ON THE DIRECT AND INVERSE ENERGY TRANSFER IN ROTATING TURBULENCE

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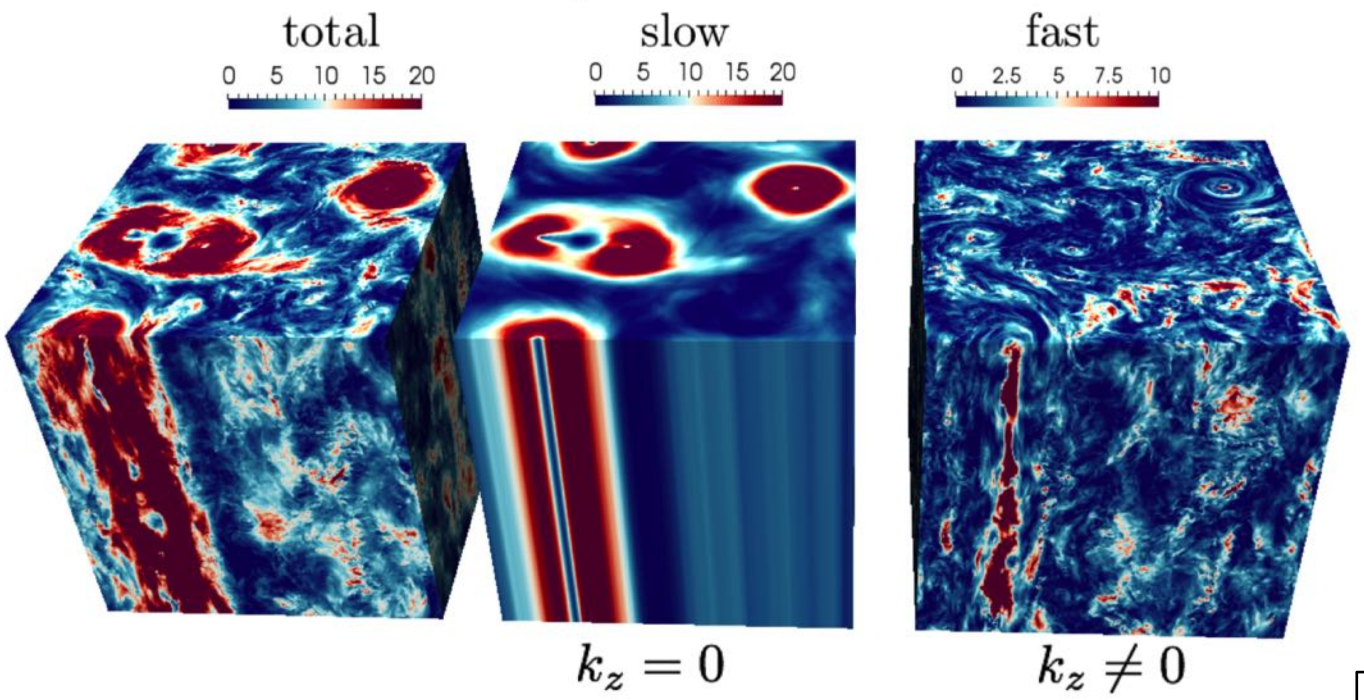
Kinetic and Related Models in the Natural Sciences (29 April – 2 May, 2018)

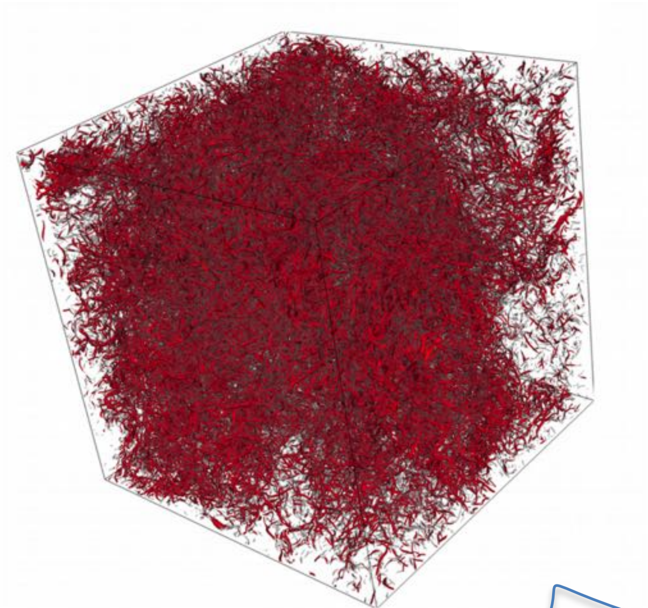


F. Bonaccorso, M. Linkmann, M. Buzzicotti,
P. Clark di Leoni (Rome, Italy);
H. Aluie (Rochester, USA);
A. Alexakis (Paris, France)

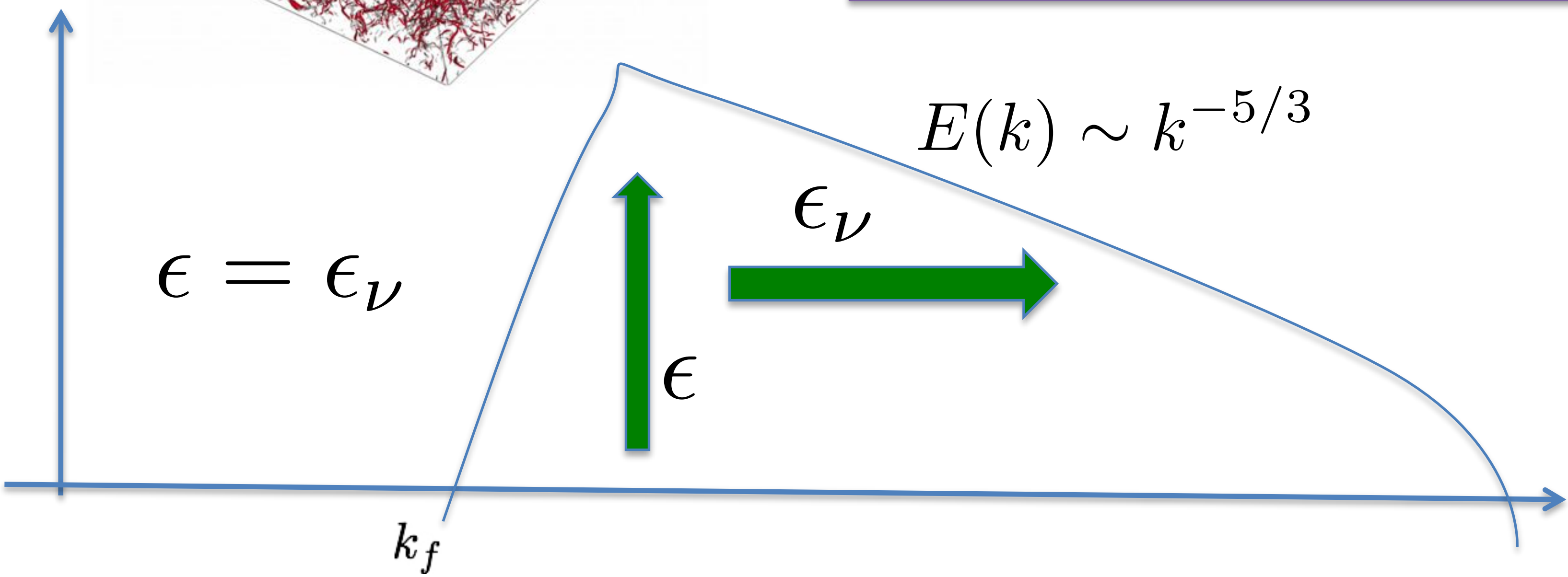
PRACE 09_2256
ROTATING TURBULENCE
2015 – 55MH

MOTIVATION:

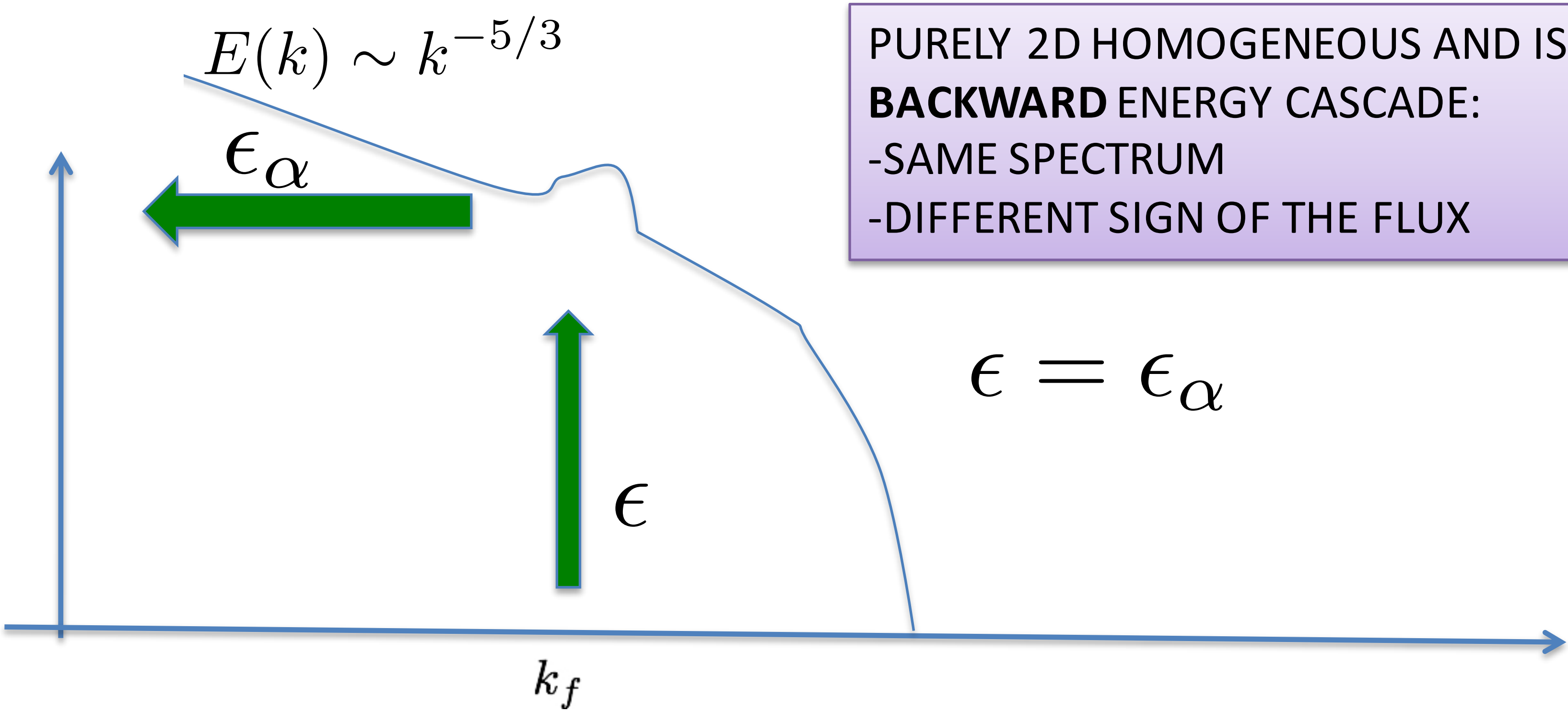
WHAT DO WE KNOW ABOUT THE DIRECTION(S) AND INTENSITY(IES) OF THE TURBULENT ENERGY TRANSFER?



3D HOMOGENEOUS AND ISOTROPIC
FORWARD ENERGY CASCADE



MOTIVATION:

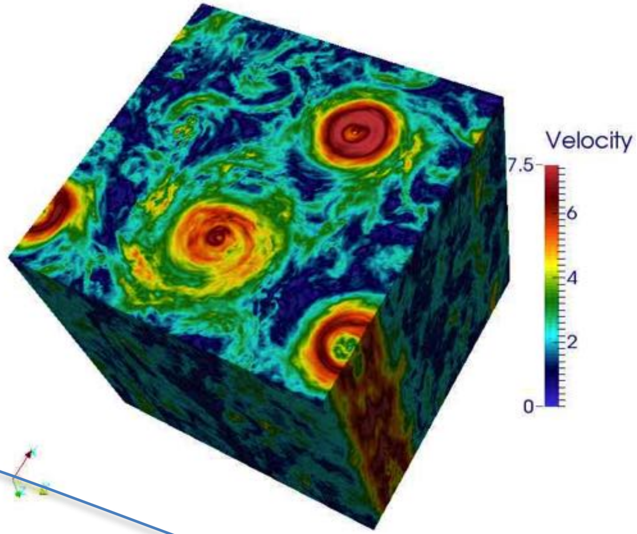


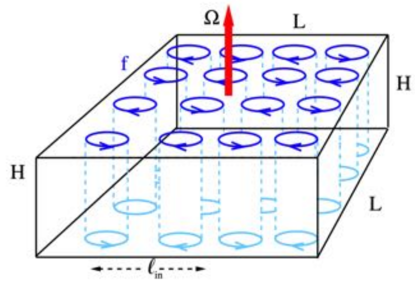
PURELY 2D HOMOGENEOUS AND ISOTROPIC
BACKWARD ENERGY CASCADE:
-SAME SPECTRUM
-DIFFERENT SIGN OF THE FLUX

$$\epsilon = \epsilon_\alpha$$

MOTIVATION:

WHAT DO WE KNOW ABOUT THE DIRECTION(S) AND INTENSITY(IES) OF THE TURBULENT ENERGY TRANSFER?

