Inverse energy transfer and large-scale dynamo action in helically projected MHD flows

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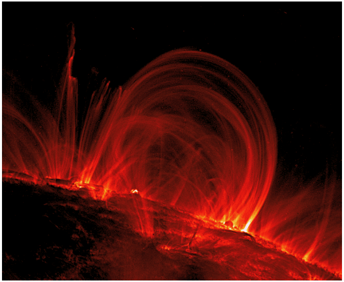
Large-scale magnetic fields in MHD



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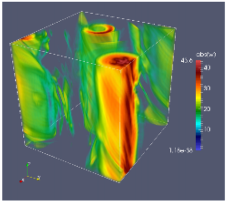
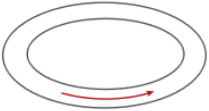
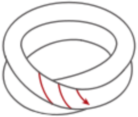


Dallas & Alexakis PoF 2015

Minnini, Annu. Rev. Fluid Mech. 2011





$$\partial_t \boldsymbol{u} = -\frac{1}{\rho}\nabla P - (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \frac{1}{\rho}(\nabla\times\boldsymbol{b})\times\boldsymbol{b} + \nu\Delta\boldsymbol{u}$$

$$\partial_t \boldsymbol{b} = (\boldsymbol{b}\cdot\nabla)\boldsymbol{u} - (\boldsymbol{u}\cdot\nabla)\boldsymbol{b} + \eta\Delta\boldsymbol{b}$$

$$\nabla\cdot\boldsymbol{u} = 0 \quad \text{and} \quad \nabla\cdot\boldsymbol{b} = 0$$

$$H_m(t) = \int_V d\boldsymbol{x}\ \boldsymbol{a}(\boldsymbol{x},t)\cdot\boldsymbol{b}(\boldsymbol{x},t) \rightarrow \text{ inverse cascade}$$

$$H_k(t) = \int_V d\boldsymbol{x}\ \boldsymbol{u}(\boldsymbol{x},t)\cdot\boldsymbol{\omega}(\boldsymbol{x},t) \rightarrow \text{ dynamo action (e.g. } \alpha\text{-effect)}$$

$$\partial_t \boldsymbol{u} = -\frac{1}{\rho}\nabla P - (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \frac{1}{\rho}(\nabla\times\boldsymbol{b})\times\boldsymbol{b} + \nu\Delta\boldsymbol{u}$$

$$\partial_t \boldsymbol{b} = (\boldsymbol{b}\cdot\nabla)\boldsymbol{u} - (\boldsymbol{u}\cdot\nabla)\boldsymbol{b} + \eta\Delta\boldsymbol{b}$$

$$\nabla\cdot\boldsymbol{u} = 0 \text{ and } \nabla\cdot\boldsymbol{b} = 0$$

Theory

Fourier transform

helical decomposition

reduction to triads

stability analysis

Simulations

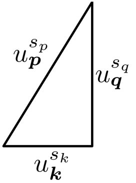
pseudospectral

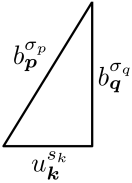
helical decomposition

full system (all triads)

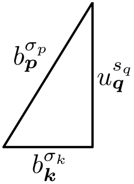
helical projection

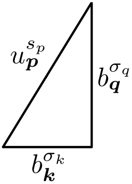
Generic minimal triadic interaction





$$\partial_t u_{\boldsymbol{k}}^{s_k *} = \frac{1}{2} \left(g_{s_p s_q}^{s_k} (s_p p - s_q q) u_{\boldsymbol{p}}^{s_p} u_{\boldsymbol{q}}^{s_q} - g_{\sigma_p \sigma_q}^{s_k} (\sigma_p p - \sigma_q q) b_{\boldsymbol{p}}^{\sigma_p} b_{\boldsymbol{q}}^{\sigma_q} \right)$$





$$\partial_t b_{\boldsymbol{k}}^{\sigma_k *} = \frac{\sigma_k k}{2} \left(g_{\sigma_p s_q}^{\sigma_k} b_{\boldsymbol{p}}^{\sigma_p} u_{\boldsymbol{q}}^{s_q} - g_{s_p \sigma_q}^{\sigma_k} u_{\boldsymbol{p}}^{s_p} b_{\boldsymbol{q}}^{\sigma_q} \right)$$

