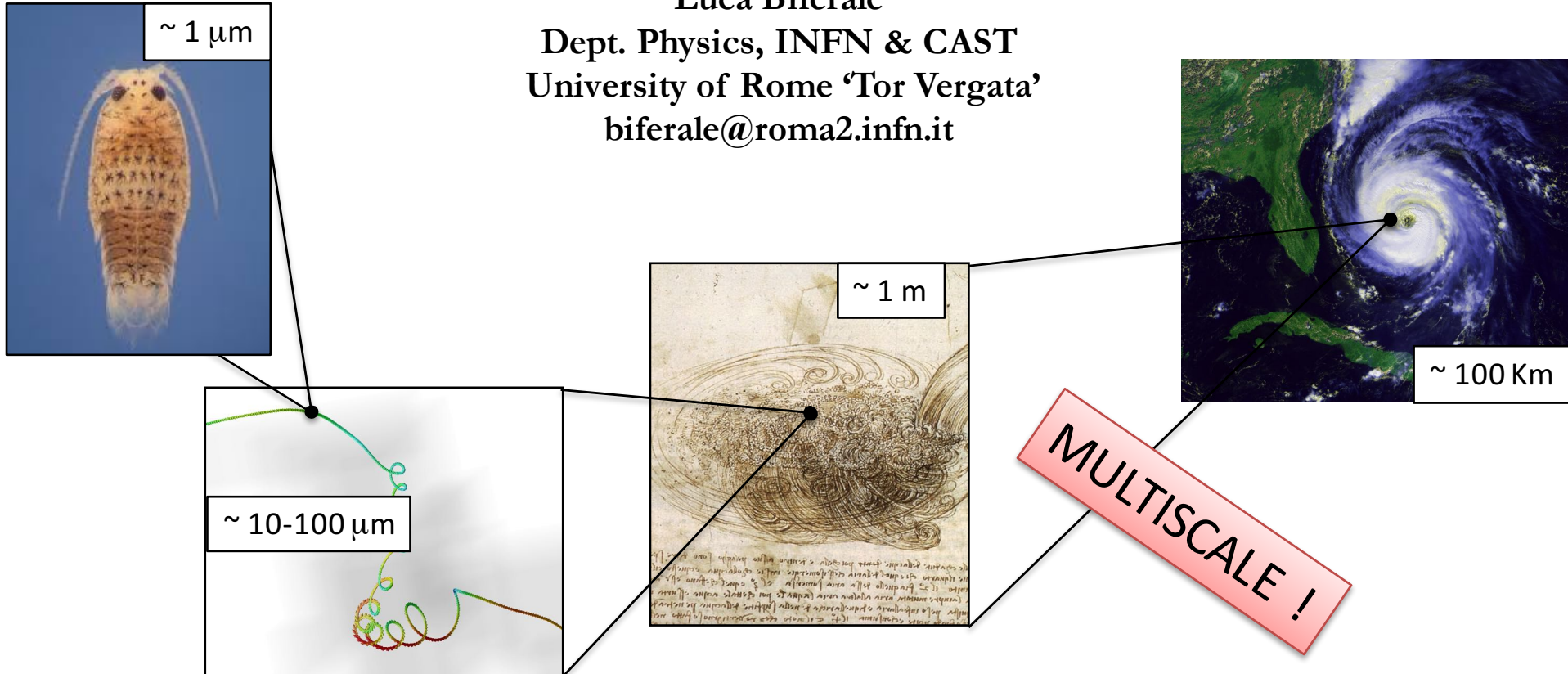


ENERGY TRANSFER AND ENERGY DISSIPATION IN TURBULENT FLOWS

Πάντα ρει (everything flows)

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WHERE DOES ENERGY GO ?

WHAT CAN WE SAY ABOUT THE STATISTICAL PROPERTIES OF TURBULENT FLOWS AT LARGE/SMALL SCALES ?

NAVIER-STOKES EQUATIONS:

$$m\vec{a} = \vec{F}$$

$$\underbrace{\partial_t \vec{v}}_{\text{acceleration}} + \underbrace{(\vec{v} \cdot \vec{\partial}) \vec{v}}_{\text{pressure}} = - \underbrace{\vec{\partial} P}_{\text{viscosity}} + \underbrace{\nu \Delta \vec{v}}_{\text{external forcing}} + \underbrace{\vec{f}}_{\text{external forcing}}$$

Leonardo da Vinci (~ 1500): “doue la turbolenza de si genera **[injected]**; doue la turbolenza dell aqua si mantiene **[advected]** plugho; doue la turbolenza dell aqua si posa **[dissipated]**”



Leonardo da Vinci (~ 1500): “doue la turbolenza de si genera [injected]; doue la turbolenza dell aqua si mantiene [advected] plugho; doue la turbolenza dell aqua si posa [dissipated]”

$$\partial_t \vec{v} + (\vec{v} \cdot \vec{\partial}) \vec{v} = -\vec{\partial} P + \nu \Delta \vec{v} + \vec{f}$$

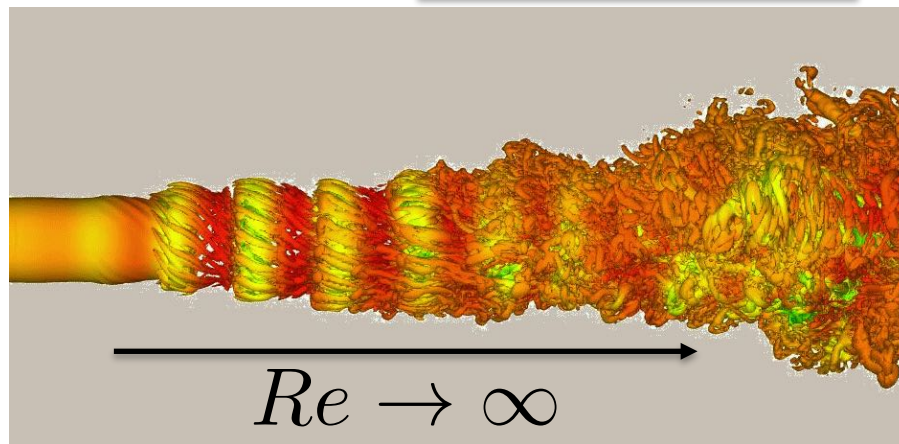
control parameter:

$$Re = \frac{l_0 v_0}{\nu}$$

$$\partial_t \vec{v} + (\vec{v} \cdot \vec{\partial}) \vec{v} = -\vec{\partial} P + \frac{1}{Re} \Delta \vec{v} + \vec{f}$$

$$Re \rightarrow \infty$$

FULLY NON-LINEAR



NAVIER-STOKES 3D \leftrightarrow 2D

(NASA - Space Flight Center Scientific Visualization Studio)

(Vortices within vortices - APS Gallery of Fluid Motions)



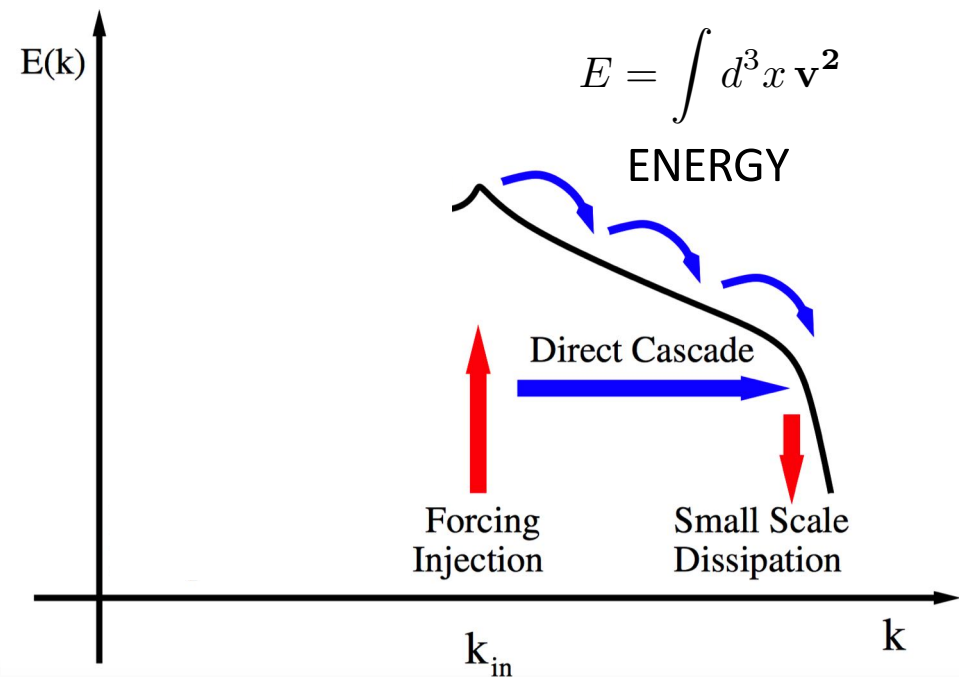
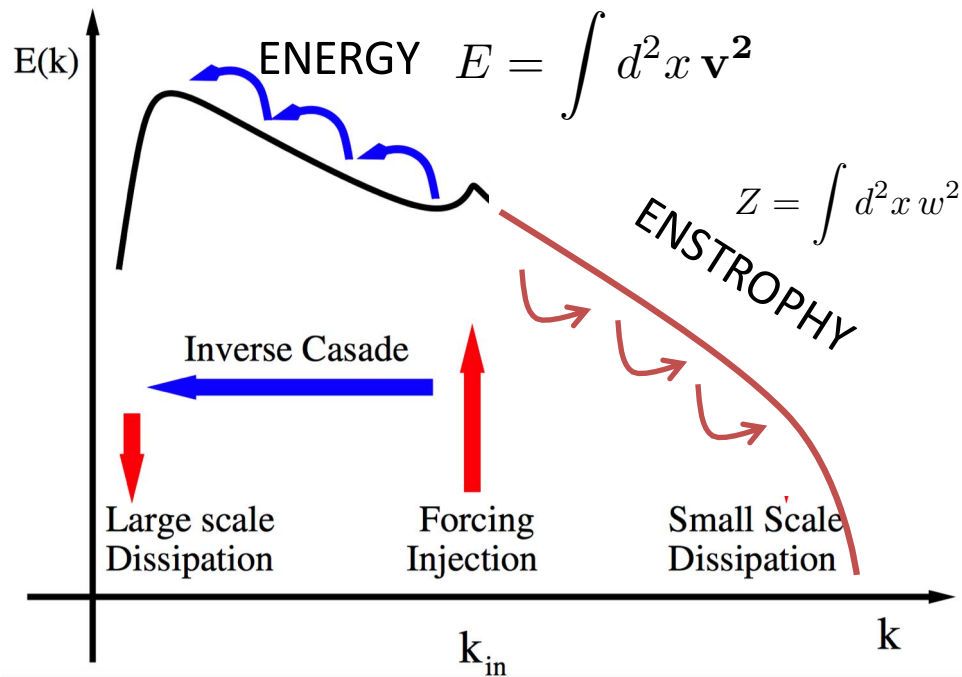
Entry #: 84174

Vortices within vortices: hierarchical nature of vortex tubes in turbulence

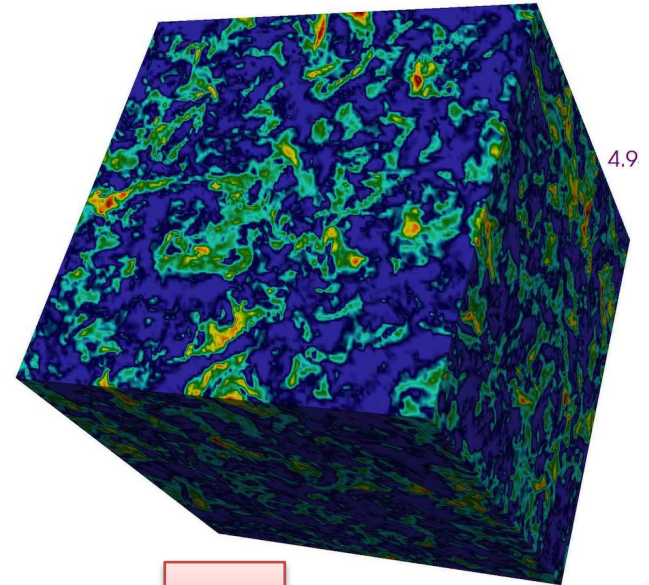
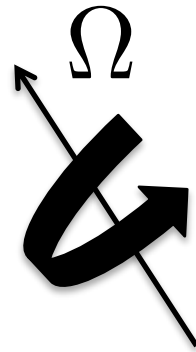
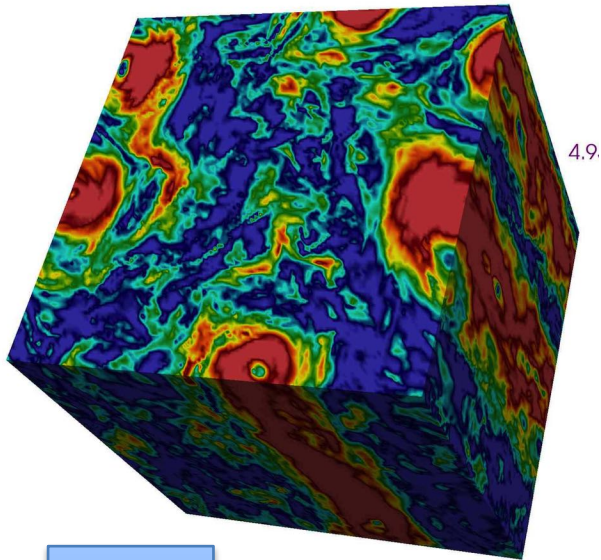
Kai Bürger¹, Marc Treib¹, Rüdiger Westermann¹,
Suzanne Werner², Cristian C Lalescu³,
Alexander Szalay², Charles Meneveau⁴, Gregory L Eyink^{2,3,4}

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² Department of Physics & Astronomy, The Johns Hopkins University
³ Department of Applied Mathematics & Statistics, The Johns Hopkins University
⁴ Department of Mechanical Engineering, The Johns Hopkins University

