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Slide of the Seminar

Energy fluxes, scale energy and turbulent separation

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ERC Advanced Grant (N. 339032) “NewTURB”
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PRACE

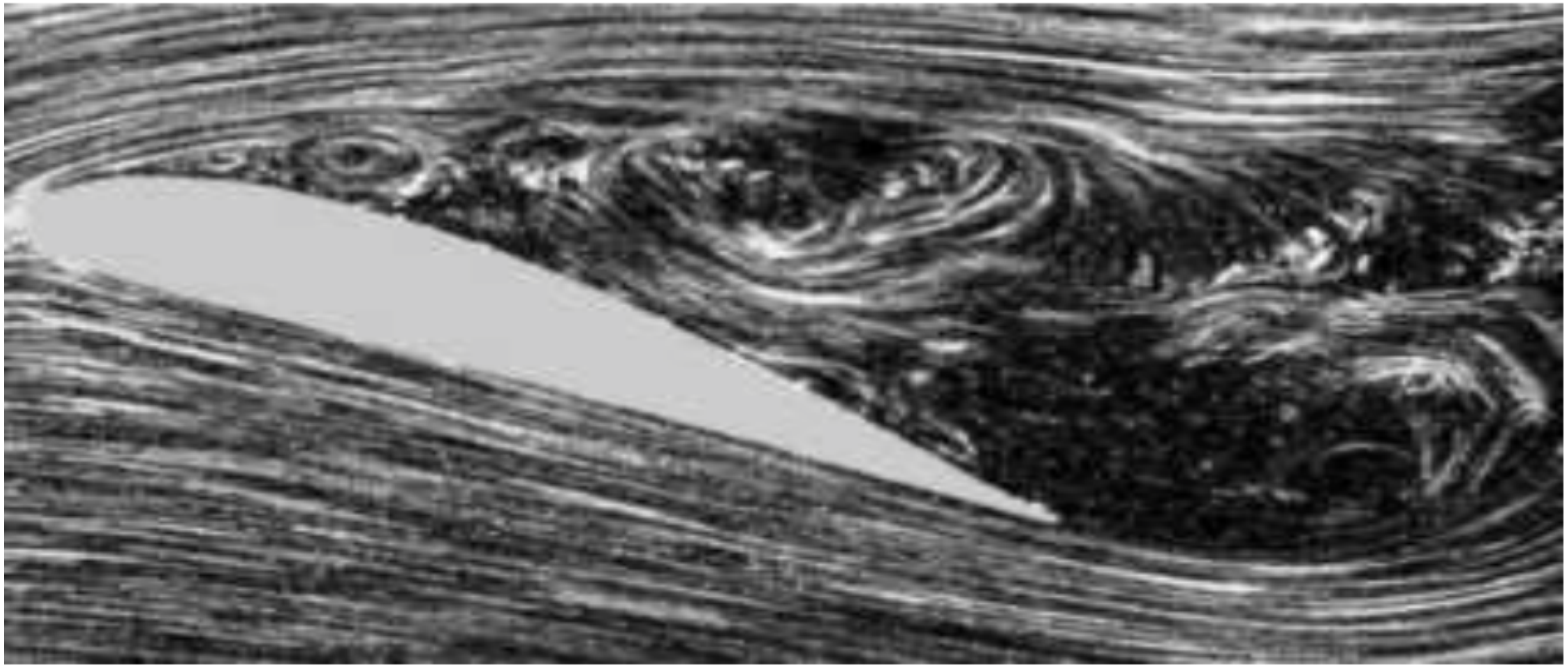
Energy fluxes, scale energy and turbulent separation

C.M. CASCIOLA

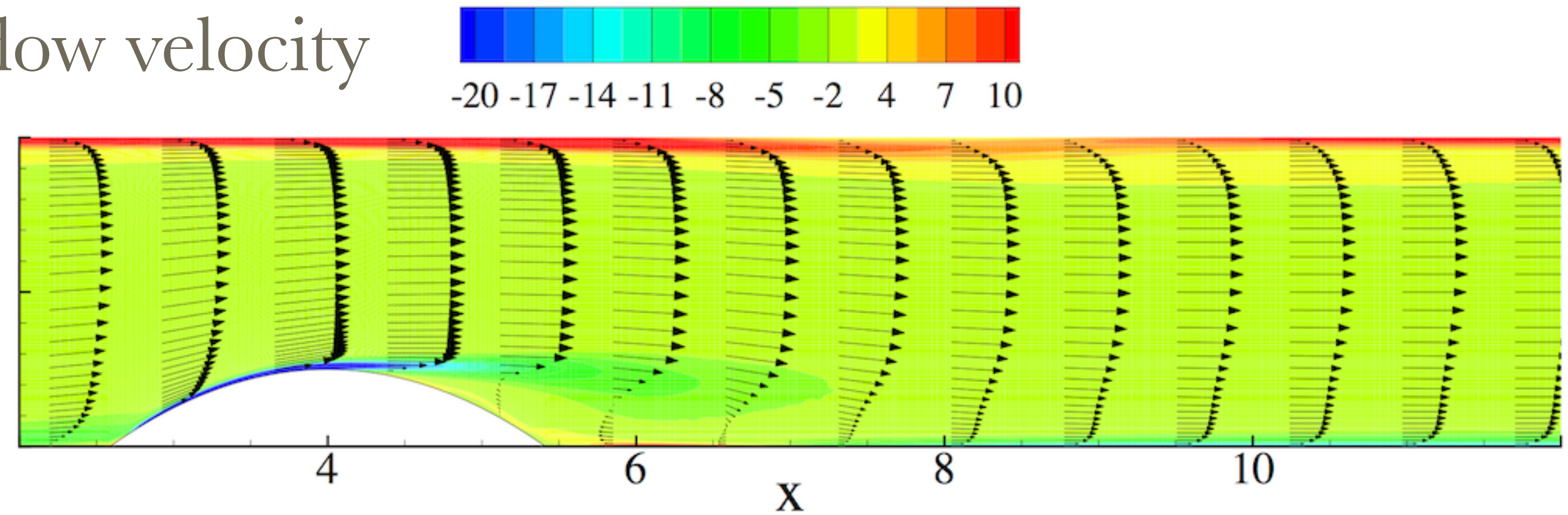
DEPT. OF MECHANICAL AND AEROSPACE ENGINEERING
SAPIENZA UNIVERSITY

NewTURB Meeting

July 18 2016, Dept. of Physics, Tor Vergata

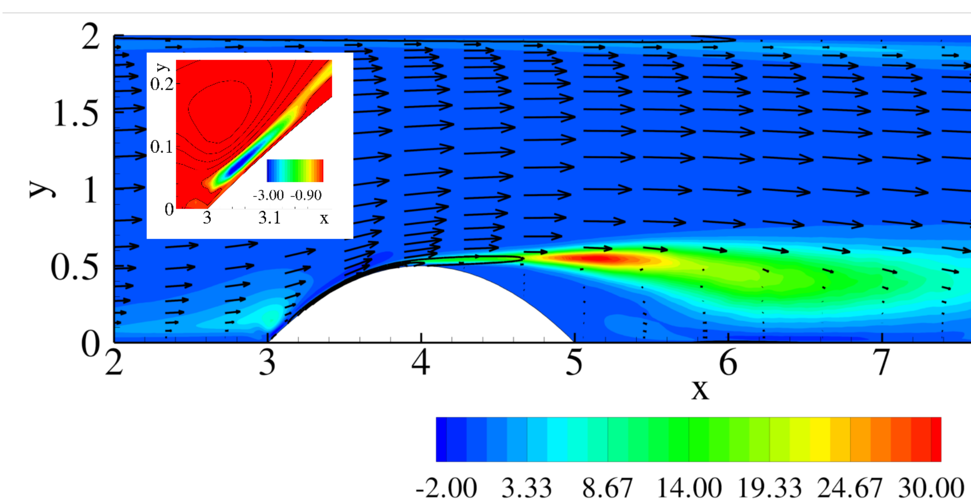


Mean flow velocity



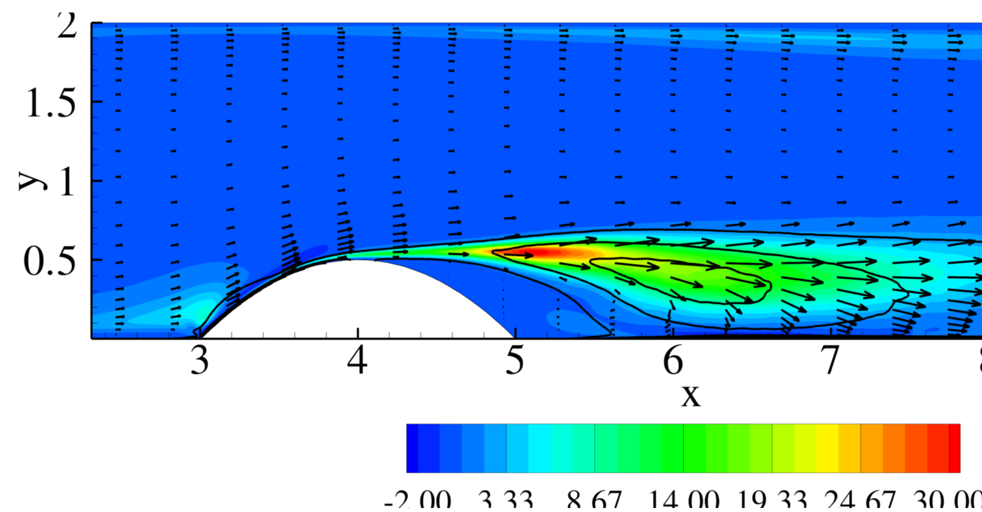
$$\frac{\partial \Phi_{Mj}}{\partial x_j} = -\varepsilon_M - \Pi + \frac{\Delta P}{L_x} U_x \qquad \frac{\partial \Phi_{Tj}}{\partial x_j} = -\varepsilon_T + \Pi + \langle \frac{\Delta p'}{L_x} u'_x \rangle$$