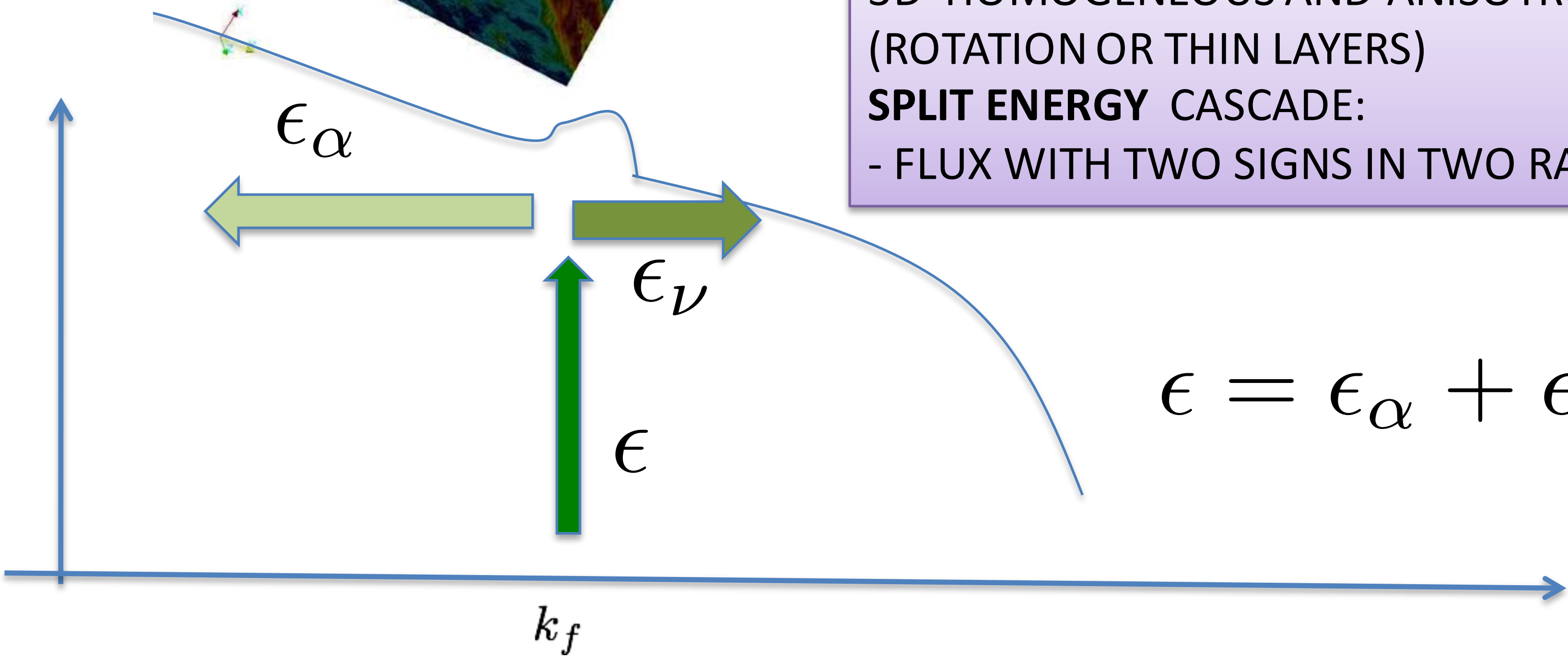




3D HOMOGENEOUS AND ANISOTROPIC (ROTATION OR THIN LAYERS)

SPLIT ENERGY CASCADE:

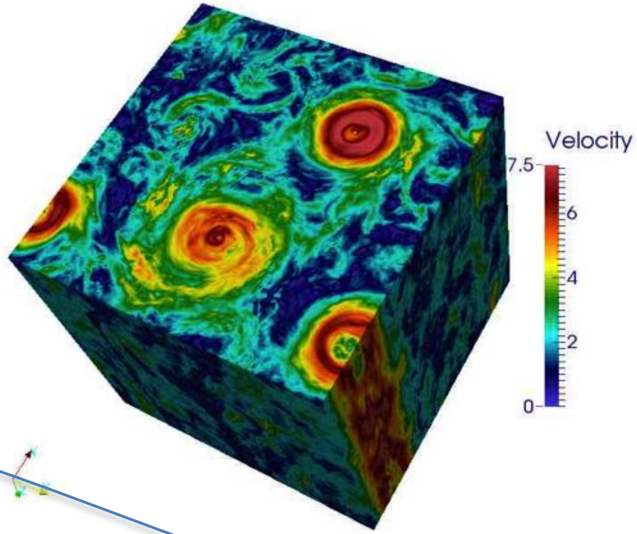
- FLUX WITH TWO SIGNS IN TWO RANGES



$$\epsilon = \epsilon_\alpha + \epsilon_\nu$$

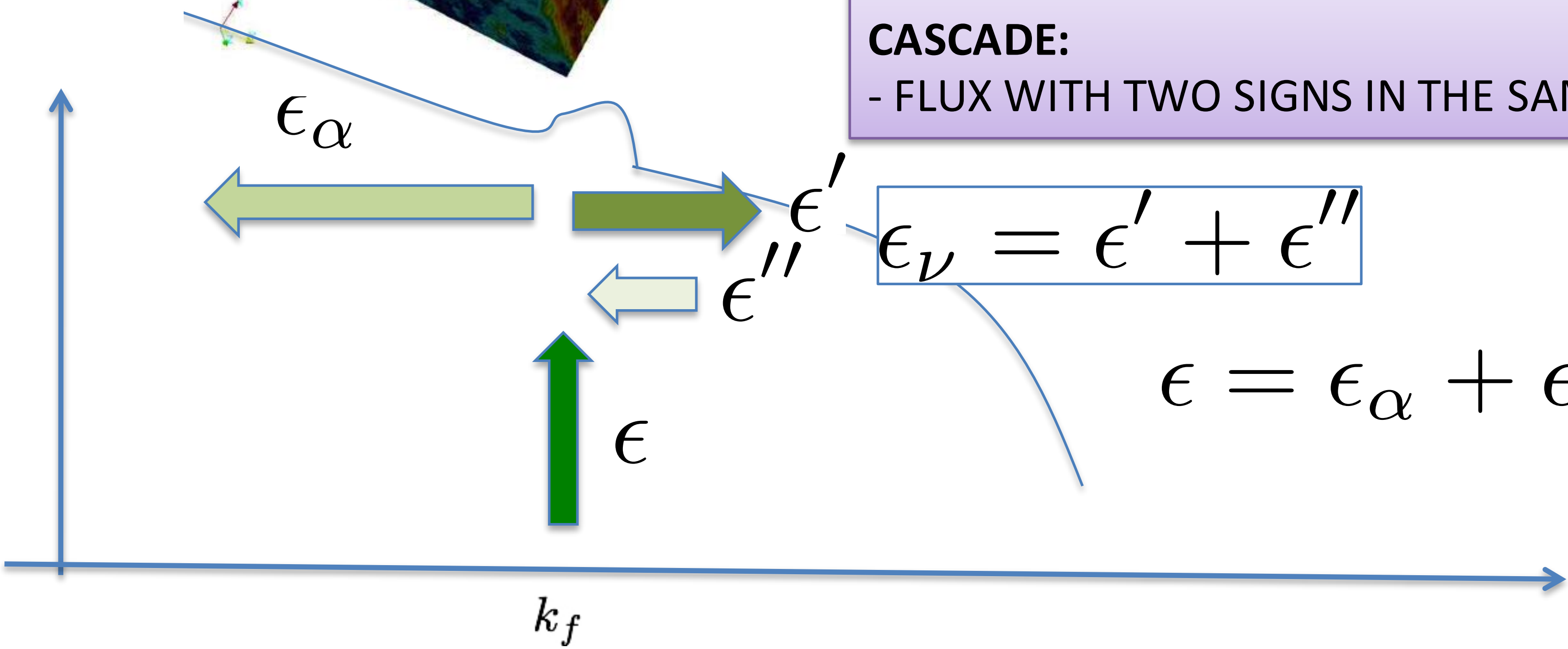
MOTIVATION:

WHAT DO WE KNOW ABOUT THE DIRECTION(S) AND INTENSITY(IES) OF THE TURBULENT ENERGY TRANSFER?



SPLIT + BIDIRECTIONAL CASCADE:

- FLUX WITH TWO SIGNS IN THE SAME RANGE



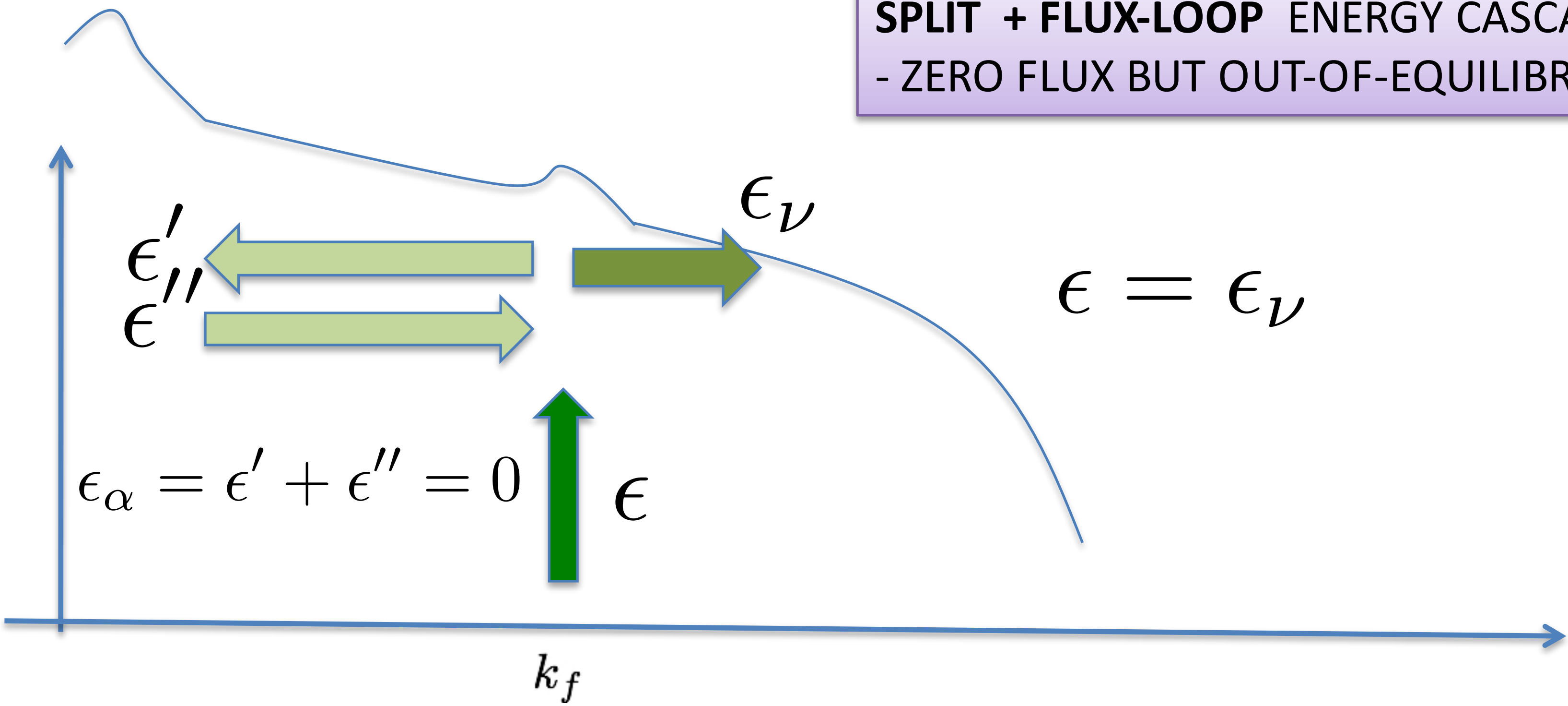
$$\epsilon_\nu = \epsilon' + \epsilon''$$

$$\epsilon = \epsilon_\alpha + \epsilon_\nu$$

MOTIVATION:

WHAT DO WE KNOW ABOUT THE DIRECTION(S) AND INTENSITY(IES) OF THE TURBULENT ENERGY TRANSFER?

SPLIT + FLUX-LOOP ENERGY CASCADE:
- ZERO FLUX BUT OUT-OF-EQUILIBRIUM!



TAKE HOME MESSAGE: NEITHER THE SPECTRUM NOR THE SIGN OF THE ENERGY FLUX ARE ENOUGH TO CONTROL THE UNDERLYING PHYSICS

DNS OF NAVIER_STOKES EQS IN A ROTATING FRAME (NO BOUNDARIES)

DNS: L. Smith, F. Waleffe, A. Pouquet, P. Mininni, A. Alexakis, S. Chen, G. Eyink ....