3D

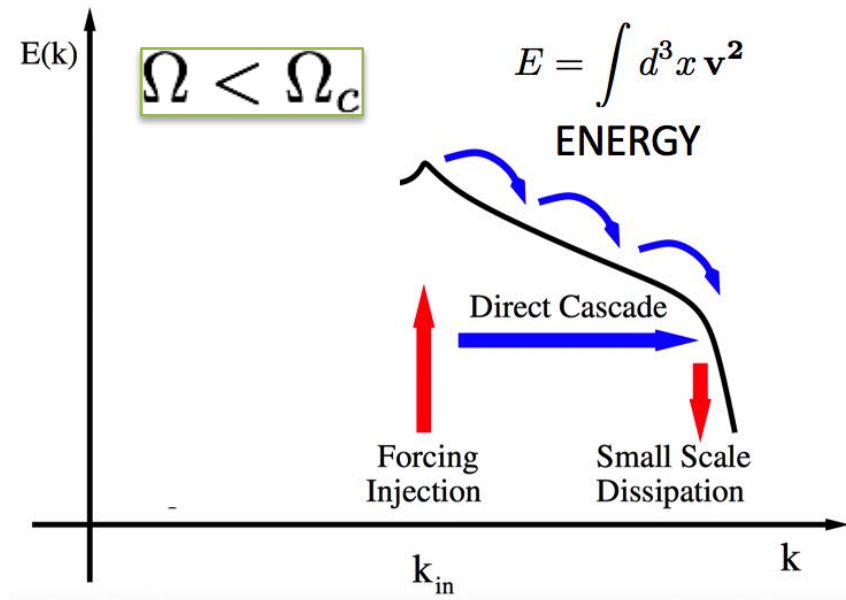
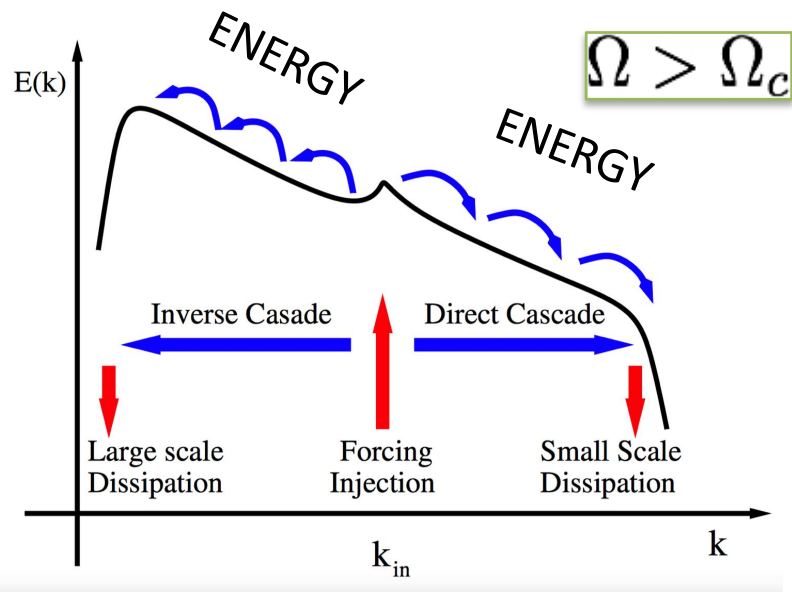


(PHASE) TRANSITIONS IN THE ENERGY TRANSFER:
ROTATING FLOWS



~ 2D

3D



COMPLEX FLUID & COMPLEX FLOWS

$$\left\{ \begin{array}{l} \partial_t v + v \partial v = -\partial p + \nu \Delta v + F(B, B) + g\theta + \sum_i c_0(u_i, v) \delta(r - r_i) + f \\ \partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta \\ \partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B \\ \partial \cdot v = 0 \\ + \text{boundary conditions} \end{array} \right.$$

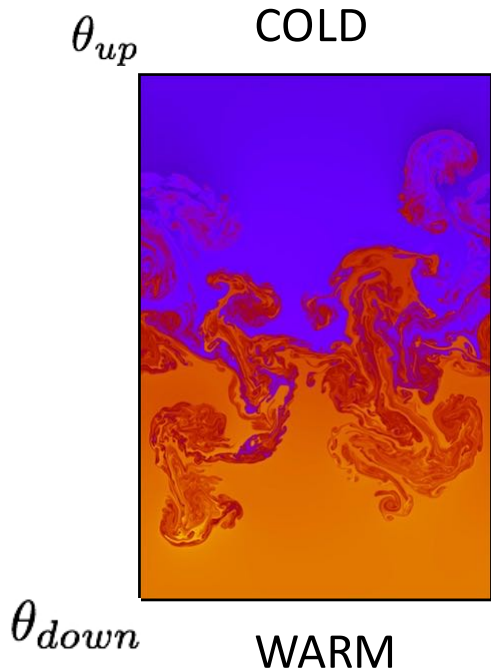
control parameter:

$$Re = \frac{l_0 v_0}{\nu}$$

$Re \rightarrow \infty$

FULLY NON-LINEAR

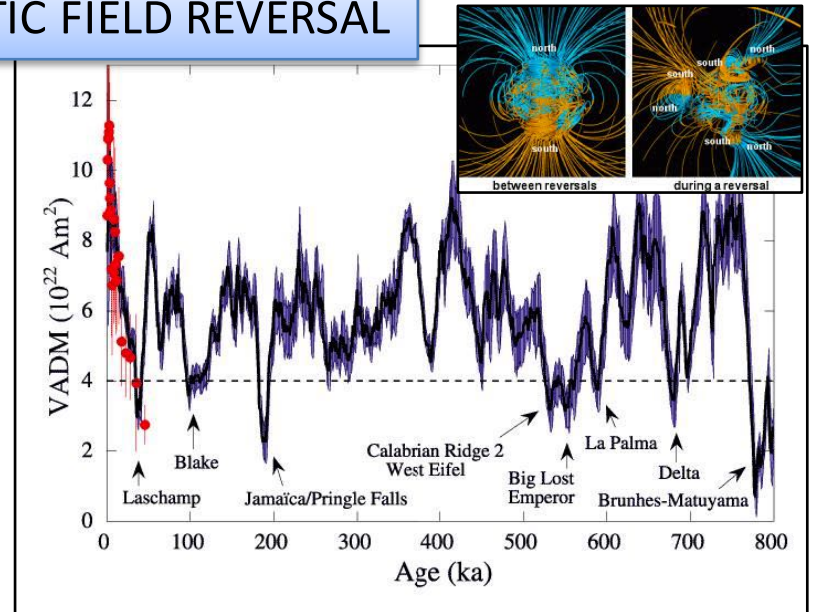
$$\left\{ \begin{array}{l} \frac{du_i(r_i, t)}{dt} = -\rho_f |u_i - v| (u_i - v) \\ + \rho_f \left(\frac{Dv}{Dt} - \frac{Du_i}{Dt} \right) + (u_i - v) \times \omega \end{array} \right.$$



CONVECTION



MAGNETIC FIELD REVERSAL



COMPLEX FLUID & COMPLEX FLOWS

$$\left\{ \begin{aligned} \partial_t v + v \partial v &= -\partial p + \nu \Delta v + F(B, B) + g\theta + \sum_i c_0(u_i, v) \delta(r - r_i) + f \\ \partial_t \theta + v \cdot \partial \theta &= \chi \partial^2 \theta \\ \partial_t B + v \cdot \partial B &= B \cdot \partial v + \chi \partial^2 B \\ \partial \cdot v &= 0 \\ &+ \text{boundary conditions} \end{aligned} \right.$$

control parameter:

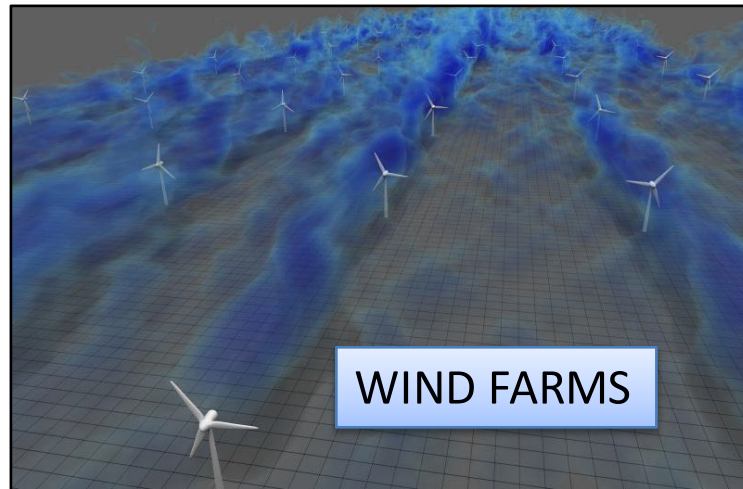
$$Re = \frac{l_0 v_0}{\nu}$$

$$Re \rightarrow \infty$$

FULLY NON-LINEAR

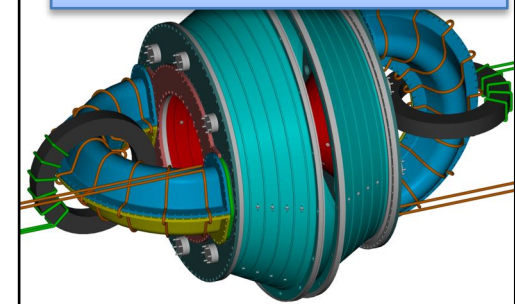
$$\left\{ \begin{aligned} \frac{du_i(r_i, t)}{dt} &= -\rho_f |u_i - v| (u_i - v) \\ &+ \rho_f \left(\frac{Dv}{Dt} - \frac{Du_i}{Dt} \right) + (u_i - v) \times \omega \end{aligned} \right.$$

ROTATING CONVECTION



WIND FARMS

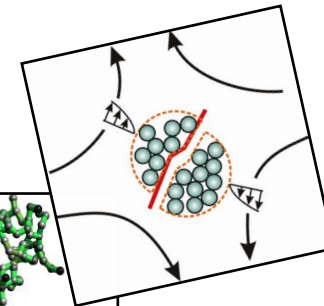
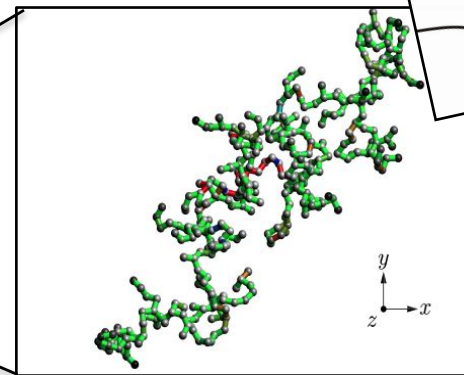
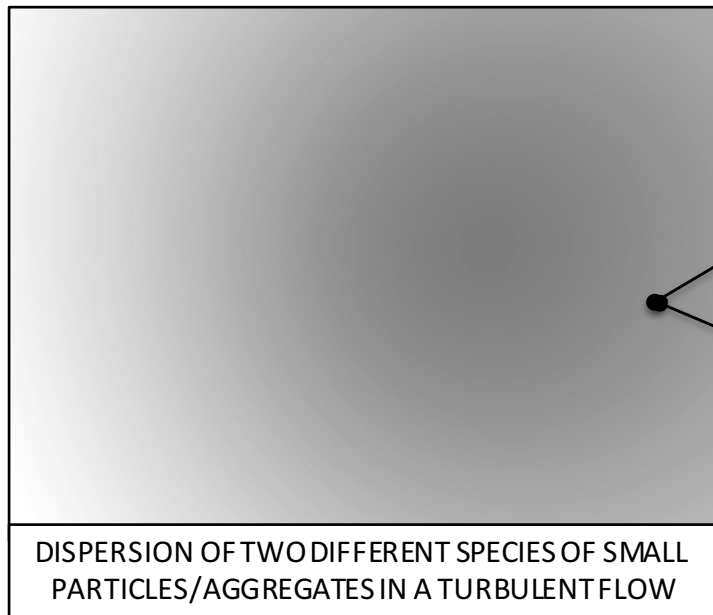
MAGNETIC PLASMA



COMPLEX FLUID & COMPLEX FLOWS

$$\left\{ \begin{aligned} \partial_t \mathbf{v} + \mathbf{v} \cdot \partial \mathbf{v} &= -\partial P + \nu \partial^2 \mathbf{v} + F(B, B) + g\theta + \sum_i c_0(\mathbf{u}_i, \mathbf{v}) \delta(\mathbf{r} - \mathbf{r}_i) + \mathbf{f} \\ \partial_t \theta + \mathbf{v} \cdot \partial \theta &= \chi \partial^2 \theta \quad \leftarrow \text{temperature} \\ \partial_t \mathbf{B} + \mathbf{v} \cdot \partial \mathbf{B} &= \mathbf{B} \cdot \partial \mathbf{v} + \chi \partial^2 \mathbf{B} \quad \leftarrow \text{magnetic field} \\ \partial \cdot \mathbf{v} &= 0 \\ &+ \text{boundary conditions} \end{aligned} \right.$$

$$\left\{ \begin{aligned} \frac{d\mathbf{u}_i(\mathbf{r}_i, t)}{dt} &= -\rho_f |\mathbf{u}_i - \mathbf{v}| (\mathbf{u}_i - \mathbf{v}) \quad \leftarrow \text{small particles/colloidal aggregates:} \\ &\quad \text{Stokes drag, added mass, lift force, etc...} \\ + \rho_f \left(\frac{D\mathbf{v}}{Dt} - \frac{D\mathbf{u}_i}{Dt} \right) &+ (\mathbf{u}_i - \mathbf{v}) \times \boldsymbol{\omega} \end{aligned} \right.$$