Φ_M

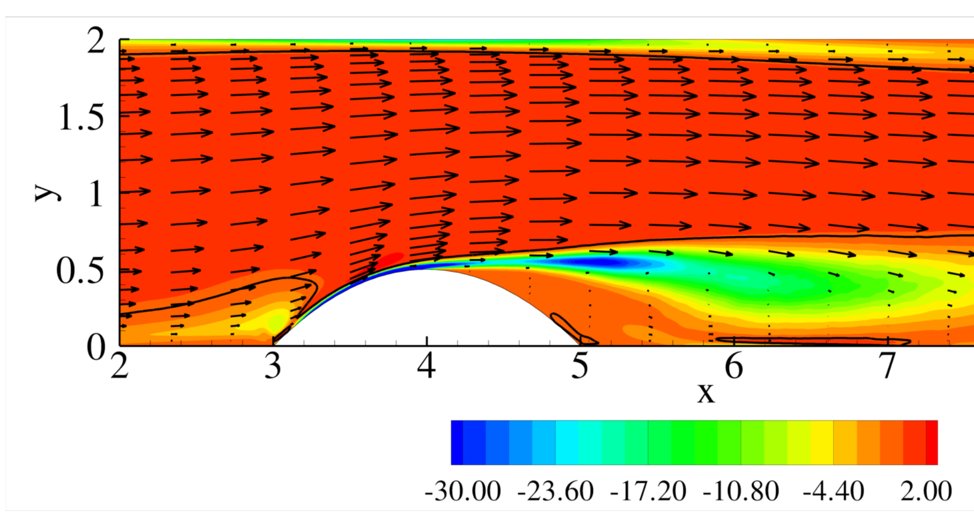


Π

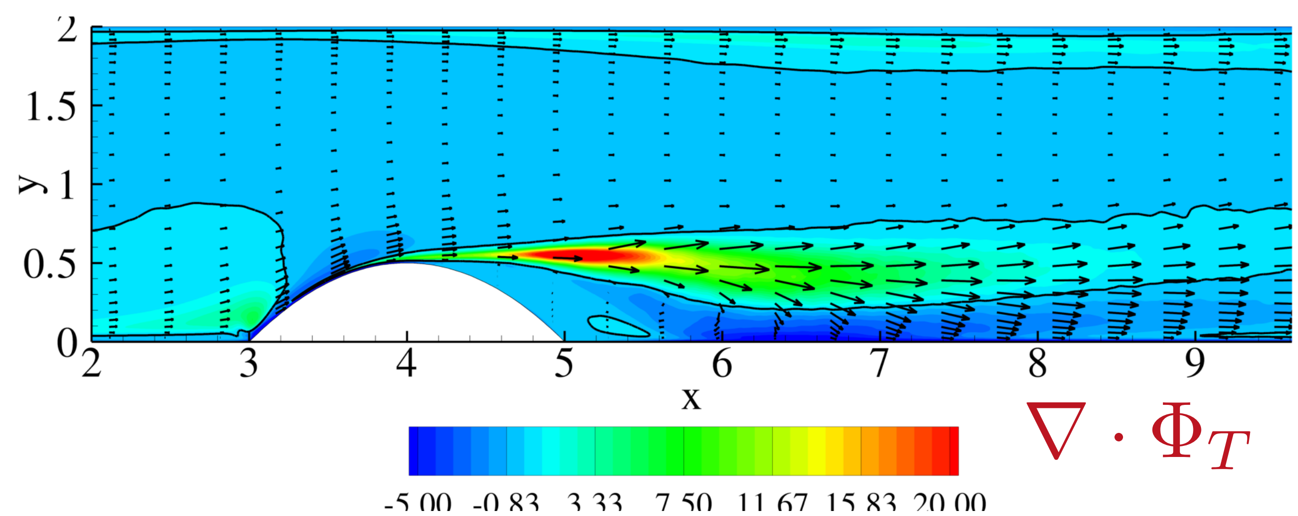
Φ_T



Π

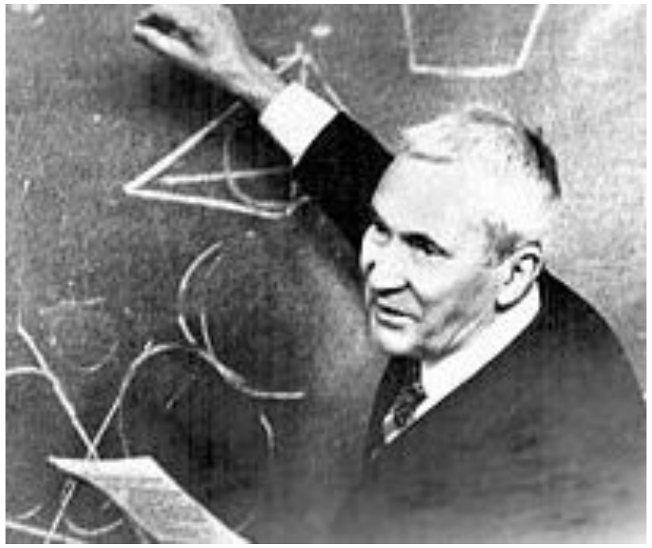


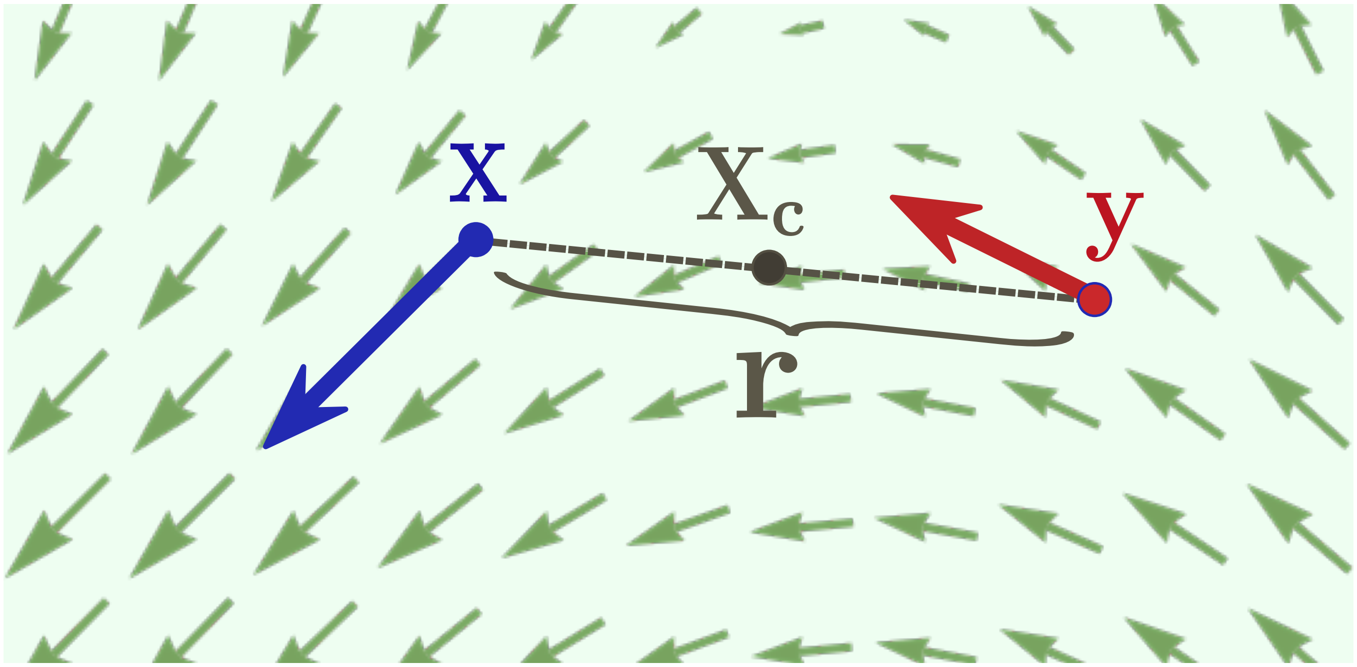
$\nabla \cdot \Phi_M$



$\nabla \cdot \Phi_T$

Kolmogorov's four-fifths law





$$\mathbf{r} = \mathbf{y} - \mathbf{x}$$

$$\mathbf{X}_c = \frac{\mathbf{y} + \mathbf{x}}{2}$$

$$\delta \mathbf{u} = \mathbf{u}(\mathbf{y}) - \mathbf{u}(\mathbf{x}), \; \delta u^2 = [\mathbf{u}(\mathbf{y}) - \mathbf{u}(\mathbf{x})] \cdot [\mathbf{u}(\mathbf{y}) - \mathbf{u}(\mathbf{x})]$$

$$\nabla_r \cdot \langle \delta u^2 \delta \mathbf{u} \rangle = -4\epsilon + \frac{2}{\mathrm{Re}} \nabla_r^2 \langle \delta u^2 \rangle + \frac{2}{\mathrm{Fr}^2} \langle \delta \mathbf{f} \cdot \delta \mathbf{u} \rangle$$

Extension to homogeneous anisotropic flows $\mathbf{S} = S\, \hat{\mathbf{e}}_y \times \hat{\mathbf{e}}_x$

(Townsend, 1976; CMC, Gualtieri, Benzi, Piva, 2003. J. Fluid Mech. 476.)

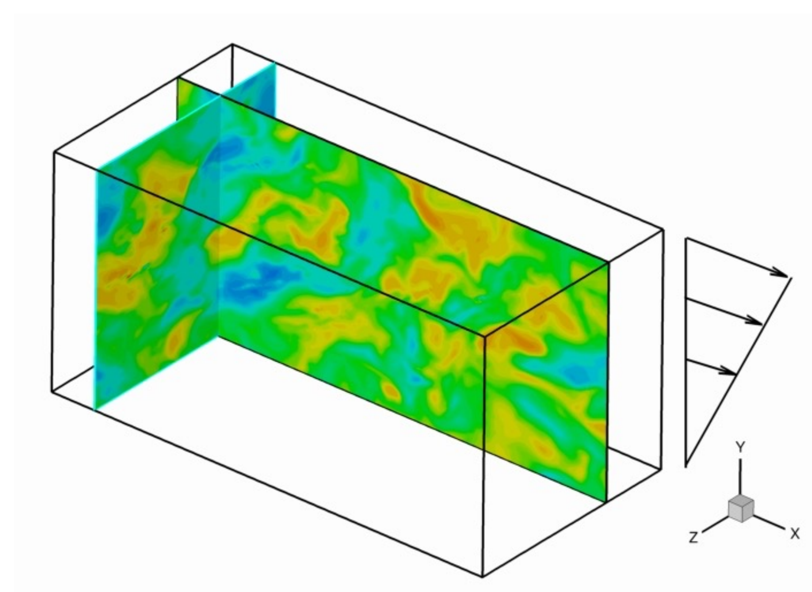
$$\nabla_r \cdot \langle \delta u^2 \delta \mathbf{u} \rangle + \nabla_r \cdot \left(\mathbf{r} \cdot \mathbf{S} \langle \delta u^2 \rangle \right) + 2\mathbf{S} : \langle \delta \mathbf{u} \otimes \delta \mathbf{u} \rangle = -4\epsilon + \frac{2}{\mathrm{Re}} \nabla_r^2 \langle \delta u^2 \rangle$$

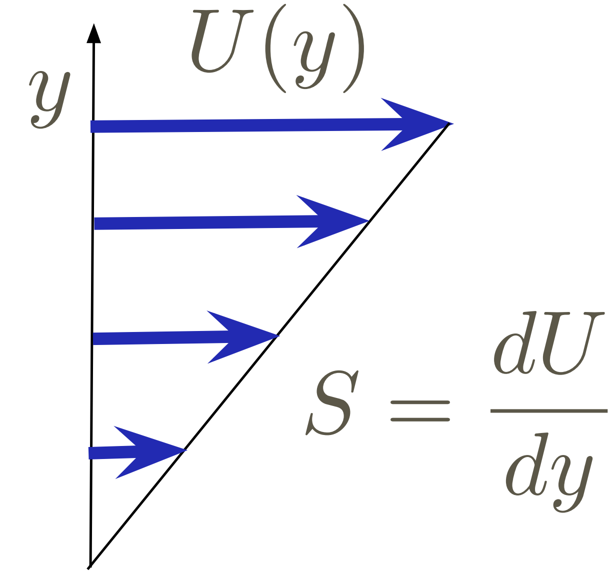
$$\frac{\partial}{\partial r_i} \langle \delta u^2 \delta u_i \rangle + S \frac{\partial}{\partial r_x} \left(r_y \langle \delta u^2 \rangle \right) + 2S \langle \delta u_x \delta u_y \rangle = -4\epsilon + \frac{2}{\mathrm{Re}} \frac{\partial^2}{\partial r_i \partial r_i} \langle \delta u^2 \rangle$$

$$\boldsymbol{\Phi}(\mathbf{r}) = \langle \delta u^2 \delta \mathbf{u} \rangle + \mathbf{r} \cdot \mathbf{S} \langle \delta u^2 \rangle - \frac{2}{\mathrm{Re}} \nabla_r \langle \delta u^2 \rangle$$

$$\Pi = -\frac{1}{2} \mathbf{S} : \langle \delta \mathbf{u} \otimes \delta \mathbf{u} \rangle$$

$$\nabla_r \cdot \boldsymbol{\Phi} = -4 (\epsilon - \Pi)$$





Dimensional analysis: scaling *à la* Kolmogorov $(\delta u \propto \epsilon^{1/3} r^{1/3})$

Inertial flux balances production: $\epsilon r \simeq \epsilon^{2/3} r^{5/3} S$

$L_S = \sqrt{\epsilon / S^3}$ (Shear scale) (CMC, Gualtieri, Jacob, Piva, Phys. Rev. Lett '05)

In statistically homogeneous flows a flux of "scale-energy" $\langle \delta u^2 \rangle$ occurs in in scale-space $\mathbf{r}$

$$\mathbf{\Phi}_r^{(HI)} = \langle \delta u^2 \delta \mathbf{u} \rangle - \frac{2}{\mathrm{Re}} \nabla_r^2 \langle \delta u^2 \rangle$$

← homogeneous isotropic

$$\mathbf{\Phi}_r^{(HS)} = \langle \delta u^2 \delta \mathbf{u} \rangle + \mathbf{r} \cdot \mathbf{S} \langle \delta u^2 \rangle - \frac{2}{\mathrm{Re}} \nabla_r^2 \langle \delta u^2 \rangle$$

← homogeneous shear

- The flux has convective and diffusive contributions

- In presence of shear the mean flow $\delta \mathbf{U} = \mathbf{r} \cdot \mathbf{S}$ contributes to the convective flux

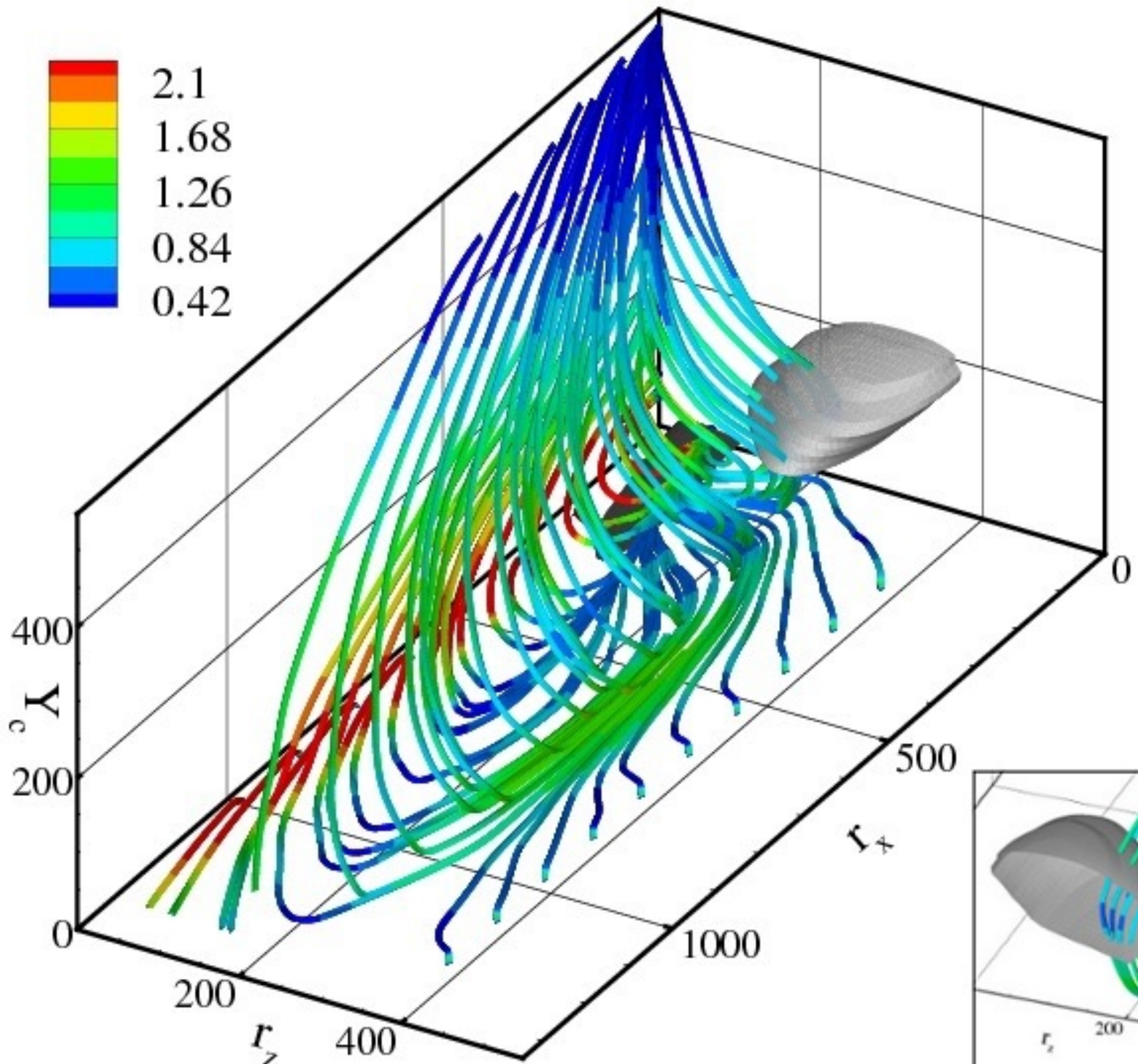
- Production $\Pi = \begin{cases} -\frac{1}{2} \mathbf{S} : \langle \delta \mathbf{u} \otimes \delta \mathbf{u} \rangle & \leftarrow \text{homogeneous shear} \\ \frac{2}{\mathrm{Fr}^2} \langle \delta \mathbf{f} \cdot \delta \mathbf{u} \rangle & \leftarrow \text{homogeneous isotropic} \end{cases}$

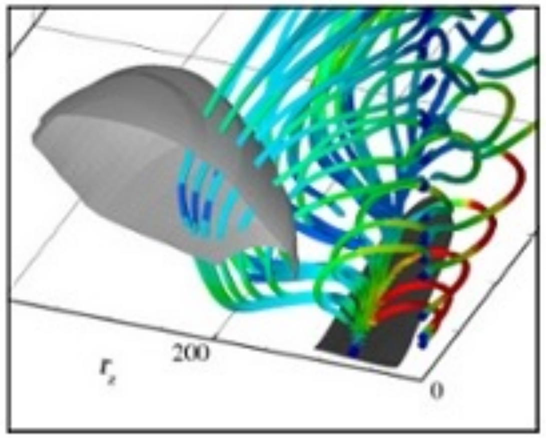
Note i): in the inertial range of HI turbulence $\mathbf{\Phi}_r^{(HI)} = \dfrac{\langle \delta u_\parallel^3 \rangle}{r} \hat{\mathbf{r}}$

Note ii): No spatial flux due to homogeneity

Channel Flow

Cimarelli, De Angelis, CMC, JFM 2013
Cimarelli, De Angelis, Jimenez, CMC, JFM 2016





Computational Aspects

Small Reynolds number 8192 Cores

Large Reynolds number 32768 Cores

1GB RAM/ core

6 TB stored data for statistics

400 million grid points

40 million core hours (PRACE)

Fermi@CINECA Blue Blue/Q

Spectral element method (NEK5000, Fisher et al., Argonne Nat. Lab)

Direct Numerical Simulation (Incompressible Navier-Stokes eqns.)

