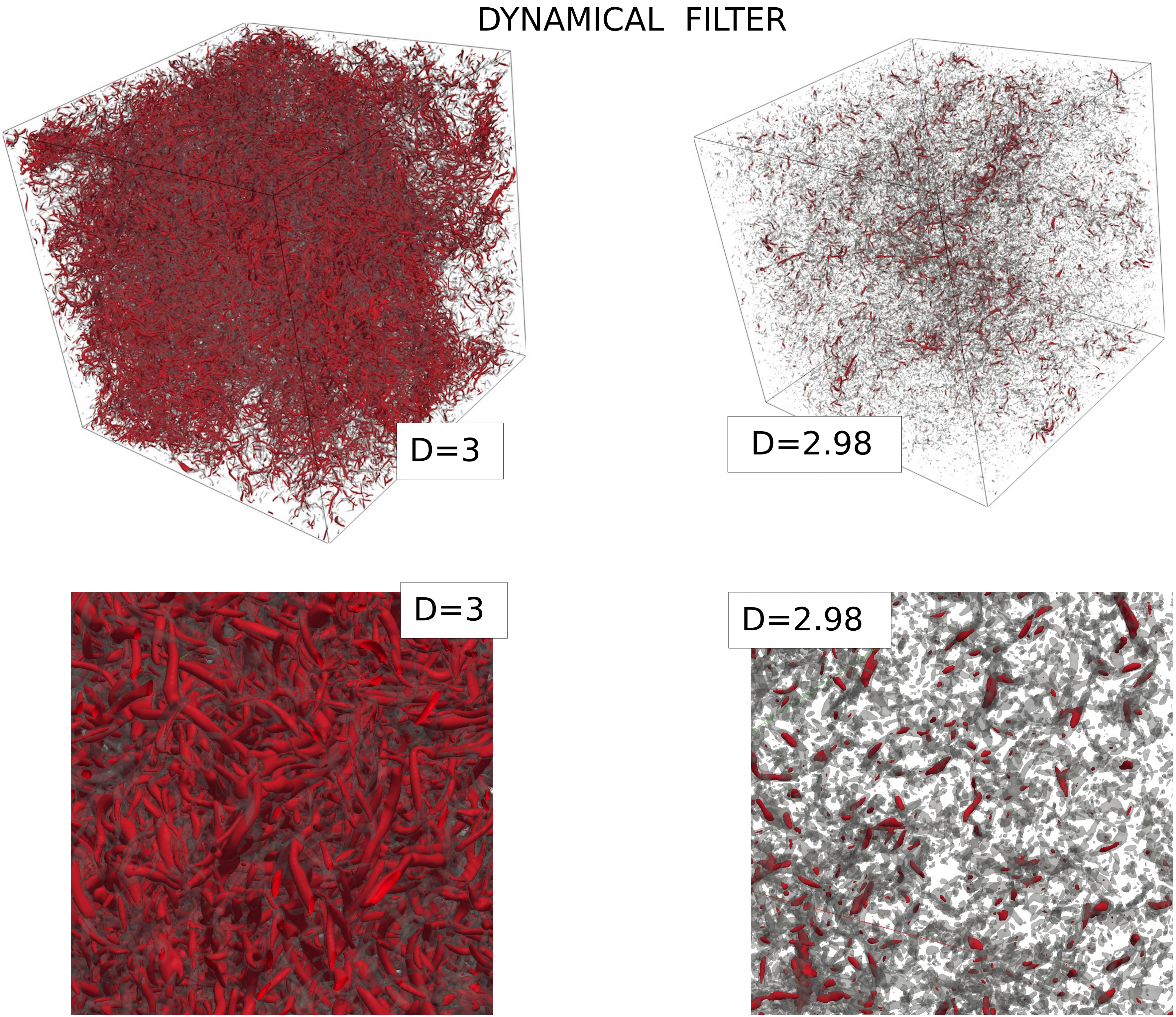
Intermittency is washed out.