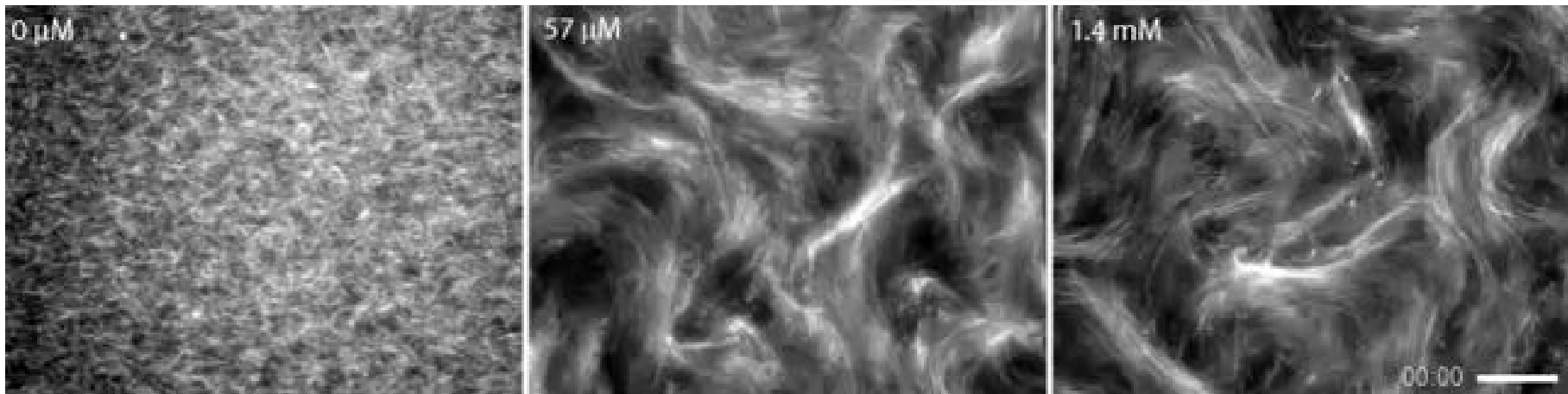
+ Stokesian dynamics

COAGULATION/FRAGMENTATION OF COLLOIDAL AGGREGATES IN TURBULENT FLOWS

COMPLEX FLUID & COMPLEX FLOWS

$$\left\{ \begin{array}{l} \partial_t v + v \cdot \partial v = -\partial P + \nu \partial^2 v + F(B, B) + g\theta + \sum_i c_0(u_i, v) \delta(r - r_i) + f(V_{active}) \\ \partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta \quad \leftarrow \text{temperature} \\ \partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B \quad \leftarrow \text{magnetic field} \\ \partial \cdot v = 0 \\ + \text{boundary conditions} \\ \frac{du_i(r_i, t)}{dt} = -\rho_f |u_i - v| (u_i - v) \\ + \rho_f \left(\frac{Dv}{Dt} - \frac{Du_i}{Dt} \right) + (u_i - v) \times \omega \end{array} \right.$$

ACTIVE MATTER



Sanchez et al Nature 2012 "Microtubules activated by Kinesin Motor Proteins"

$$\left\{ \begin{array}{l} \partial_t v + v \partial v = -\partial p + \nu \Delta v \\ \partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta \\ \partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B \\ \partial \cdot v = 0 \\ + \text{boundary conditions} \\ \frac{du_i(r_i, t)}{dt} = -\rho_f |u_i - v| (u_i - v) \\ + \rho_f \left(\frac{Dv}{Dt} - \frac{Du_i}{Dt} \right) + (u_i - v) \times \omega \end{array} \right. + F(B, B) + g\theta + \sum_i c_0(u_i, v) \delta(r - r_i) + f$$

control parameter:

$$Re = \frac{l_0 v_0}{\nu}$$

$$\left\{ \begin{array}{l} Re \rightarrow \infty \\ \nu \rightarrow 0 \end{array} \right.$$

Too many turbulences? NO! -> UNIVERSALITY

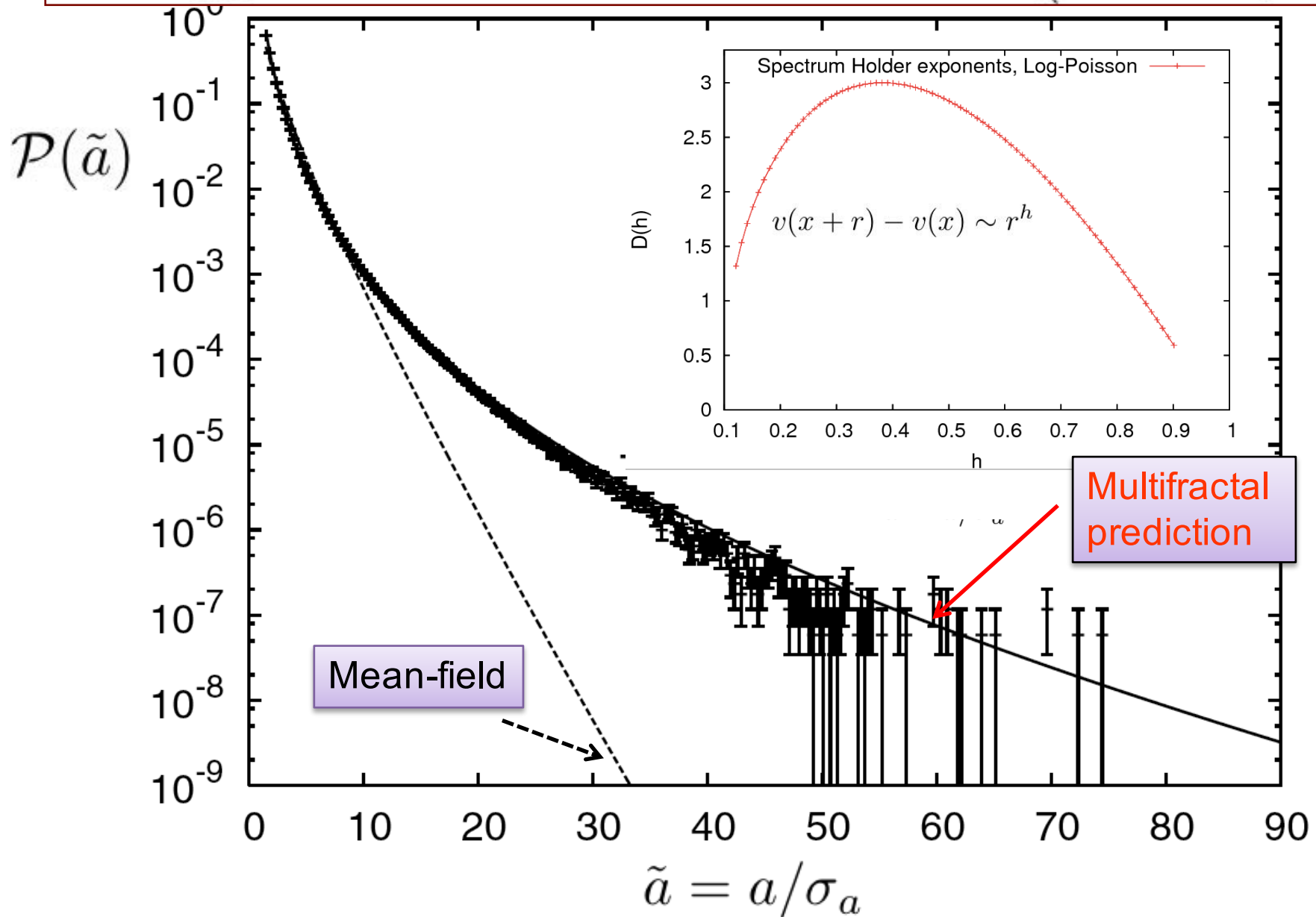
ALL TURBULENT FLOWS RECOVER ISOTROPY AND HOMOGENEITY (AT SCALES SMALL ENOUGH)

- Homogeneous & Isotropic Turbulence
- Fully periodic 3D domain
- Gaussian delta-correlated forcing
- Incompressible

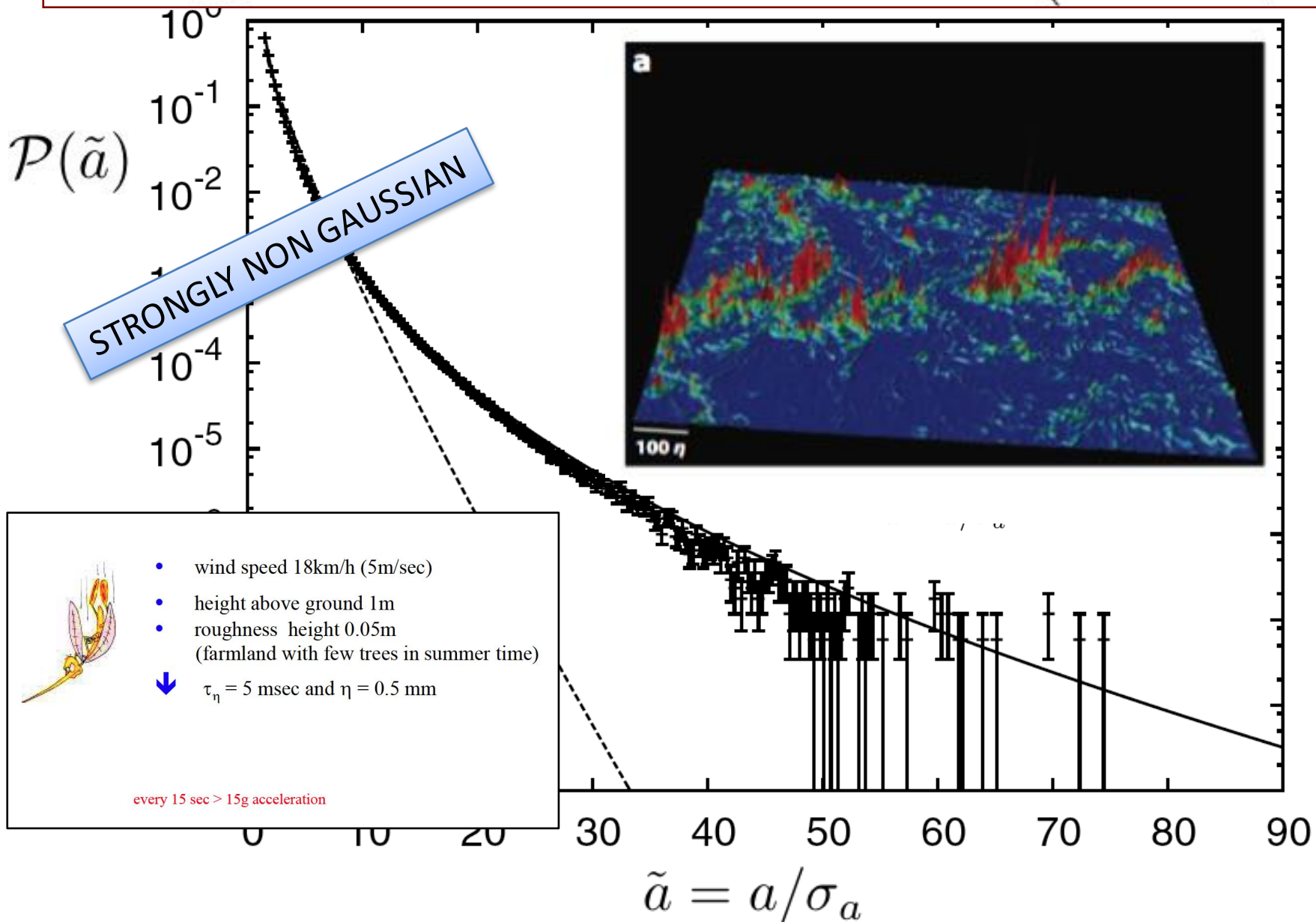
Homogeneous and Isotropic turbulence: the (UNSOLVED) hydrogen atom of fluid dynamics

WHY STILL UNSOLVED?
(EQUATIONS ARE KNOWN SINCE 250 YEARS AGO!)

$$\mathcal{P}(a) \sim \int_{h \in I} dh a^{\frac{h-5+D(h)}{3}} \nu^{\frac{7-2h-2D(h)}{3}} L_0^{D(h)+h-3} \sigma_v^{-1} \times \exp\left(-\frac{a^{\frac{2(1+h)}{3}} \nu^{\frac{2(1-2h)}{3}} L_0^{2h}}{2\sigma_v^2}\right)$$



$$\mathcal{P}(a) \sim \int_{h \in I} dh a^{\frac{h-5+D(h)}{3}} \nu^{\frac{7-2h-2D(h)}{3}} L_0^{D(h)+h-3} \sigma_v^{-1} \times \exp\left(-\frac{a^{\frac{2(1+h)}{3}} \nu^{\frac{2(1-2h)}{3}} L_0^{2h}}{2\sigma_v^2}\right)$$



Turbulent luminance in impassioned van Gogh paintings

J.L. Aragón

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Instituto de Física Aplicada, Consejo Superior de Investigaciones Científicas, Serrano 144, 28006 Madrid, Spain.

P.K. Maini

Centre for Mathematical Biology, Mathematical Institute, 24-29 St Giles Oxford OX1 3LB, U.K.

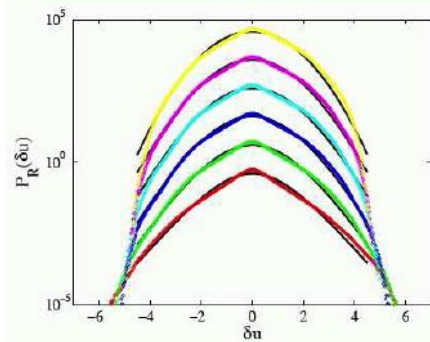


FIG. 2: Semilog plot of the probability density $P_R(\delta u)$ of luminance changes δu for pixel separations $R = 60, 240, 400, 600, 800, 1200$ (from bottom to top). Curves have been vertically shifted for better visibility. Data points were fitted, according to Ref. [13], and the results are shown in full lines; parameter values are $\lambda = 0.2, 0.15, 0.12, 0.11, 0.09, 0.0009$ (from bottom to top).



