Inverse and direct cascades in rotating turbulence

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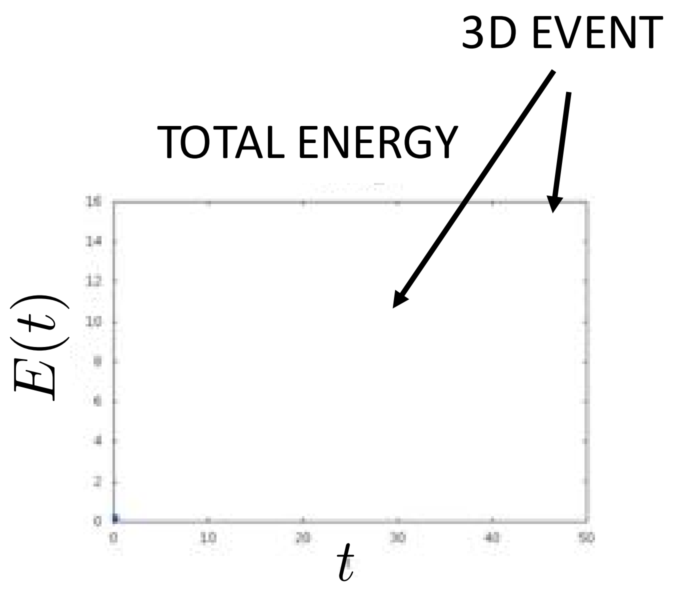
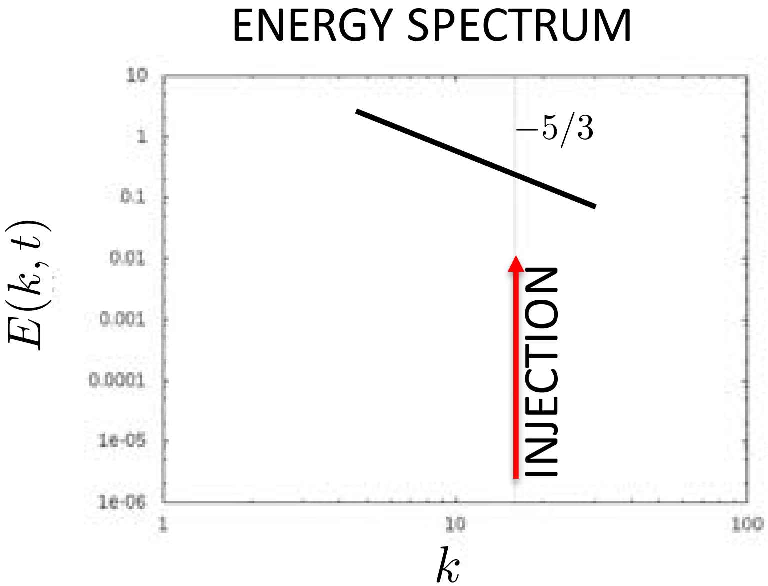
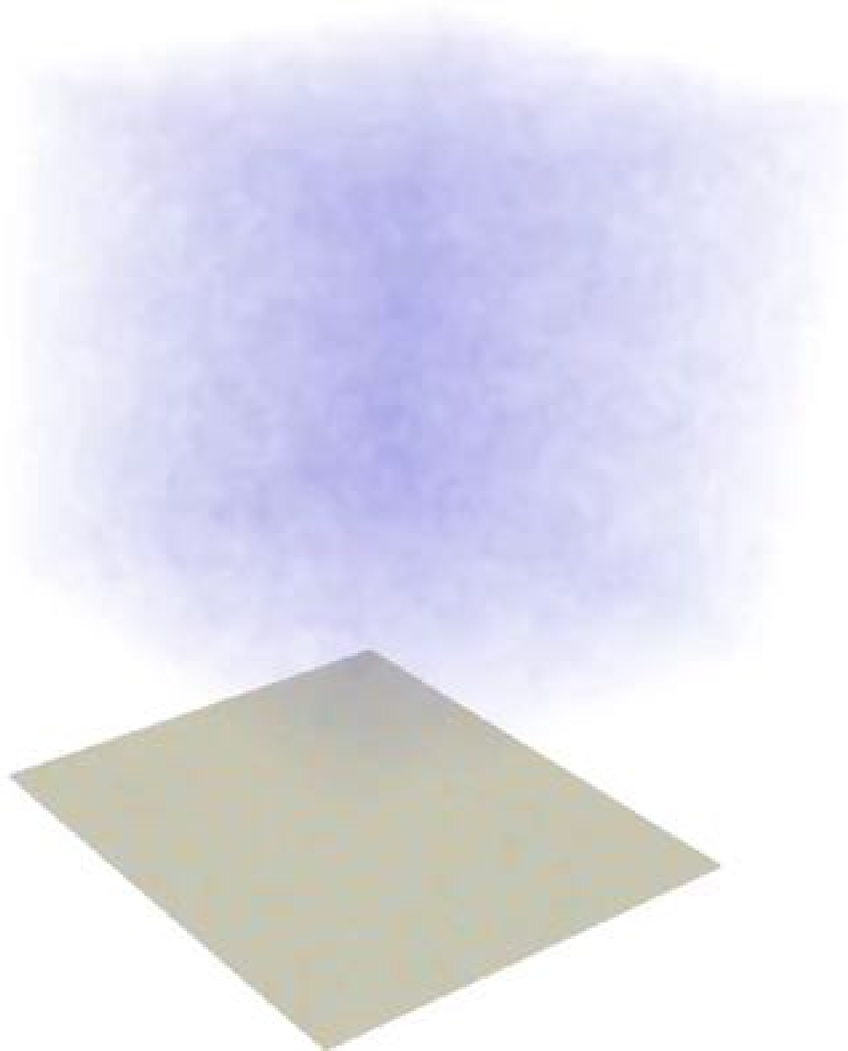
PARIS 2018
WAVE INTERACTIONS AND TURBULENCE