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{\Omega} \times \mathbf{v} = \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{F} - \alpha \mathbf{v}$$

$\mathbf{\Omega}$ =rotation

$$P = P_0 + \frac{1}{2} |\mathbf{\Omega} \times \mathbf{r}|^2$$

$\mathbf{F}$ =large scale Forcing

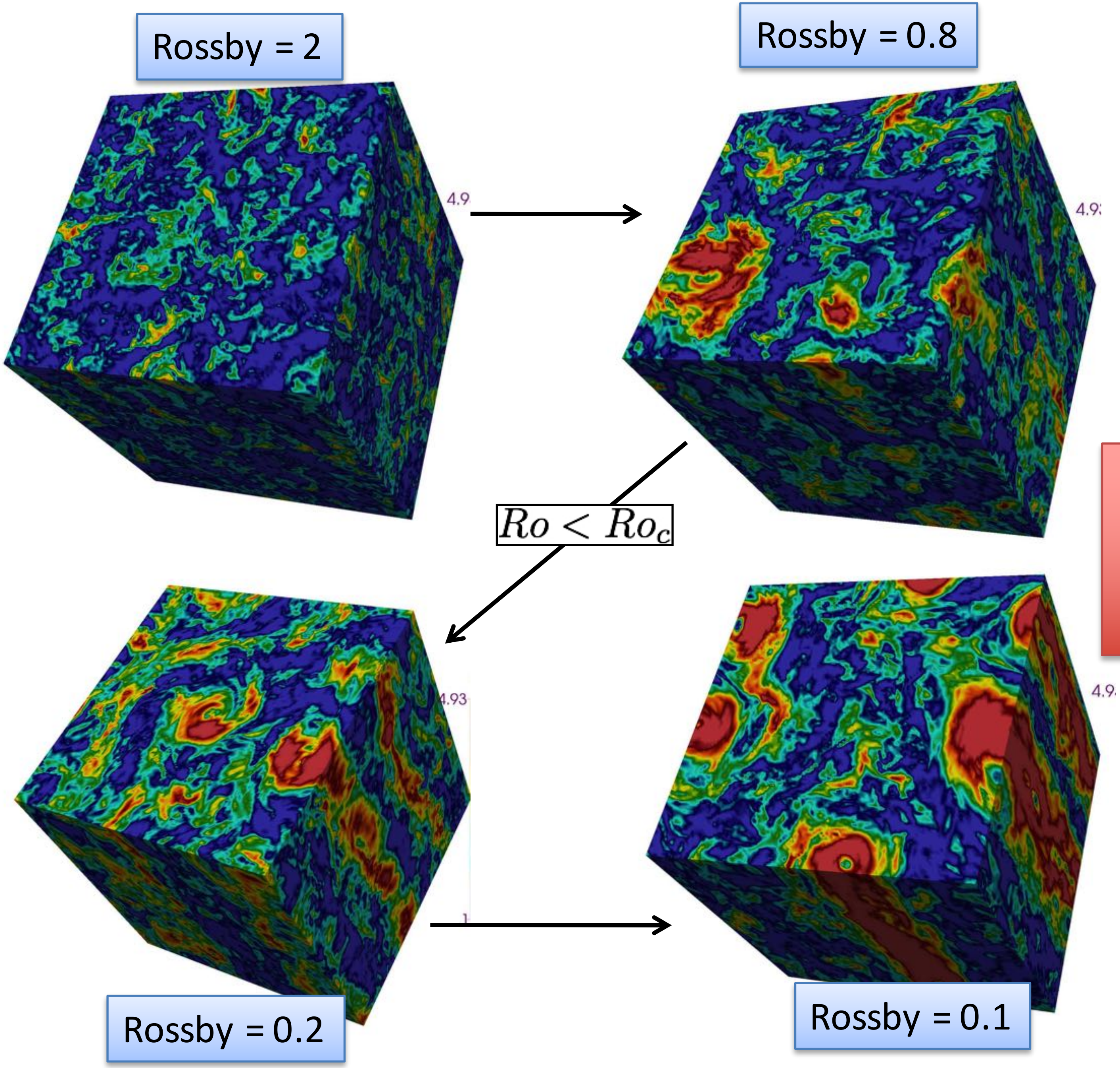
α = large scale energy sink

ROSSBY NUMBER ~ NON-LINEAR/ROTATION

$$\mathrm{Ro} \sim \frac{v_0}{\Omega L_0}$$

$\mathrm{Ro} \geq Ro_c \rightarrow$ FORWARD ENERGY TRANSFER

$\mathrm{Ro} \leq Ro_c \rightarrow$ SPLIT ENERGY TRANSFER



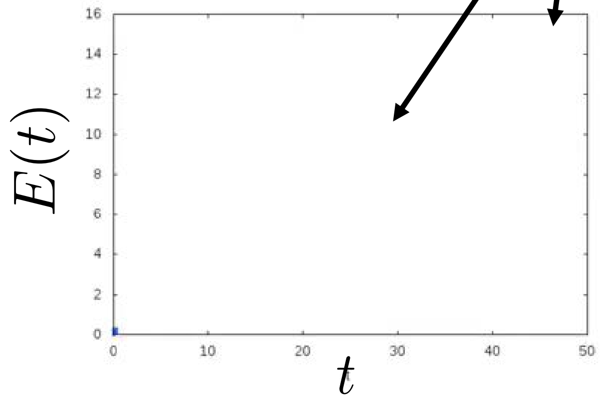
HOMOGENEOUS
ANISOTROPIC
2D & 3D PHYSICS
CHOERENT -STRUCTURES

A.Sen et al Jour Atmos Science 68, 2757 (2011); E. Yarom et al PoF 25, 085105 (2013); A. Campagne et al PoF 26, 125112 (2014); L.B. F. Bonaccorso et al PRX 6, 041036 (2016); E. Deusebio et al PRE 90, 023005 (2014); A. Alexakis JFM 769, 46 (2015)

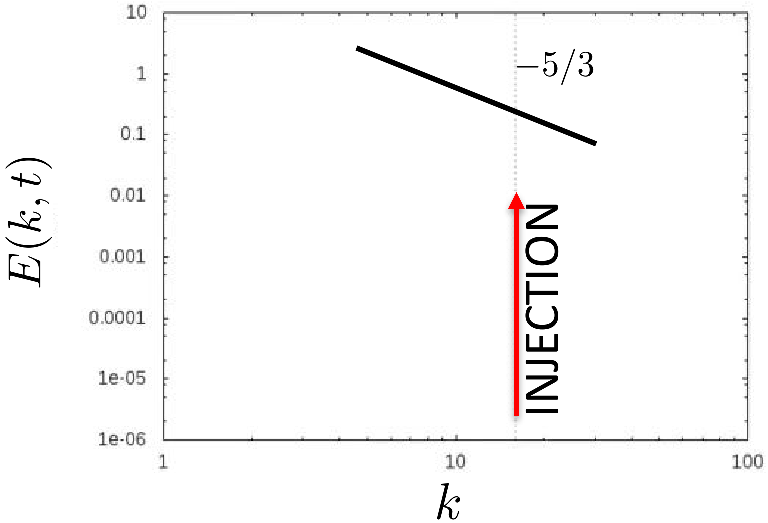
TURBULENCE UNDER ROTATION

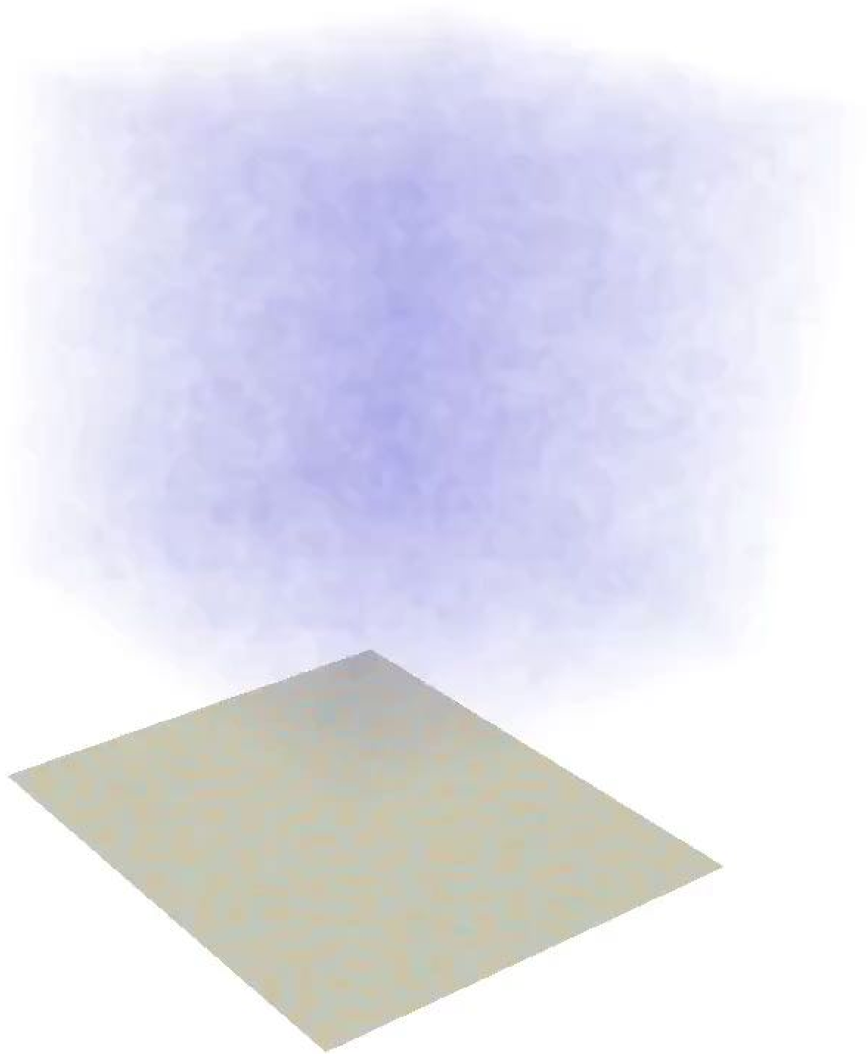
3D EVENT

TOTAL ENERGY



ENERGY SPECTRUM





Courtesy of M. Buzzicotti & F. Bonaccorso

PHYSICS OF FLUIDS VOLUME 11, NUMBER 6 JUNE 1999

Transfer of energy to two-dimensional large scales in forced, rotating three-dimensional turbulence

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$$\boldsymbol{u}(\boldsymbol{x},t) = \sum_{\boldsymbol{k},s_k} \mathbf{h}_{s_k}(\boldsymbol{k}) e^{i[\boldsymbol{k}\cdot\boldsymbol{x} - \omega_{s_k}(\boldsymbol{k})t]}$$

$$i\boldsymbol{k} \times \mathbf{h}_{s_k} = s_k k\, \mathbf{h}_{s_k}$$

$$\omega_{s_k}(\boldsymbol{k}) = s_k 2\Omega \frac{k_z}{|\boldsymbol{k}|},$$

$$\frac{d}{dt} u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}} (s_p p - s_q q)$$

$$e^{i(\omega^{s_k} + \omega^{s_p} + \omega^{s_q})t/Ro} \times [u^{s_p}(\mathbf{p}) u^{s_q}(\mathbf{q})]^*.$$

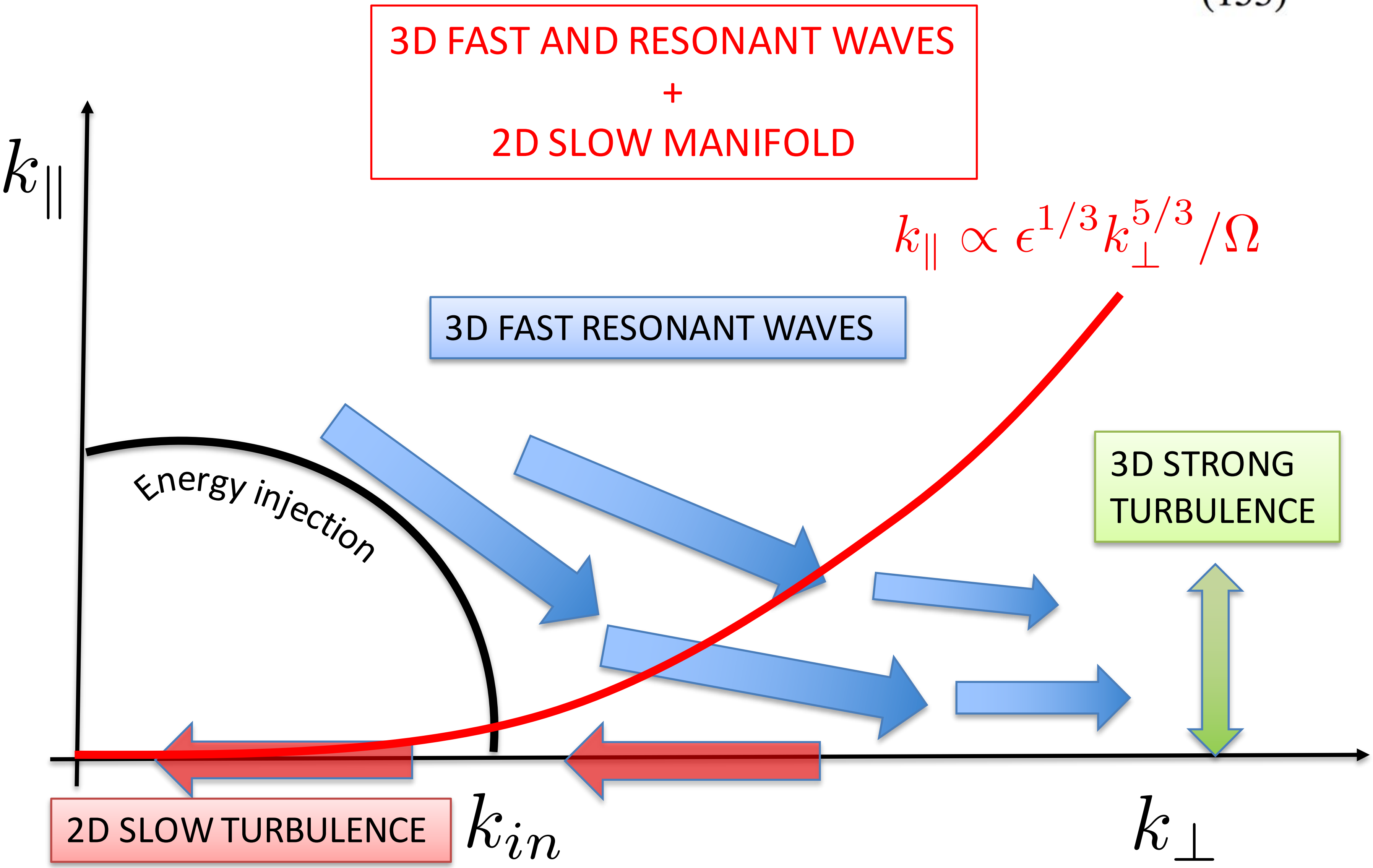
$$\Omega \to \infty$$

$$\omega_{s_k}(\boldsymbol{k}) + \omega_{s_p}(\boldsymbol{p}) + \omega_{s_q}(\boldsymbol{q}) = 0,$$