Starry night

Road with Cypress and Star

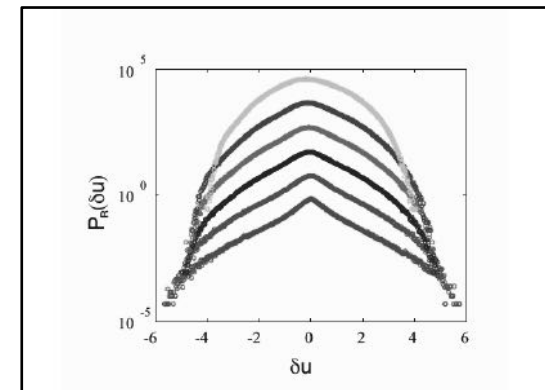
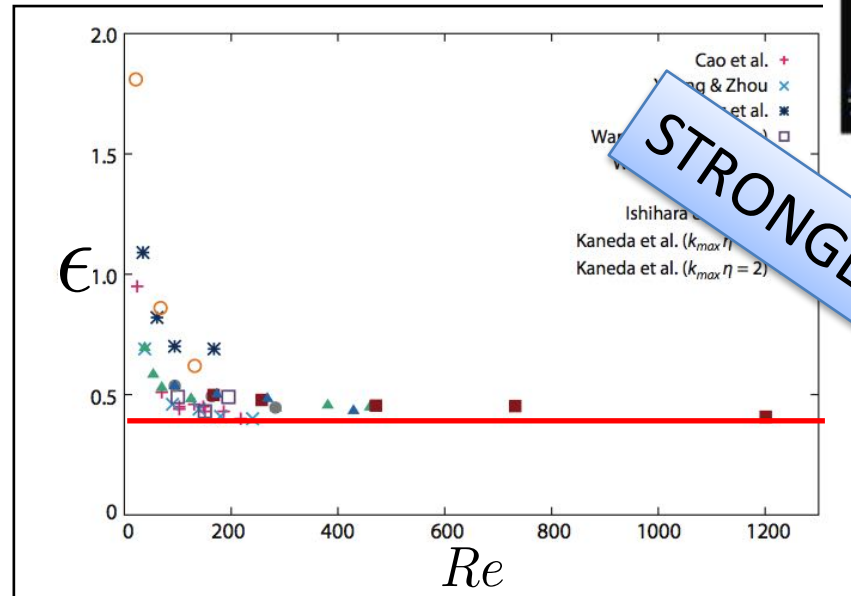
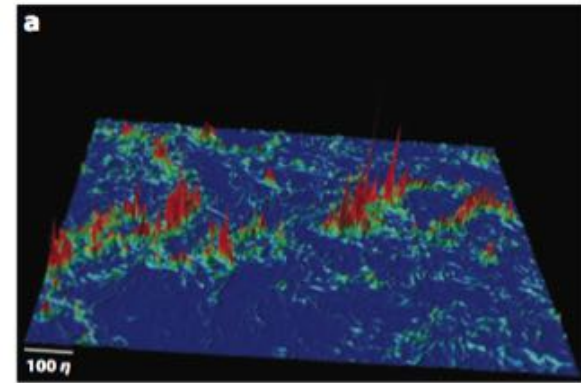


FIG. 5: Left: *Road with Cypress and Star* (Rijksmuseum Kröller-Müller, Otterlo). Right: PDF for pixel separations $R = 2, 5, 15, 20, 30, 60$ (from bottom to top). The studied image was taken from the WebMuseum-Paris, webpage.

NAVIER-STOKES 3D

ENERGY DISSIPATION



STRONGLY OUT-OF-EQUILIBRIUM!

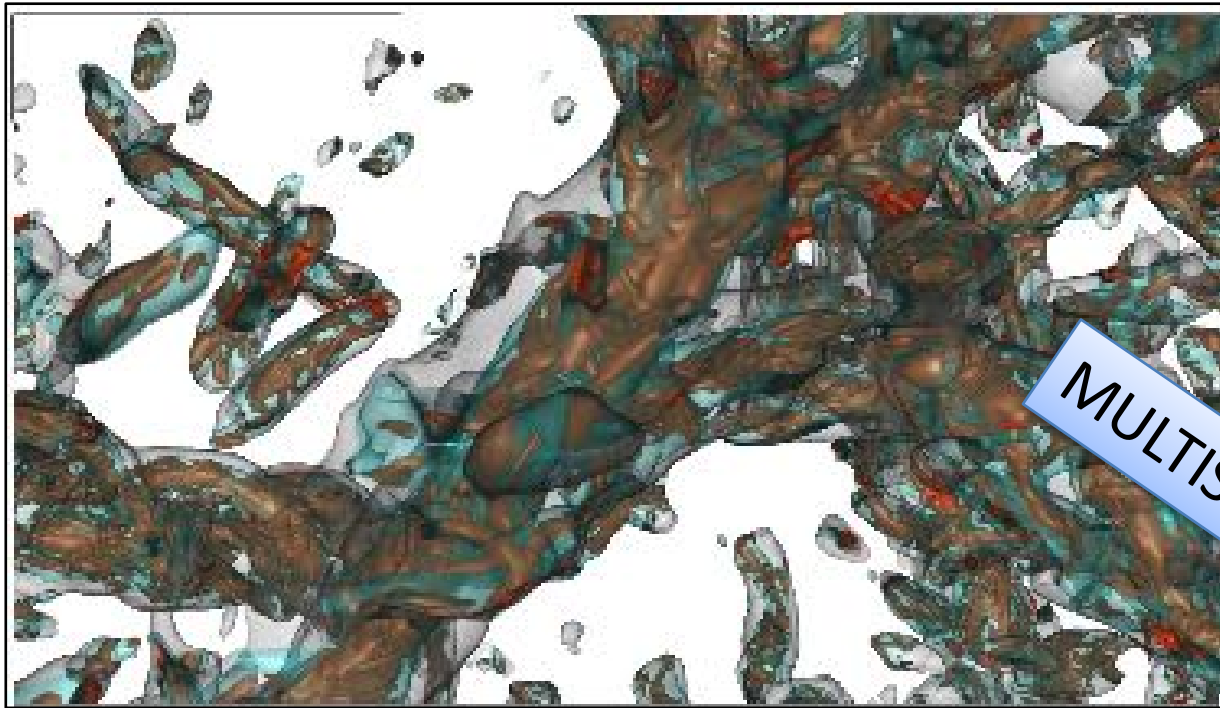
$$\partial_t \vec{v} + (\vec{v} \cdot \vec{\partial}) \vec{v} = -\vec{\partial} P + \frac{1}{Re} \Delta \vec{v} + \vec{f}$$

$$\lim_{Re \rightarrow \infty} \epsilon = \lim_{\nu \rightarrow 0} \nu \langle (\partial v)^2 \rangle \rightarrow const.$$

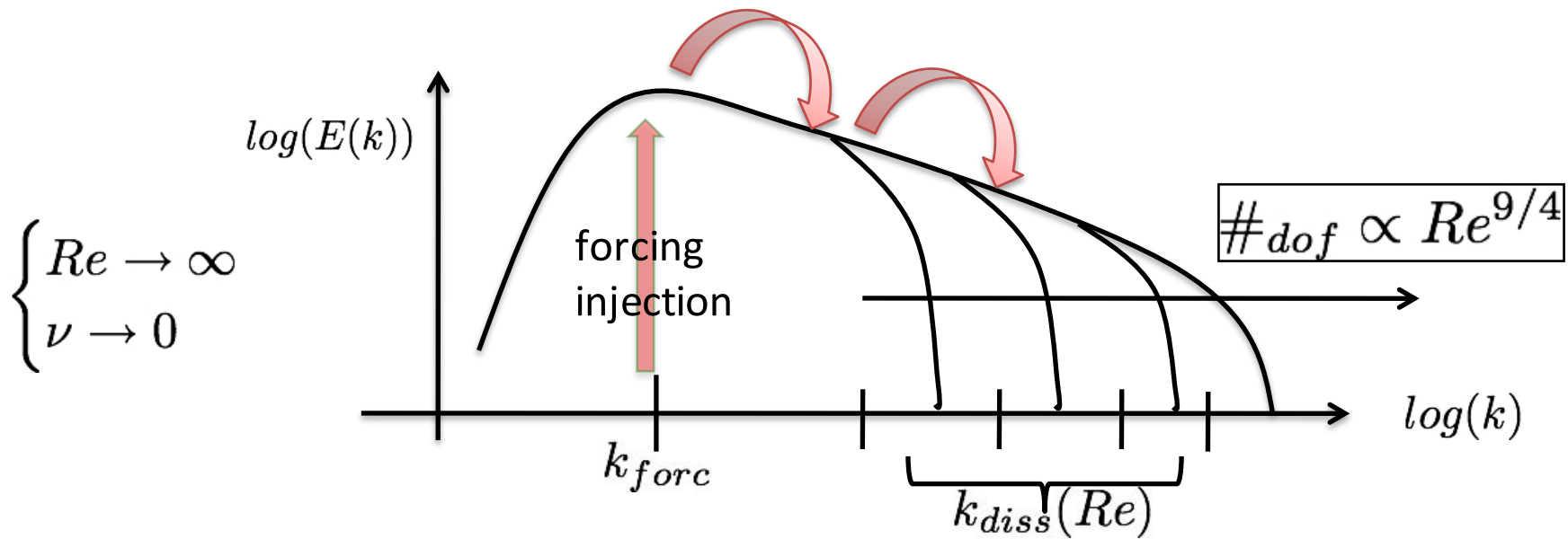
DISSIPATIVE ANOMALY

NO ROOM FOR QUASI-EQUILIBRIUM STAT MECH!

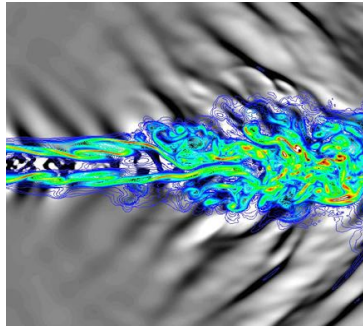
NAVIER-STOKES 3D



MULTISCALE !



$$\#_{dof} = \left(\frac{k_{diss}}{k_{forc}}\right)^3 \sim Re^{9/4}$$



laboratory flow

$$Re \sim 10^5 - 10^9$$

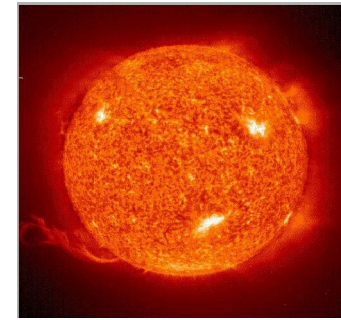
$$\#_{dof} \sim 10^{11} - 10^{20}$$



atmosph. flow

$$Re \sim 10^8 - 10^{12}$$

$$\#_{dof} \sim 10^{18} - 10^{30}$$



astrophys. flow

$$Re > 10^{15}$$

$$\#_{dof} \sim \infty$$

state-of-the-art Direct Numerical Simulation:

Isotropic, homogeneous Fully Periodic Flows

Pseudo-Spectral Methods.

Resolution 12000^3 (Y. Kaneda, APS 2017)

Reynolds: 10^8 ,

Storage of 1 velocity configuration (double precision): 40 Tbyte

RAM requirements for time marching \sim 160 Tbyte

**Moral: brute force Direct Numerical Simulations
able to saturate any computing power
(present and/or future): Computo ergo sum?**

J. von NEUMANN (1949)

These considerations justify the view that a considerable mathematical effort towards a detailed understanding of the mechanism of turbulence is called for. The entire experience with the subject indicates that the purely analytical approach is beset with difficulties, which at this moment are still prohibitive. The reason for this is probably as was indicated above: That our intuitive relationship to the subject is still too loose — not having succeeded at anything like deep mathematical penetration in any part of the subject, we are still quite disoriented as to the relevant factors, and as to the proper analytical machinery to be used.

Under these conditions there might be some hope to 'break the deadlock' by extensive, but well-planned, computational efforts. It must be admitted that the problems in question are too vast to be solved by a direct computational attack, that is, by an outright calculation of a representative family of special cases. There are, however, strong indications that one could name certain strategic points in this complex, where relevant information must be obtained by direct calculations. If this is properly done, and the operation is then repeated on the basis of broader information then becoming available, etc., there is a reasonable chance of effecting real penetrations in this complex of problems and gradually developing a useful, intuitive relationship to it. This should, in the end, make an attack with analytical methods, that is truly more mathematical, possible.¹

HOW TO USE UNCONVENTIONAL NUMERICS TO UNDERSTAND TURBULENCE

$$\left\{ \begin{array}{l} \partial_t \mathbf{v} + (\mathbf{v} \cdot \partial) \mathbf{v} = -\partial P + \nu \Delta \mathbf{v} + \mathbf{F} \\ \partial \cdot \mathbf{v} = 0 \\ + \textit{Boundary Conditions} \end{array} \right.$$