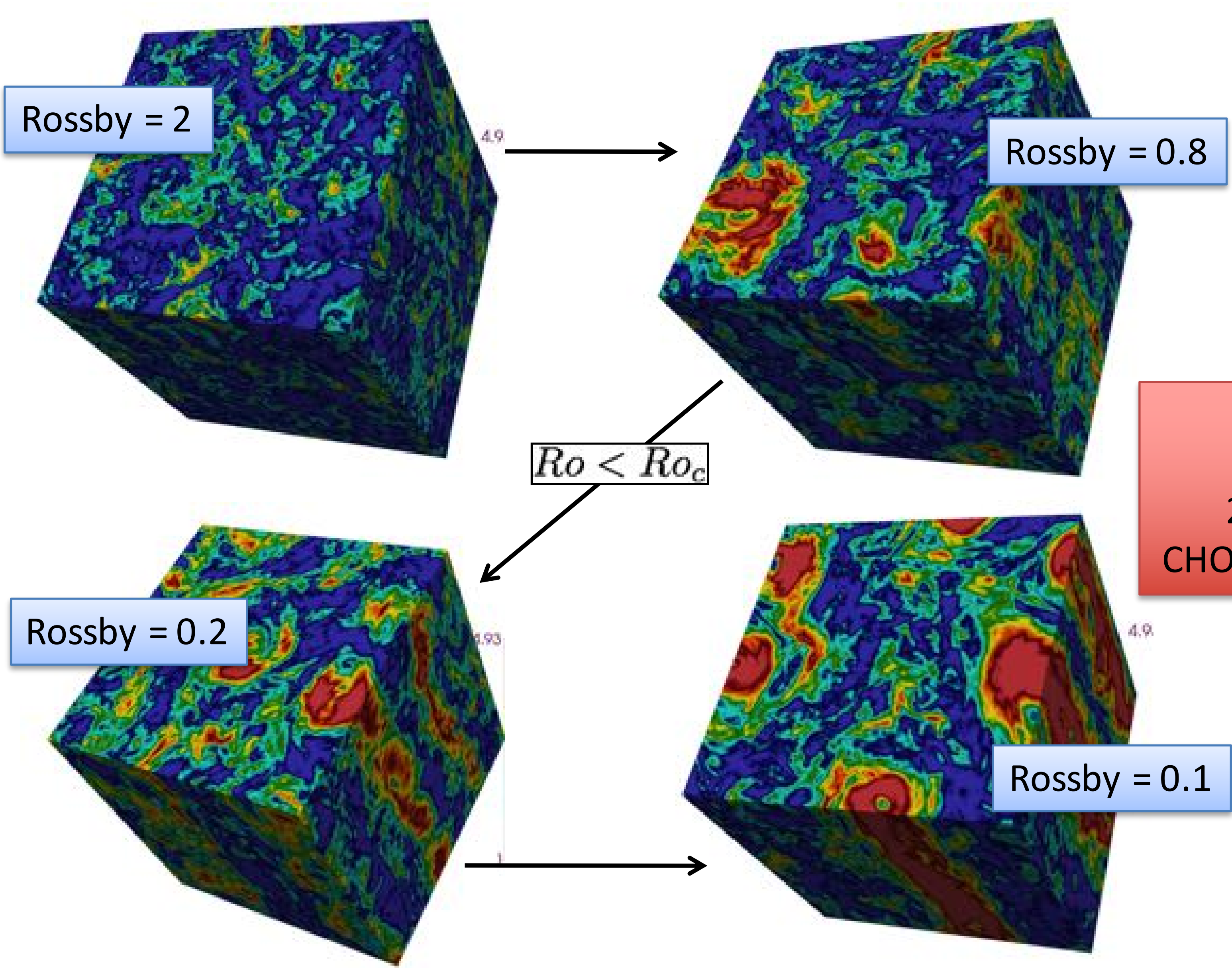
Credits [in order of appearance]: **F. Bonaccorso, M. Buzzicotti** (Univ. of Roma 'Tor Vergata', Italy), **M. Linkmann** (Univ. of Marburg, Germany); **A. Alexakis** (ENS-Paris, France); **P. Clark di Leoni** (Univ. of Roma 'Tor Vergata', Italy),

TURBULENCE UNDER ROTATION









HOMOGENEOUS
ANISOTROPIC
2D & 3D PHYSICS
CHOERENT -STRUCTURES

H.P. Greenspan *The Theory of Rotating Fluids* (Cambridge Univ. Press 1968); Clark di Leoni and P.D. Minnini JFM, 809, 821 (2016); L.M. Smith and Waleffe PoF 11, 1608 (1999); S. Galtier PRE 68, 015301 (2003). S. Nazarenko *Wave Turbulence* (Springer 2011); B. Gallet JFM 783, 412 (2015), L.B. F. Bonaccorso et al PRX 6, 041036 (2016)

(Fast-3D) Inertial Waves

$$\partial_t (\nabla \times \boldsymbol{u}) = 2\,(\boldsymbol{\Omega} \cdot \nabla)\,\boldsymbol{u}$$