RESONANT WAVES

$$\partial_t \mathbf{u}_{2D} + \mathbf{u}_{2D} \cdot \nabla \mathbf{u}_{2D} = -\overline{\mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{3D}} - \overline{\nabla P} + \nu \Delta \mathbf{u}_{2D} - \alpha \mathbf{u}_{2D} + \mathbf{f}_{2D}, \tag{152}$$

$$\partial_t \mathbf{u}_{3D} + \mathbf{u}_{2D} \cdot \nabla \mathbf{u}_{3D} + 2\Omega \hat{\mathbf{e}}_{\mathbf{z}} \times \mathbf{u}_{3D} = -\mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{2D} + (\overline{\mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{3D}} - \mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{3D}) - \nabla P + \nu \Delta \mathbf{u}_{3D} + \mathbf{f}_{3D}, \tag{153}$$

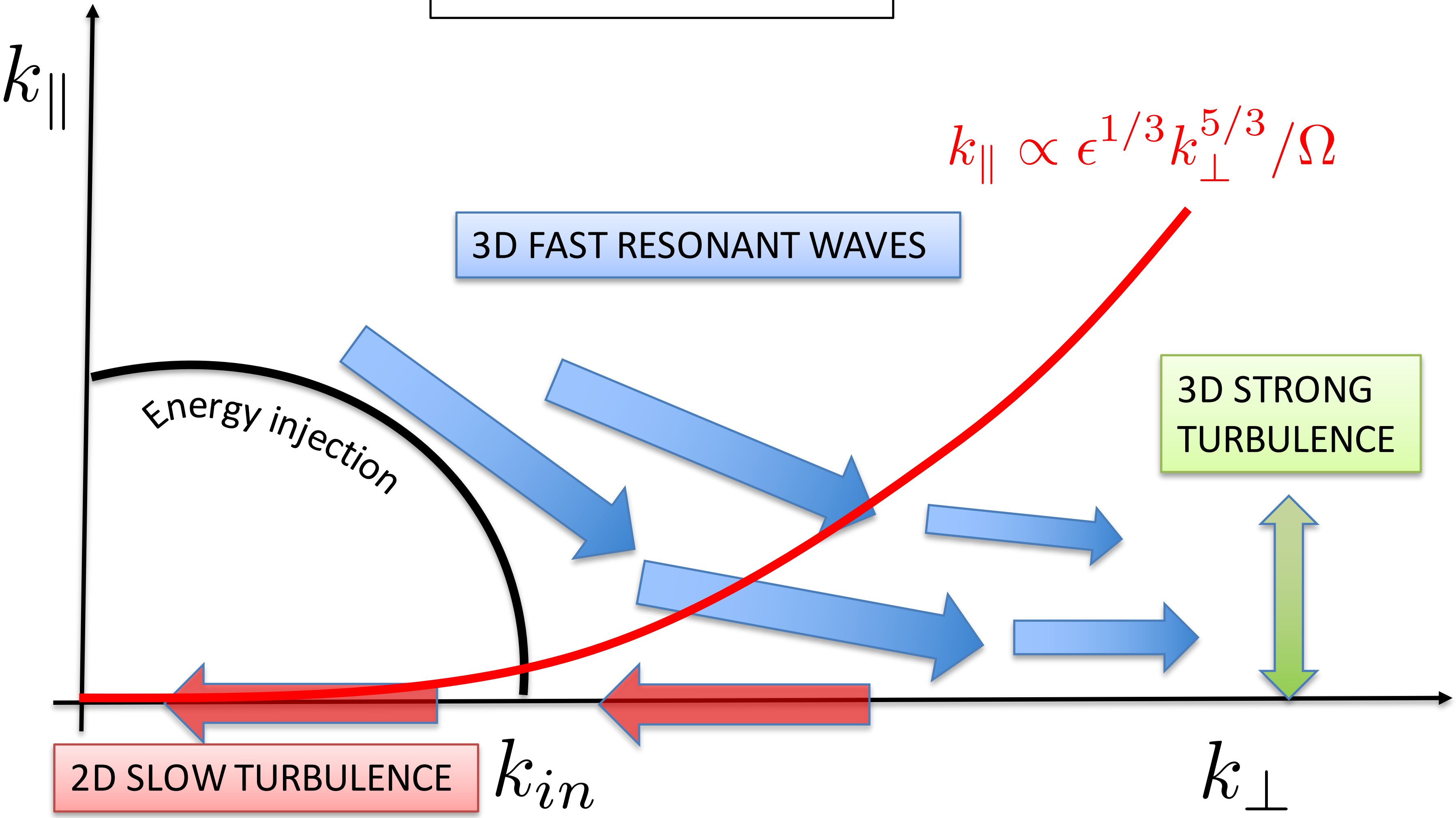


A.C. Newell and B. Rumpf ARFM 43, 59 (2011); Clark di Leoni and P.D. Minnini JFM, 809, 821 (2016); L.M. Smith and Waleffe PoF 11, 1608 (1999); S. Galtier PRE 68, 015301 (2003). S. Nazarenko *Wave Turbulence* (Springer 2011); B. Gallet JFM 783, 412 (2015)

H FIXED

$\Omega \to \infty$

2D/3D DECOUPLING:
SPLIT ENERGY TRANSFER
2D: BACKWARD
3D: FORWARD



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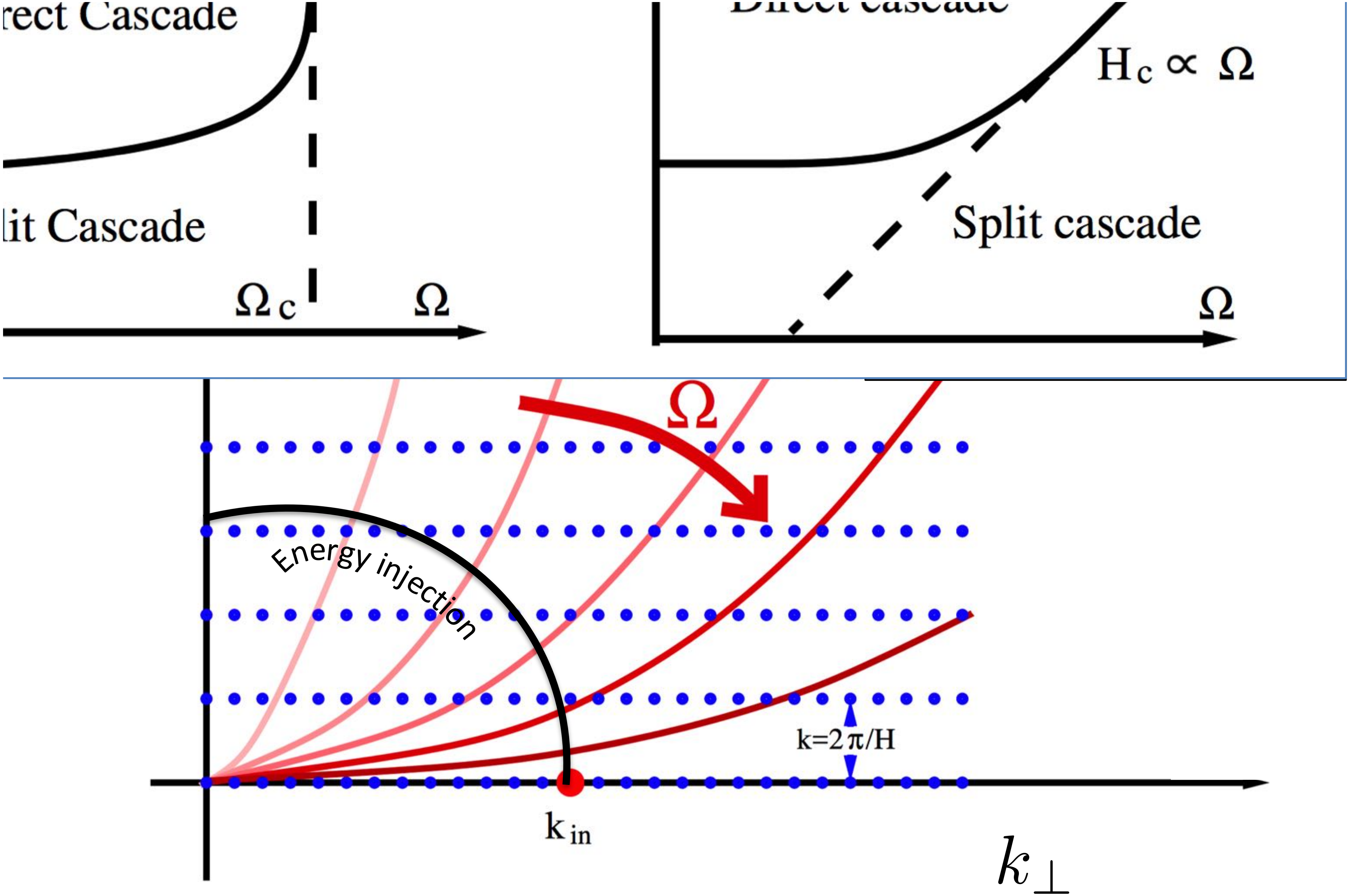
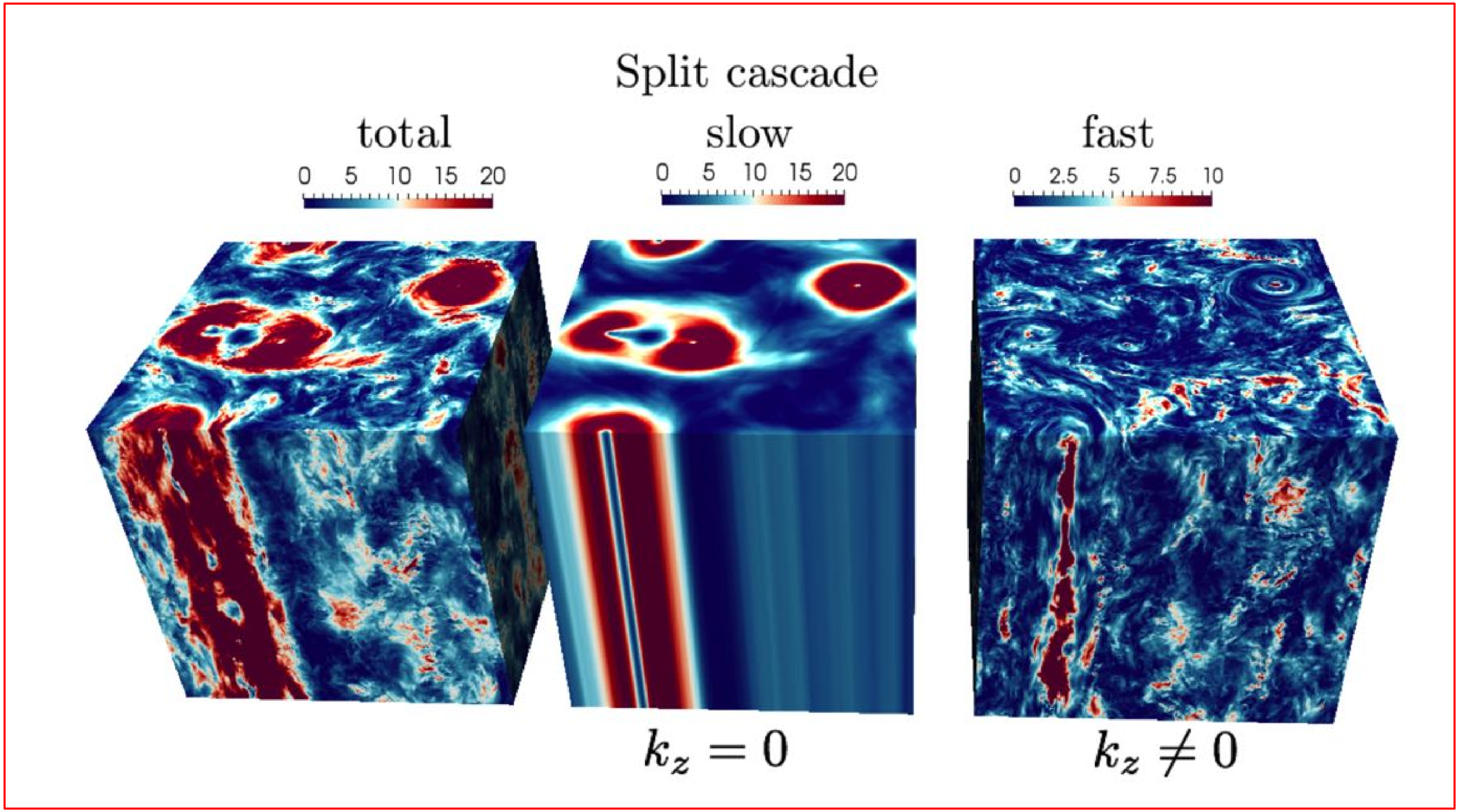


Ω_c
Ω
$H_c \propto \Omega$
Split cascade
Ω
Ω
Energy injection
$k=2\pi/H$
k_{in}
$k_\perp$