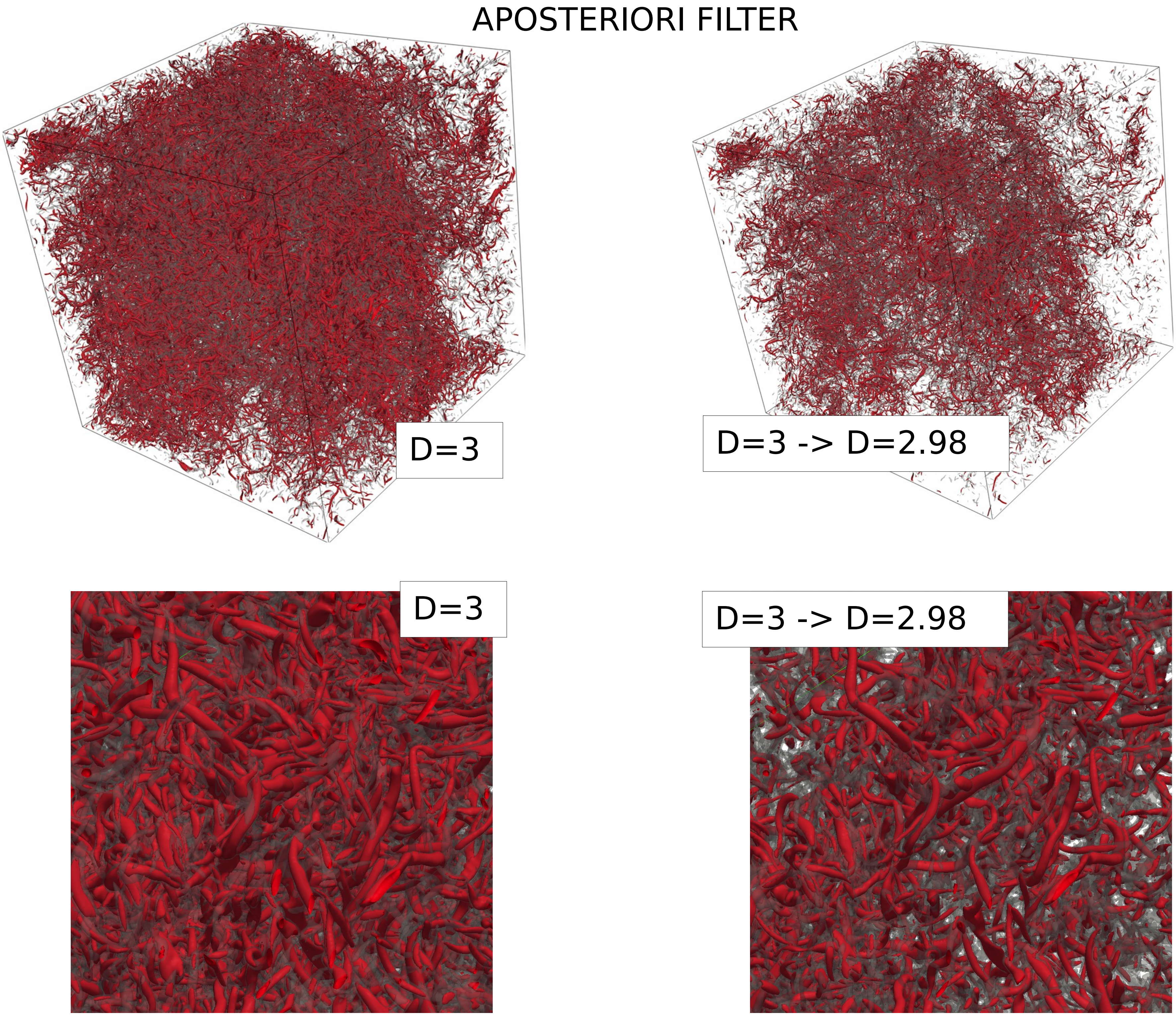
SELF- SIMILAR SURGERY OF NAVIER-STOKES INTERACTIONS

$$\partial_t \hat{u}_n^{D_f}(\mathbf{k},t) + \left(\delta_{nm} - \frac{k_n k_m}{|k|^2}\right) P_{D_f} N L_m^{D_f}(\mathbf{k},t) = -\nu |\mathbf{k}|^2 \hat{u}_n^{D_f}(\mathbf{k},t) + \hat{f}_n^{D_f}(\mathbf{k},t)$$