This equation is satisfied by plane waves of the form:

$$\boldsymbol{u}(\boldsymbol{x},t) = \sum_{\boldsymbol{k},\pm} \hat{a}_\pm(\boldsymbol{k},t)\mathbf{h}_\pm(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}}e^{i\omega_\pm(\boldsymbol{k})t}$$

eigenvector of the curl operator:

$$i\boldsymbol{k} \times \boldsymbol{h}_\pm = \pm k \boldsymbol{h}_\pm$$

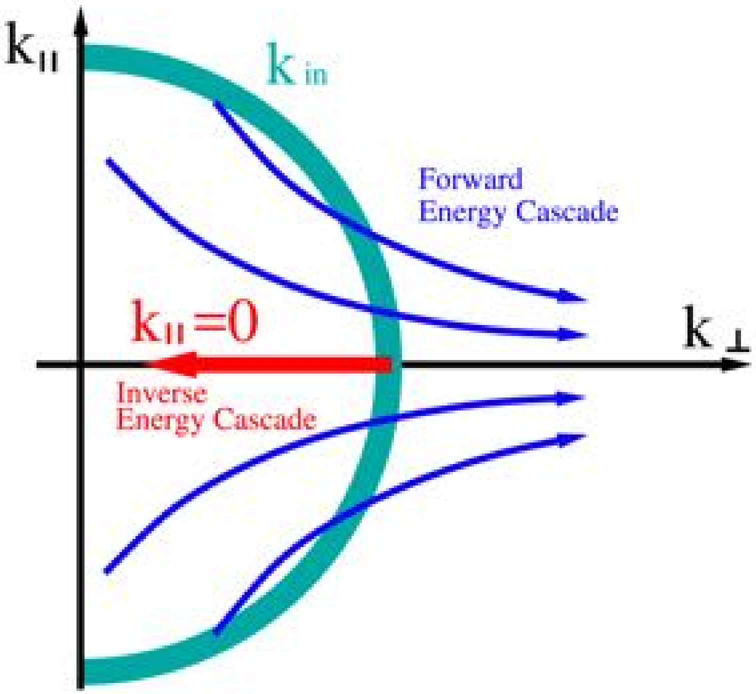
dispersion relation:

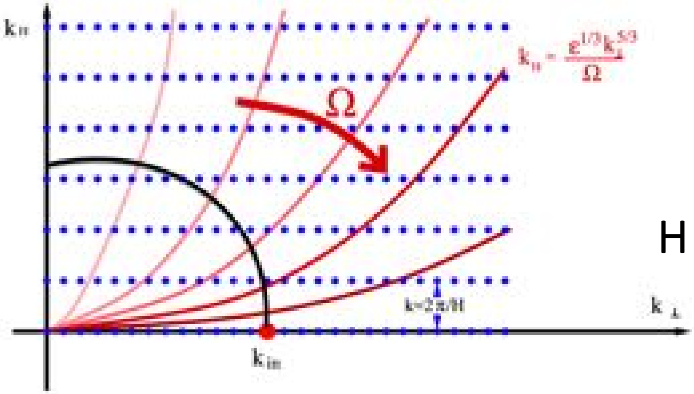
$$\omega_\pm(\boldsymbol{k}) = \pm 2\Omega \frac{k_z}{|\boldsymbol{k}|}$$

Inertial waves are intrinsically helical

A rotating flow = 2D3C $(k_z = 0, \omega_\pm = 0)$ + 3D helical $(k_z \neq 0, \omega_\pm \neq 0)$

A rotating flow =	2D3C $(k_z = 0, \omega_\pm = 0)$ Slow-2D manifold "geostrophic" flow	+ 3D helical $(k_z \neq 0, \omega_\pm \neq 0)$ Fast-3D manifold "non-geostrophic" flow





INFINITE VOLUME LIMIT

C. Cambon et al. J. Fluid Mech., 337:303332, 1997. A.Sen et al Jour Atmos Science 68, 2757 (2011), E. Yarom et al PoF 25, 085105 (2013), A. Campagne et al PoF 26, 125112 (2014), E. Deusebio et al PRE 90, 023005 (2014), A. Alexakis JFM 769, 46 (2015)