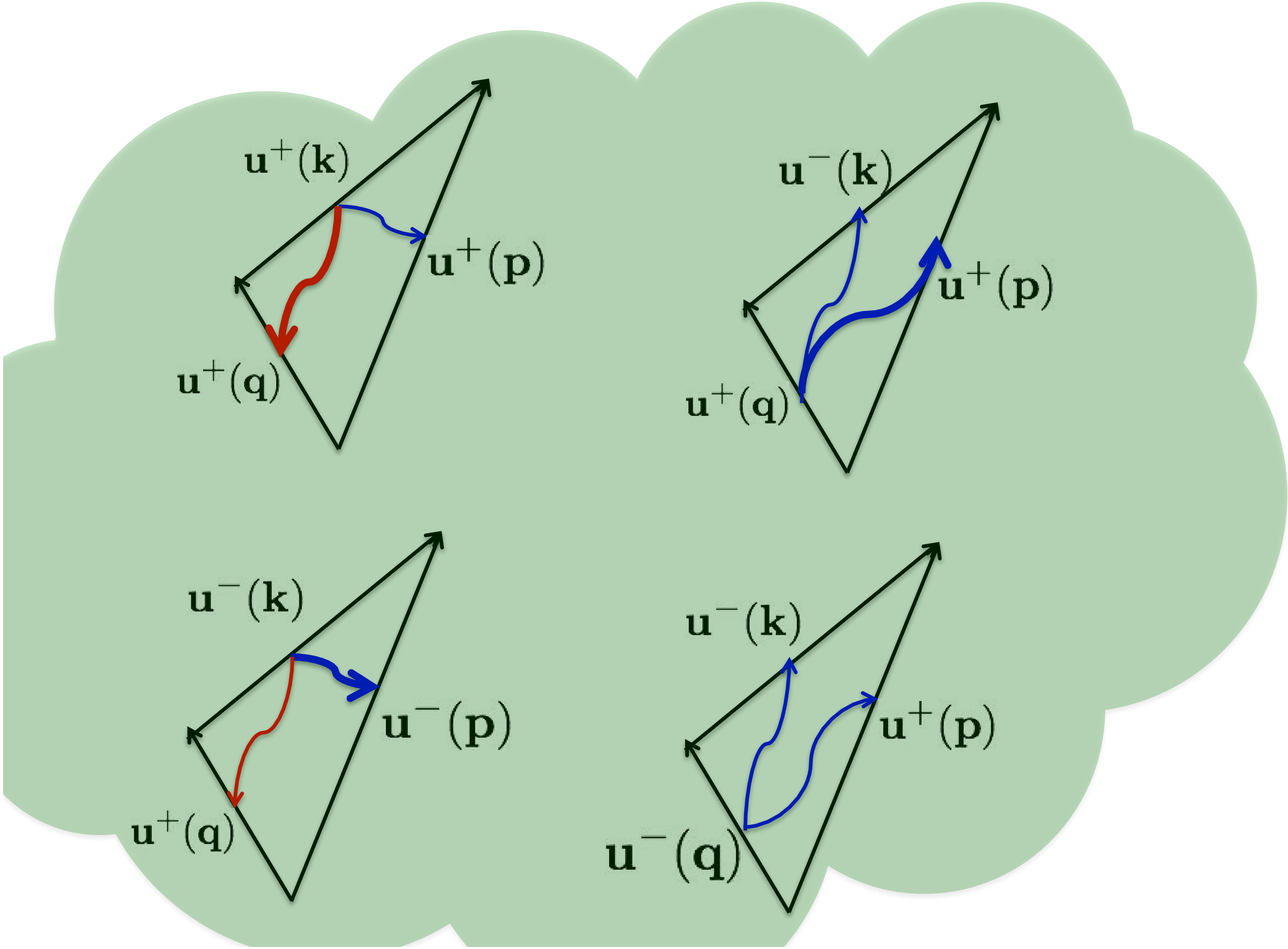
$$\partial_t \mathbf{u}_{2D} + \mathbf{u}_{2D} \cdot \boldsymbol{\nabla} \mathbf{u}_{2D} = -\overline{\mathbf{u}_{3D} \cdot \boldsymbol{\nabla} \mathbf{u}_{3D}} - \overline{\boldsymbol{\nabla}} P + \nu \Delta \mathbf{u}_{2D} - \alpha \mathbf{u}_{2D} + \mathbf{f}_{2D}, \tag{152}$$

$$\partial_t \mathbf{u}_{3D} + \mathbf{u}_{2D} \cdot \boldsymbol{\nabla} \mathbf{u}_{3D} + 2\Omega \hat{\mathbf{e}}_{\mathbf{z}} \times \mathbf{u}_{\mathbf{3D}} = -\mathbf{u}_{3D} \cdot \boldsymbol{\nabla} \mathbf{u}_{2D} + (\overline{\mathbf{u}_{3D} \cdot \boldsymbol{\nabla} \mathbf{u}_{3D}} - \mathbf{u}_{3D} \cdot \boldsymbol{\nabla} \mathbf{u}_{3D}) - \boldsymbol{\nabla} P + \nu \Delta \mathbf{u}_{3D} + \mathbf{f}_{3D}, \tag{153}$$



IS THE 2D CHANNEL THE ONLY ONE AVAILABLE TO TRANSFER ENERGY UP?

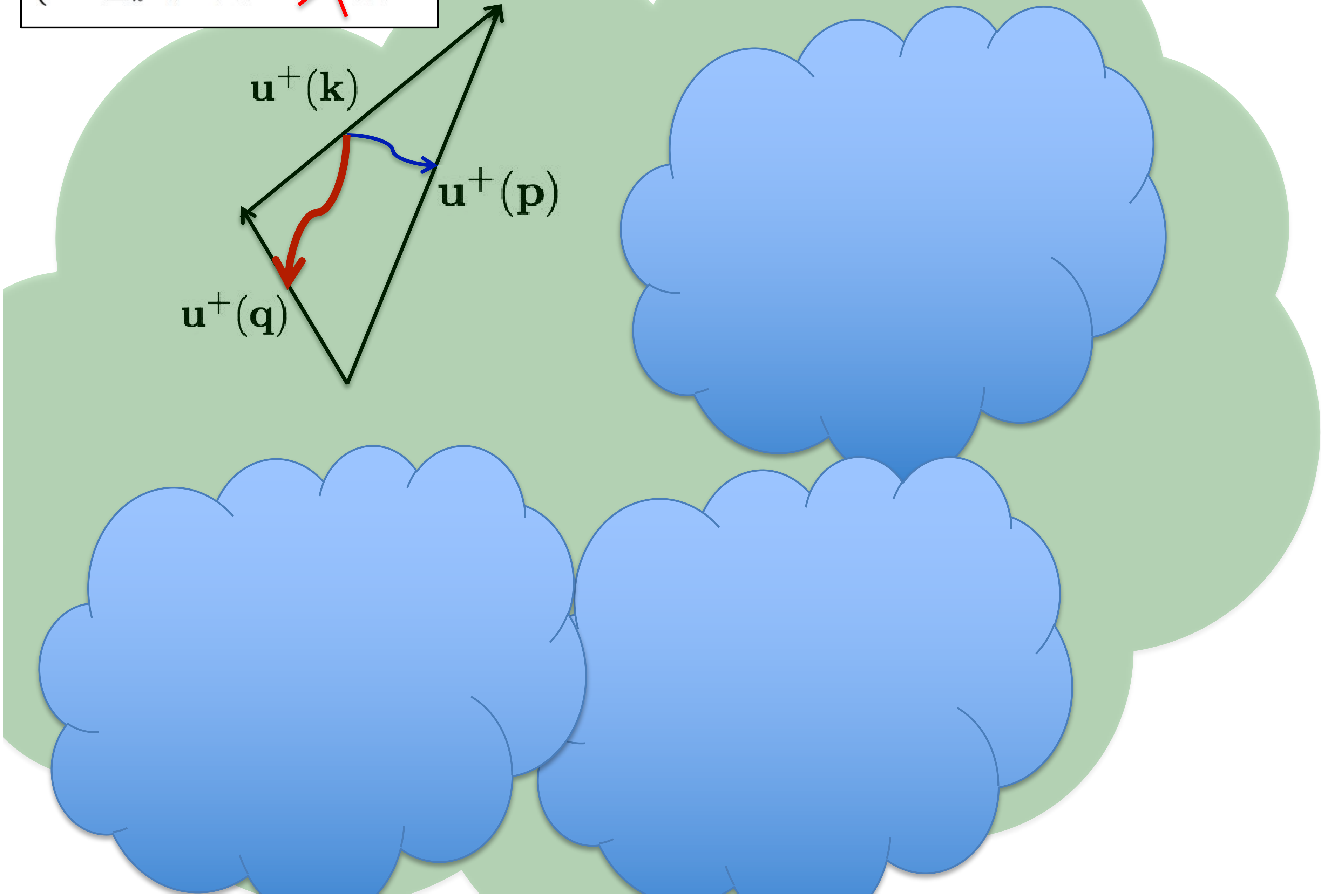
HELICAL TRIADIC INTERACTION IN THE NSE: HOMOCHIRAL-HETEROCHIRAL DECOMPOSITION



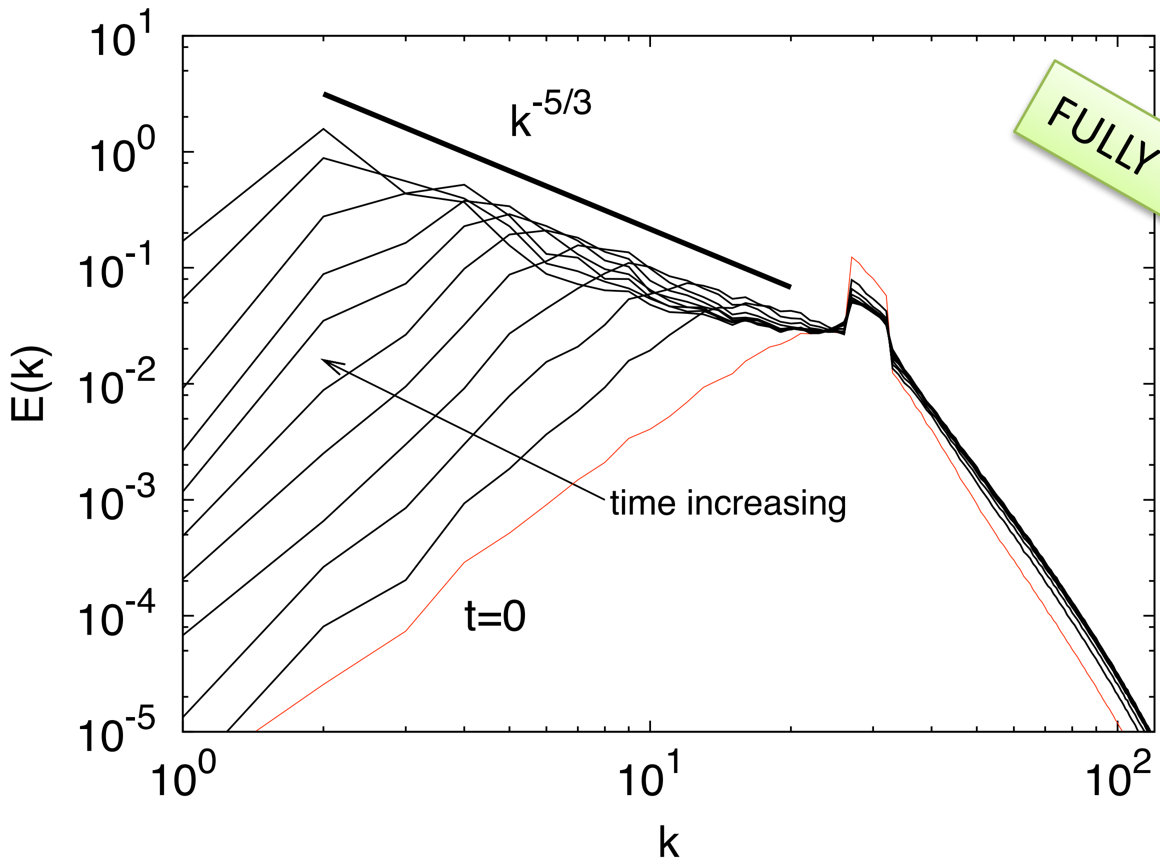
TRIADIC INTERACTION IN DECIMATED NAVIER_STOKES EQS

HOMOCHIRAL

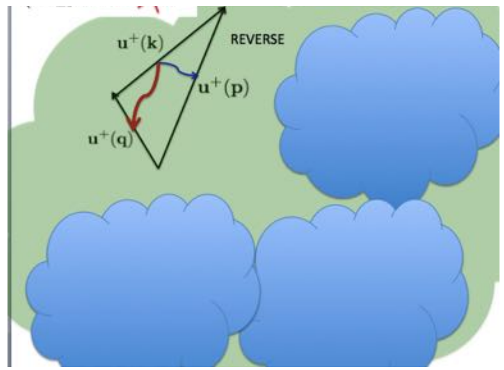
$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$