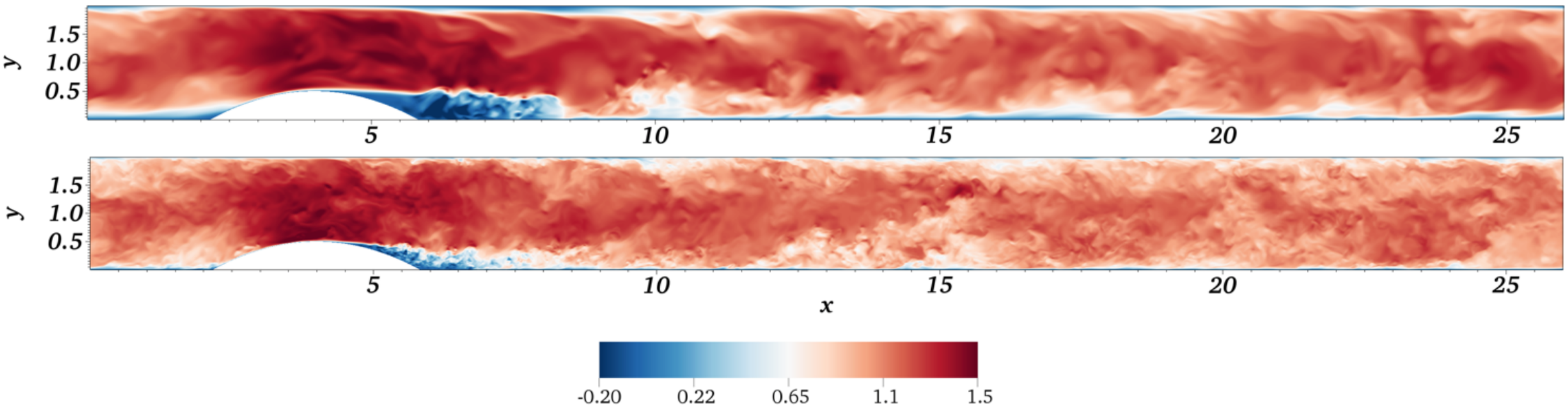
$$\nabla \cdot \mathbf{u} = 0$$

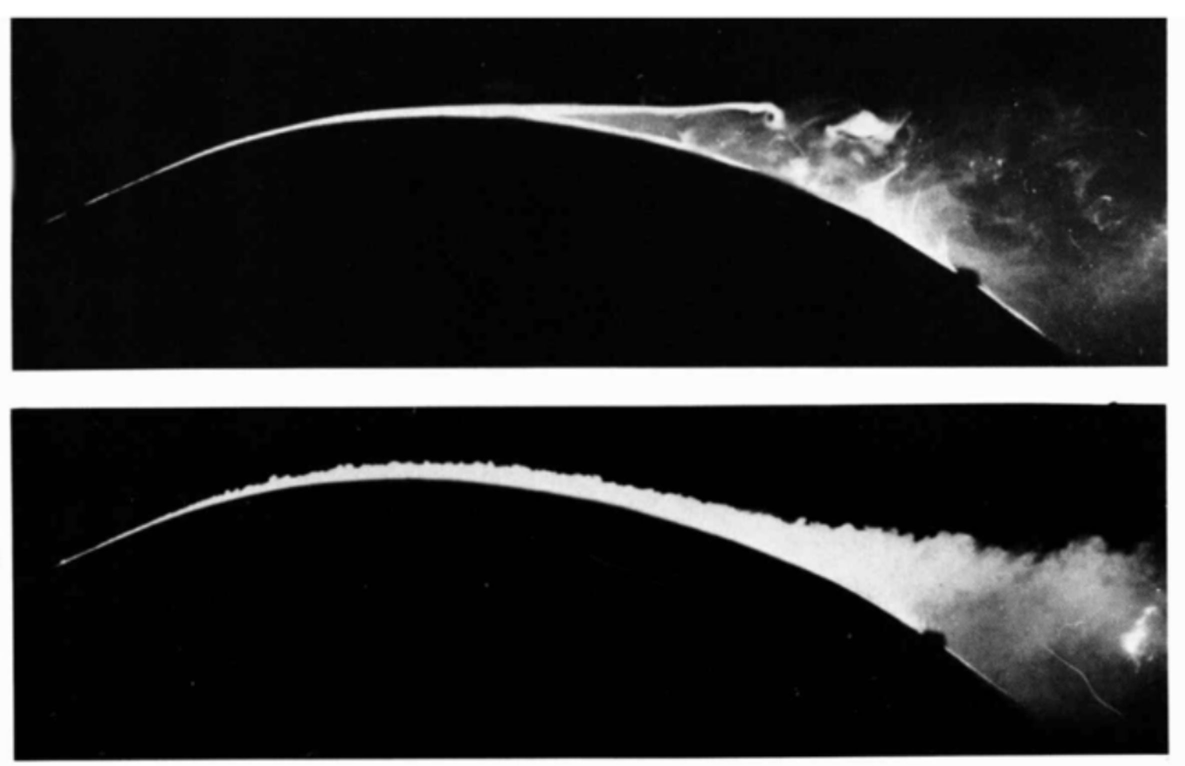
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}_h} \nabla^2 \mathbf{u}$$

$$\mathrm{Re}_h = \frac{\rho_0 U_b h_0}{\mu_0} = 2500,\ 5000,\ 10000$$

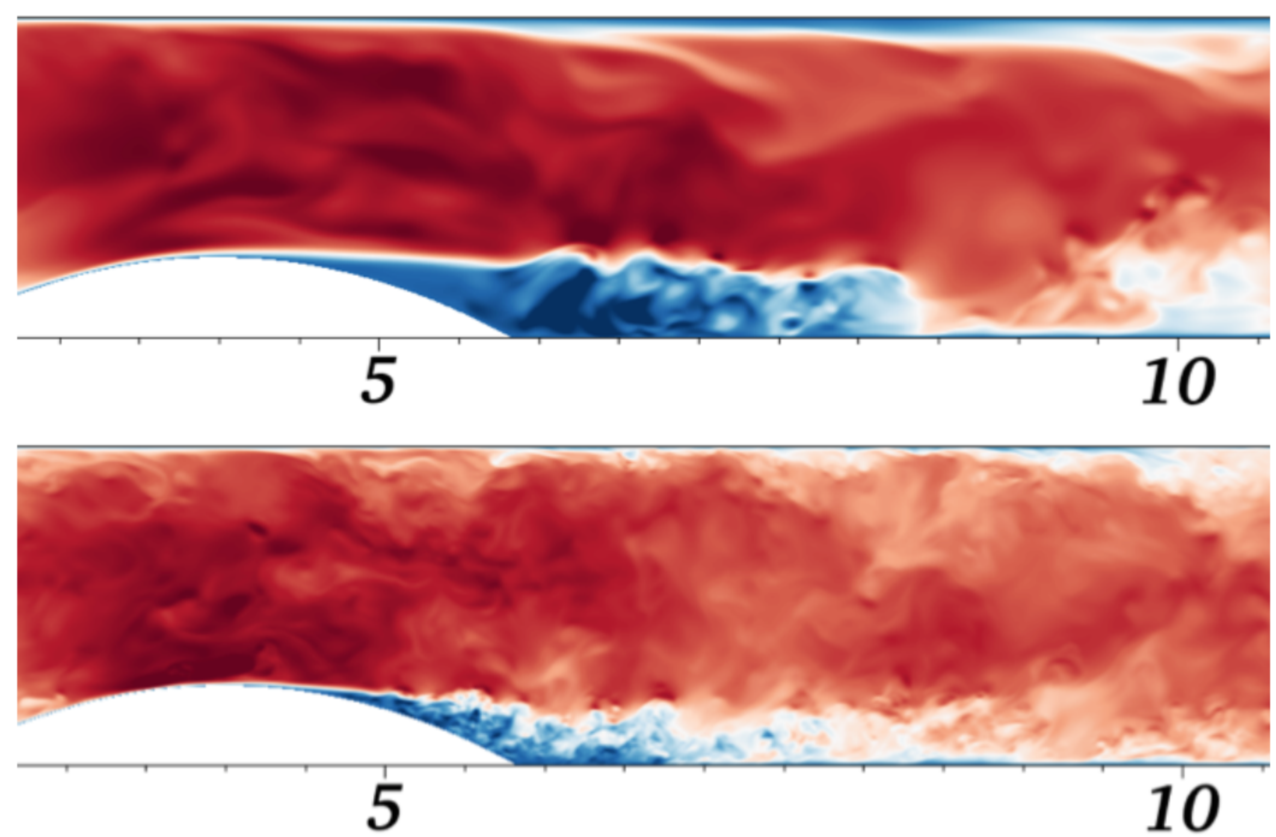
$$(L_x \times L_y \times L_z) = (26 \times 2 \times 2\pi) h_0$$

Simulation	Re_N	$\langle Re_\tau \rangle$	$Re_{\tau MAX}$	Δx^+	Δz^+	$\Delta y^+{}_{max}$	$\Delta y^+{}_{min}$
A1	2500	158	300	2.8	2.8	3.7	0.5
A2	5000	278	550	4.4	5.0	6.0	0.7
A3	10000	541	900	6.5	7.0	9.5	0.9





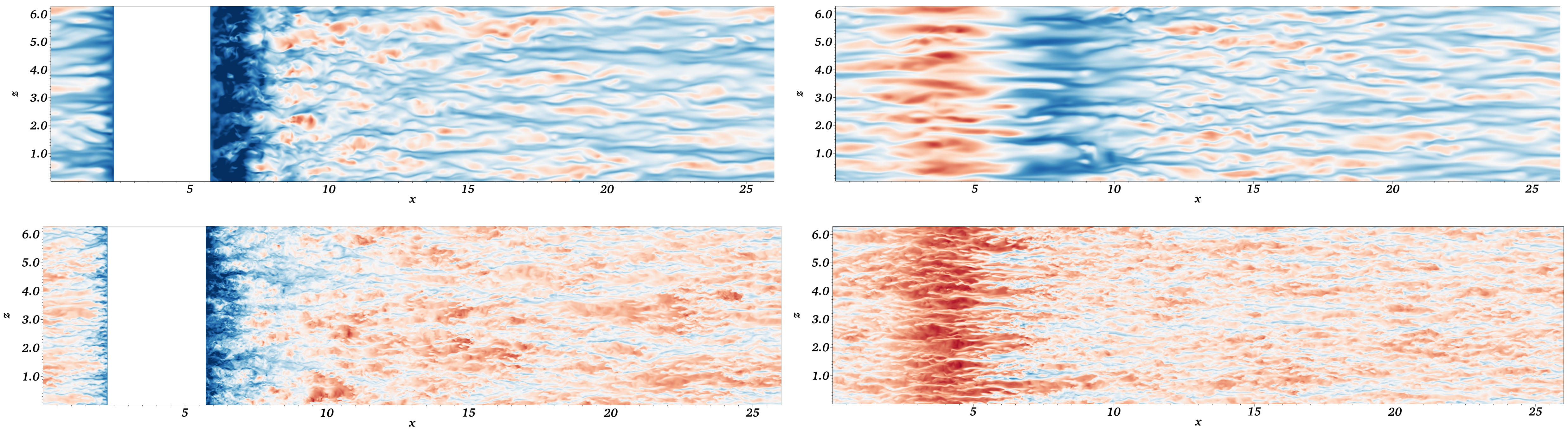
Van Dyke, M. (1982). An album of fluid motion.