HELICAL-FOURIER DECOMPOSITION

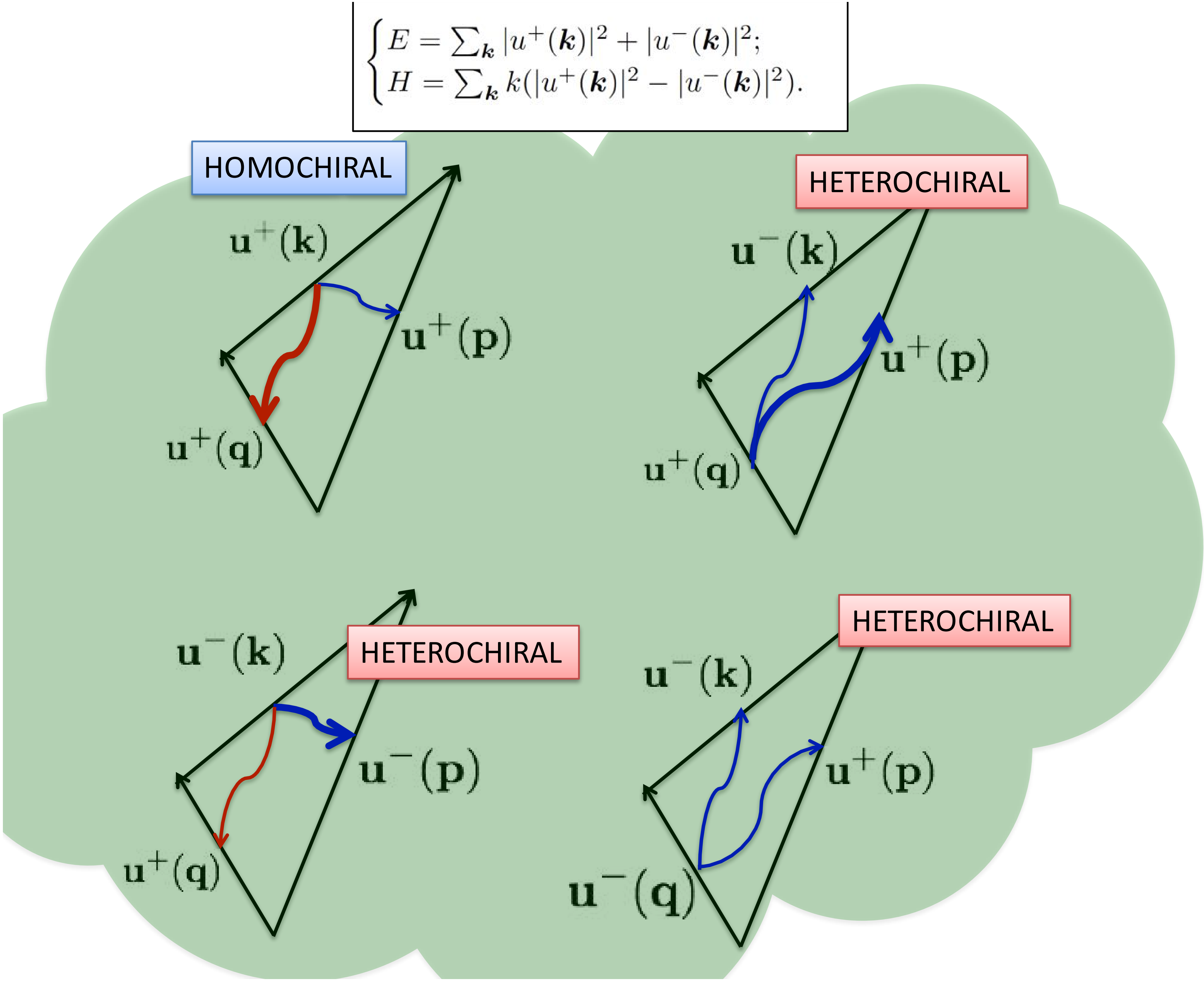
P. Contantin and A. Majda Comm Mat Phys 115, 435 (1988)
F. Waleffe Phys Fluids A 4, 350 (1992)

$$\boldsymbol{u}(\boldsymbol{k}) = u^{+}(\boldsymbol{k})\boldsymbol{h}^{+}(\boldsymbol{k}) + u^{-}(\boldsymbol{k})\boldsymbol{h}^{-}(\boldsymbol{k})$$

$$i\boldsymbol{k} \times \boldsymbol{h}^{\pm} = \pm k\boldsymbol{h}^{\pm}$$

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_p p - s_q q) \times [u^{s_p}(\mathbf{p}) u^{s_q}(\mathbf{q})]^*.$$

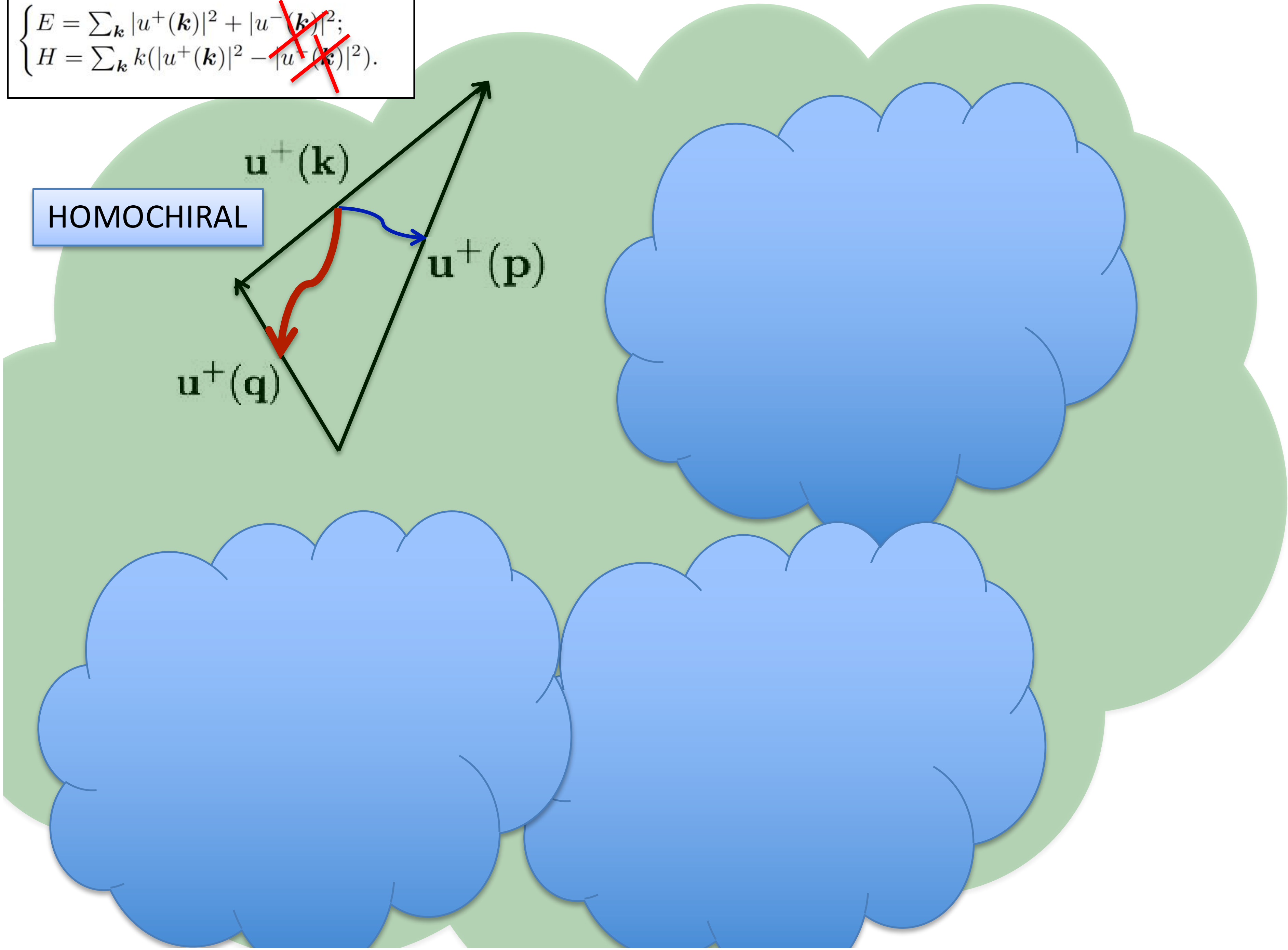
$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^{+}(\boldsymbol{k})|^2 + |u^{-}(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^{+}(\boldsymbol{k})|^2 - |u^{-}(\boldsymbol{k})|^2). \end{cases}$$