$$
\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}
$$



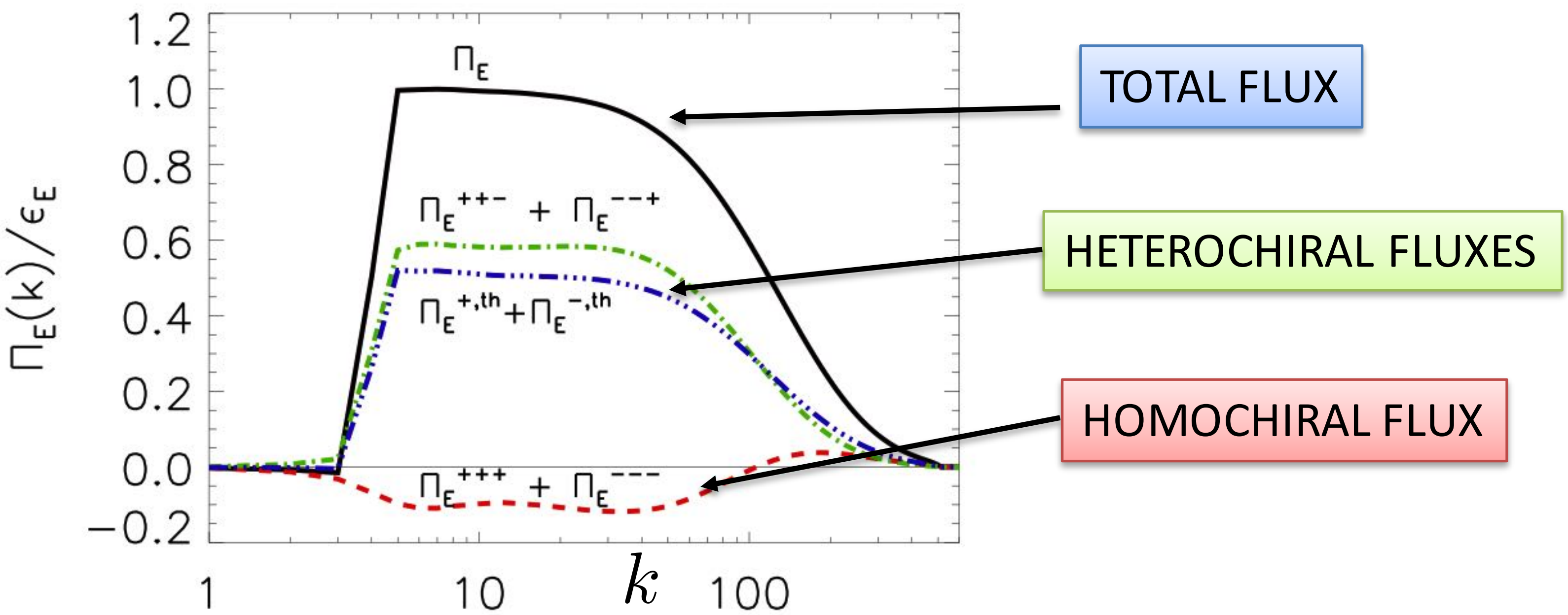
FULLY 3D AND ISOTROPIC



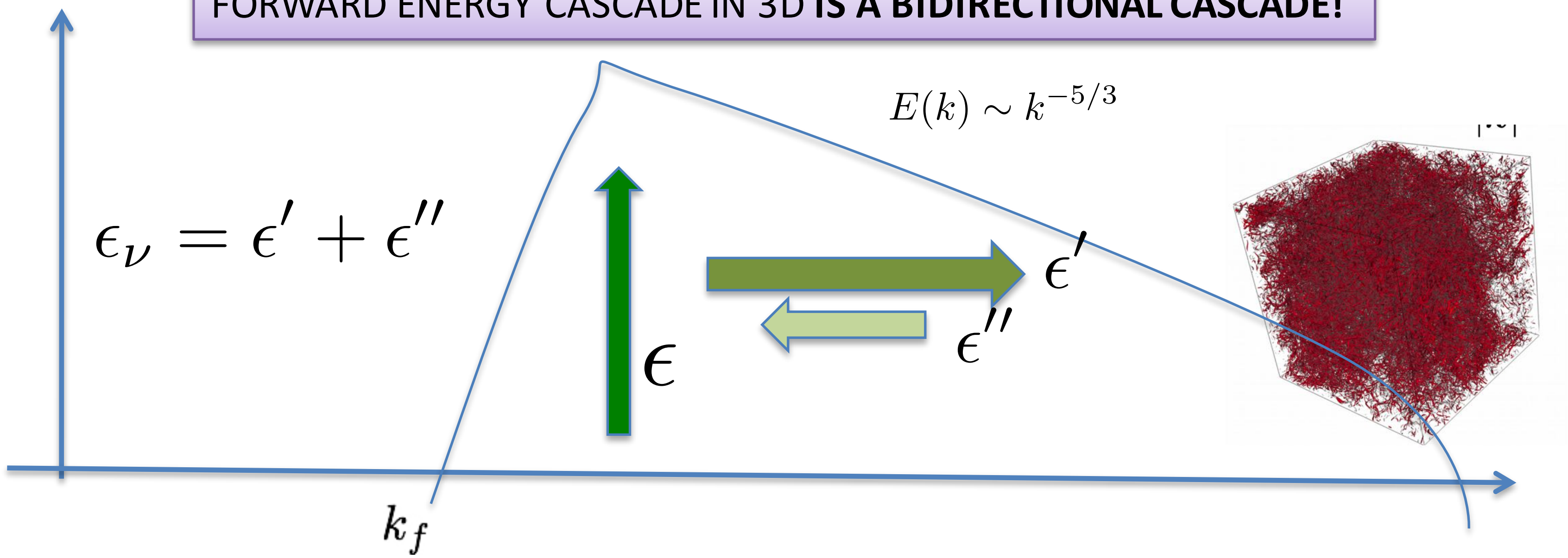
L.B., S. Musacchio & F. Toschi Phys. Rev. Lett. 108 164501, (2012); see also F. Waleffe PoF 4,350 (1992)

FULL 3D NAVIER STOKES EQS.

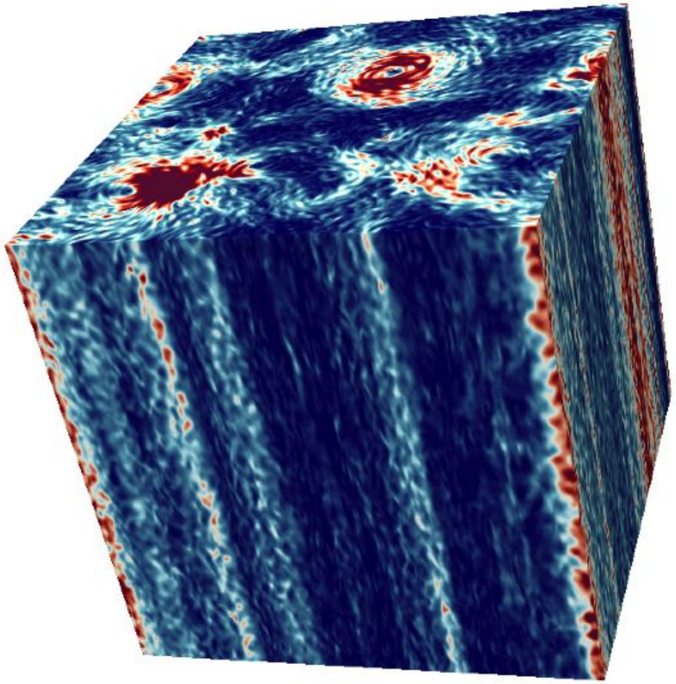
A. Alexakis JFM 812, 752 (2017); G. Sahoo and L.B. Fluid Dyn Res 50, 011420 (2018)

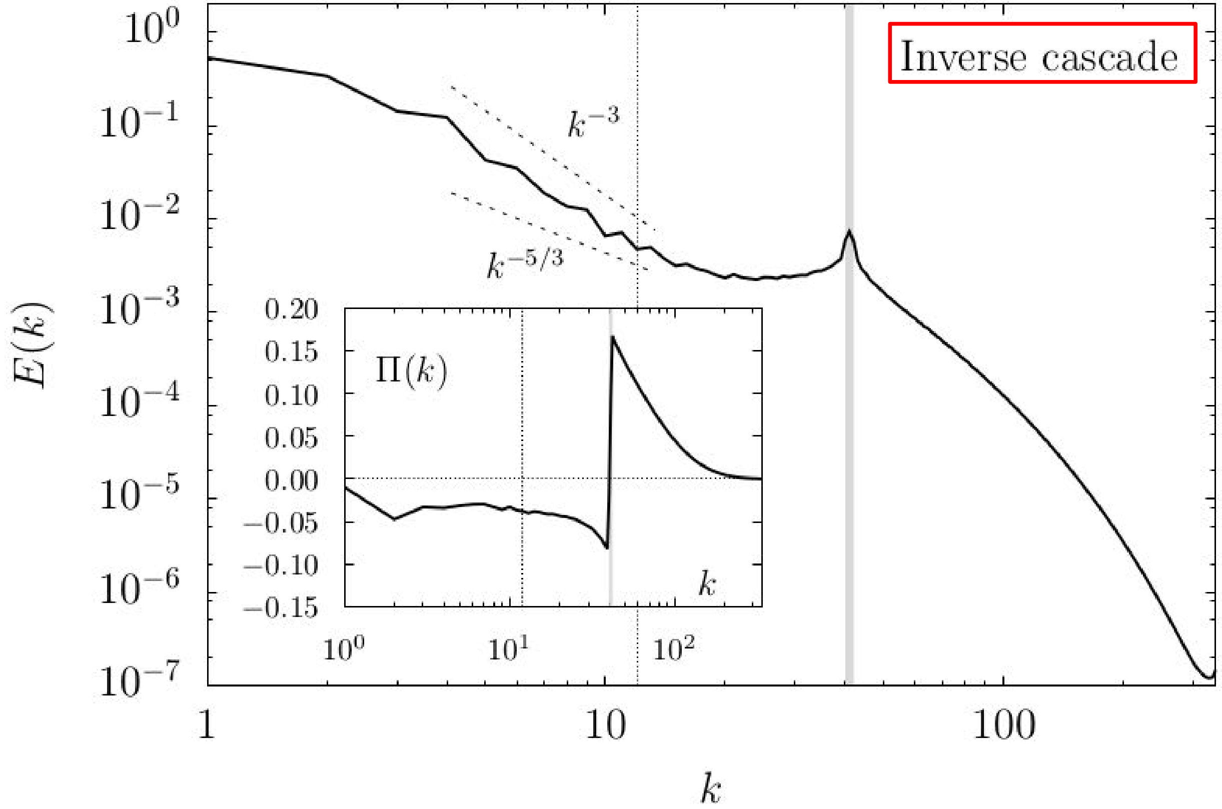


FORWARD ENERGY CASCADE IN 3D **IS A BIDIRECTIONAL CASCADE!**



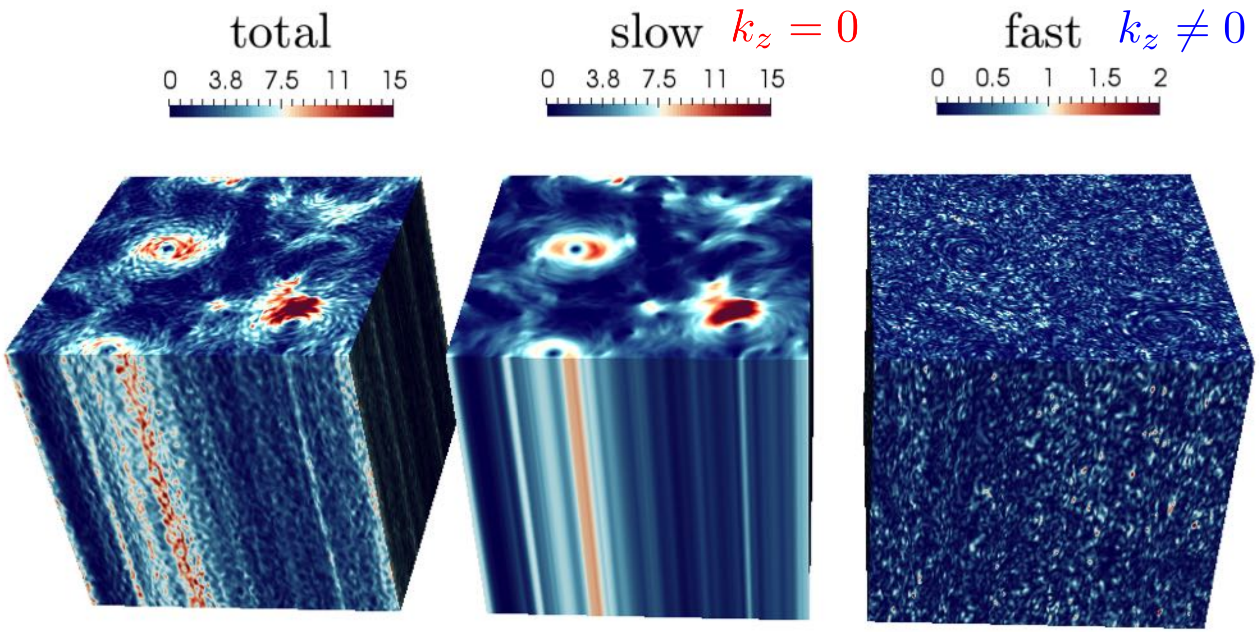
COMPARING 2D/3D SLOW/FAST DECOMPOSITION
WITH
FULLY 3D HETERO/HOMOCHIRAL DYNAMICS





M. Buzzicotti, H. Aluie, L.B. and M. Linkmann
Phys. Rev. Fluids 3, 034802 (2018)

2D/3D SLOW/FAST DECOMPOSITION



$$\hat{\boldsymbol{u}}(\boldsymbol{k}) = \begin{cases} \hat{\boldsymbol{u}}_F(\boldsymbol{k}) & \text{if } k_z \neq 0 \\ \hat{\boldsymbol{u}}_S(\boldsymbol{k}) & \text{if } k_z = 0 \end{cases}$$

$$\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{u}_S(\boldsymbol{x}) + \boldsymbol{u}_F(\boldsymbol{x})$$