$y = 0.05$ (bottom wall) $y = 1.95$ (top wall)

2500

10000

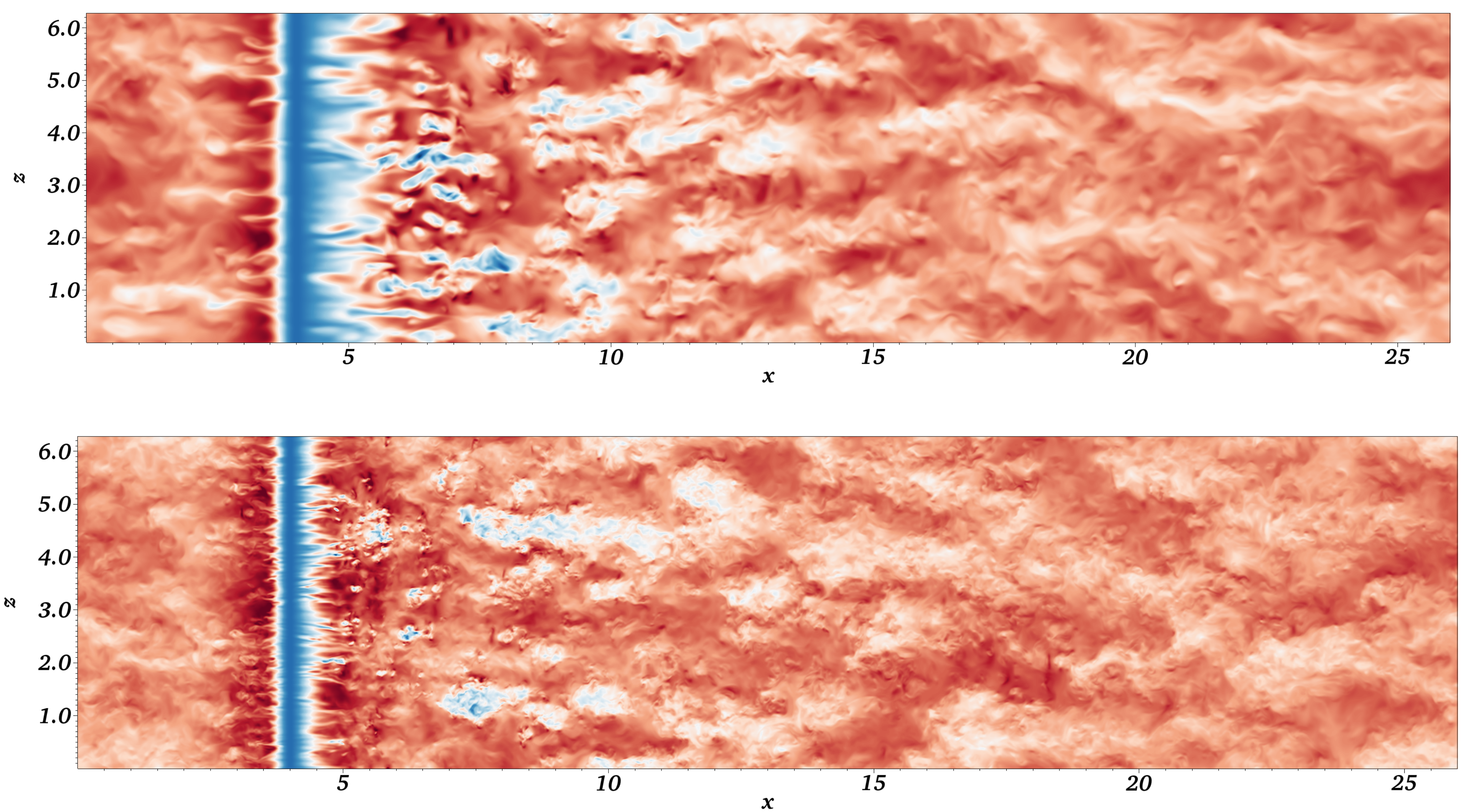


2500

$y = 0.5$ (bump tip)

10000

$y = 0.5$ (bump tip)



Generalized Kolmogorov's equation

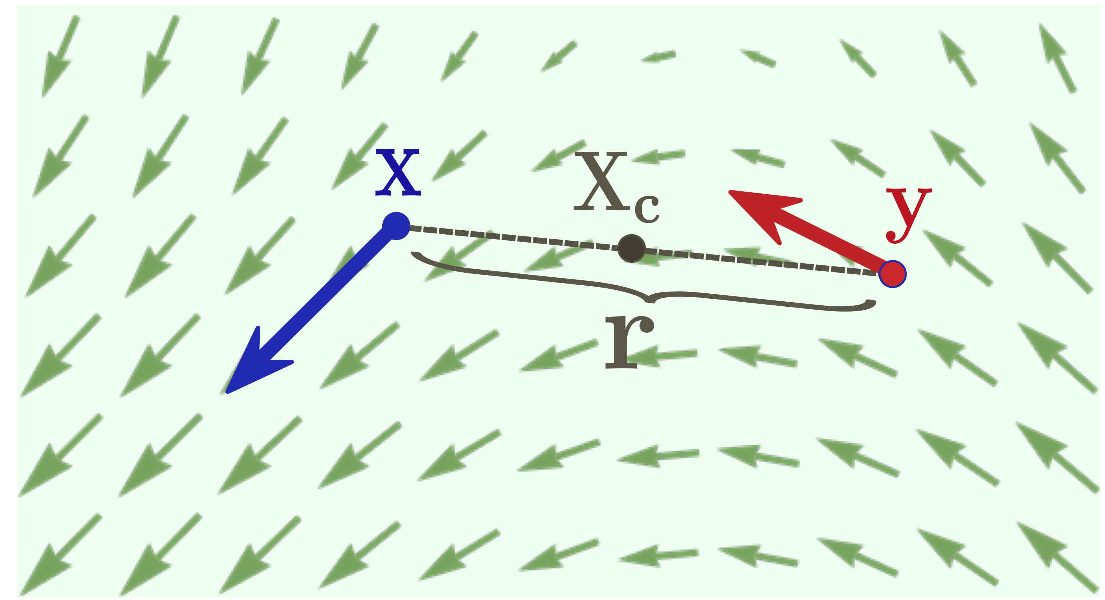
(Hill, JFM 468, 2002; Marati, CMC, Piva, JFM 521, 2004)

$$\mathbf{r} = \mathbf{y} - \mathbf{x} \qquad \mathbf{X}_c = (\mathbf{y} + \mathbf{x})/2$$

$$\nabla_r = -\nabla_x + \nabla_y \qquad \nabla_{X_c} = \nabla_x/2 + \nabla_y/2$$

$$\mathbf{x} = \mathbf{X}_c - \mathbf{r}/2 \qquad \mathbf{y} = \mathbf{X}_c + \mathbf{r}/2$$

$$\nabla_x = \nabla_{X_c}/2 - \nabla_r \qquad \nabla_y = \nabla_{X_c}/2 + \nabla_r$$



$$\delta Q = Q(\mathbf{y}) - Q(\mathbf{x}) \qquad Q^* = (Q(\mathbf{y}) + Q(\mathbf{x}))/2$$

inertial flux

$$\nabla_r \cdot \langle \delta u^2 \delta \mathbf{u} \rangle + \nabla_r \cdot \langle \delta u^2 \delta \mathbf{U} \rangle + \nabla_{X_c} \cdot \langle \delta u^2 \mathbf{u}^* \rangle + \nabla_{X_c} \cdot \langle \delta u^2 \mathbf{U}^* \rangle +$$

$$2\langle \delta \mathbf{u} \otimes \delta \mathbf{u} \rangle : \nabla_r \delta \mathbf{U} + 2\langle \delta \mathbf{u} \otimes \mathbf{u}^* \rangle : \nabla_{X_c} \delta \mathbf{U} =$$

production

$$-4\epsilon^* + \frac{2}{\mathrm{Re}} \nabla_r^2 \langle \delta u^2 \rangle + \frac{1}{2\,\mathrm{Re}} \nabla_{X_c}^2 \langle \delta u^2 \rangle + 2\nabla_{X_c} \cdot \langle \delta p \delta \mathbf{u} \rangle$$

dissipation

diffusion

pressure term

Conservative form of the GKE

$$\mathbf{\Phi}_r = \langle \delta u^2 \delta \mathbf{u} \rangle + \langle \delta u^2 \delta \mathbf{U} \rangle - \frac{2}{\mathrm{Re}} \nabla_r^2 \langle \delta u^2 \rangle$$

scale-space flux

$$\mathbf{\Phi}_c = \langle \delta u^2 \mathbf{u}^* \rangle + \langle \delta u^2 \mathbf{U}^* \rangle - \frac{1}{2\,\mathrm{Re}} \langle \delta u^2 \rangle - 2 \langle \delta p\, \delta \mathbf{u} \rangle$$

physical-space flux

$$\Pi = -2 \langle \delta \mathbf{u} \otimes \delta \mathbf{u} \rangle : \nabla_r \delta \mathbf{U} + 2 \langle \delta \mathbf{u} \otimes \mathbf{u}^* \rangle : \nabla_{X_c} \delta \mathbf{U}$$

production

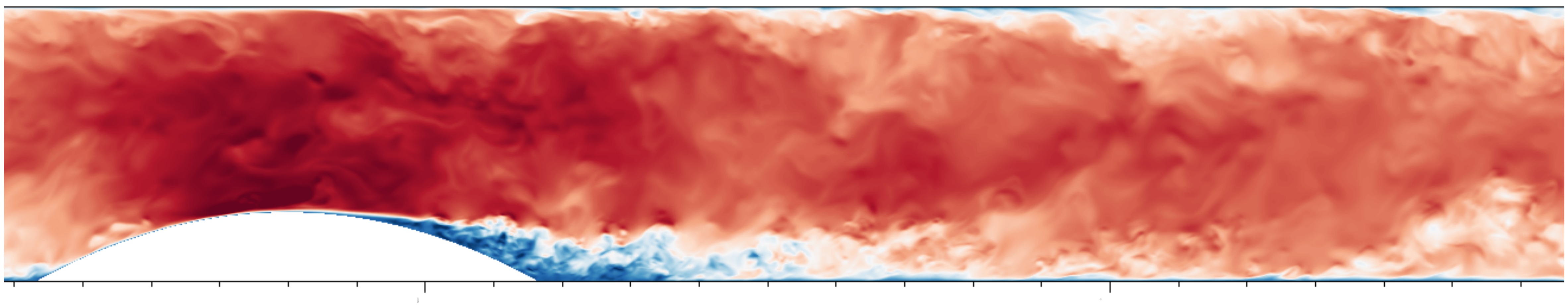
$$E = 4\epsilon^*$$

dissipation

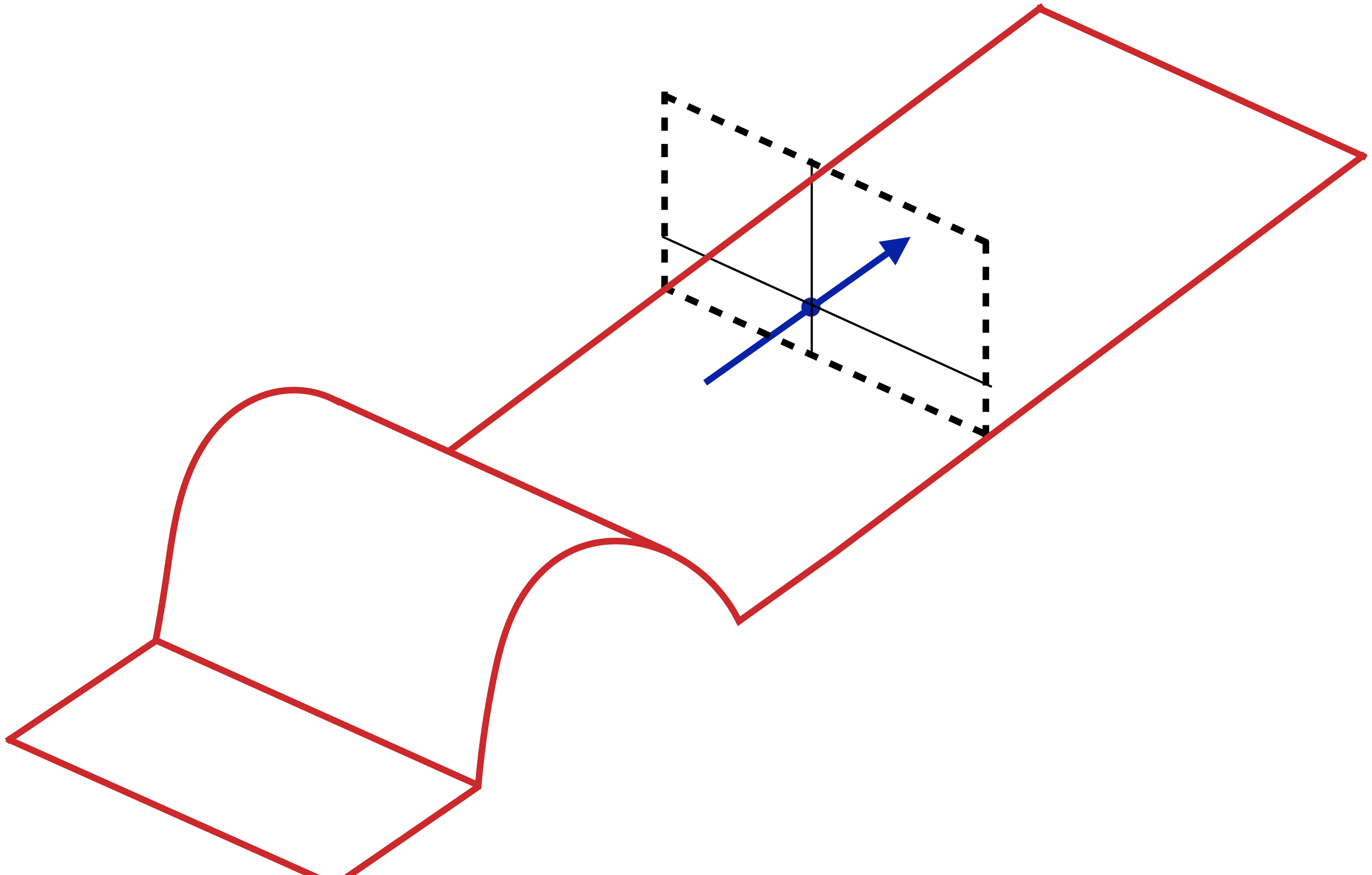
$$\nabla_r \cdot \mathbf{\Phi}_{\boldsymbol{r}} + \nabla_c \cdot \mathbf{\Phi}_{\boldsymbol{c}} = \Pi - E$$