$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^{+}(\boldsymbol{k})|^2 + |u^{-}(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^{+}(\boldsymbol{k})|^2 - |u^{-}(\boldsymbol{k})|^2). \end{cases}$$
HOMOCHIRAL
$\mathbf{u}^{+}(\mathbf{k})$
$\mathbf{u}^{+}(\mathbf{p})$
$\mathbf{u}^{+}(\mathbf{q})$
HETEROCHIRAL
$\mathbf{u}^{-}(\mathbf{k})$
$\mathbf{u}^{+}(\mathbf{p})$
$\mathbf{u}^{+}(\mathbf{q})$
HETEROCHIRAL
$\mathbf{u}^{-}(\mathbf{k})$
$\mathbf{u}^{-}(\mathbf{p})$
$\mathbf{u}^{+}(\mathbf{q})$
HETEROCHIRAL
$\mathbf{u}^{-}(\mathbf{k})$
$\mathbf{u}^{+}(\mathbf{p})$
$\mathbf{u}^{-}(\mathbf{q})$

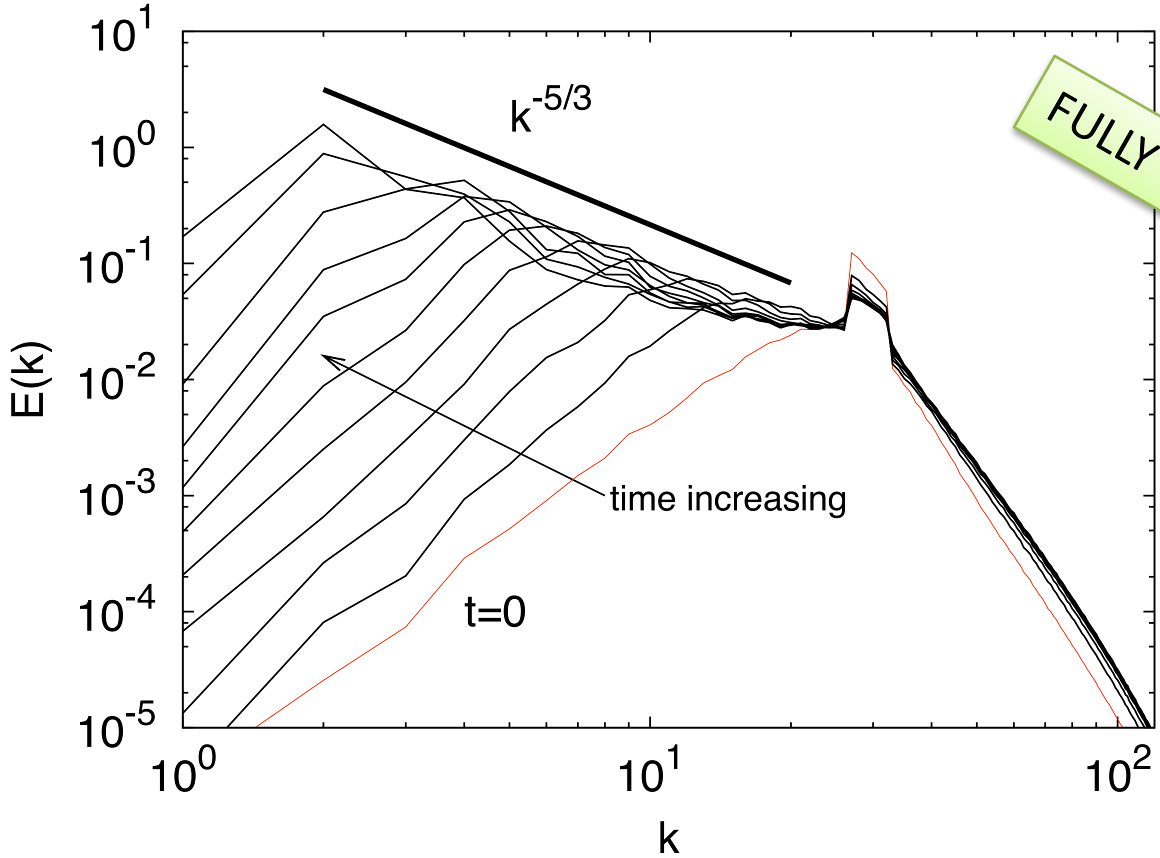
TRIADIC INTERACTION IN DECIMATED HOMOCHIRAL NAVIER_STOKES EQS

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + \cancel{|u^-(\boldsymbol{k})|^2}; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - \cancel{|u^-(\boldsymbol{k})|^2}). \end{cases}$$