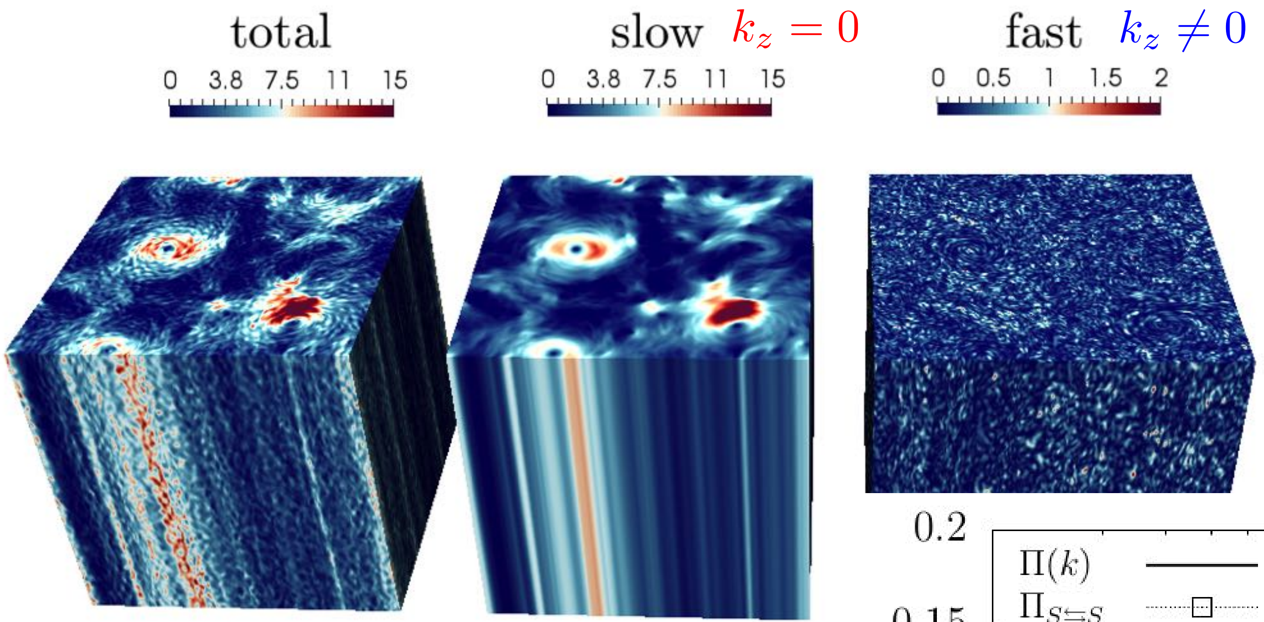
We can rewrite the Navier-Stokes equations for the Slow and Fast manifolds separately:

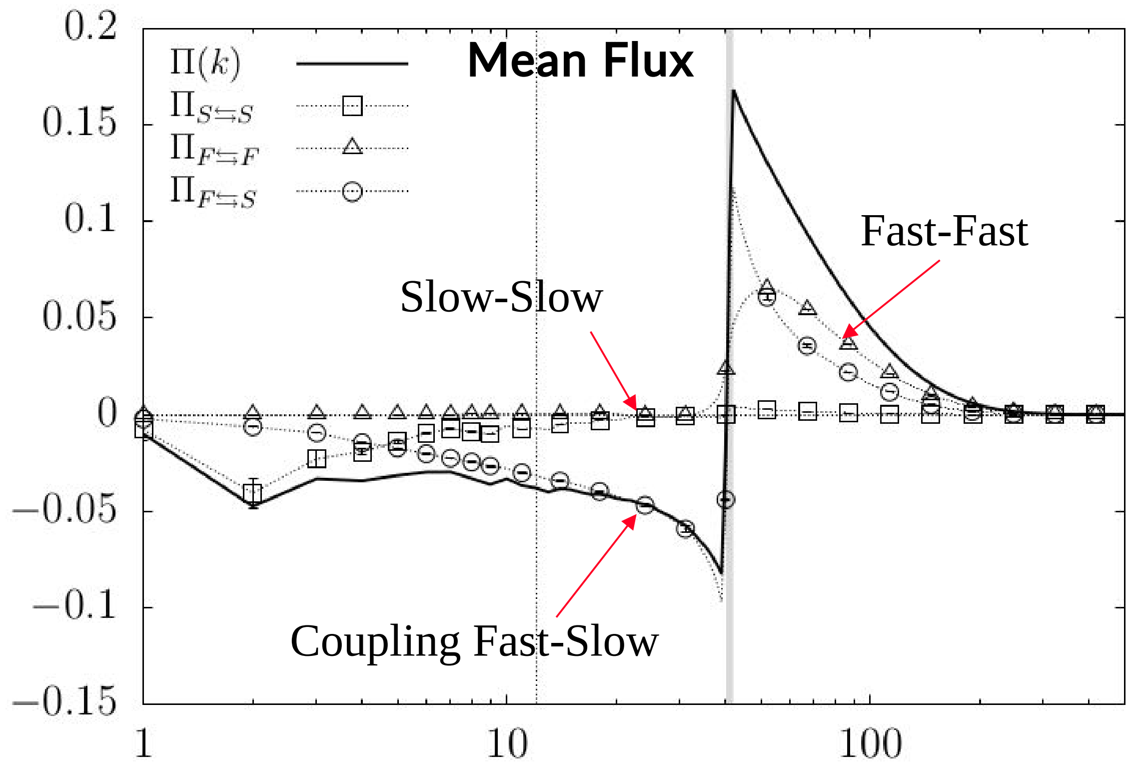
$$\begin{cases} \partial_t \frac{1}{2}\boldsymbol{u}_S^2 + \boldsymbol{u}_S \mathrm{P}_S(\nabla p) = -\boldsymbol{u}_S \cdot (\boldsymbol{u}_S \cdot \nabla \boldsymbol{u}_S) - \boldsymbol{u}_S \cdot (\mathrm{P}_S\left[\boldsymbol{u}_F \cdot \nabla \boldsymbol{u}_F\right]) \\ \partial_t \frac{1}{2}\boldsymbol{u}_F^2 + \boldsymbol{u}_F \mathrm{P}_F(\nabla p) = -\boldsymbol{u}_F \cdot (\boldsymbol{u}_F \cdot \nabla \boldsymbol{u}_S) - \boldsymbol{u}_F \cdot (\boldsymbol{u}_S \cdot \nabla \boldsymbol{u}_F) - \boldsymbol{u}_F \cdot (\mathrm{P}_F[\boldsymbol{u}_F \cdot \nabla \boldsymbol{u}_F]) \end{cases}$$

$$\sum_{k'=1}^{k} \partial_t E(k', t) = \Pi_{S \leftrightarrows S}(k, t) + \Pi_{F \leftrightarrows S}(k, t) + \Pi_{F \leftrightarrows F}(k, t)$$

Slow-Slow Coupling Fast-Fast

2D/3D SLOW/FAST DECOMPOSITION





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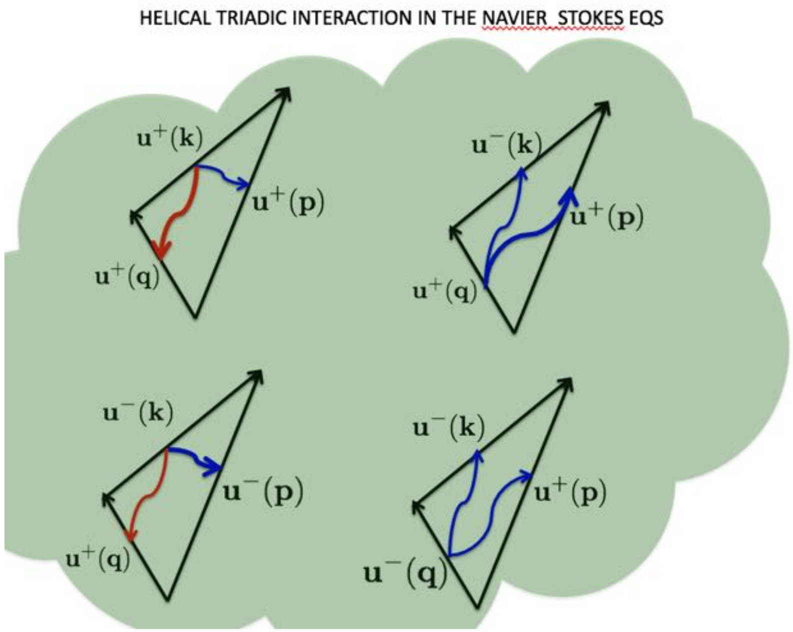
3D HOMO-HETEROCHIRAL DECOMPOSITION

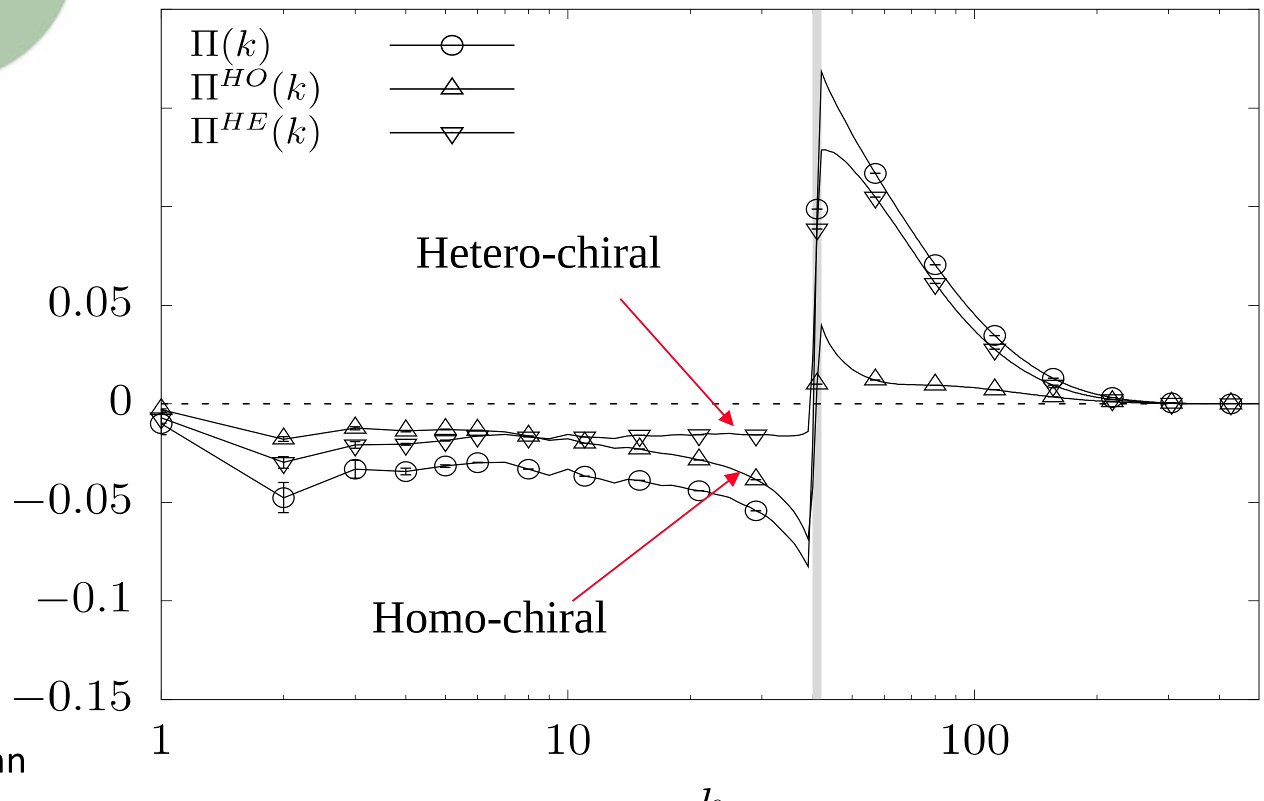


Homo-chiral flux

$$\Pi^{HO}(k) = \sum_{|\mathbf{k}|<k} \bar{\mathbf{u}}^+_{\mathbf{k}} \sum_{\mathbf{p}+\mathbf{q}=\mathbf{k}} (i\mathbf{k}\cdot\mathbf{u}^+_{\mathbf{p}})\mathbf{u}^+_{\mathbf{q}}$$

Hetero-chiral flux

$$\Pi^{\mathrm{HE}}(k) = \Pi(k) - \Pi^{\mathrm{HO}}(k)$$



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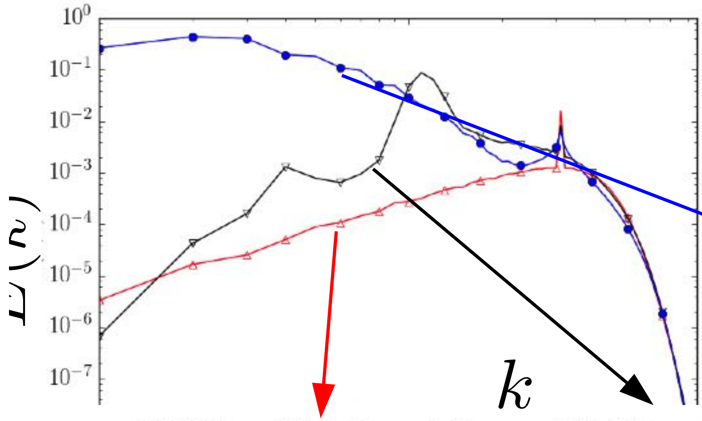
Phys. Rev. Fluids 3, 034802 (2018)