Stability analysis of single-type equilibria

Large wavenumber (small-scale) equilibria:

$$U_{\boldsymbol{p}_0}^{s_{p_0}} \text{ and } B_{\boldsymbol{p}_0}^{s_{p_0}} \text{ with } k < q < p_0$$

Small wavenumber (large-scale) perturbations:

$$\begin{aligned}
\partial_t \tilde{u}_{\boldsymbol{k}}^{s_k *} &= g_{s_{p_0} s_q}^{s_k} \left(s_p p_0 - s_q q \right) U_{\boldsymbol{p}_0}^{s_{p_0}} \tilde{u}_{\boldsymbol{q}}^{s_q} - g_{\sigma_{p_0} \sigma_q}^{s_k} \left(\sigma_{p_0} p_0 - \sigma_q q \right) B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \tilde{b}_{\boldsymbol{q}}^{\sigma_q} \ , \\
\partial_t \tilde{u}_{\boldsymbol{q}}^{s_q *} &= g_{s_k s_{p_0}}^{s_q} \left(s_k k - s_{p_0} p_0 \right) \tilde{u}_{\boldsymbol{k}}^{s_k} U_{\boldsymbol{p}_0}^{s_{p_0}} - g_{\sigma_k \sigma_{p_0}}^{s_q} \left(\sigma_k k - \sigma_{p_0} p_0 \right) \tilde{b}_{\boldsymbol{k}}^{\sigma_k} B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \ , \\
\partial_t \tilde{b}_{\boldsymbol{k}}^{\sigma_k *} &= \sigma_k k \ \left(g_{\sigma_{p_0} s_q}^{\sigma_k} B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \tilde{u}_{\boldsymbol{q}}^{s_q} - g_{s_{p_0} \sigma_q}^{\sigma_k} U_{\boldsymbol{p}_0}^{s_{p_0}} \tilde{b}_{\boldsymbol{q}}^{\sigma_q} \right) \ , \\
\partial_t \tilde{b}_{\boldsymbol{q}}^{\sigma_q *} &= \sigma_q q \ \left(g_{\sigma_k s_{p_0}}^{\sigma_q} \tilde{b}_{\boldsymbol{k}}^{\sigma_k} U_{\boldsymbol{p}_0}^{s_{p_0}} - g_{s_k \sigma_{p_0}}^{\sigma_q} \tilde{u}_{\boldsymbol{k}}^{s_k} B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \right) \ ,
\end{aligned}$$

$U_{\boldsymbol{p}_0}^{+} \longrightarrow$ kinematic dynamo
$B_{\boldsymbol{p}_0}^{+} \longrightarrow$ magnetic self-interaction

Stability analysis of single-type equilibria

Large wavenumber (small-scale) equilibria:

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Small wavenumber (large-scale) perturbations:

$$\partial_t \tilde{u}_{\boldsymbol{k}}^{s_k *} = g^{s_k}_{s_{p_0} s_q} \left(s_p p_0 - s_q q \right) U_{\boldsymbol{p}_0}^{s_{p_0}} \tilde{u}_{\boldsymbol{q}}^{s_q} - g^{s_k}_{\sigma_{p_0} \sigma_q} \left(\sigma_{p_0} p_0 - \sigma_q q \right) B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \tilde{b}_{\boldsymbol{q}}^{\sigma_q} \ ,$$

$$\partial_t \tilde{u}_{\boldsymbol{q}}^{s_q *} = g^{s_q}_{s_k s_{p_0}} \left(s_k k - s_{p_0} p_0 \right) \tilde{u}_{\boldsymbol{k}}^{s_k} U_{\boldsymbol{p}_0}^{s_{p_0}} - g^{s_q}_{\sigma_k \sigma_{p_0}} \left(\sigma_k k - \sigma_{p_0} p_0 \right) \tilde{b}_{\boldsymbol{k}}^{\sigma_k} B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \ ,$$