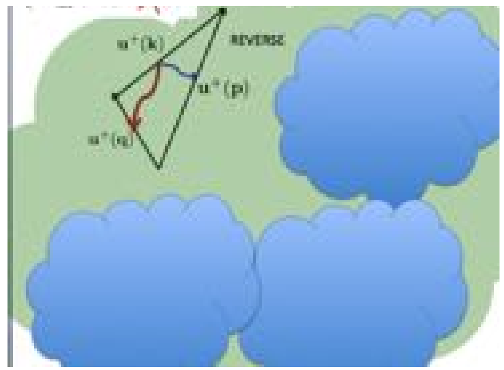


HOMOCHIRAL 3D NAVIER STOKES EQS.

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + \cancel{|u^-(\boldsymbol{k})|^2}; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - \cancel{|u^-(\boldsymbol{k})|^2}). \end{cases}$$



FULLY 3D AND ISOTORPIC

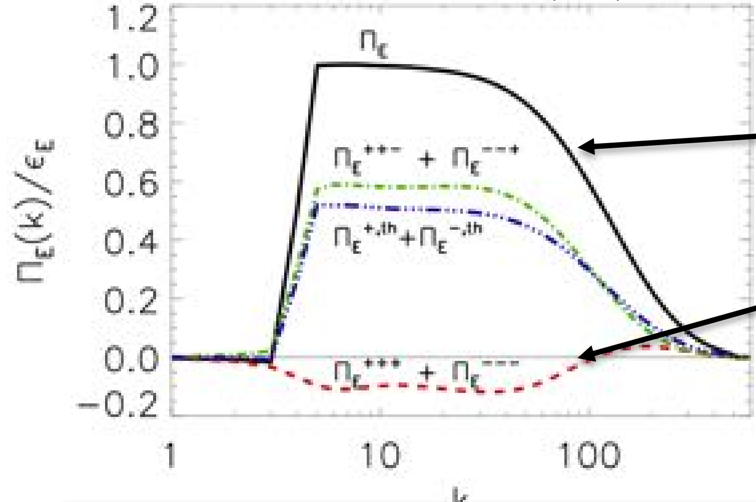


L.B., S. Musacchio & F. Toschi Phys. Rev. Lett. 108 164501, (2012); JFM 730, 309 (2013)

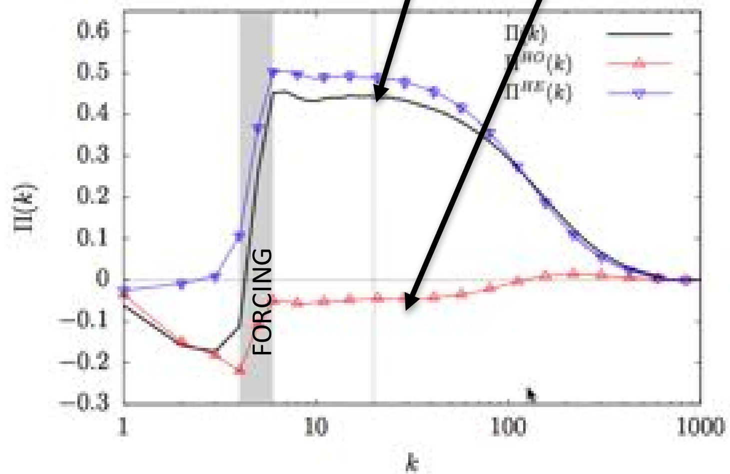
FULL 3D NAVIER STOKES EQS.

$$\partial_t \mathbf{u}^{s_1} = \mathcal{P}^{s_1} \sum_{s_2,s_3} [\mathbf{u}^{s_2} \times \omega^{s_3}] + \nu\Delta\mathbf{u}^{s_1} - \alpha\mathbf{u}^{s_1} + \mathbf{f}^{s_1}.$$

A. Alexakis JFM 812, 752 (2017)



FULL 3D NAVIER STOKES EQS. + ROTATION:

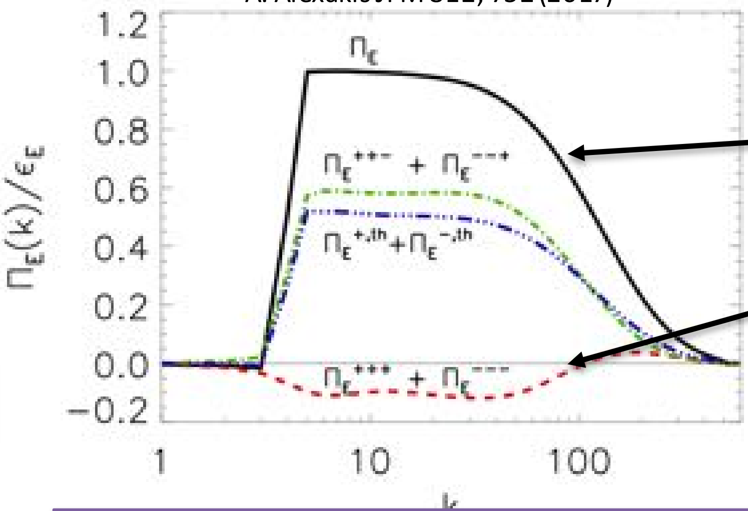


M. Buzzicotti, H. Aluie, L. B., M. Linkmann Phys. Rev. Fluids 3 (3) (2018) 034802

FULL 3D NAVIER STOKES EQS.