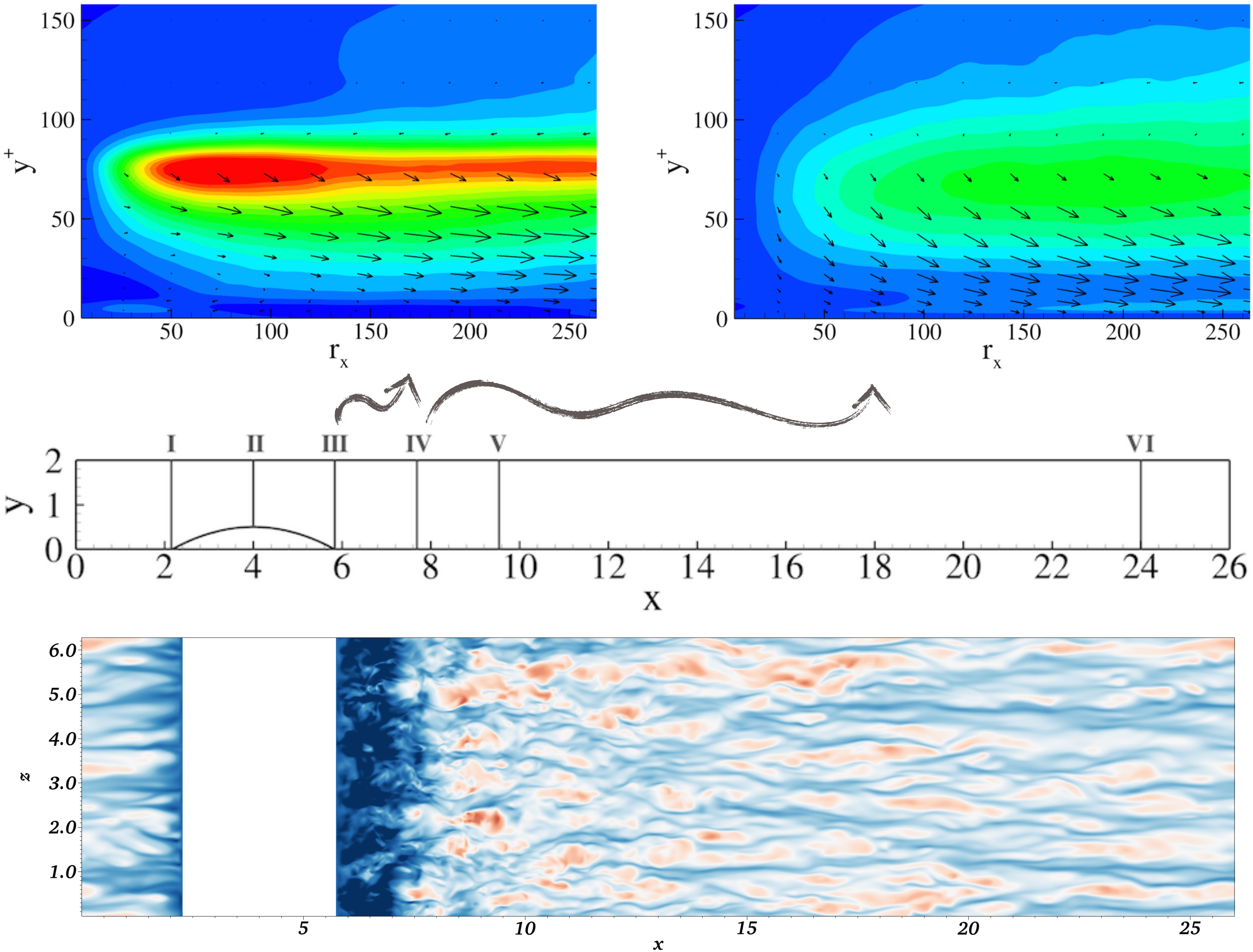
The second order structure function $\langle \delta u^2 \rangle = S_2(\mathbf{r}, \mathbf{X_c})$ is governed by an equation in conservative form

Two different fluxes: in the space of separations (scales) and in physical space

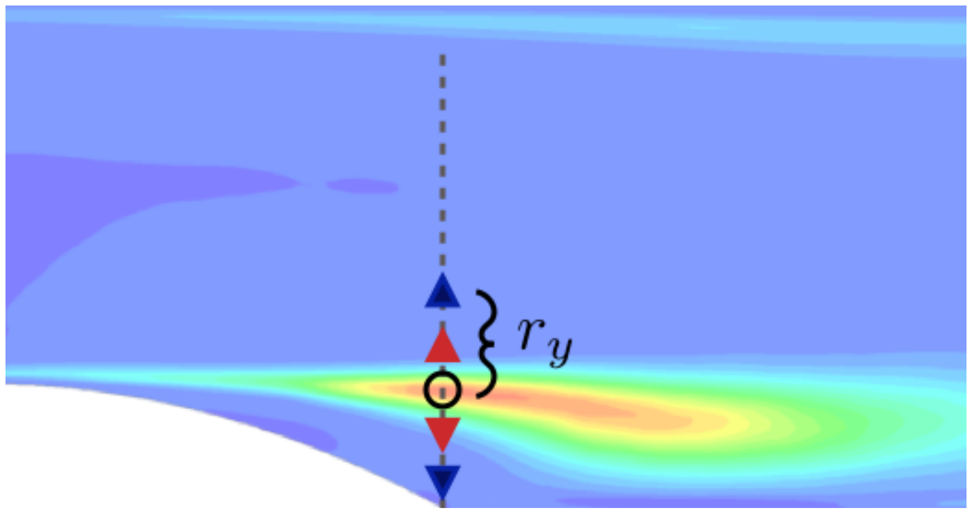


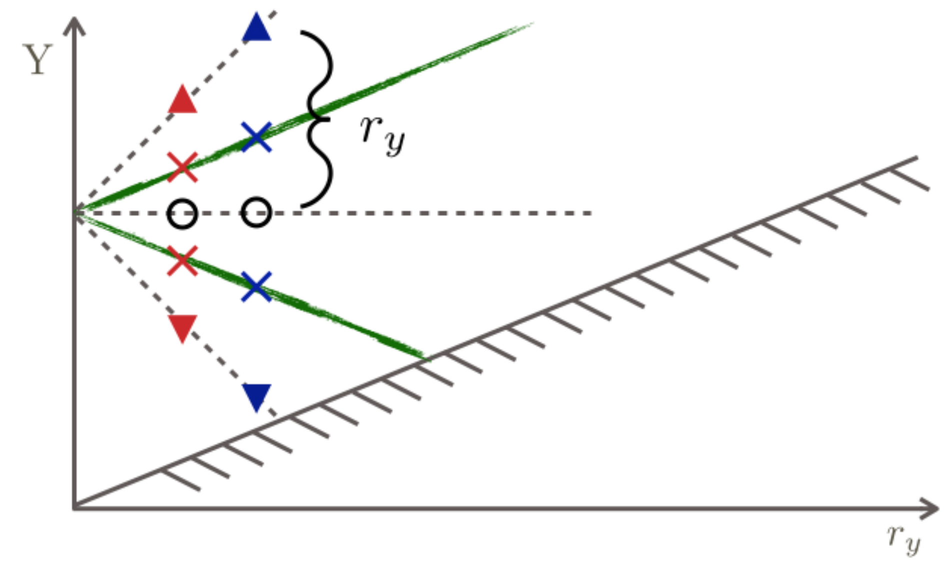
$$\mathbf{X}_c = \left(X_c^0, Y_c, Z_c^0\right) \qquad \mathbf{r} = (r_x, 0, 0)$$

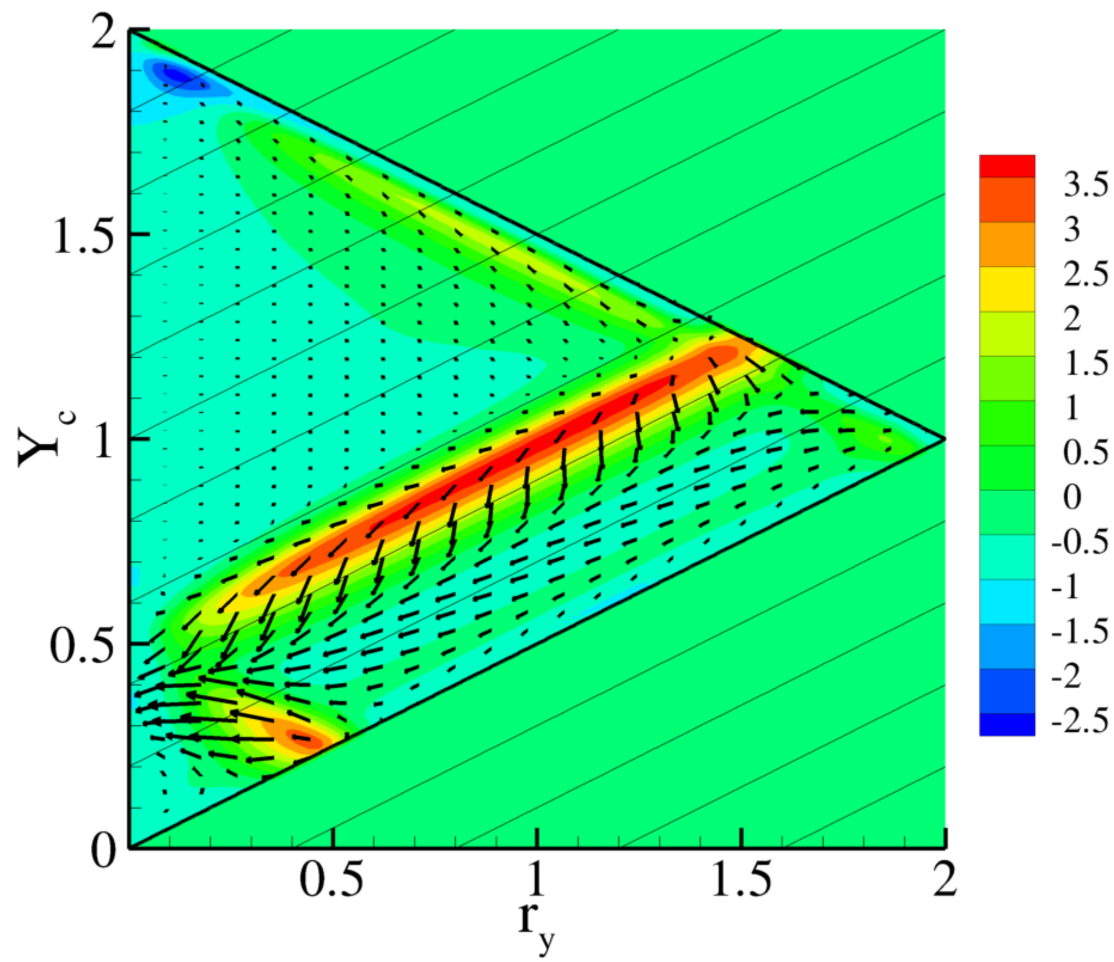


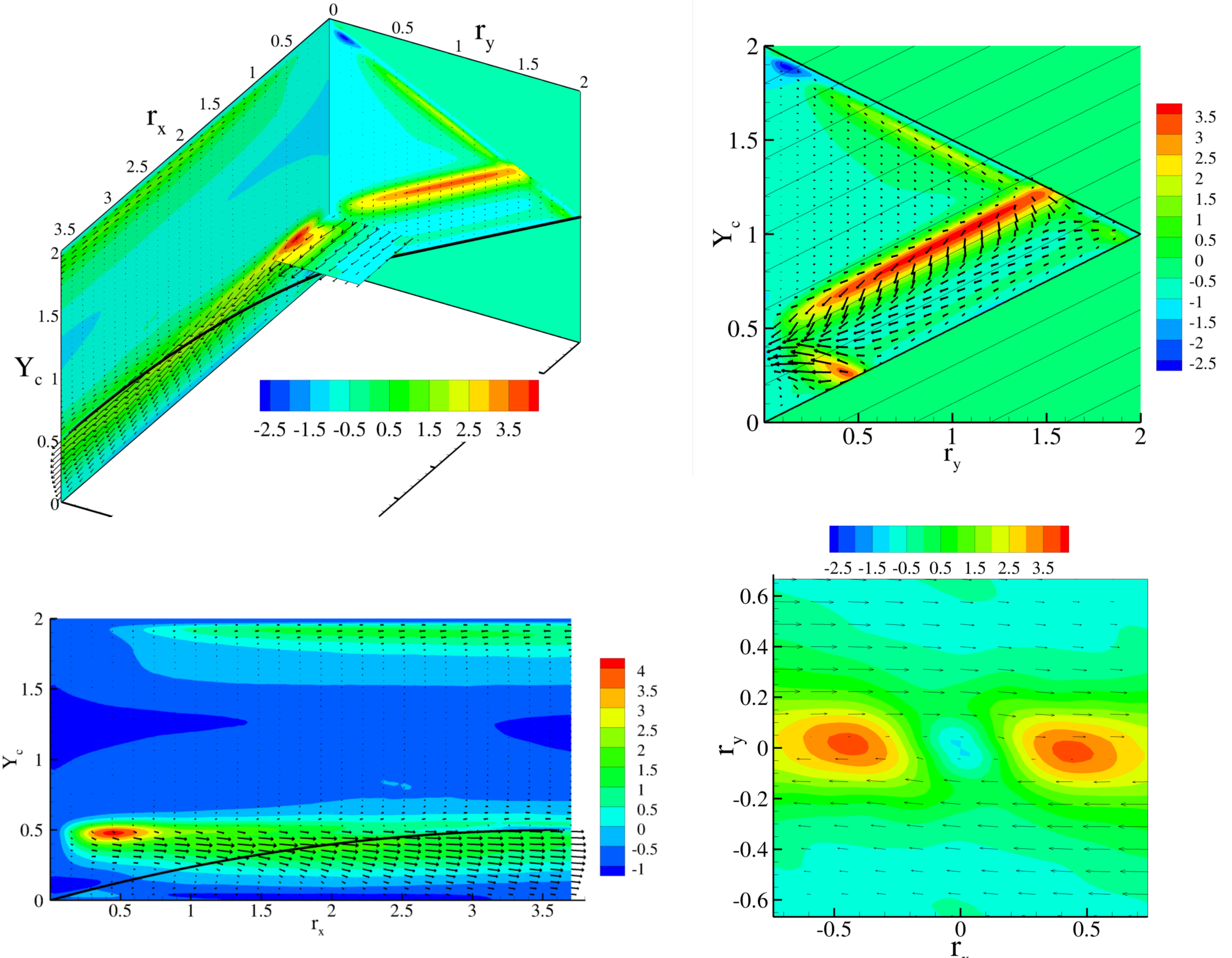


150
100
50
0
y^+
50 100 150 200 250
r_x
I II III IV V VI
2 1 0
y
0 2 4 6 8 10 12 14 16 18 20 22 24 26
x
6.0 5.0 4.0 3.0 2.0 1.0
z
5 10 15 20 25
x