$$\partial_t \tilde{b}_{\boldsymbol{k}}^{\sigma_k *} = \sigma_k k \left(g^{\sigma_k}_{\sigma_{p_0} s_q} B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \tilde{u}_{\boldsymbol{q}}^{s_q} - g^{\sigma_k}_{s_{p_0} \sigma_q} U_{\boldsymbol{p}_0}^{s_{p_0}} \tilde{b}_{\boldsymbol{q}}^{\sigma_q} \right) \ ,$$

$$\partial_t \tilde{b}_{\boldsymbol{q}}^{\sigma_q *} = \sigma_q q \left(g^{\sigma_q}_{\sigma_k s_{p_0}} \tilde{b}_{\boldsymbol{k}}^{\sigma_k} U_{\boldsymbol{p}_0}^{s_{p_0}} - g^{\sigma_q}_{s_k \sigma_{p_0}} \tilde{u}_{\boldsymbol{k}}^{s_k} B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \right) \ ,$$

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$$\begin{aligned}
\partial_t \tilde{u}_{\boldsymbol{k}}^{s_k *} &= g^{s_k}_{s_{p_0} s_q} \left(s_p p_0 - s_q q\right) U_{\boldsymbol{p}_0}^{s_{p_0}} \tilde{u}_{\boldsymbol{q}}^{s_q} - g^{s_k}_{\sigma_{p_0} \sigma_q} \left(\sigma_{p_0} p_0 - \sigma_q q\right) B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \tilde{b}_{\boldsymbol{q}}^{\sigma_q} \ , \\
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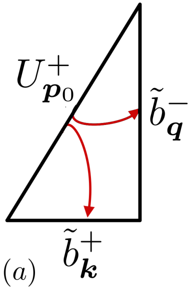
\partial_t \tilde{b}_{\boldsymbol{k}}^{\sigma_k *} &= \sigma_k k \left(g^{\sigma_k}_{\sigma_{p_0} s_q} B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \tilde{u}_{\boldsymbol{q}}^{s_q} - g^{\sigma_k}_{s_{p_0} \sigma_q} U_{\boldsymbol{p}_0}^{s_{p_0}} \tilde{b}_{\boldsymbol{q}}^{\sigma_q} \right) \ , \\
\partial_t \tilde{b}_{\boldsymbol{q}}^{\sigma_q *} &= \sigma_q q \left(g^{\sigma_q}_{\sigma_k s_{p_0}} \tilde{b}_{\boldsymbol{k}}^{\sigma_k} U_{\boldsymbol{p}_0}^{s_{p_0}} - g^{\sigma_q}_{s_k \sigma_{p_0}} \tilde{u}_{\boldsymbol{k}}^{s_k} B_{\boldsymbol{p}_0}^{\sigma_{p_0}} \right) \ ,
\end{aligned}$$

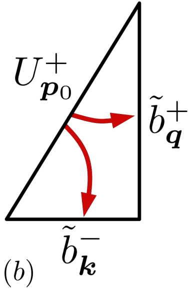
$U_{\boldsymbol{p}_0}^{+} \longrightarrow$ kinematic dynamo
$B_{\boldsymbol{p}_0}^{+} \longrightarrow$ magnetic self-interaction

Large-scale kinematic dynamo



$$\partial_t^2 \tilde{b}^+_{\boldsymbol{k}} = |g^{++-}|^2 \, kq |U^+_{\boldsymbol{p}_0}|^2 \tilde{b}^+_{\boldsymbol{k}} \qquad U^+_{\boldsymbol{p}_0} \xrightarrow{\tilde{b}^-_{\boldsymbol{q}}} \tilde{b}^+_{\boldsymbol{k}}$$