DF	2.5	2.8	2.98	2.99	2.999	3.0
1024^3	**3%**	**25%**	**87%**	**93%**	**99%**	**100%**



DYNAMICAL FILTER
D=3
D=2.98
D=3
D=2.98


APOSTERIORI FILTER
D=3
D=3 -> D=2.98
D=3
D=3 -> D=2.98

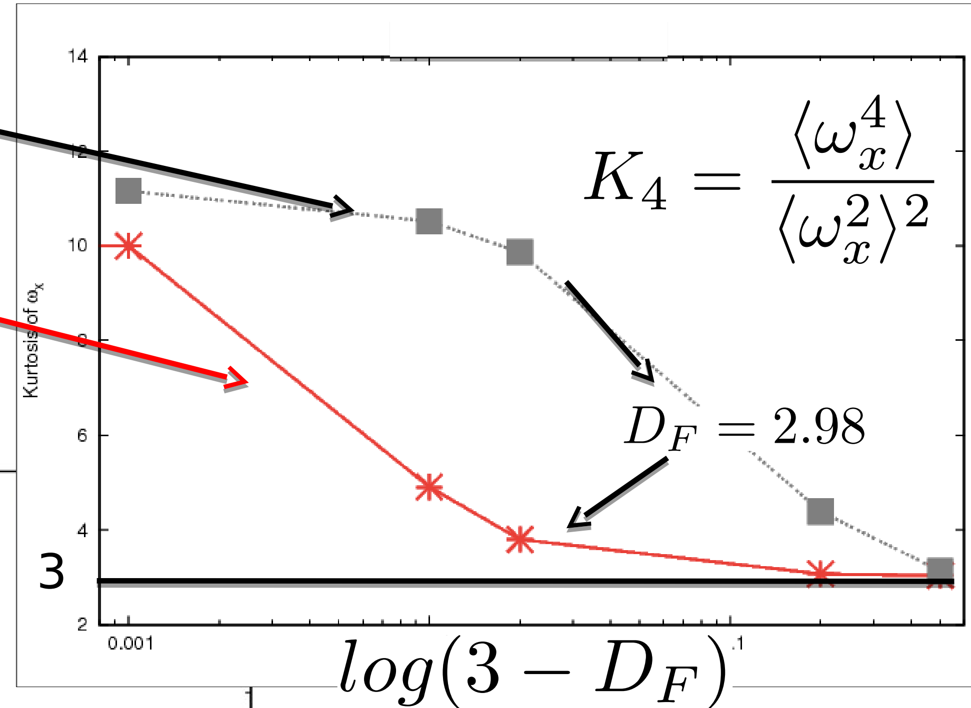
PRL 115, 264502 (2015) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2015