r_y

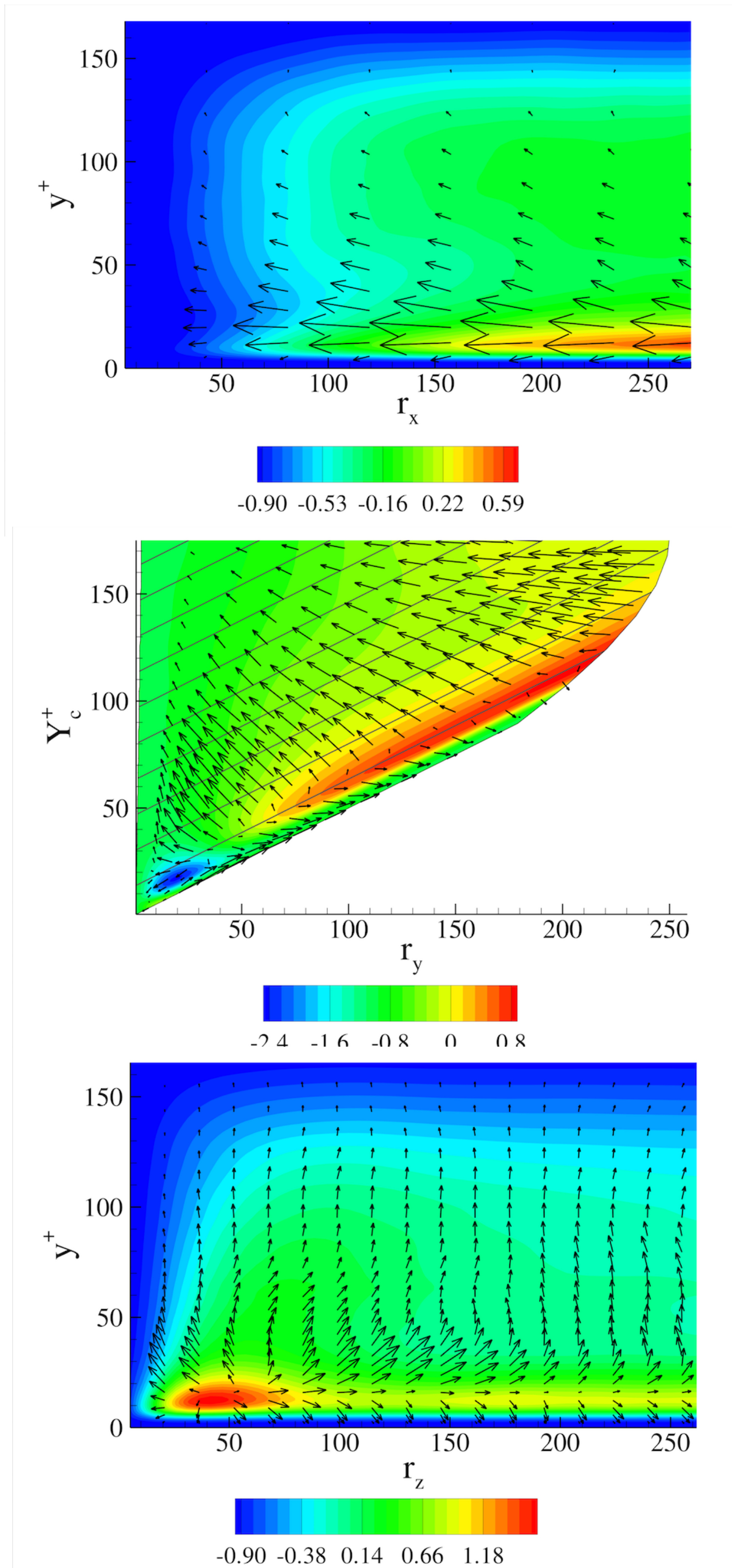

Y
r_y
r_y


2
1.5
1
0.5
0
0.5
1
1.5
2
Y_c
r_y
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

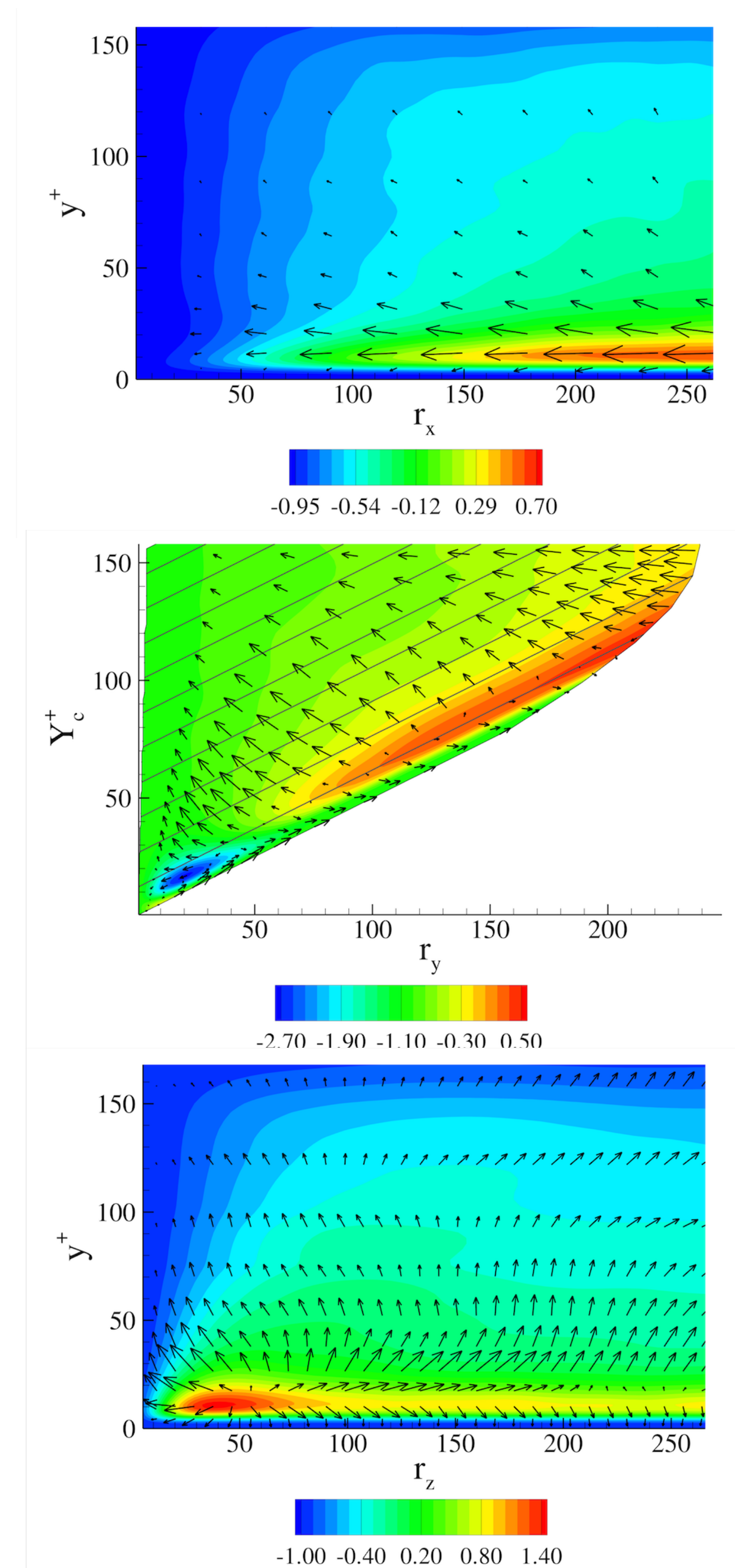


r_x
r_y
Y_c
-2.5 -1.5 -0.5 0.5 1.5 2.5 3.5
0.5 1 1.5 2
3.5 3 2.5 2 1.5 1 0.5 0

Planar channel

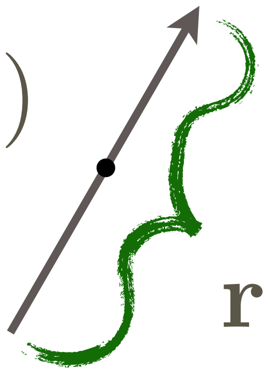


Bumpy channel
(far downstream)



$$\dot{\boldsymbol{\xi}} = \mathbf{w}^* = \frac{\langle \mathbf{u}_T^* |\delta\mathbf{u}|^2\rangle}{\langle|\delta\mathbf{u}|^2\rangle} \qquad \dot{\boldsymbol{\zeta}} = \delta\mathbf{w} = \frac{\langle \delta\mathbf{u}_T |\delta\mathbf{u}|^2\rangle}{\langle|\delta\mathbf{u}|^2\rangle}$$

$\mathbf{X}_c(t)$

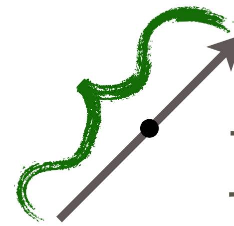


$\mathbf{r}(t)$

$\dot{\mathbf{X}}_c = \mathbf{w}^*$

$\dot{\mathbf{r}} = \delta\mathbf{w}$

$\mathbf{r}(t+dt) = \mathbf{r}(t) + \delta\mathbf{w}dt$

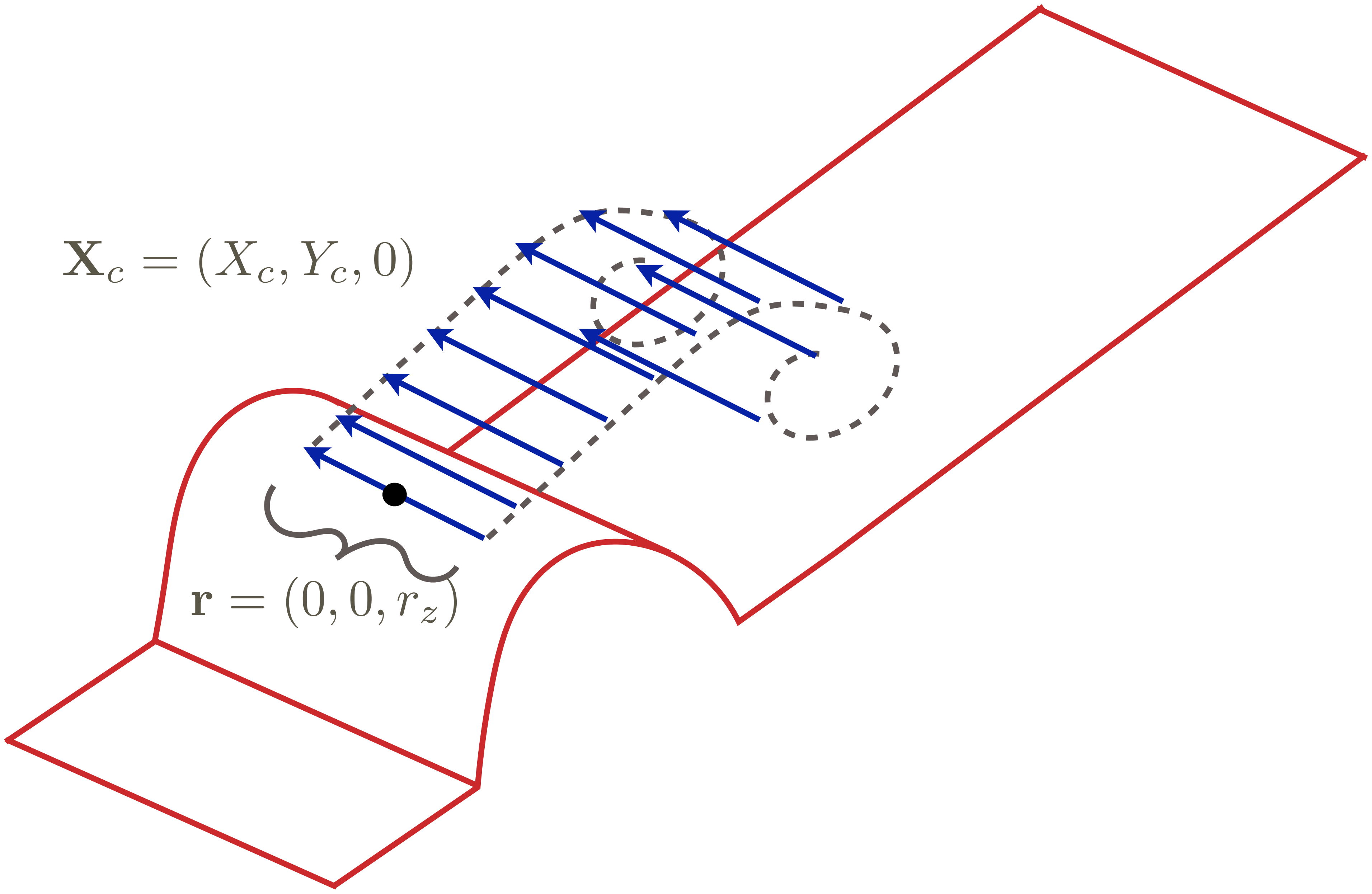


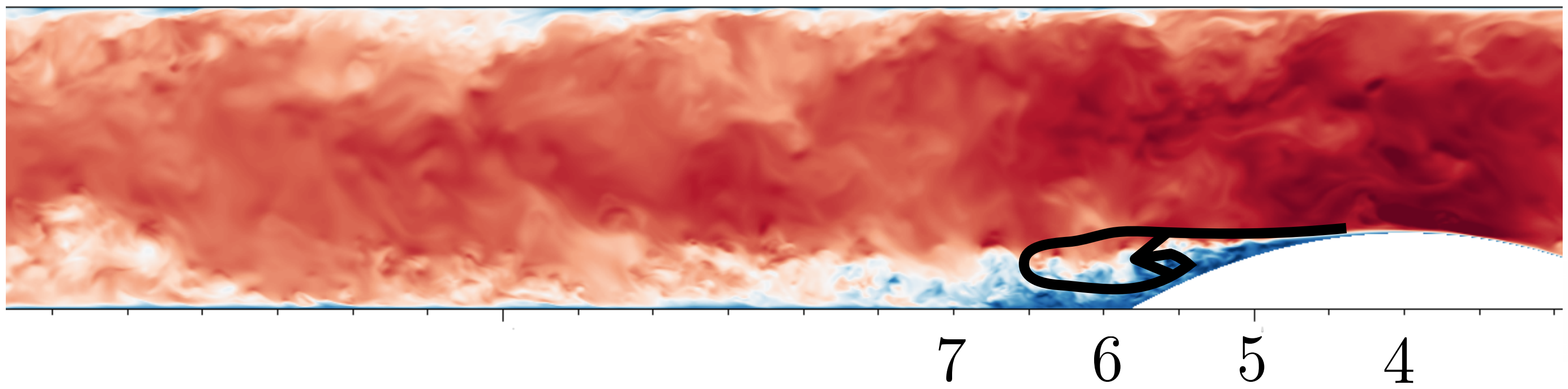
$\mathbf{X}_c(t+dt) =$
$\mathbf{X}_c(t) + \mathbf{w}^* dt$