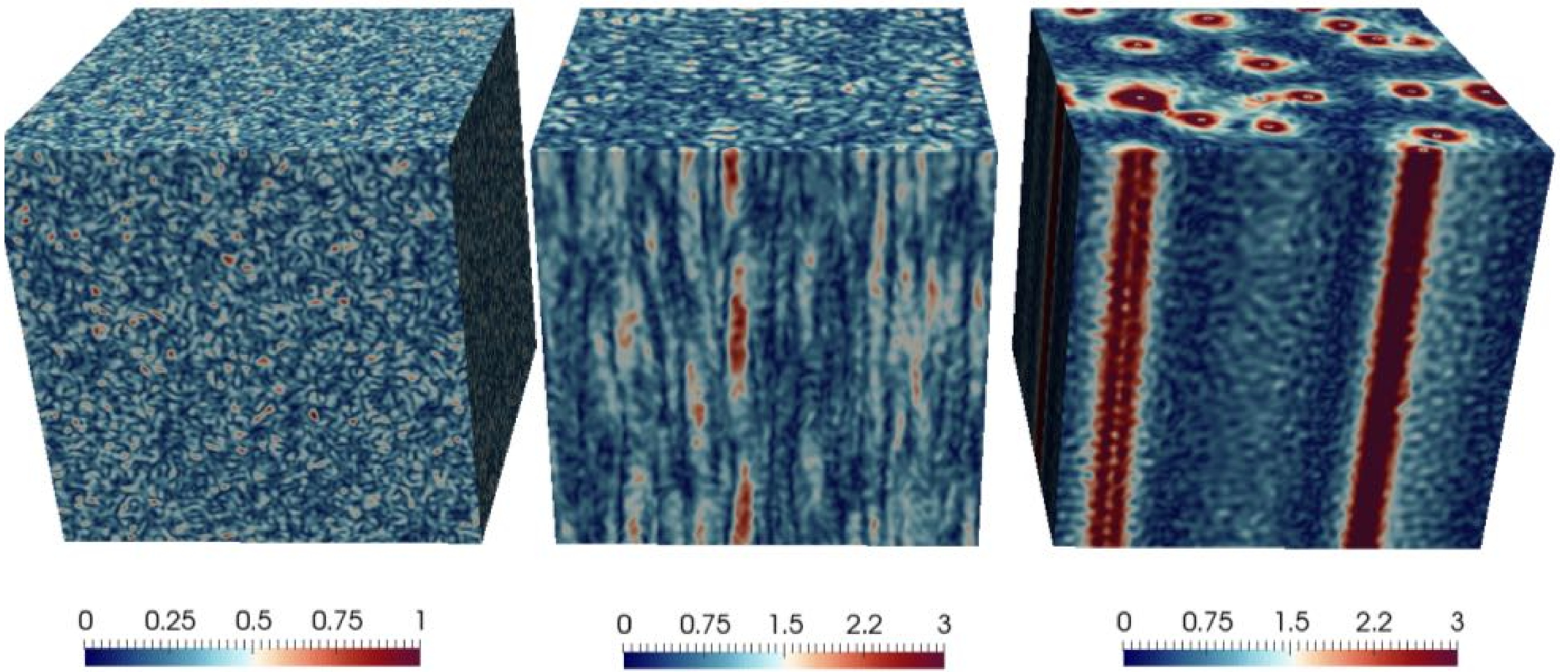
REMOVING THE 2D DYNAMICS

$$\boldsymbol{v}(\boldsymbol{x},t) = \mathcal{P}\,\boldsymbol{u}(\boldsymbol{x},t) = \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,\gamma_{\boldsymbol{k}}\hat{\boldsymbol{u}}(\boldsymbol{k},t)\,, \qquad \gamma_{\boldsymbol{k}} = \begin{cases} 1, & \text{if } \boldsymbol{k} \in \boldsymbol{k}_{3D} \\ 0, & \text{if } \boldsymbol{k} \in \boldsymbol{k}_{2D}\,. \end{cases}$$



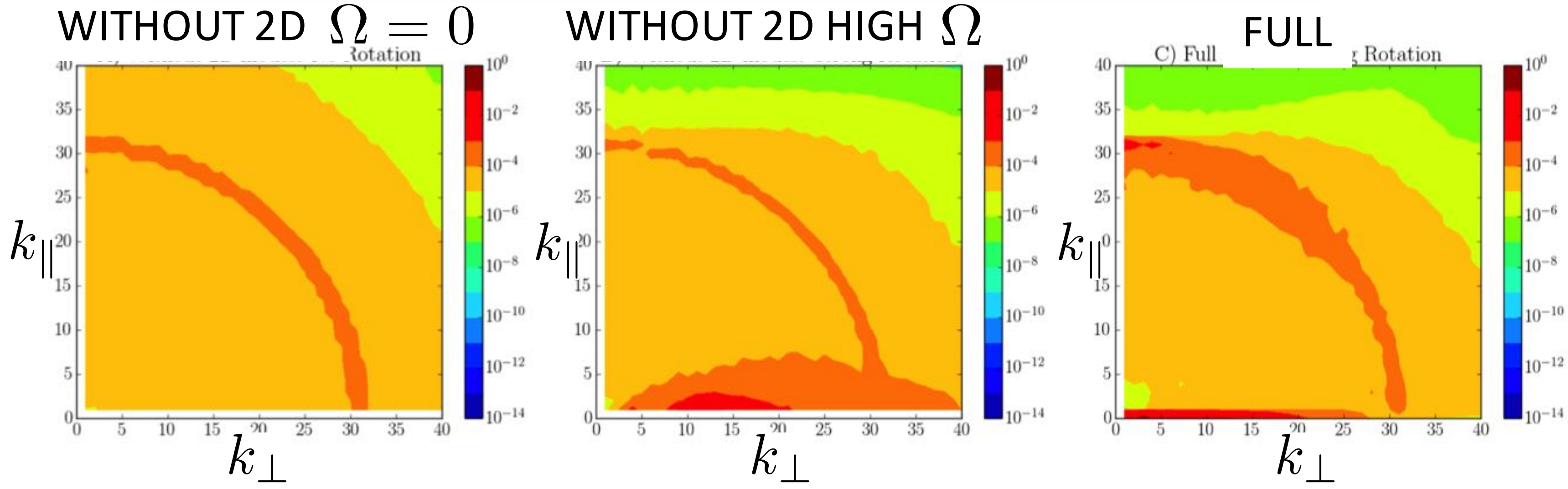
$$\begin{cases} \partial_t \boldsymbol{v} = \mathcal{P}[-\nabla p - (\boldsymbol{v}\cdot\nabla\boldsymbol{v})] - 2\boldsymbol{\Omega}\times\boldsymbol{v} + \nu\Delta\boldsymbol{v} + \mathcal{P}\boldsymbol{f} \\ \nabla\cdot\boldsymbol{v} = 0. \end{cases}$$

A) Without 2D: No rotation **B) Without 2D: Strong rotation** **C) Full system: Strong rotation**

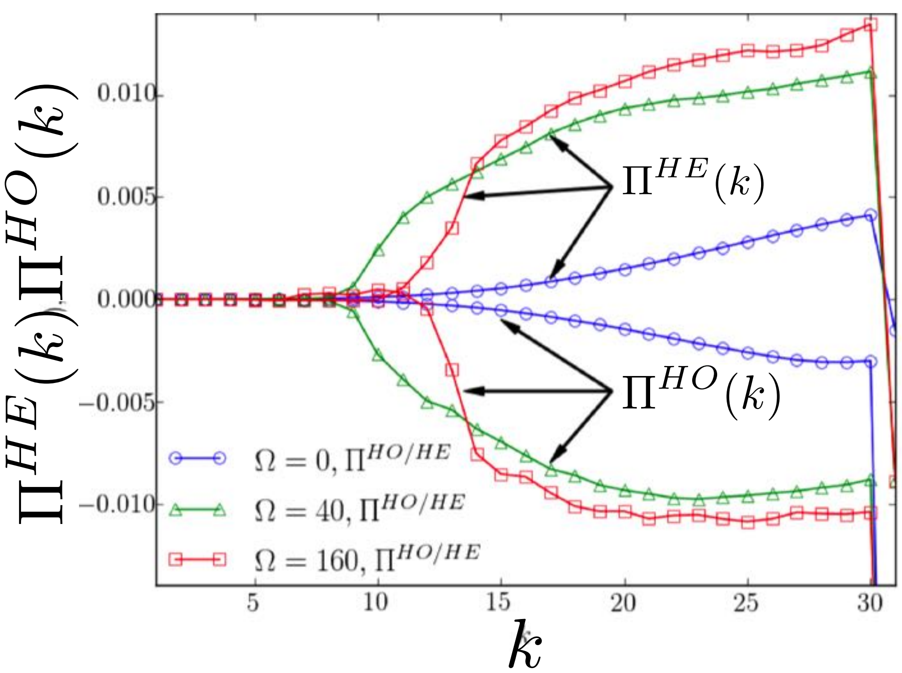


M. Buzzicotti, Di Leoni P. Clark, L.B. arXiv:1804.07687

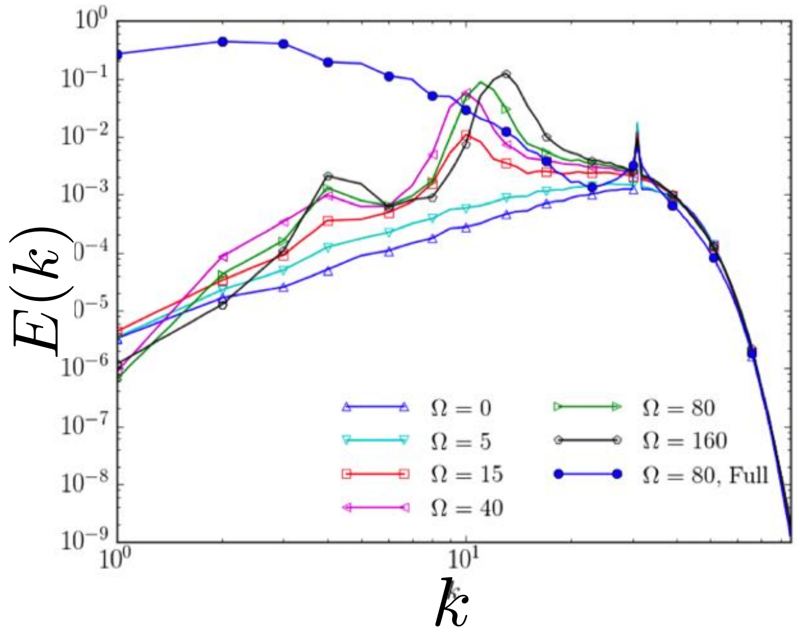
FLUX-LOOP CASCADE: ZERO FLUX BUT OUT-OF-EQUILIBRIUM



ENERGY FLUX



SPECTRUM

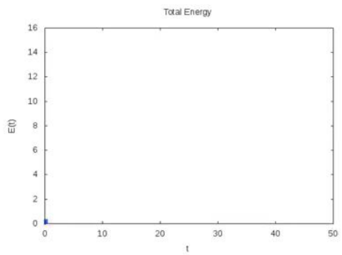


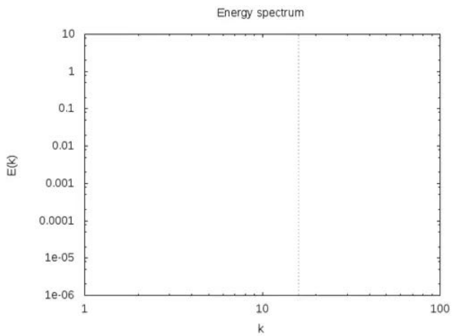
M. Buzzicotti, Di Leoni P. Clark, L.B. arXiv:1804.07687

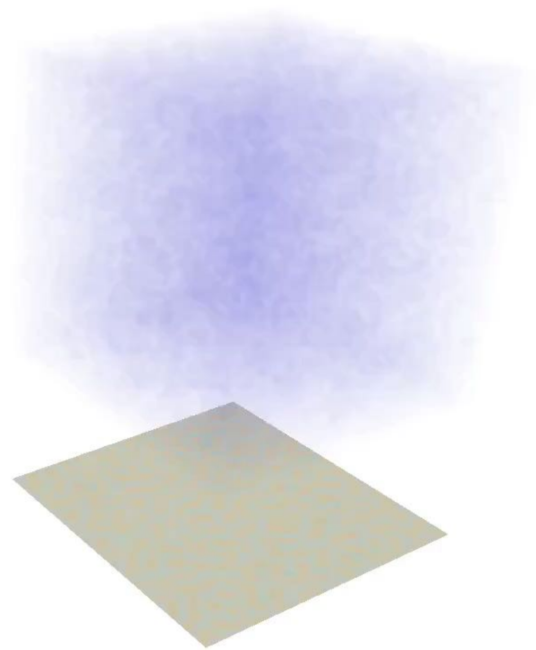
ROTATING TURBULENCE IS THE RESULT OF A COMPLICATED AND INTRICATED DYNAMICAL INTERACTIONDS AMONG 3D FAST WAVES AND 2D AND 3D TURBULENT MODES

BOTH 2D MODES AND SOME (HOMOCHIRAL) 3D MODES CAN TRANSFER ENERGY BACKWARD.

A DESCRIPTION BASED ON A SIMPLE (LINEAR) SUPERPOSITION OF 2D INVERSE ENERGY CASCADE PLUS 3D WAVE-TURBULENCE DIRECT-CASCADE MIGHT BE CORRECT ONLY IN SOME ASYMPTOTIC.







L.B., S. Musacchio, F. Toschi PRL. 108 164501, (2012)
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