$$\partial_t \mathbf{u}^{s_1} = \mathcal{P}^{s_1} \sum_{s_2,s_3} [\mathbf{u}^{s_2} \times \omega^{s_3}] + \nu \Delta \mathbf{u}^{s_1} - \alpha \mathbf{u}^{s_1} + \mathbf{f}^{s_1}.$$

A. Alexakis JFM 812, 752 (2017)

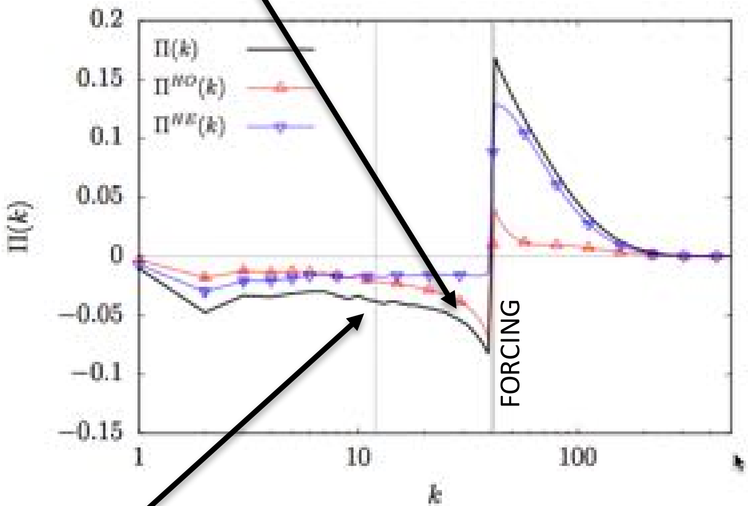


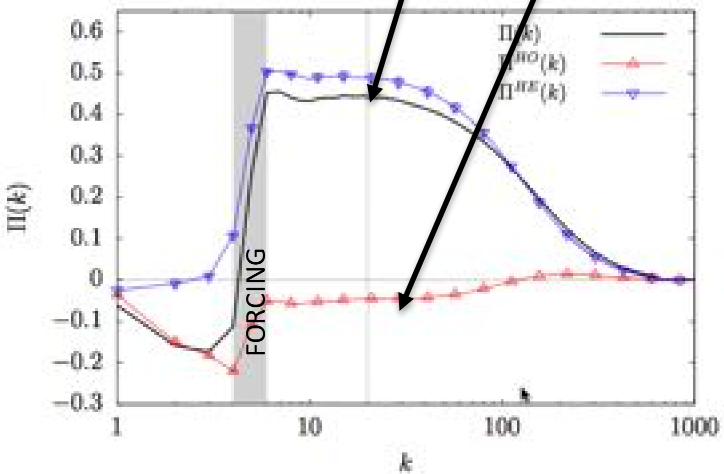
TOTAL FLUX

HOMOCHIRAL FLUX

HOMOCHIRAL FLUX

FULL 3D NAVIER STOKES EQS. + ROTATION



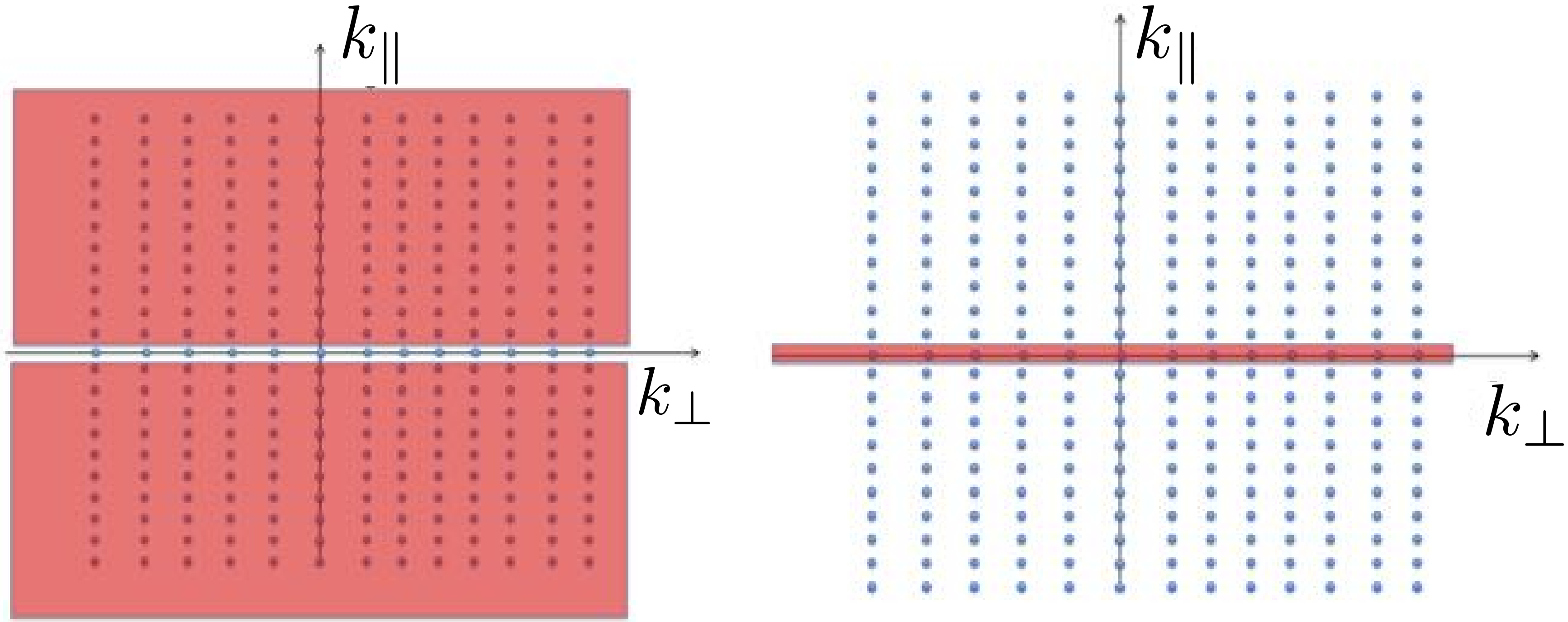


TOTAL FLUX

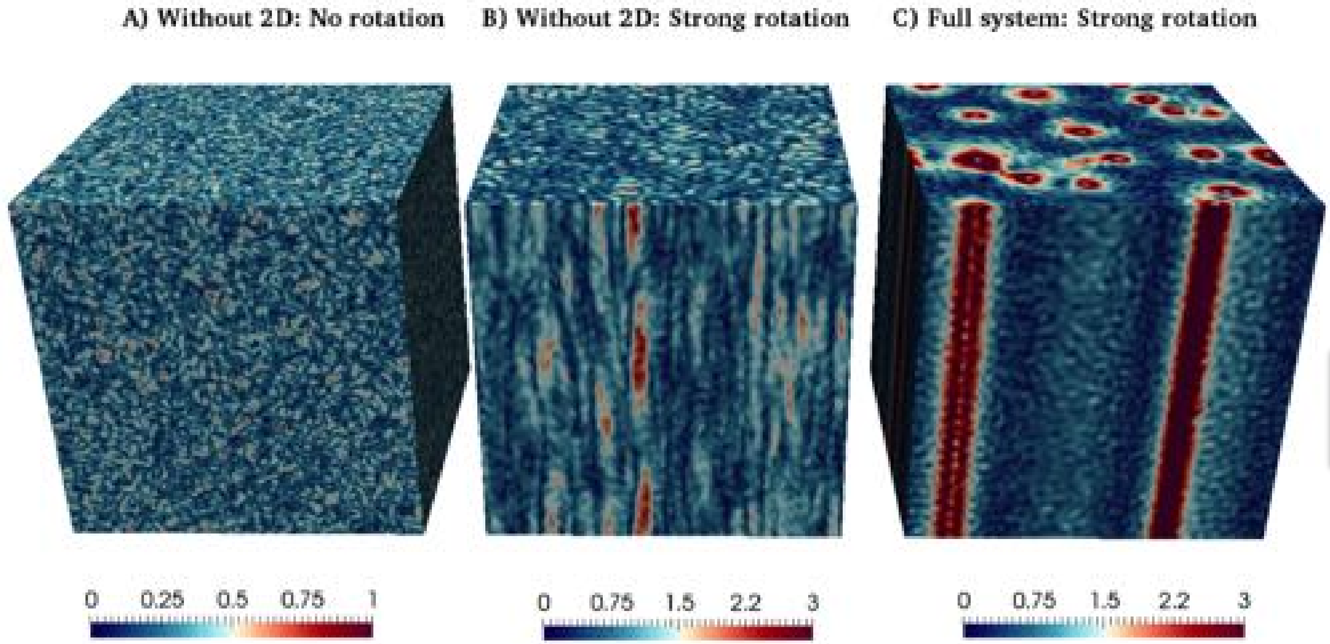