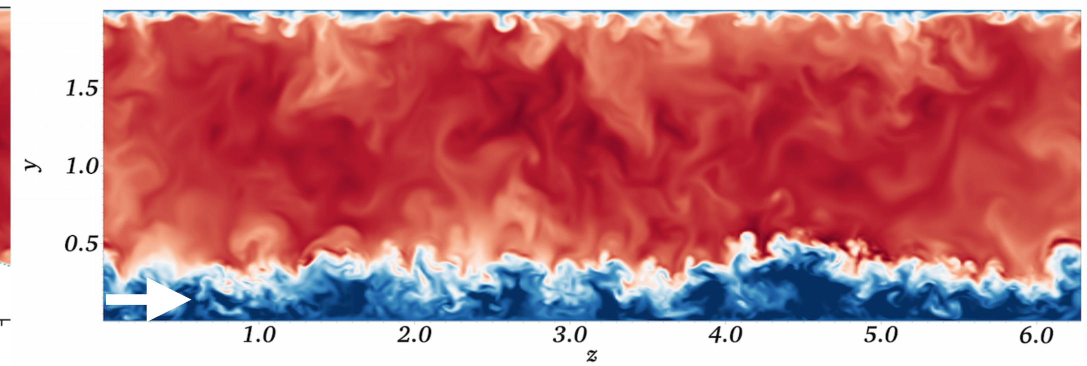
$$\frac{d\langle|\delta\mathbf{u}|^2\rangle}{dt} = \frac{\partial\langle|\delta\mathbf{u}|^2\rangle}{\partial t} + \dot{\boldsymbol{\xi}}\cdot\nabla_{\mathbf{X}_c}\langle|\delta\mathbf{u}|^2\rangle + \dot{\boldsymbol{\zeta}}\cdot\nabla_{\mathbf{r}}\langle|\delta\mathbf{u}|^2\rangle$$

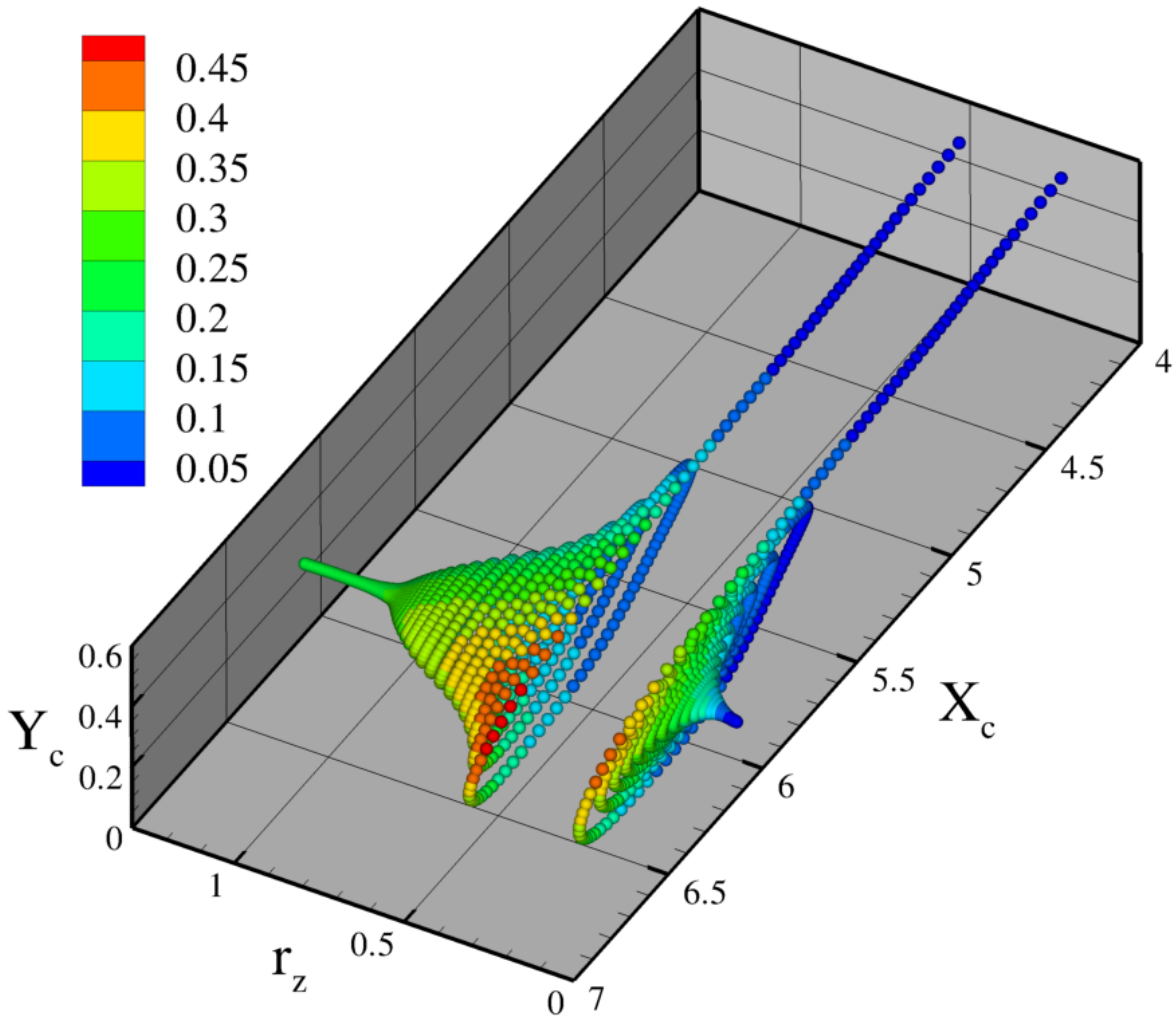
$$\frac{1}{2}\frac{d\langle|\delta\mathbf{u}|^2\rangle}{dt} = -\nabla_{\mathbf{X}_c}\cdot\langle\delta p\,\delta\mathbf{u}\rangle + \frac{\nabla^2_{\mathbf{X}_c}\langle|\delta\mathbf{u}|^2\rangle}{4\mathrm{Re}} + \frac{\nabla^2_{\mathbf{r}}\langle|\delta\mathbf{u}|^2\rangle}{\mathrm{Re}}$$
$$-\frac{1}{2}\langle|\delta\mathbf{u}|^2\rangle\left(\nabla_{\mathbf{X}_c}\cdot\mathbf{w}^*\right) - \frac{1}{2}\langle|\delta\mathbf{u}|^2\left(\rangle\nabla_{\mathbf{r}}\cdot\delta\mathbf{w}\right) + \Pi_{\mathbf{X}_c} + \Pi_{\mathbf{r}} - 2\langle\varepsilon^*\rangle$$

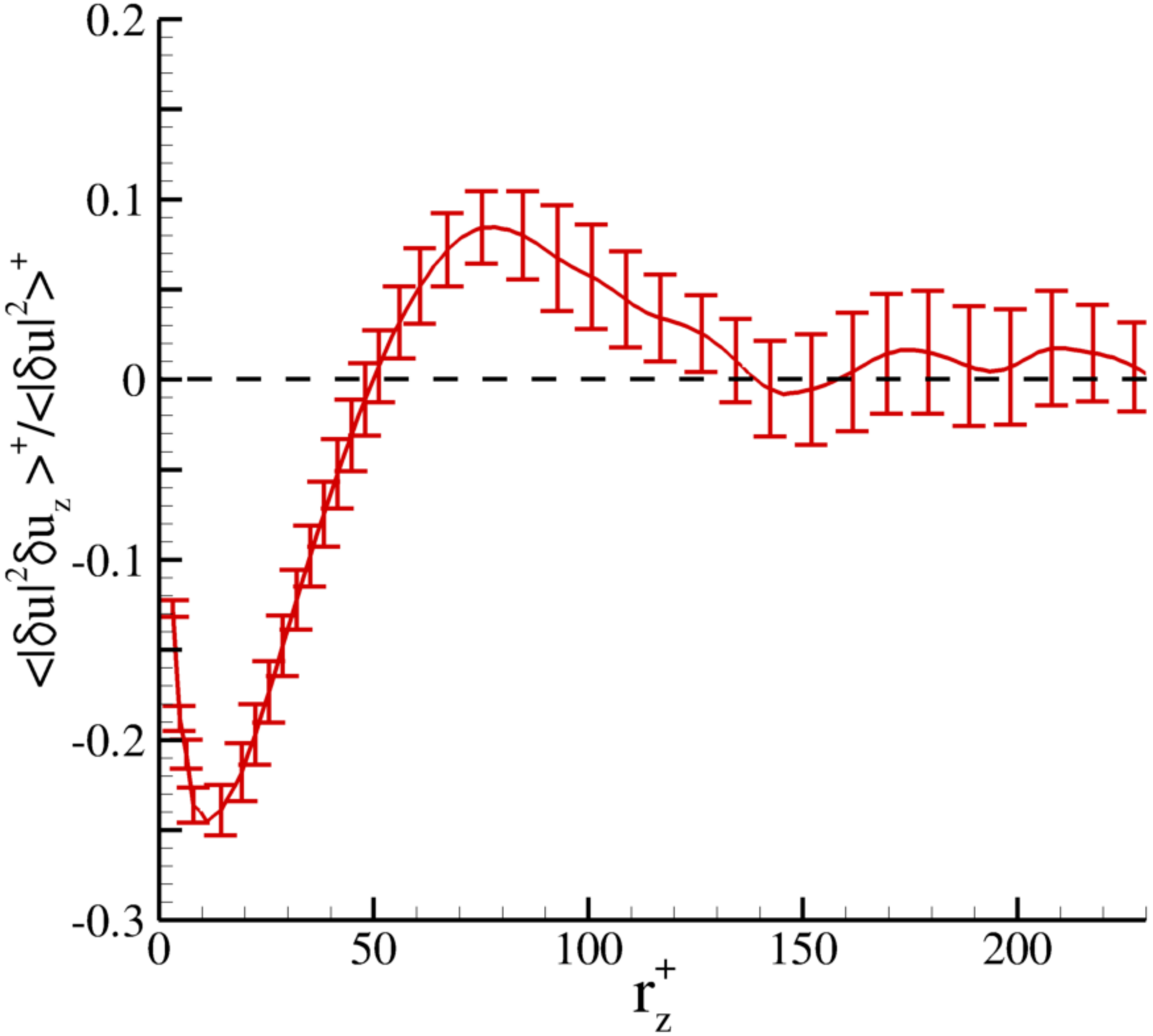
Lagrangian evolution @ $Z_c = r_x = r_y = 0$




7
6
5
4


1.5
y 1.0
0.5
1.0
2.0
3.0
z
4.0
5.0
6.0

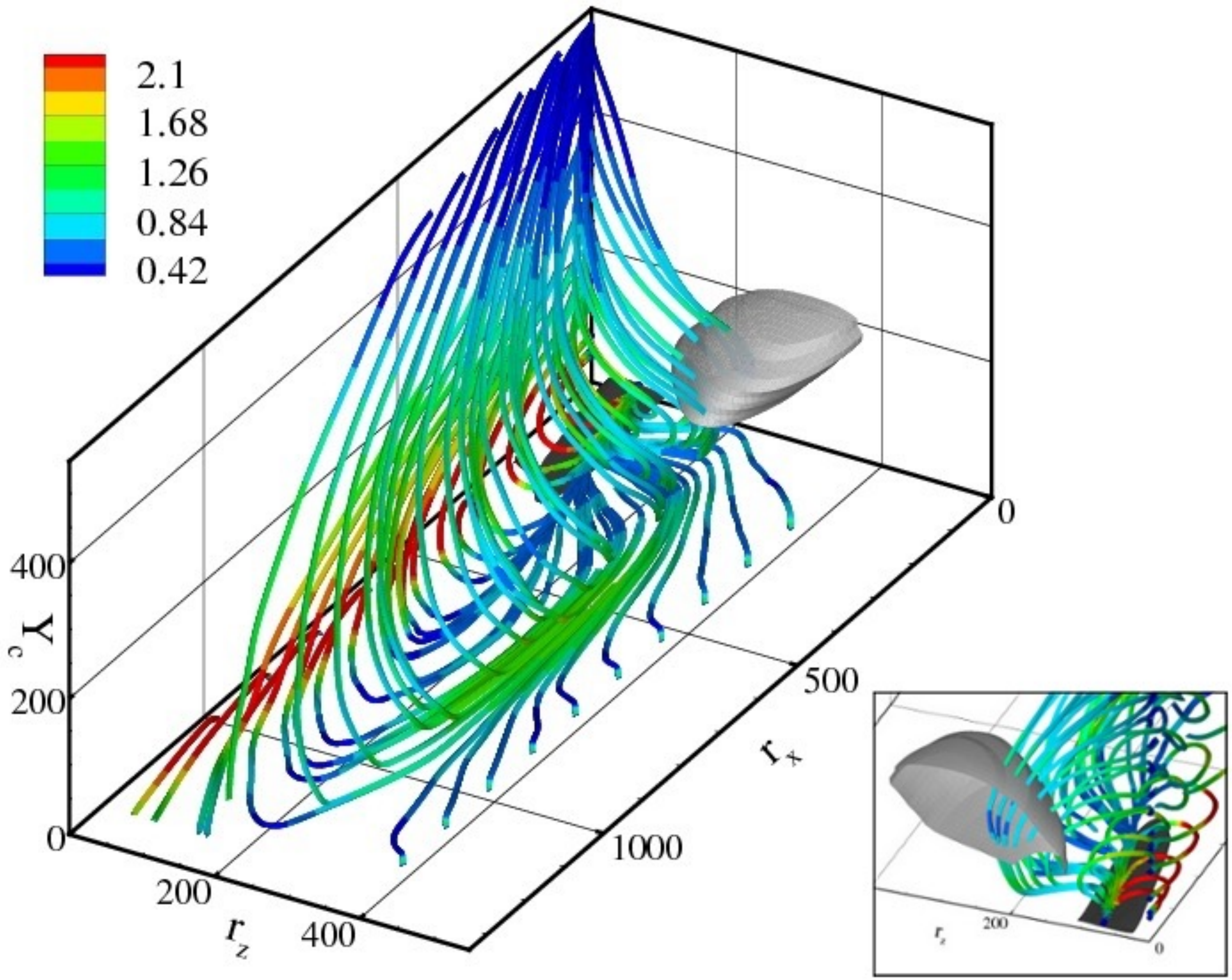

0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
Y_c
0.6
0.4
0.2
0
r_z
1
0.5
0
X_c
4
4.5
5
5.5
6
6.5
7



$$\delta w_z = \frac{\langle \delta u_z |\delta \mathbf{u}|^2 \rangle}{\langle |\delta \mathbf{u}^2| \rangle} \qquad \dot{r}_z = \delta w_z$$