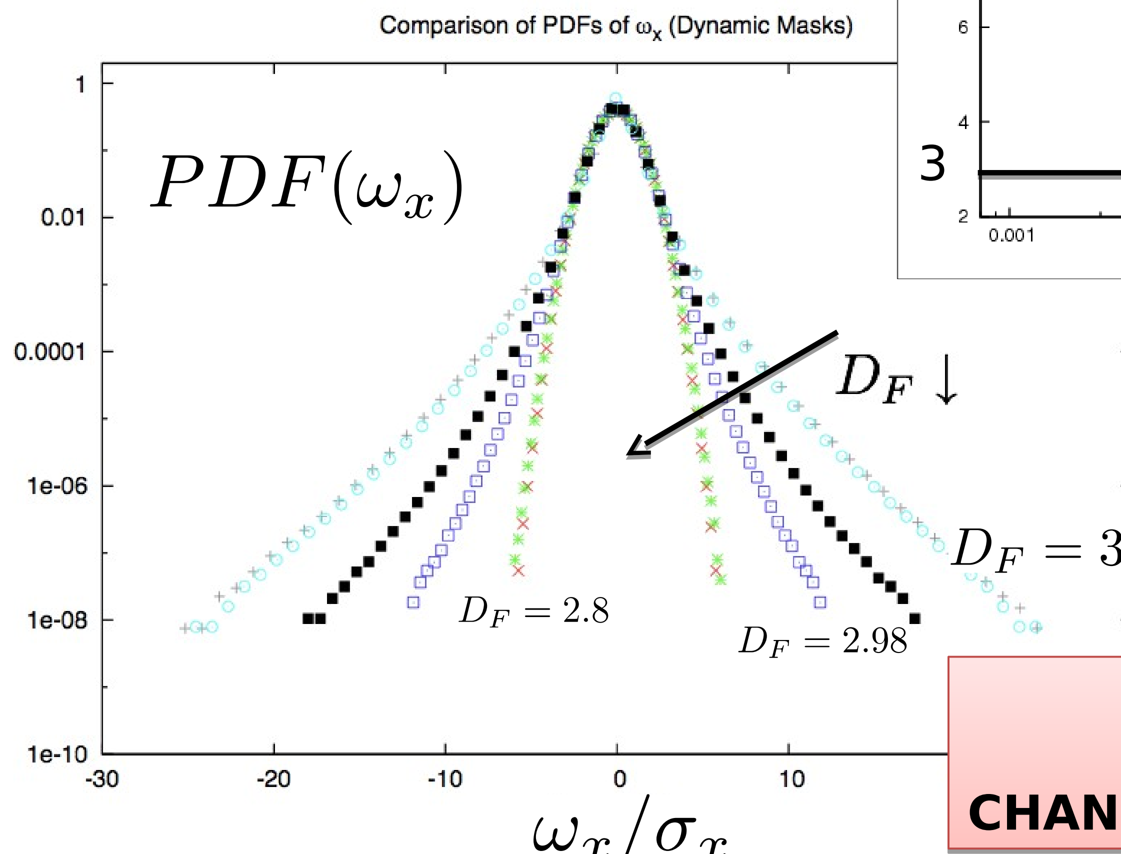
Turbulence on a Fractal Fourier Set

Alessandra S. Lanotte,[1,*] Roberto Benzi,[2] Shiva K. Malapaka,[2,3] Federico Toschi,[4] and Luca Biferale[2]

$$\begin{cases} \partial_t \mathbf{u} = B(\mathbf{u},\mathbf{u}) + \Delta\mathbf{u} + \mathbf{f} \\ \mathbf{u} \to P^{D_F}\mathbf{u} \end{cases}$$

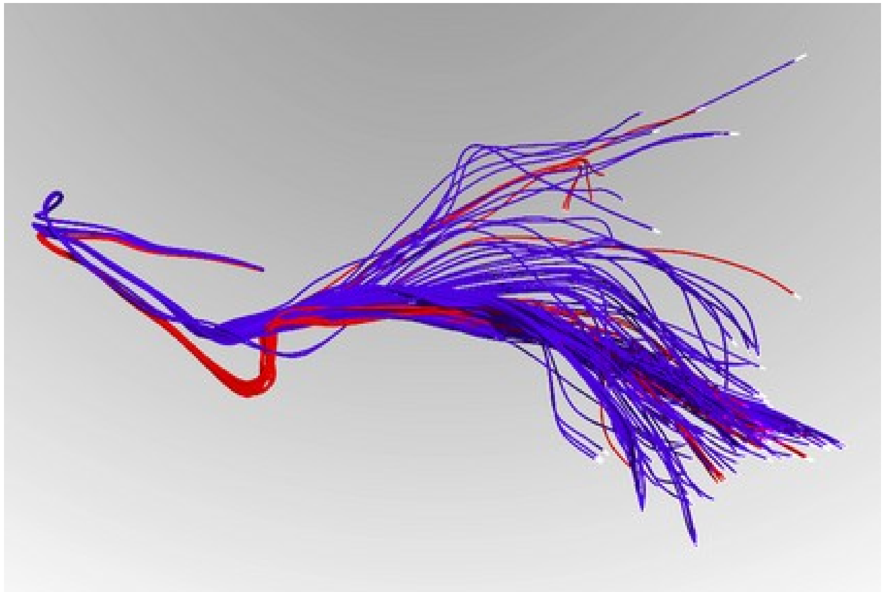
$$\partial_t \mathbf{u}^{D_F} = P^{D_F} B(\mathbf{u}^{D_F}, \mathbf{u}^{D_F}) + \Delta\mathbf{u}^{D_F} + \mathbf{f}^{D_F}$$