Prob. 1: STRONGLY OUT-OF-EQUILIBRIUM

Prob. 2: STRONGLY NON-GAUSSIAN STATISTICS

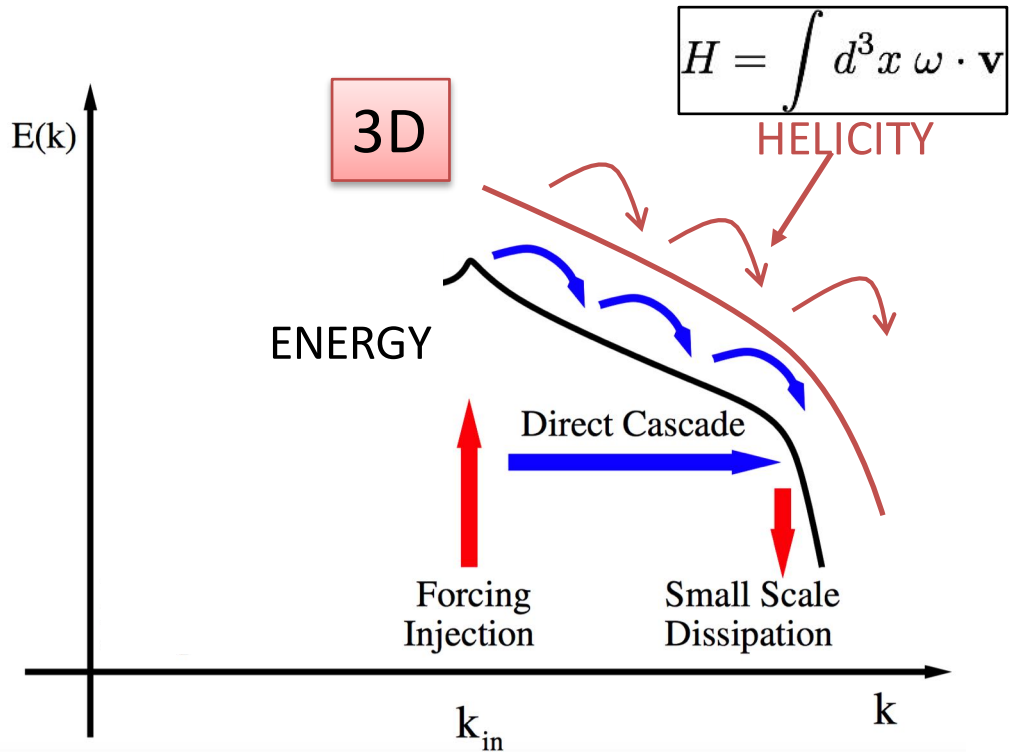
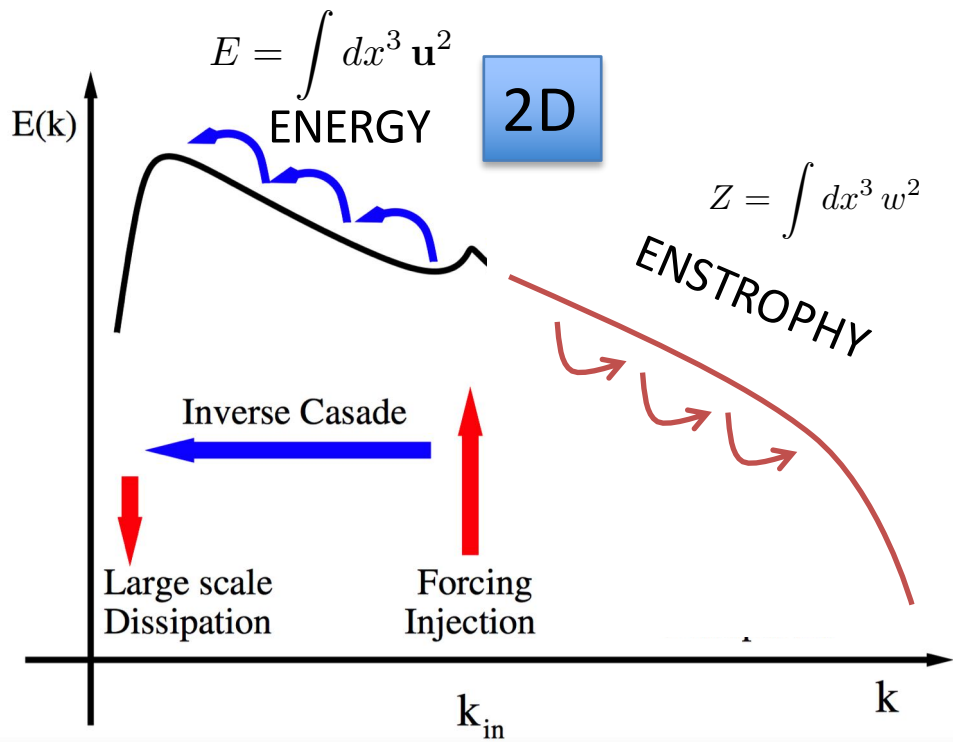
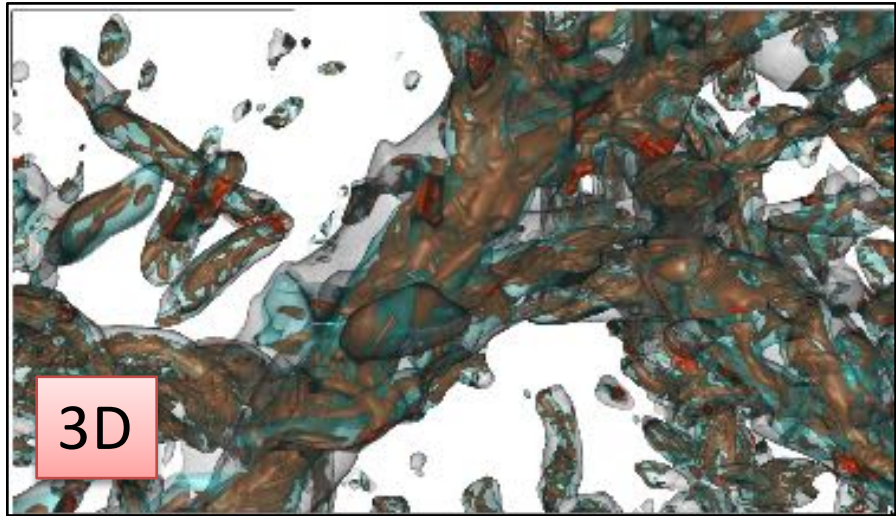
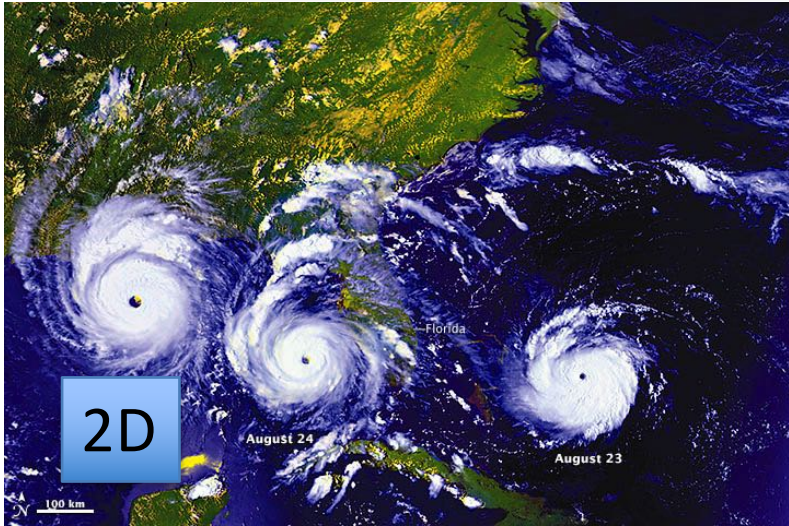
Prob. 3: MULTI-SCALE: 'INFINITE' NUMBER OF DOF

Q1: CAN WE DISSECT (AND RECONSTRUCT) NS EQUATIONS TO EXTRACT INTERESTING INFORMATION FROM ITS ELEMENTARY CONSTITUENTS?

Q2: CAN WE UNDERSTAND THE ORIGIN OF THE STRONG FLUCTUATIONS EMPIRICALLY OBSERVED IN THE ENERGY TRANSFER RATE?

Q3: CAN WE UNDERSTAND THE ORIGIN OF ENERGY-FLUX REVERSAL OBSERVED IN MANY GEO-FLOWS?

ON THE ROLE OF INVISCID INVARIANTS (HELICITY & ENERGY) IN 3D FORWARD/BACKWARD ENERGY CASCADES



ON THE ROLE OF INVISCID INVARIANTS (HELICITY & ENERGY) IN 3D
FORWARD/BACKWARD ENERGY CASCADES

$$H = \int d^3x \boldsymbol{\omega} \cdot \mathbf{v}$$

$$\mathbf{u}(\mathbf{k}) = u^+(\mathbf{k})\mathbf{h}^+(\mathbf{k}) + u^-(\mathbf{k})\mathbf{h}^-(\mathbf{k})$$

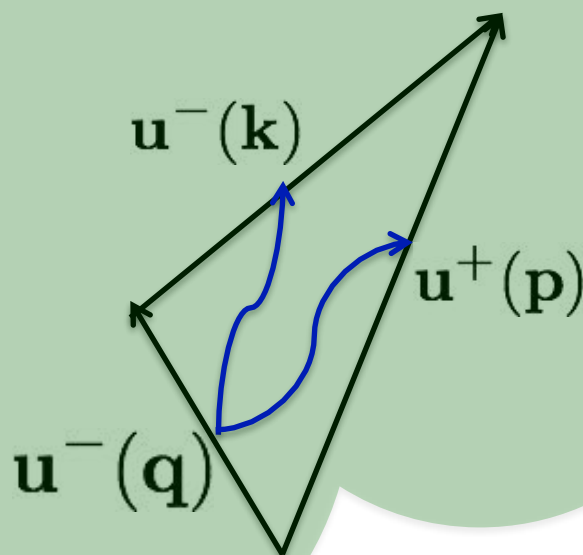
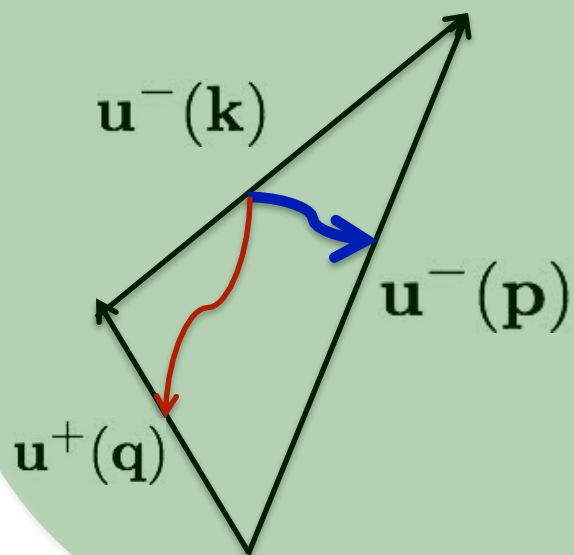
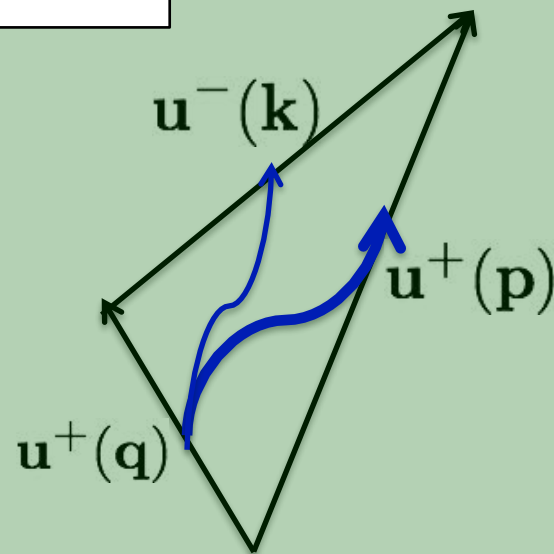
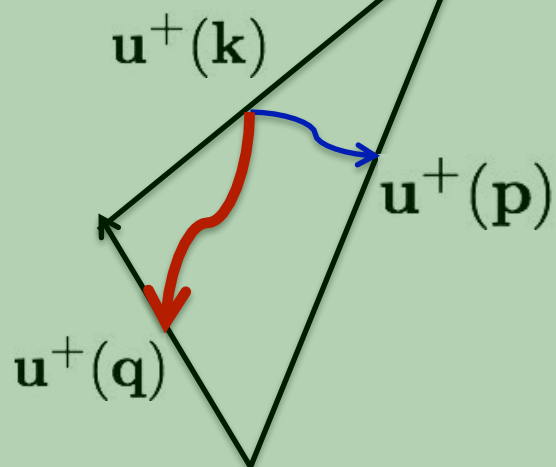
$$i\mathbf{k} \times \mathbf{h}^\pm = \pm k\mathbf{h}^\pm$$

$$\begin{cases} E = \sum_{\mathbf{k}} |u^+(\mathbf{k})|^2 + |u^-(\mathbf{k})|^2; \\ H = \sum_{\mathbf{k}} k(|u^+(\mathbf{k})|^2 - |u^-(\mathbf{k})|^2). \end{cases}$$

$$\frac{d}{dt} u^{\mathbf{s}_k}(\mathbf{k}) = \sum_{\mathbf{s}_p=\pm, \mathbf{s}_q=\pm} g_{\mathbf{s}_k, \mathbf{s}_p, \mathbf{s}_q} \sum_{\mathbf{p}+\mathbf{q}=\mathbf{k}} u^{\mathbf{s}_p}(\mathbf{p}) u^{\mathbf{s}_q}(\mathbf{q}) - \nu k^2 u^{\mathbf{s}_k}(\mathbf{k})$$

$$\frac{d}{dt}u^{sk}(\mathbf{k}) + \nu k^2 u^{sk}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_p p - s_q q) \times [u^{s_p}(\mathbf{p})u^{s_q}(\mathbf{q})]^*. \quad (15)$$

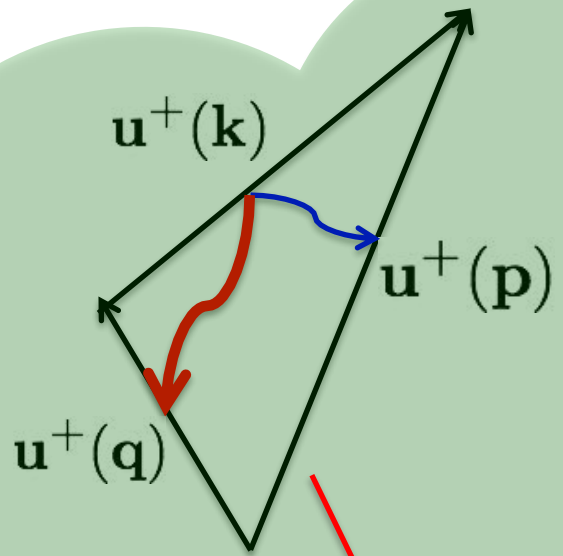
$$\begin{cases} E = \sum_{\mathbf{k}} |u^+(\mathbf{k})|^2 + |u^-(\mathbf{k})|^2; \\ H = \sum_{\mathbf{k}} k(|u^+(\mathbf{k})|^2 - |u^-(\mathbf{k})|^2). \end{cases}$$



$$\begin{cases} E = \sum_{\mathbf{k}} |u^+(\mathbf{k})|^2 + |u^-(\mathbf{k})|^2; \\ H = \sum_{\mathbf{k}} k(|u^+(\mathbf{k})|^2 - |u^-(\mathbf{k})|^2). \end{cases}$$