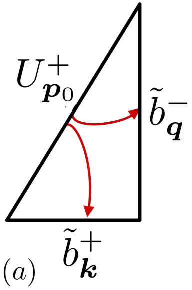
$$\partial_t^2 \tilde{b}^-_{\boldsymbol{q}} = |g^{++-}|^2 \, kq |U^+_{\boldsymbol{p}_0}|^2 \tilde{b}^-_{\boldsymbol{q}} \qquad U^+_{\boldsymbol{p}_0} \xrightarrow{\tilde{b}^+_{\boldsymbol{k}}} \tilde{b}^-_{\boldsymbol{q}}$$



$$\partial_t^2 \tilde{b}^-_{\boldsymbol{k}} = |g^{-++}|^2 \, kq |U^+_{\boldsymbol{p}_0}|^2 \tilde{b}^-_{\boldsymbol{k}} \qquad U^+_{\boldsymbol{p}_0} \xrightarrow{\tilde{b}^+_{\boldsymbol{q}}} \tilde{b}^-_{\boldsymbol{k}}$$

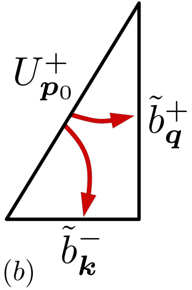
$$\partial_t^2 \tilde{b}^+_{\boldsymbol{q}} = |g^{-++}|^2 \, kq |U^+_{\boldsymbol{p}_0}|^2 \tilde{b}^+_{\boldsymbol{q}} \qquad U^+_{\boldsymbol{p}_0} \xrightarrow{\tilde{b}^-_{\boldsymbol{k}}} \tilde{b}^+_{\boldsymbol{q}}$$

Large-scale kinematic dynamo



$$\partial_t^2 \tilde{b}^+_{\boldsymbol{k}} = |g^{++-}|^2\, kq |U^+_{\boldsymbol{p}_0}|^2 \tilde{b}^+_{\boldsymbol{k}} \qquad U^+_{\boldsymbol{p}_0} \xrightarrow{\tilde{b}^-_{\boldsymbol{q}}} \tilde{b}^+_{\boldsymbol{k}}$$

$$\partial_t^2 \tilde{b}^-_{\boldsymbol{q}} = |g^{++-}|^2\, kq |U^+_{\boldsymbol{p}_0}|^2 \tilde{b}^-_{\boldsymbol{q}} \qquad U^+_{\boldsymbol{p}_0} \xrightarrow{\tilde{b}^+_{\boldsymbol{k}}} \tilde{b}^-_{\boldsymbol{q}}$$



$$\partial_t^2 \tilde{b}^-_{\boldsymbol{k}} = |g^{-++}|^2\, kq |U^+_{\boldsymbol{p}_0}|^2 \tilde{b}^-_{\boldsymbol{k}} \qquad U^+_{\boldsymbol{p}_0} \xrightarrow{\tilde{b}^+_{\boldsymbol{q}}} \tilde{b}^-_{\boldsymbol{k}}$$

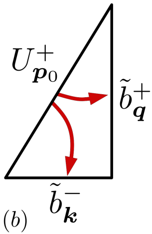
$$\partial_t^2 \tilde{b}^+_{\boldsymbol{q}} = |g^{-++}|^2\, kq |U^+_{\boldsymbol{p}_0}|^2 \tilde{b}^+_{\boldsymbol{q}} \qquad U^+_{\boldsymbol{p}_0} \xrightarrow{\tilde{b}^-_{\boldsymbol{k}}} \tilde{b}^+_{\boldsymbol{q}}$$

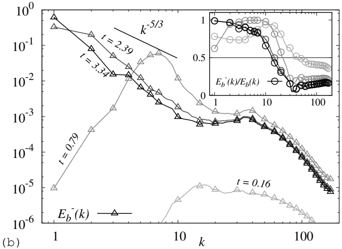
$$\partial_t \boldsymbol{B}_0 = \alpha \nabla \times \boldsymbol{B}_0 \qquad \alpha = -\frac{1}{3}\langle \boldsymbol{u} \cdot \boldsymbol{\omega} \rangle$$

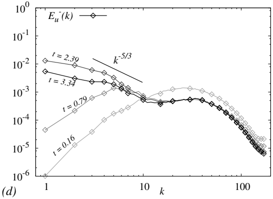
Large-scale dynamo: DNS - laminar flow ($Re_\lambda = 15$)