**PDF OF VORTICITY
AT
CHANGING FRACTAL DIMENSION**

Lagrangian Intermittency



$$\begin{cases} \dfrac{\partial \mathbf{x}(t)}{\partial t} = \mathbf{v}^{D_F}(t) = \mathbf{u}^{D_F}(\mathbf{x}(t), t) \\ \partial_t \mathbf{u}^{D_F} = P^{D_F} B(\mathbf{u}^{D_F}, \mathbf{u}^{D_F}) + \Delta \mathbf{u}^{D_F} + \mathbf{f}^{D_F} \end{cases}$$

Lagrangian Intermittency

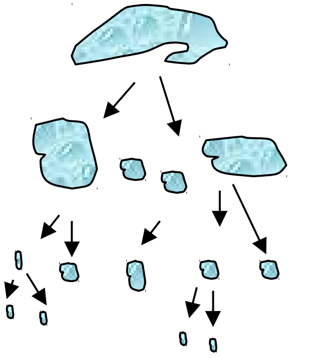
$$S_p(r) = \langle (u(x+r) - u(x))^p \rangle \sim r^{\zeta_E(p)}$$

$$S_p(\tau) = \langle (v(t+\tau) - v(t))^p \rangle \sim \tau^{\zeta_L(p)}$$

In the Multifractal terminology:

$$\delta_r u \sim r^h$$

$$P(h) \sim r^{3-D(h)}$$



$$S_p(r) \sim \int_I P(h) \delta_r u(x)^p dh \sim \int_I r^{3-D(h)} r^{hp} dh \to \zeta_E(p) = \inf_h \left[hp + 3 - D(h) \right]$$

$$\tau_r \sim r/\delta_r u \qquad \tau \sim r^{1-h} \to \zeta_L(p) = \inf_h \left[\frac{hp + 3 - D(h)}{1-h} \right]$$

Bridge Relation between Lagrangian and Eulerian increments

PRL 100, 254504 (2008) PHYSICAL REVIEW LETTERS week ending 27 JUNE 2008

Universal Intermittent Properties of Particle Trajectories in Highly Turbulent Flows