MILD SYMMETRY
BREAKING

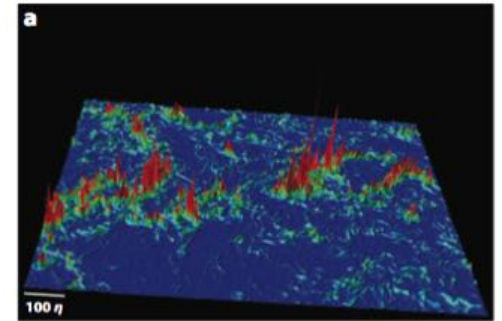
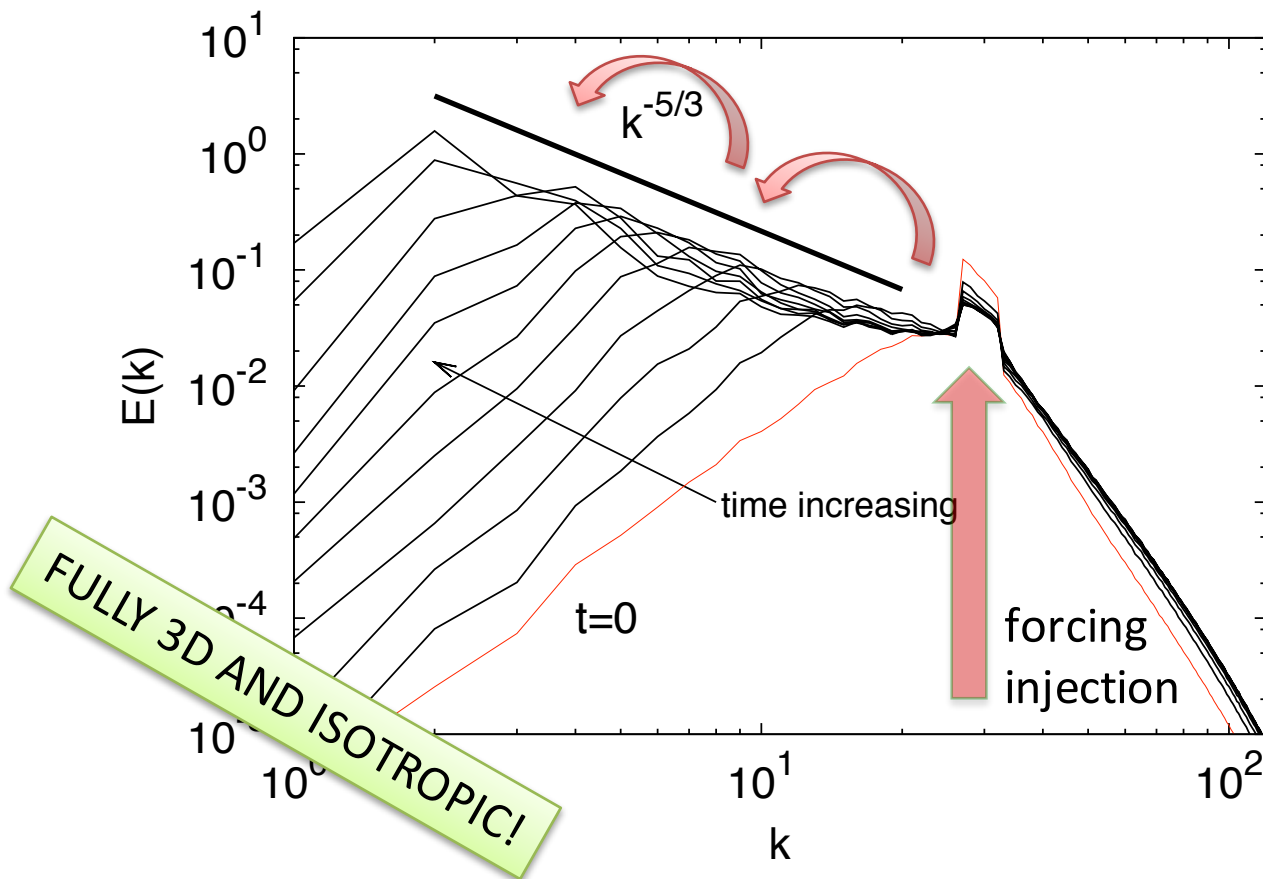
- HOMOGENEOUS OK
- ISOTROPY OK
- MIRROR SYMMETRY NO



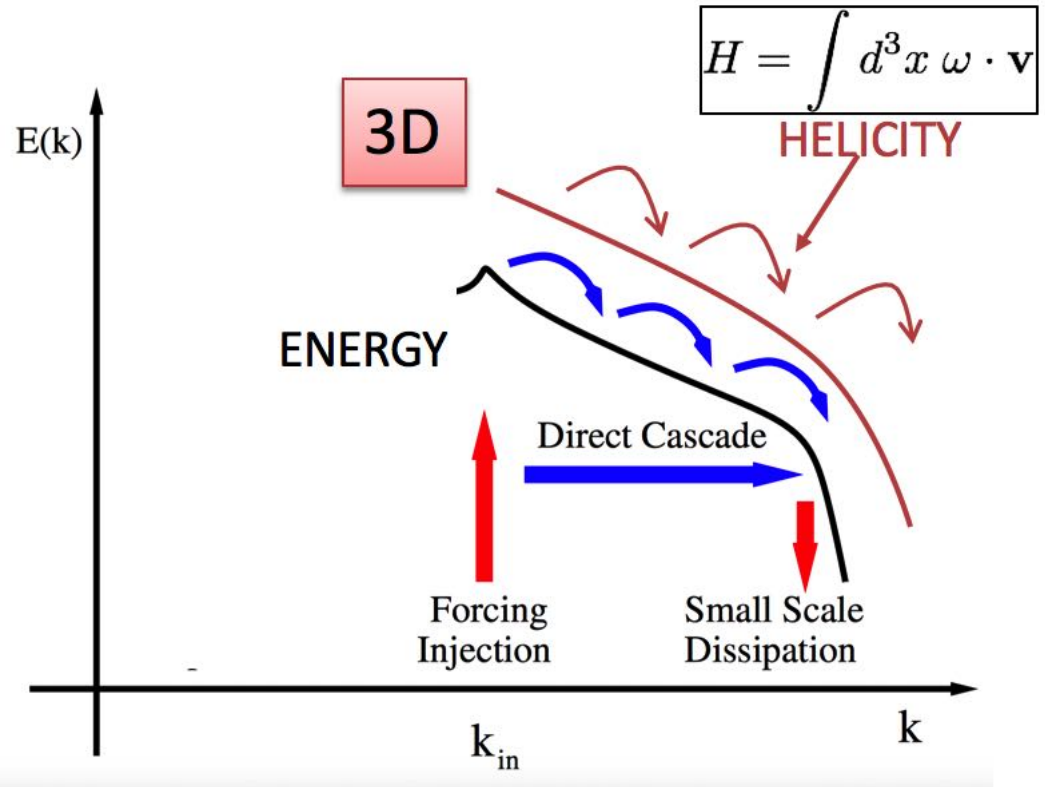
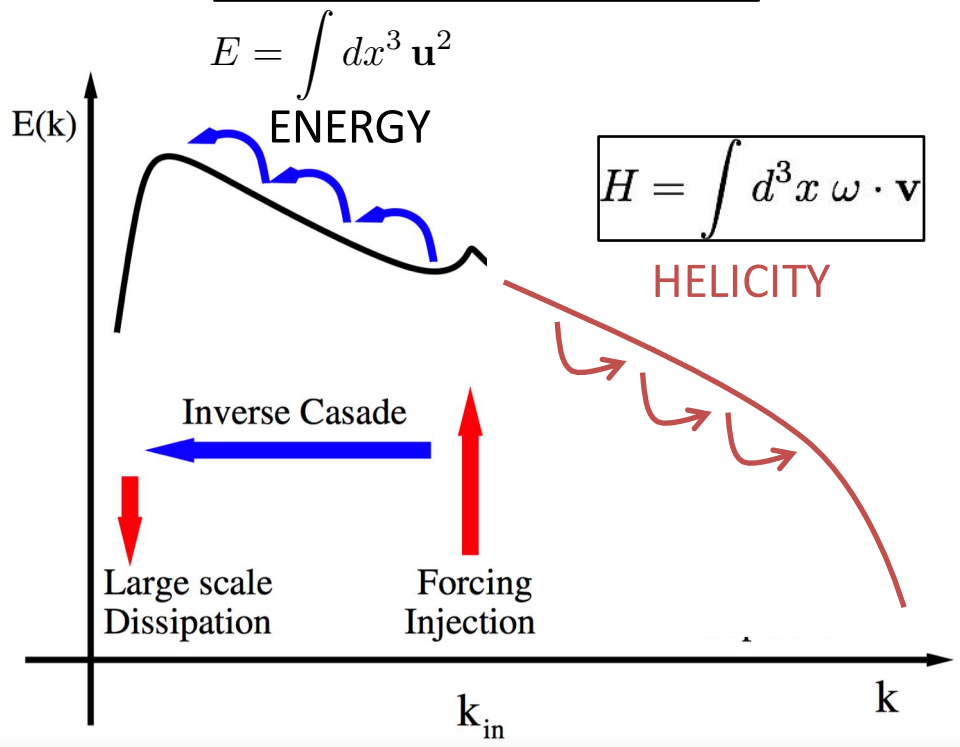
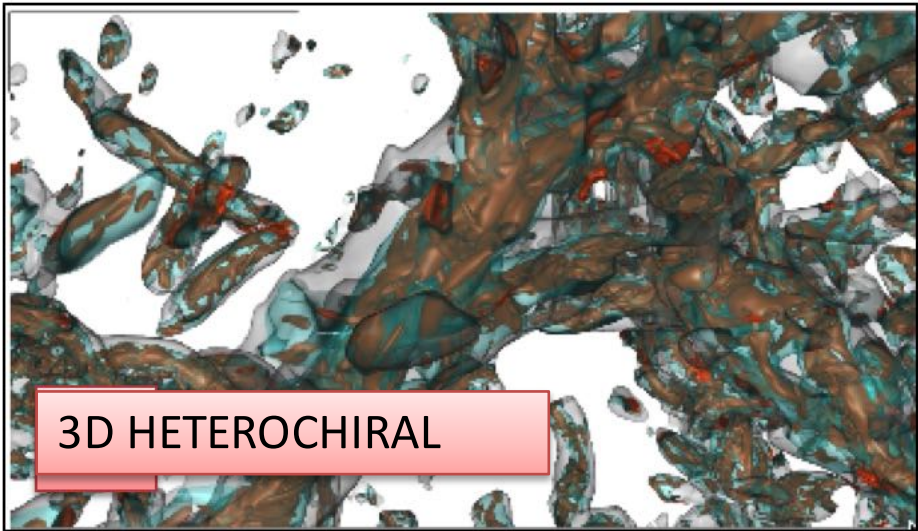
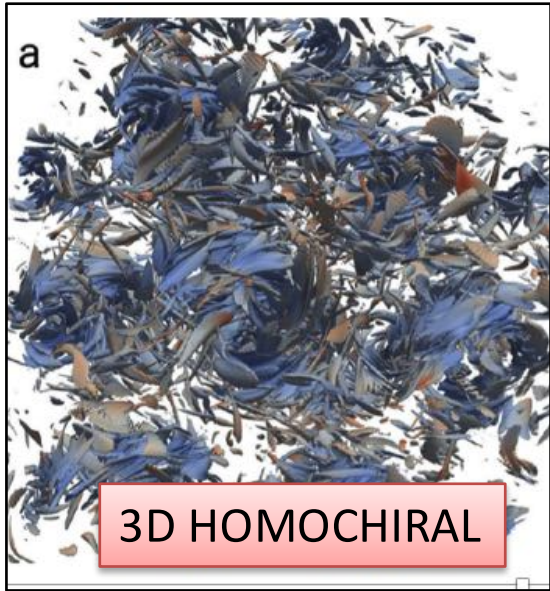
$$\begin{cases} \partial_t \mathbf{v}^+ = \mathcal{P}^+(-\mathbf{v}^+ \cdot \nabla \mathbf{v}^+ - \nabla p^+) + \nu \Delta \mathbf{v}^+ + \mathbf{f}^+ \\ \nabla \cdot \mathbf{v}^+ = 0 \end{cases}$$

INVERSE ENERGY FLUX: FROM SMALL TO LARGE SCALES in 3D!

$$\begin{cases} E = \sum_{\mathbf{k}} |u^+(\mathbf{k})|^2 + |u^-(\mathbf{k})|^2; \\ H = \sum_{\mathbf{k}} k(|u^+(\mathbf{k})|^2 - |u^-(\mathbf{k})|^2). \end{cases}$$

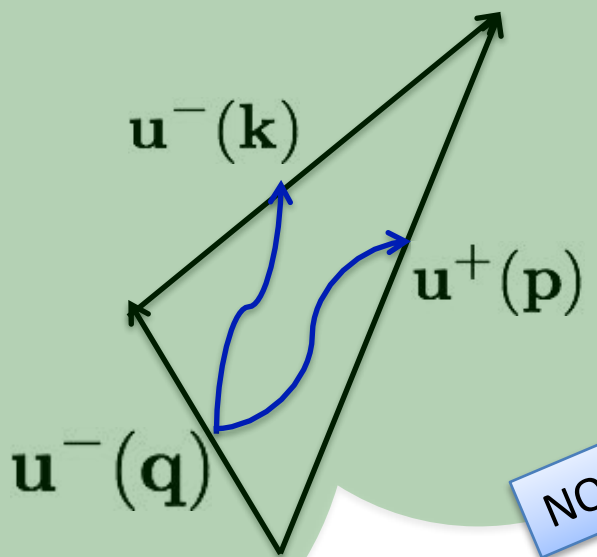
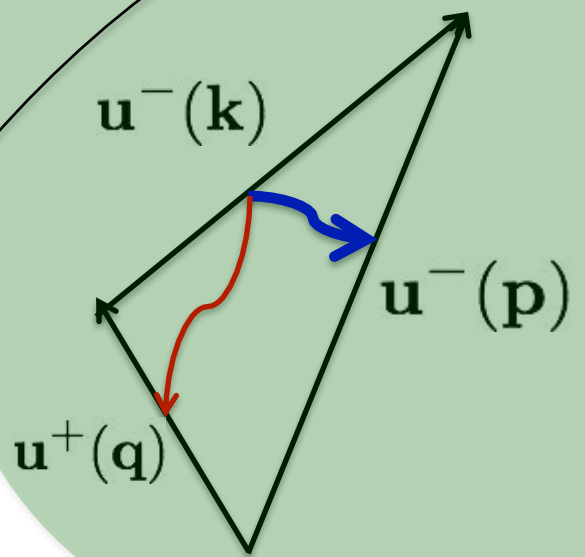
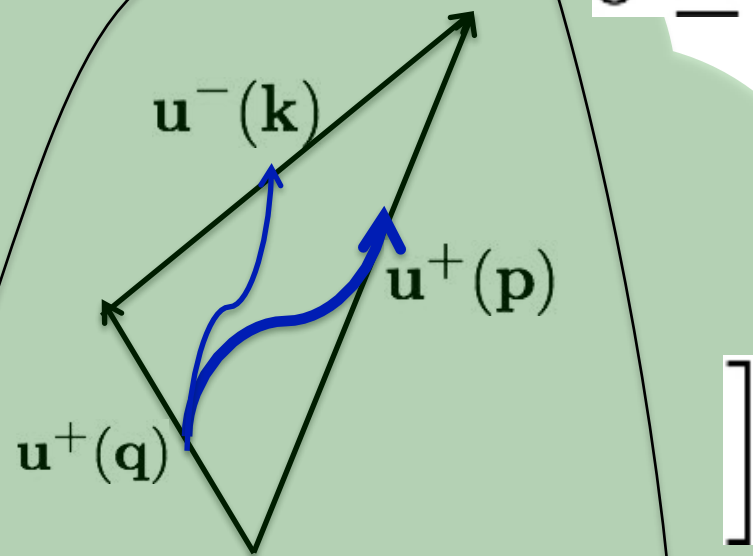
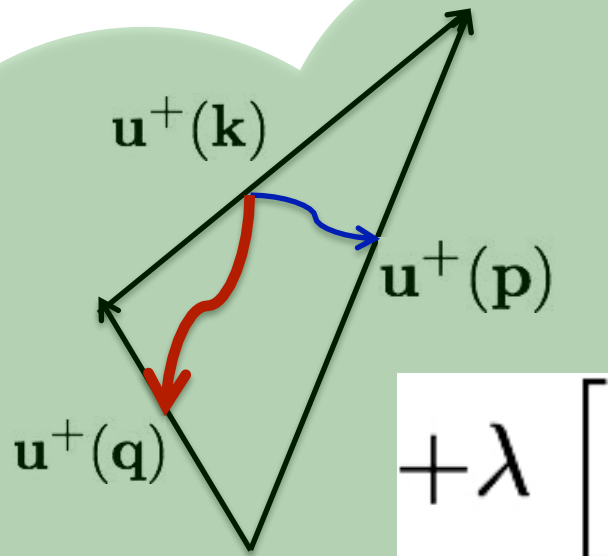