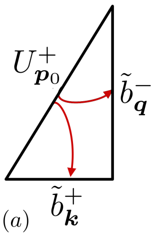
Magnetic field

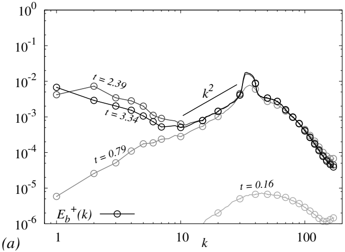
Velocity field

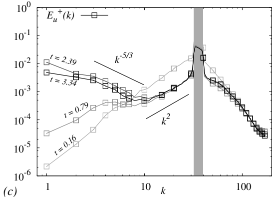




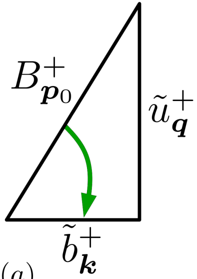






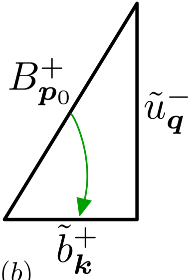


Inverse cascade of magnetic helicity



(a)

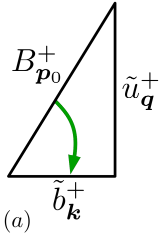
$$\partial_t^2 \tilde{b}^+_{\boldsymbol{k}} = -|g^{+++}|^2 \, k(k-p_0) \, |B^+_{\boldsymbol{p}_0}|^2 \tilde{b}^+_{\boldsymbol{k}} \qquad B^+_{\boldsymbol{p}_0} \xrightarrow{\tilde{u}^+_{\boldsymbol{q}}} \tilde{b}^+_{\boldsymbol{k}}$$



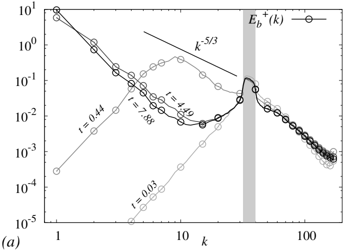
(b)

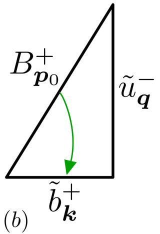
$$\partial_t^2 \tilde{b}^+_{\boldsymbol{k}} = -|g^{++-}|^2 \, k(k-p_0) |B^+_{\boldsymbol{p}_0}|^2 \tilde{b}^+_{\boldsymbol{k}} \qquad B^+_{\boldsymbol{p}_0} \xrightarrow{\tilde{u}^-_{\boldsymbol{q}}} \tilde{b}^+_{\boldsymbol{k}}$$

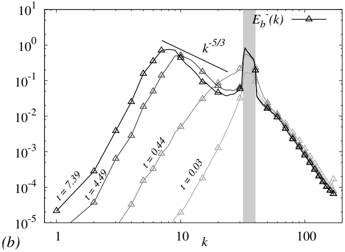
Inverse cascade of magnetic helicity: DNS ($Re_\lambda = 140$)



Magnetic field

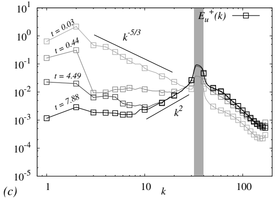


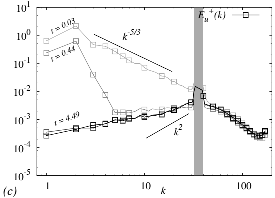




Velocity field

$\boldsymbol{u} = \boldsymbol{u}^+$





Conclusions

The triadic systems give qualitatively correct descriptions of the dynamics.

1. STF-like dynamo on the triad level:
   - α-like triadic dynamo is the dominant large-scale instability.
   - 'anti-α' triadic dynamo is the dominant small-scale instability.
2. The α-like triadic dynamo becomes more dominant with larger scale separation.
3. Inverse cascade of magnetic helicity is most efficient if H_k and H_m are of the same sign.
4. The effect of the Lorentz force on the flow is most prominent if H_k and H_m are of the same sign.

Thank you

M. Linkmann, G. Sahoo, M. McKay, A. Berera, L. Biferale, arXiv:1609.01781
M. Linkmann, A. Berera, M. McKay, J. Jäger, JFM **791**, 61-96 (2016)

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