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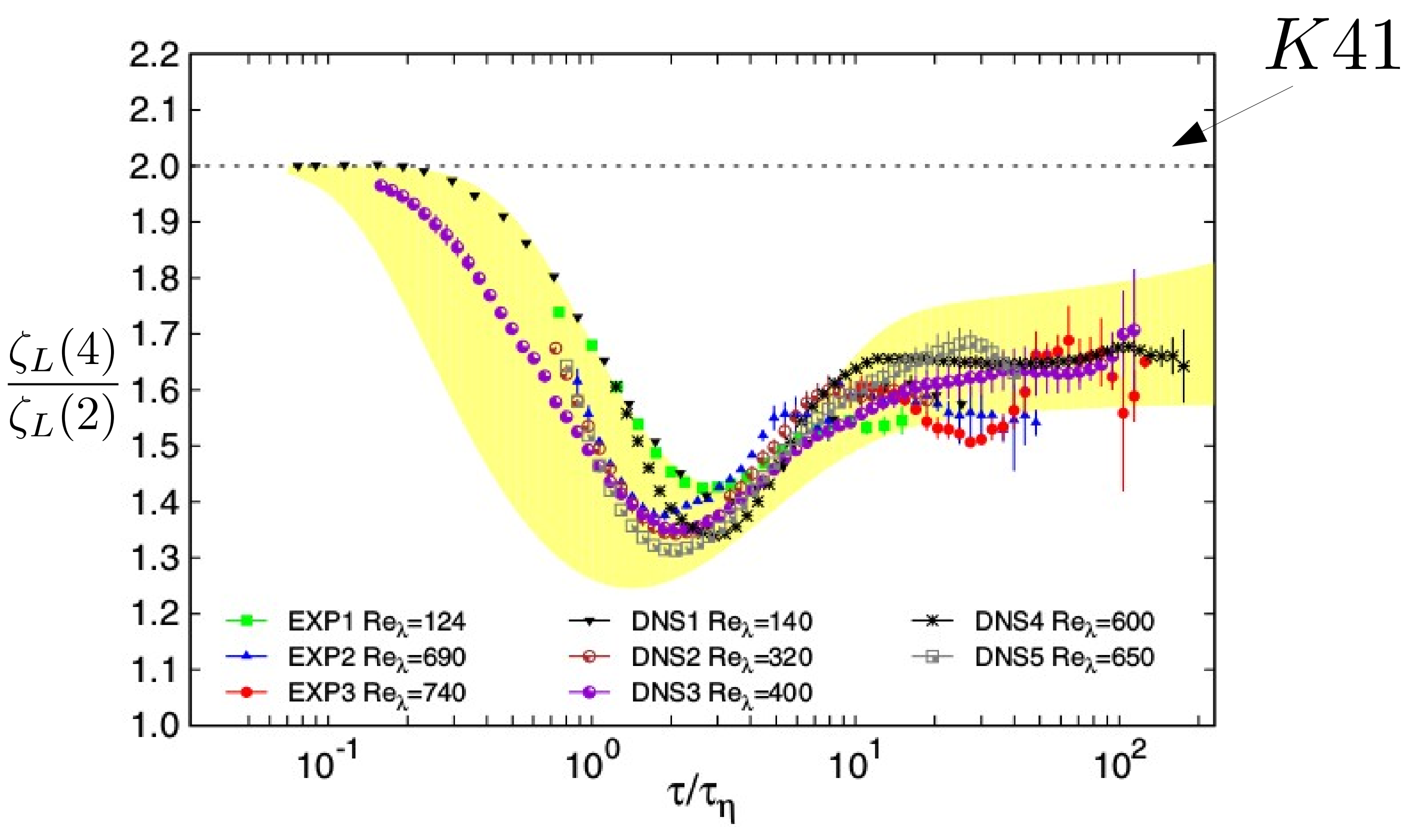
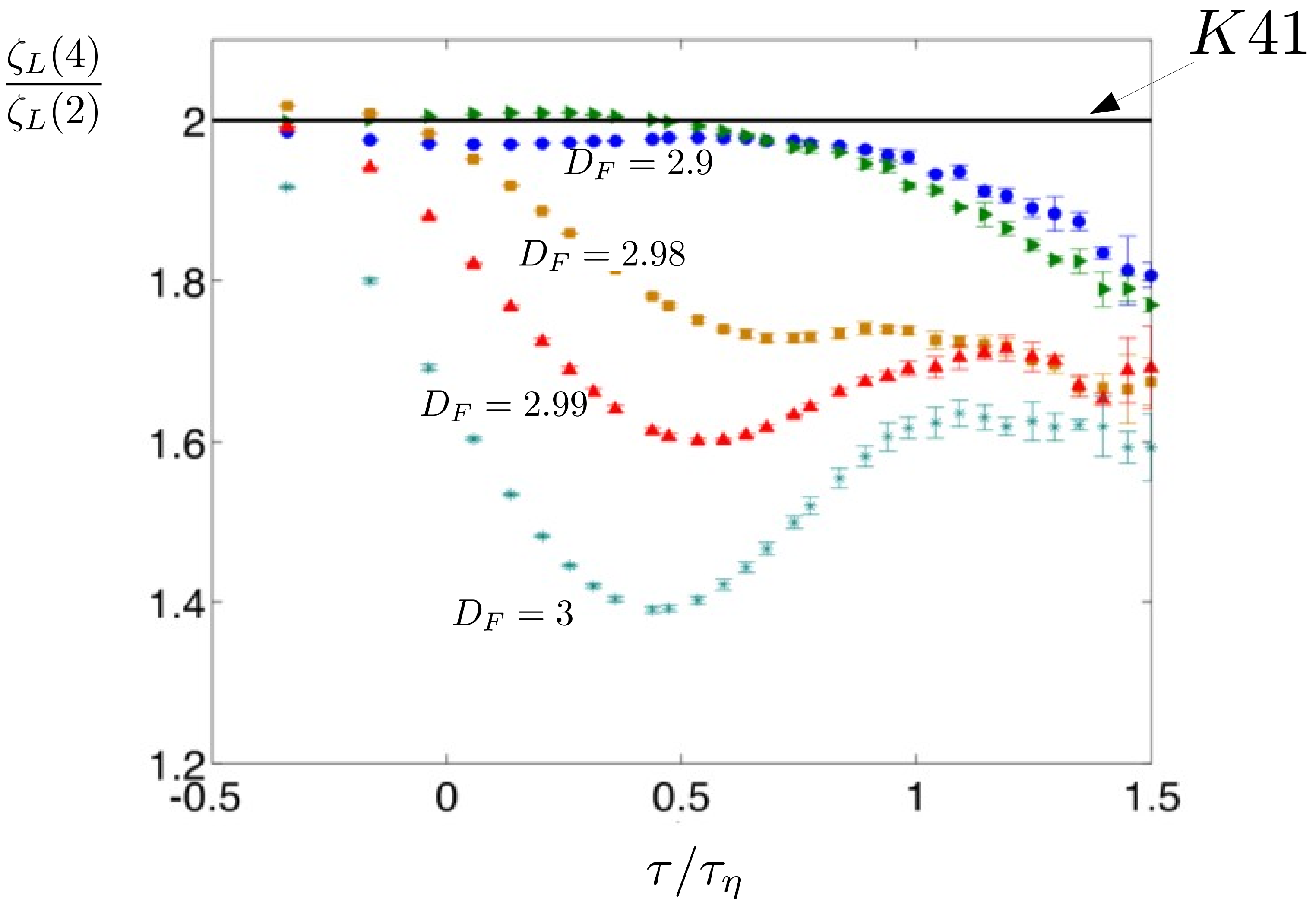
$D = 3$