HOMOCHIRAL/HETEROCHIRAL NAVIER-STOKES 3D

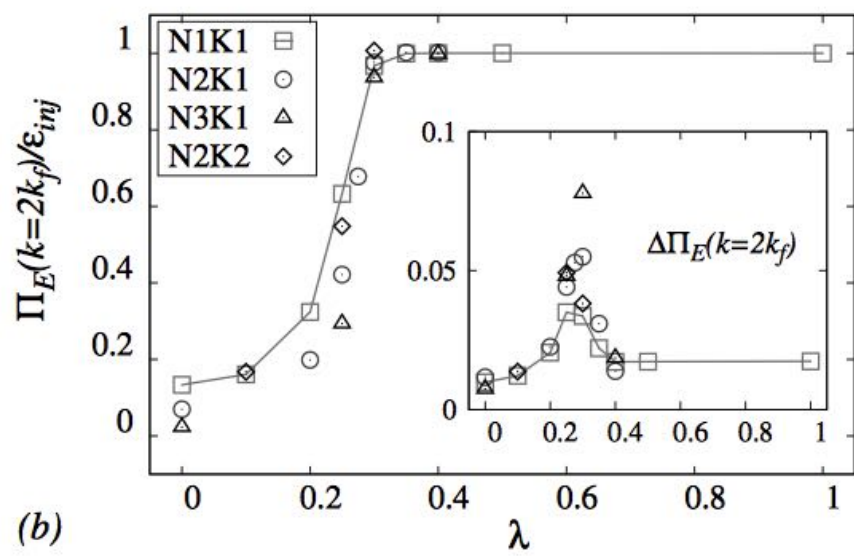
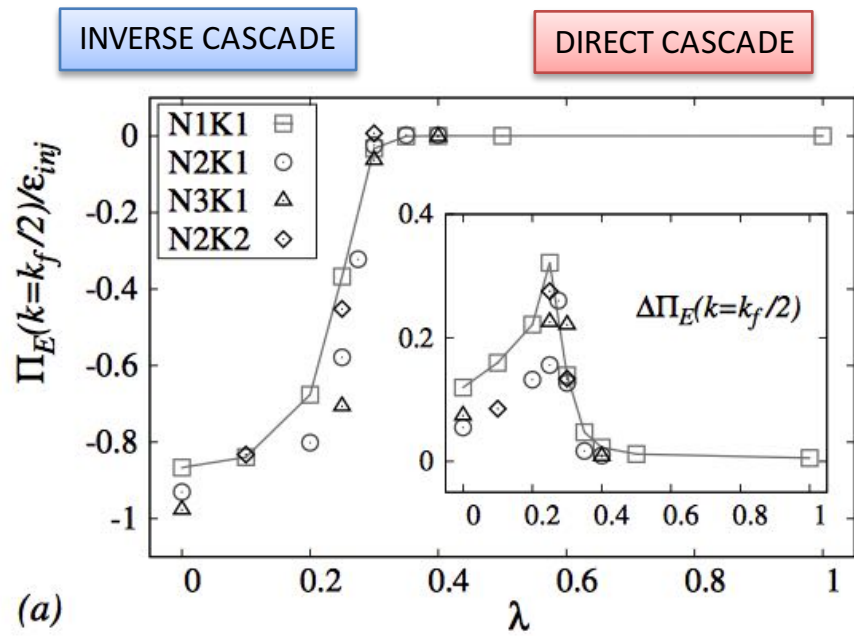
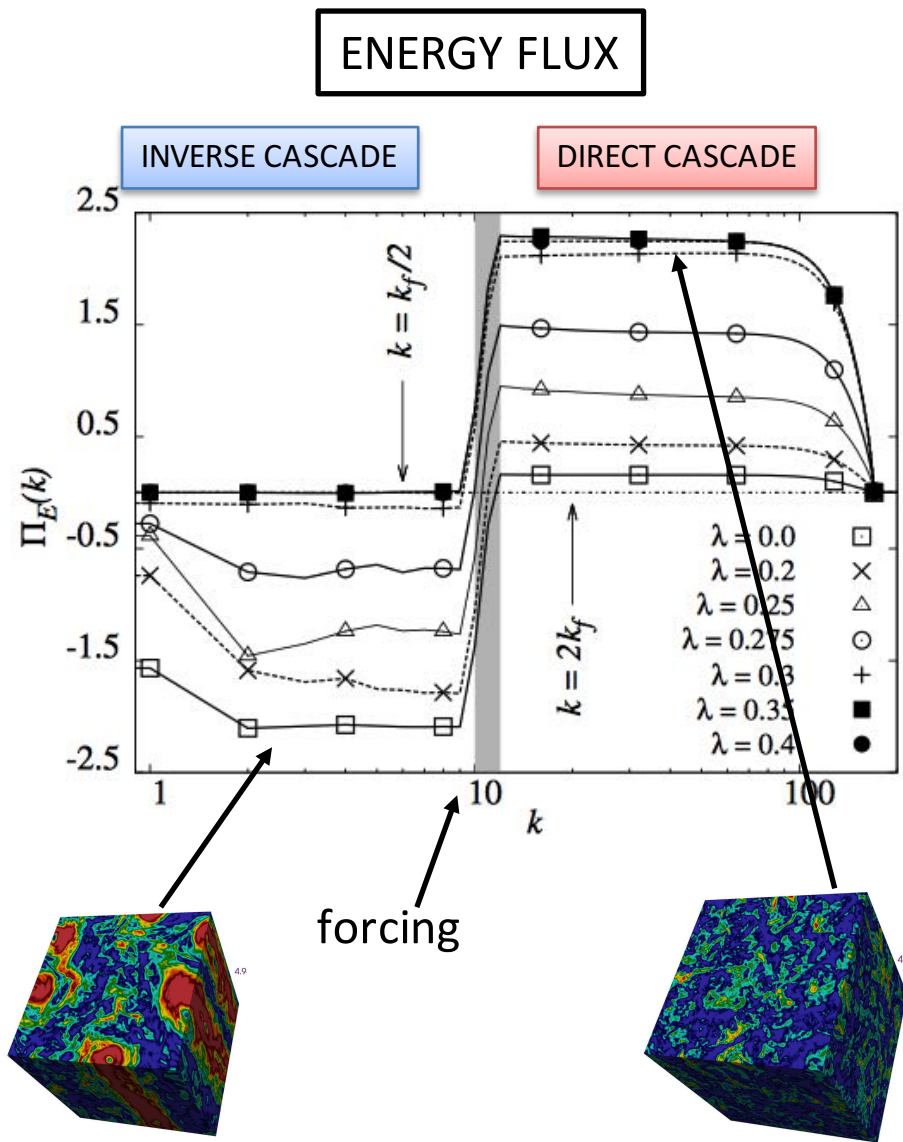


TRIADIC INTERACTION IN HOMOCHIRALE/HETEROCHIRAL NAVIER-STOKES EQS

$$0 \leq \lambda \leq 1$$



NO SYMMETRY BREAKING



Highly resolved Large Eddy Simulations

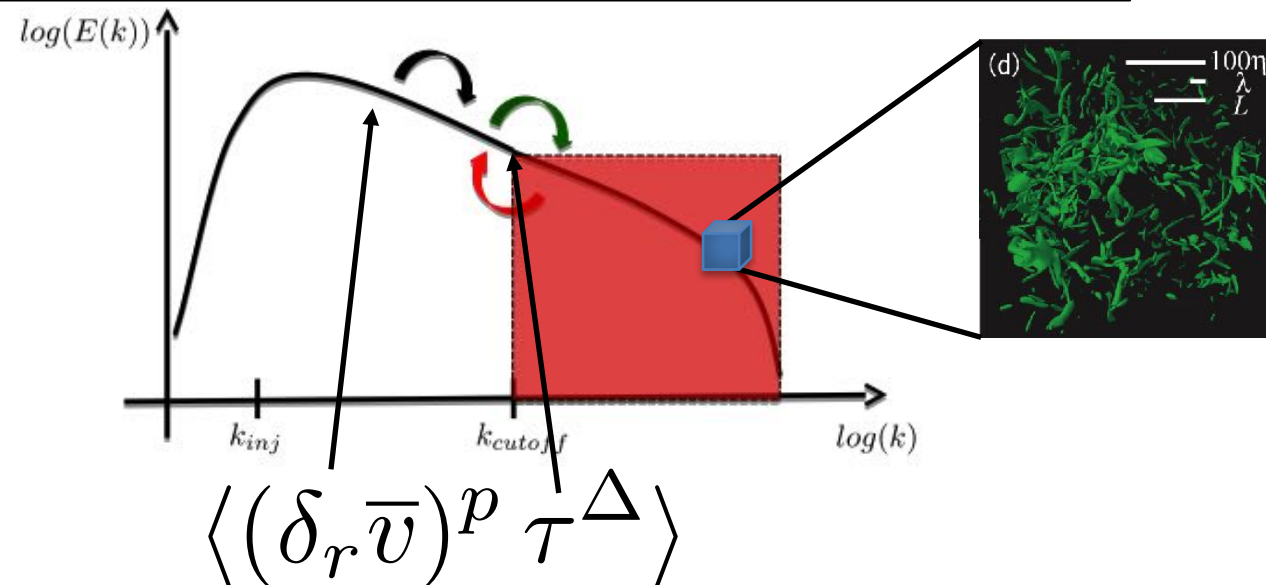
$$\partial_t \mathbf{v} + \nabla(\mathbf{v}\mathbf{v}) = -\nabla p + \nu \Delta \mathbf{v} \quad (\text{Navier Stokes eq.})$$

Filtered velocity field;

$$\bar{\mathbf{v}}(\mathbf{x}, t) \equiv \int_{\Omega} d\mathbf{y} G(|\mathbf{x} - \mathbf{y}|) \mathbf{v}(\mathbf{y}, t) = \sum_{\mathbf{k} \in \mathbb{Z}^3} G(\mathbf{k}) \hat{\mathbf{v}}(\mathbf{k}, t) e^{i\mathbf{k}\mathbf{x}}$$

$$\partial_t \bar{\mathbf{v}} + \nabla \cdot (\bar{\mathbf{v}} \otimes \bar{\mathbf{v}}) = -\nabla \bar{p} + \nabla \cdot \tau^{\Delta}(\mathbf{v}, \mathbf{v}) + \nu \Delta \bar{\mathbf{v}}$$

$$\tau_{ij}^{\Delta}(\mathbf{v}, \mathbf{v}) = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \xrightarrow{\text{LES}} \tau_{i,j}^{\Delta}(\bar{\mathbf{v}}, \bar{\mathbf{v}})$$



EFFECTS OF THE SGS-MODEL ON THE PHYSICS OF THE INERTIAL RANGE?

TAKE HOME MESSAGES

- TURBULENCE IS AT THE CORE OF MANY APPLICATIONS CROSSING HUGE RANGE OF SPATIAL AND TEMPORAL SCALES AND A HUGE SET OF SCIENTIFIC DISCIPLINES AND FIELDS.
- HOMOGENEOUS AND ISOTROPIC TURBULENCE IS THE 'UNSOLVED' HYDROGEN ATOM OF TURBULENCE.
- TURBULENCE HAS A LONG HISTORY BEHIND IT AND A LONG FUTURE AHEAD. IT HAS BEEN OBSERVED SINCE 500 YEARS AGO. WE KNOW THE EQUATIONS SINCE 250 YEARS AGO, WE STARTED TO PERFORM SYSTEMATIC EXPERIMENTS SINCE THE EARLY '900, WE HAVE A BASIC PHENOMENOLOGICAL SET-UP SINCE THE 1940, WE HAVE STARTED TO PERFORM NUMERICAL SIMULATIONS SINCE 30 YEARS AGO, BUT STILL...
- WE DO NOT HAVE THE COMPUTATIONAL TOOLS TO STUDY NUMERICALLY REALISTIC TURBULENT FLOWS (REYNOLDS NUMBER LARGE ENOUGH)
- EXPERIMENTS CAN ACCESS ONLY PARTIAL INFORMATION ABOUT THE FLOW CONFIGURATION AND FOLLOW ONLY A LIMITED SET OF EVOLVING OBJECTS.
- WE DO NOT KNOW WHAT ARE THE DYNAMICAL ORIGINS OF NON-GAUSSIAN INTENSE FLUCTUATIONS (PUT ASIDE THE POSSIBILITY TO PREDICT THEM FROM THEORY)
- WE DO NOT KNOW HOW TO MODEL THESE FLUCTUATIONS TO CONTROL/DESCRIBE THE EVOLUTION OF SMALL OBJECTS ADVECTED BY THE FLOW (LAGRANGIAN DYNAMICS)
- WE DO NOT PREDICT/CONTROL THE TRANSITION BETWEEN QUASI DIRECT AND INVERSE DYNAMICS
- ALL FLOWS POSSESS INTERACTIONS THAT ARE ABLE TO TRANSFER ENERGY FORWARD (HETEROCHIRAL) OR BACKWARD (HOMOCHIRAL)

Playing with mirror symmetry...



Piero della Francesca "Madonna del Parto". Monterchi

credits:

M. Buzzicotti, G. Sahoo, M. Linkmann, K. Gustafsson, M. De Pietro, F. Bonaccorso, R. Scatamacchia (ERC NewTURB)
S. Colabrese, G. Margazouglou, F. Milan, G. Tauzin (PhD, EJD HPC-LEAP)

F. Toschi (TuE, Eindhoven), A. S. Lanotte (CNR, Lecce), M. Cencini (CNR, Rome), A. Alexakis (ENS, Paris), A. Celani (ICTP)
R. Benzi, M. Sbragaglia (Tor Vergata, Rome)



ON THE SHOULDERS OF GIANTS...