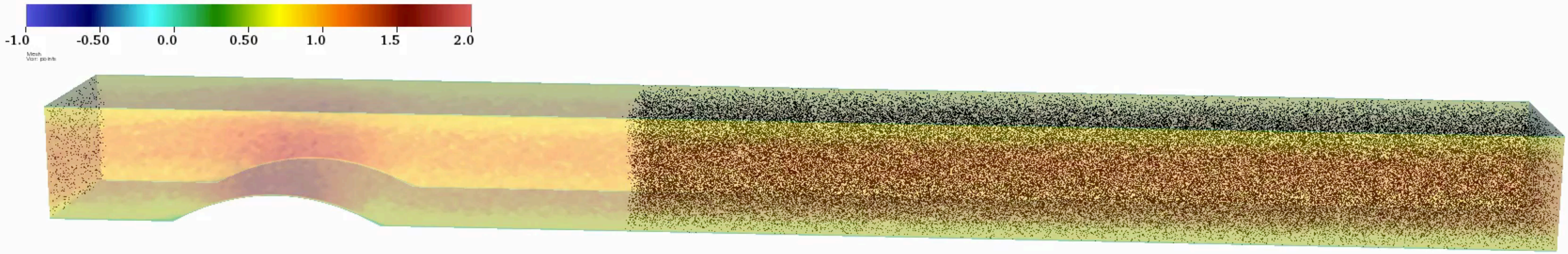
Channel Flow

Cimarelli, De Angelis, CMC, JFM 2013
Cimarelli, De Angelis, Jimenez, CMC, JFM 2016



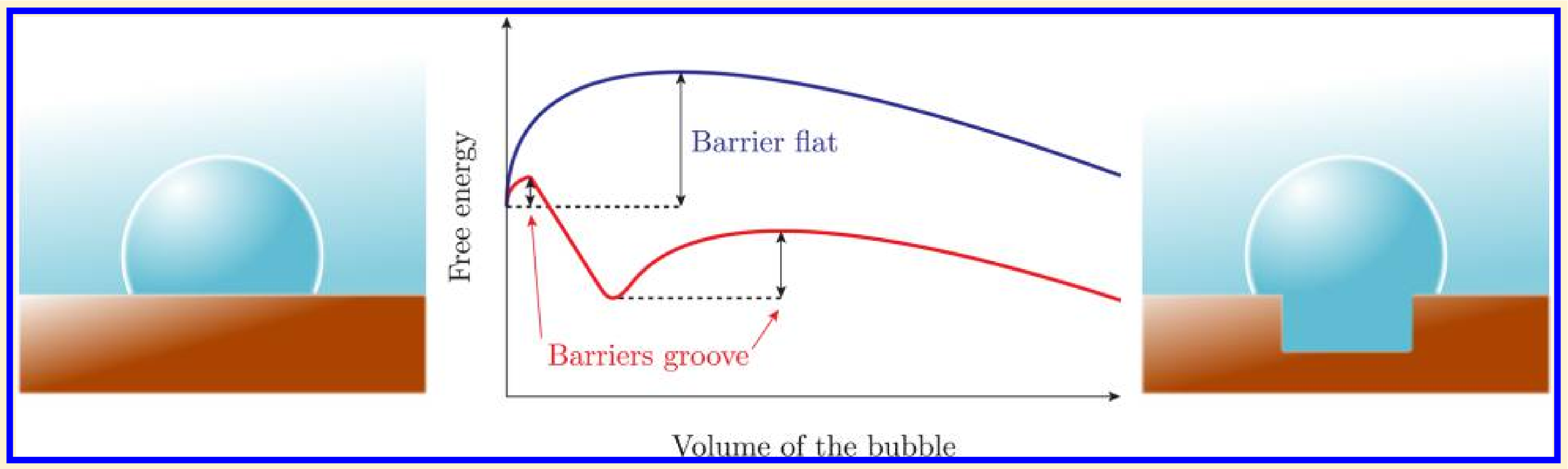
Conclusions & Outlook

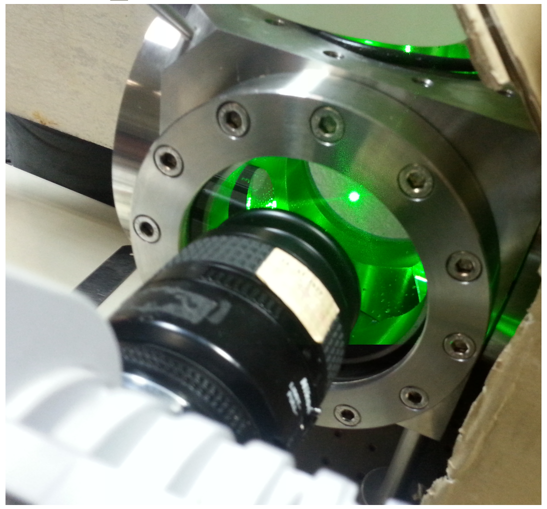
- State of the art DNS of complex turbulent separated flow
- Separation bubble and form drag
- Budget of single point kinetic energy highly non trivial
- Generalized Kolmogorov equation in five-dimensional space
- Allows to identify mechanisms of transport in the space of scales, e.g. direct & inverse cascades

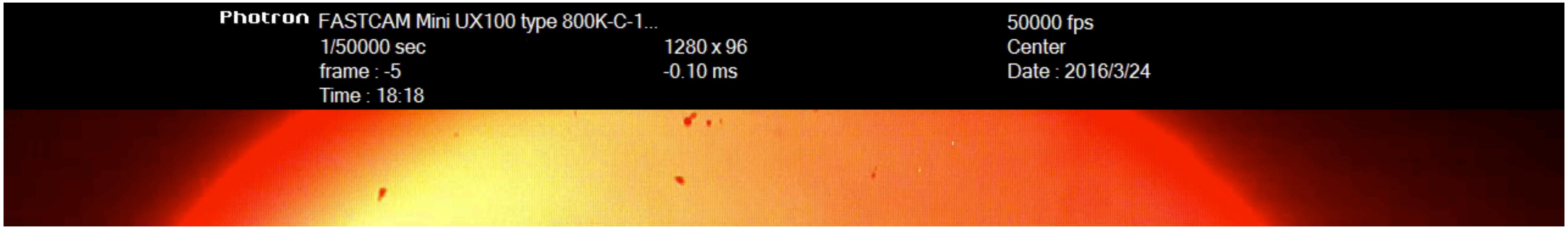


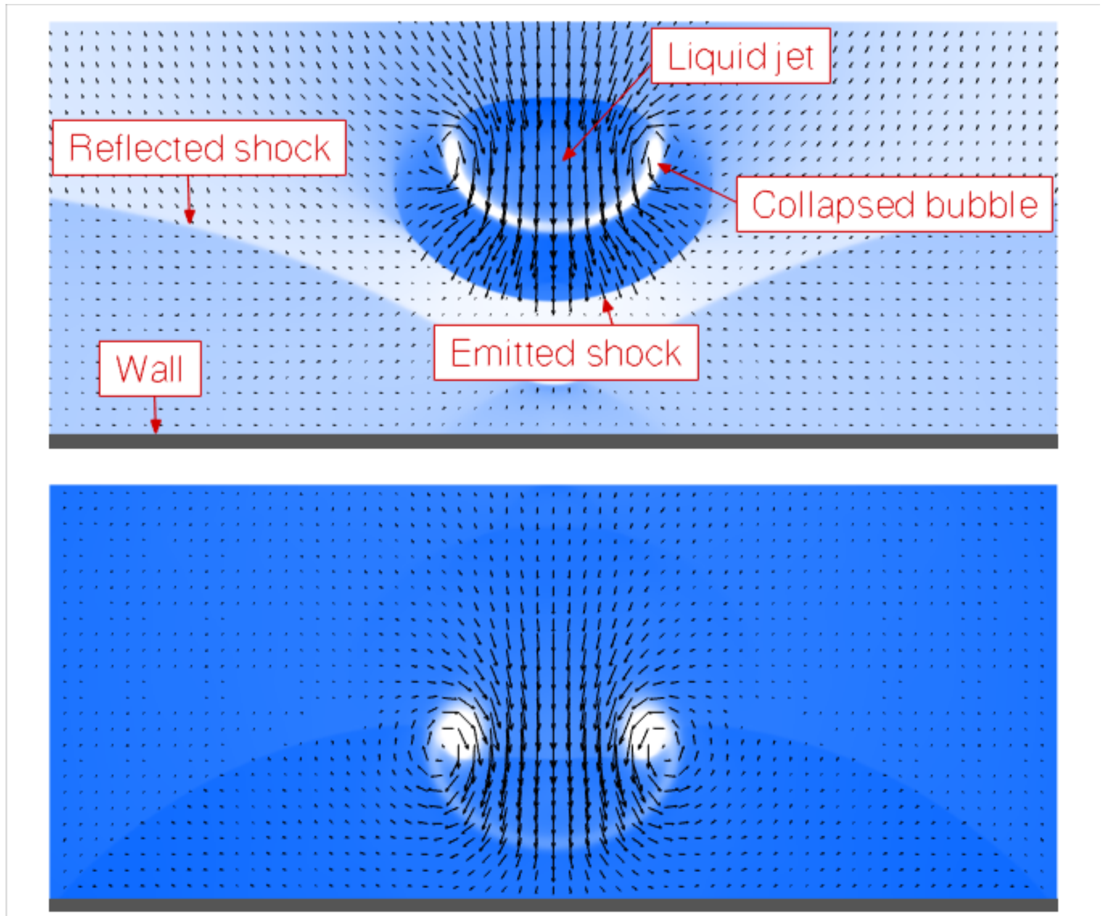
Acknowledgments
PRACE
ERC: Advanced Grant 2013 - BIC - Following Bubbles from Inception to Collapse

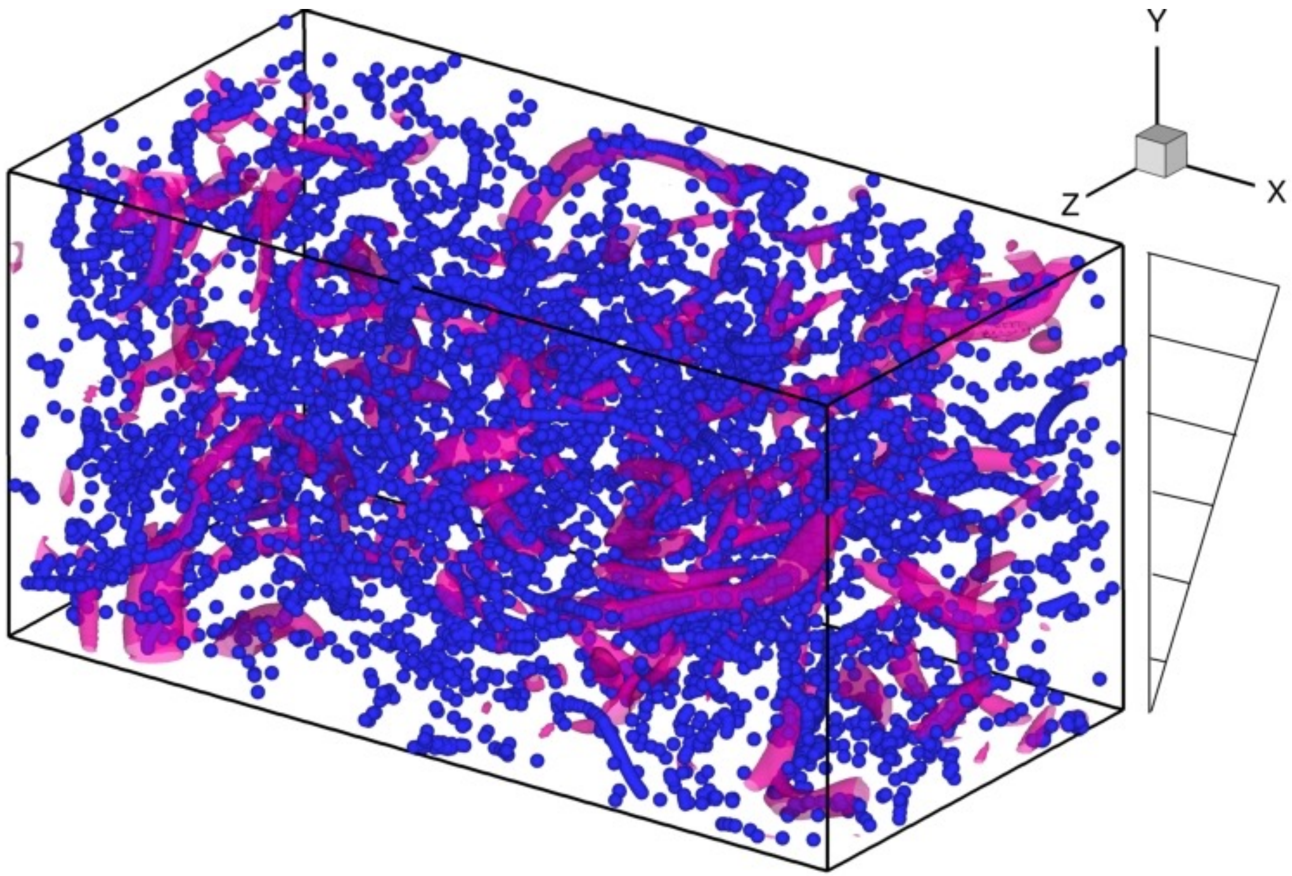
BIC - Following Bubbles from Inception to Collapse











Thank you

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