Homogeneous and Isotropic turbulence



$$S_4(\tau) \sim \tau^{\zeta_L(4)} \rightarrow log(S_4(\tau)) \sim \zeta_L(4) log(\tau) \rightarrow \zeta_L(4) = \frac{\partial\,(log(S_4(\tau)))}{\partial log(\tau)}$$

Lagrangian Intermittency in fractally decimated Turbulence



$$S_4(\tau) \sim \tau^{\zeta_L(4)} \rightarrow log(S_4(\tau)) \sim \zeta_L(4) log(\tau) \rightarrow \zeta_L(4) = \frac{\partial\,(log(S_4(\tau)))}{\partial log(\tau)}$$

Conclusions

+ QUANTIFY IMPORTANCE OF LOCAL VS NON-LOCAL TRIADIC INTERACTIONS

+ CORRECTION TO FLUCTUATIONS: HUGE. SMALL SCALE VORTICITY IS STRONGLY SENSITIVE TO DECIMATION. “CHOERENT” SMALL-SCALE STRUCTURES FEEL GLOBAL CORRELATIONS ACROSS SCALES IN FOURIER.

+ HOW TO BRING INTERMITTENCY BACK TO DECIMATED NS EQUATIONS?

+ THE INTERMITTENCY DISAPPEARS ALSO IN THE LAGRANGIAN STATISTIC, FOLLOWING THE BRIDGE RELATION.

- WE STILL MISS A CLEAR DEFINITION OF INTERMITTENCY IN FOURIER SPACE