Leonardo da Vinci (~ 1500): “doue la turbolenza de si genera [injected]; doue la turbolenza dell aqua si mantiene [advected] plugho; doue la turbolenza dell aqua si posa [dissipated]”

Sir H. Lamb (1932): “I am an old man now, and when I die and go to Heaven there are two matters on which I hope enlightenment. One is **quantum electrodynamics** (QED) and the other is **turbulence** of fluids. About the former, I am really rather optimistic.”

J. Von Neumann (1949) “[...] The entire experience with the subject indicates that the purely analytical approach is beset with difficulties, which at the moment are prohibitive. [...] Under these conditions there may be some hope to “break the deadlock” by extensive, but well-planned computational efforts.

R.P. Feynman (1970): “Certainly. I’ve spent years trying to solve some difficult problems without success. The theory of turbulence is one. In fact, it is still unsolved.”

$$\partial_t v + v \partial v = -\partial p + \nu \Delta v$$

$$+F(B, B) + g\theta + \sum_i c_0(u_i, v) \delta(r - r_i) + f$$

$$\partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta$$

$$\partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B$$

$$\partial \cdot v = 0$$

+ boundary conditions

$$\frac{du_i(r_i, t)}{dt} = -\rho_f |u_i - v| (u_i - v)$$

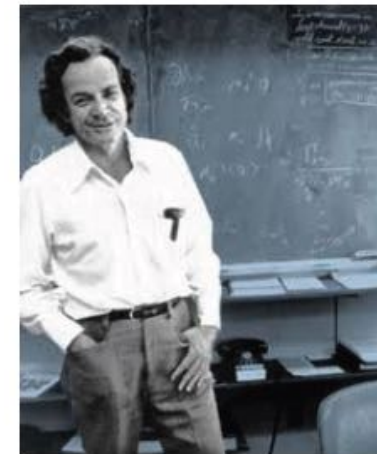
$$+\rho_f \left(\frac{Dv}{Dt} - \frac{Du_i}{Dt} \right) + (u_i - v) \times \omega$$

control parameter:

$$Re = \frac{l_0 v_0}{\nu}$$

$$Re \rightarrow \infty$$

FULLY NON-LINEAR



“With turbulence, it's not just a case of physical theory being able to handle only simple cases—we can't do any. We have no good fundamental theory at all.” (Feynman, 1979, Omni Magazine, Vol. 1, No.8).

ABOUT THE PHYSICS OF 1 CUBIC CENTIMETER OF WATER, WE CAN SAFELY SAY THAT WE CONTROL MUCH BETTER WHAT ARE THE FUNDAMENTAL INTERACTIONS AMONG ITS SUBNUCLEAR CONSTITUENTS THEN ITS **HYDRODYNAMICAL** (AND **MOLECULAR**) PROPERTIES (U. FRISCH, PHYSICS TODAY 2001)



$$\partial_t v + v \partial v = -\partial p + \nu \Delta v$$

$$\partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta$$

$$\partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B$$

$$\partial \cdot v = 0$$

+ boundary conditions

$$\frac{du_i(r_i, t)}{dt} = -\rho_f |u_i - v| (u_i - v)$$

$$+\rho_f \left(\frac{Dv}{Dt} - \frac{Du_i}{Dt} \right) + (u_i - v) \times \omega$$

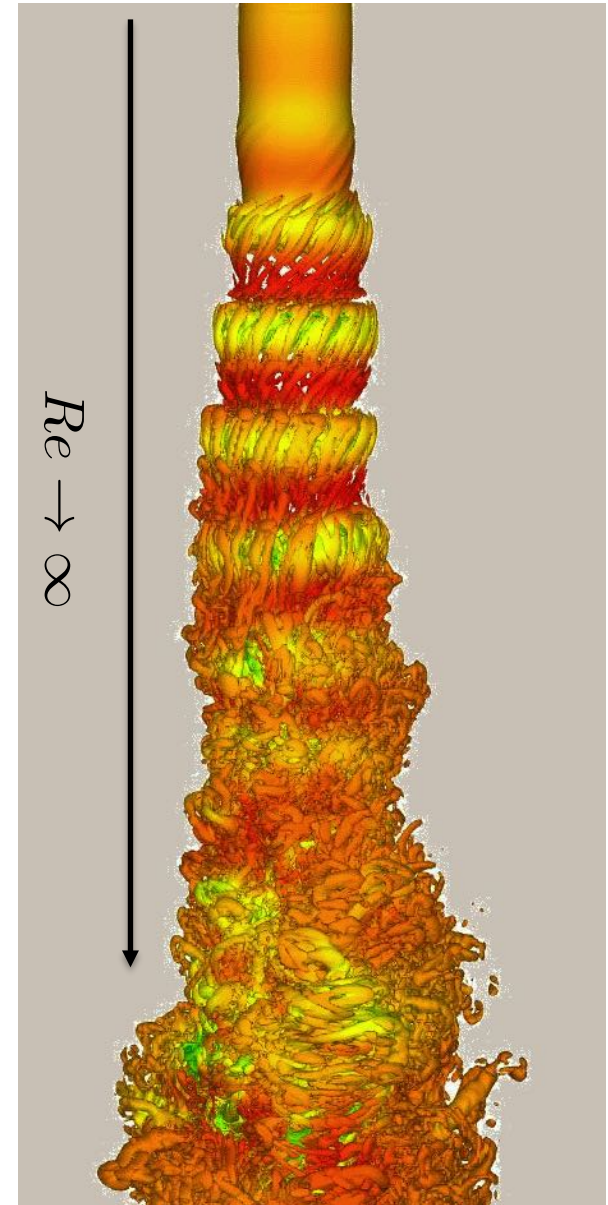
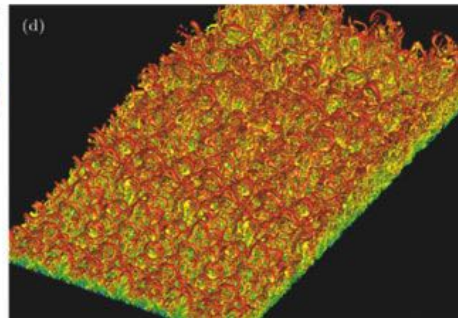
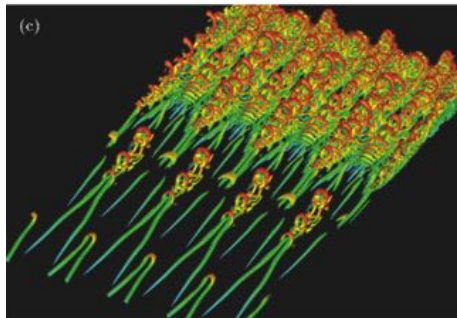
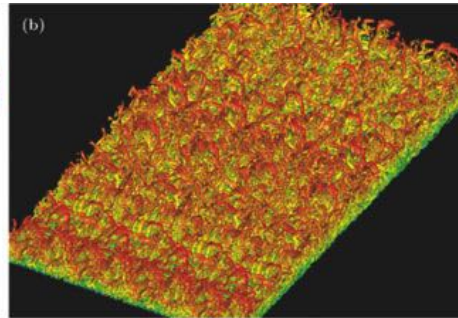
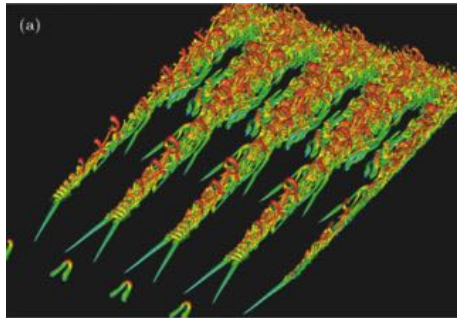
$$+F(B, B) + g\theta + \sum_i c_0(u_i, v) \delta(r - r_i) + f$$

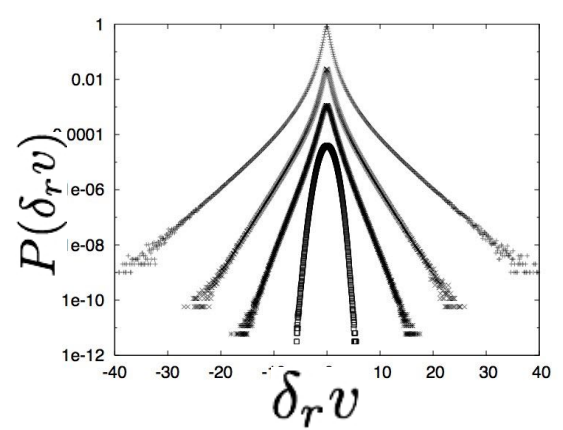
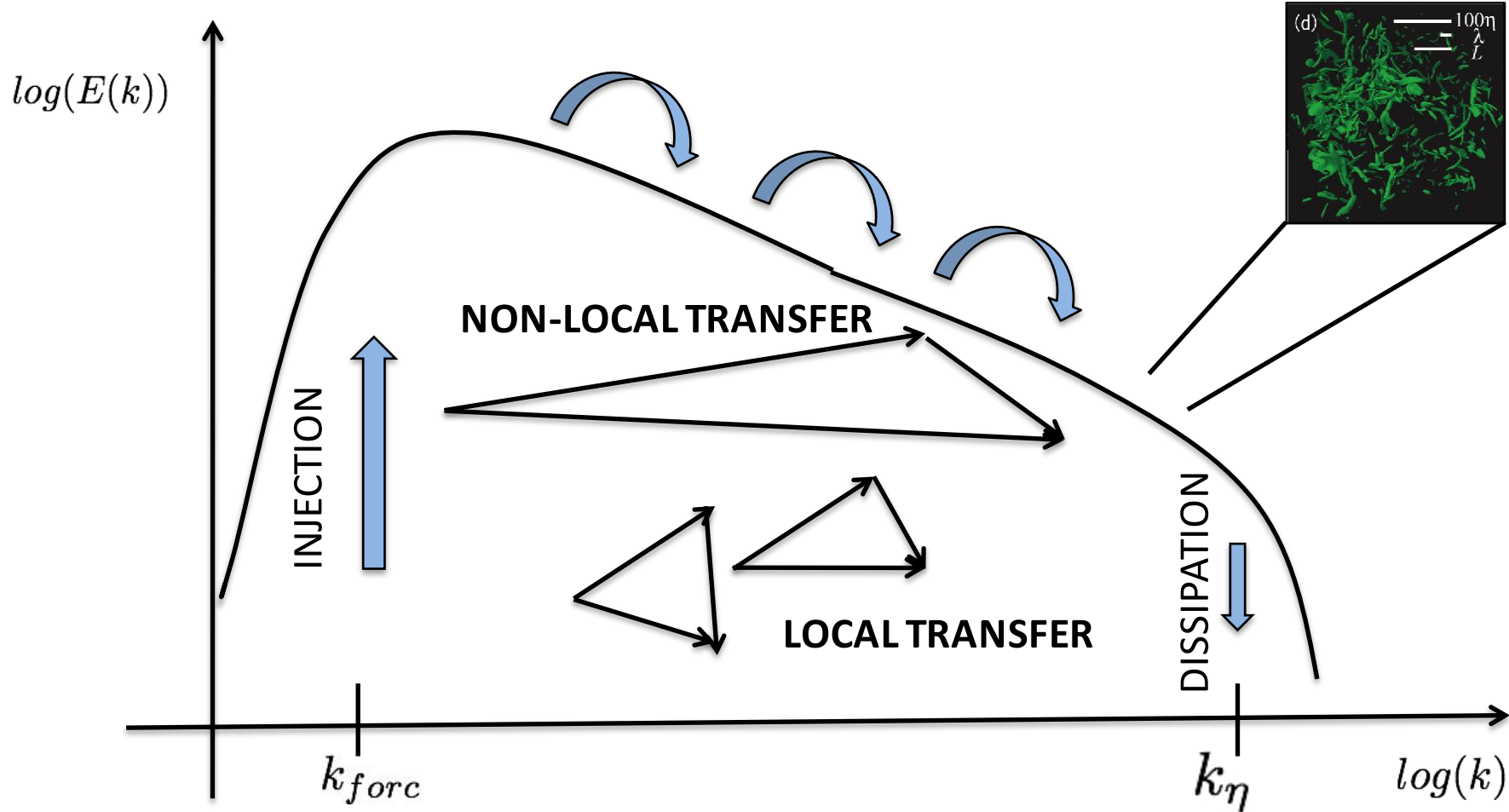
control parameter:

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$$Re \rightarrow \infty$$

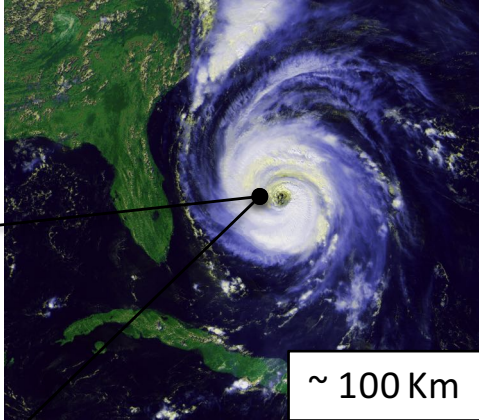
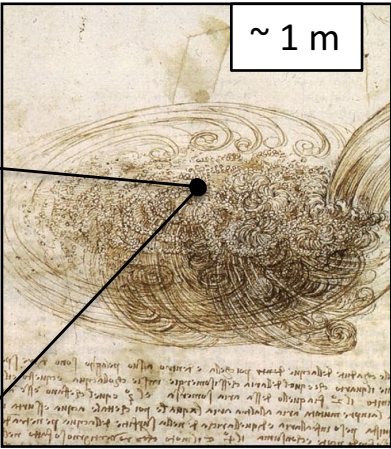
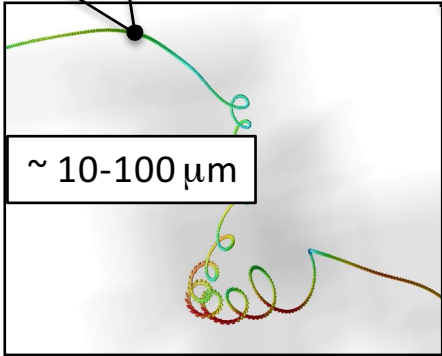
FULLY NON-LINEAR





ENERGY TRANSFER AND ENERGY DISSIPATION IN TURBULENT FLOWS

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MULTISCALE !