M. Buzzicotti, H. Aluie, L. B., M. Linkmann Phys. Rev. Fluids 3 (3) (2018) 034802

- ONLY 2D3C SLOW-MANIFOLD DYNAMICS (AND VARIATIONS THEREOF) [1]

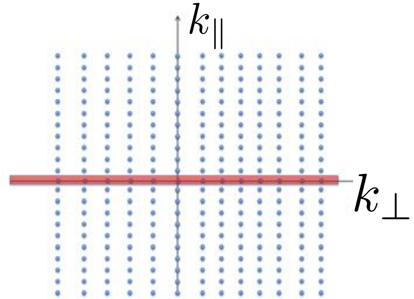
- ONLY 3D FAST-MANIFOLD [2,3]



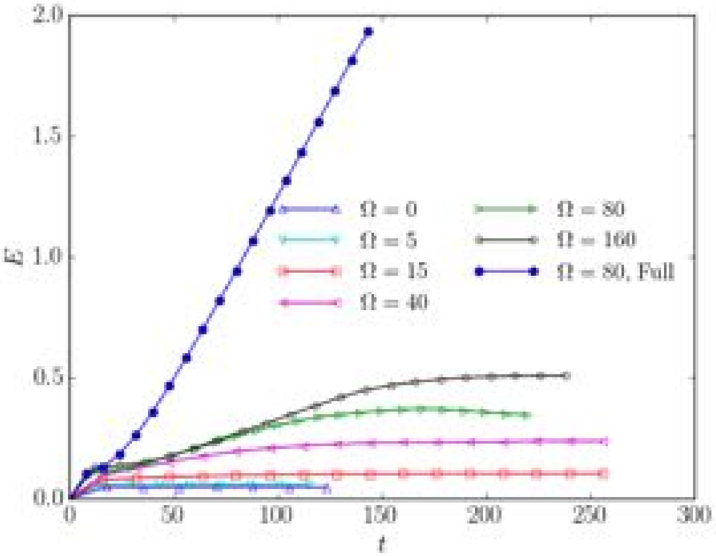
[1] L.B., M Buzzicotti, M Linkmann Physics of Fluids 29 (11), 111101 (2017)
[2] M. Buzzicotti, P. Clark di Leoni, L.B. Eur. Phys. J. E 41: 131 (2018)
[3] T. Le Reun, B. Favier, A.J. Barker, M Le Bars. PRL 119 034502, (2017)

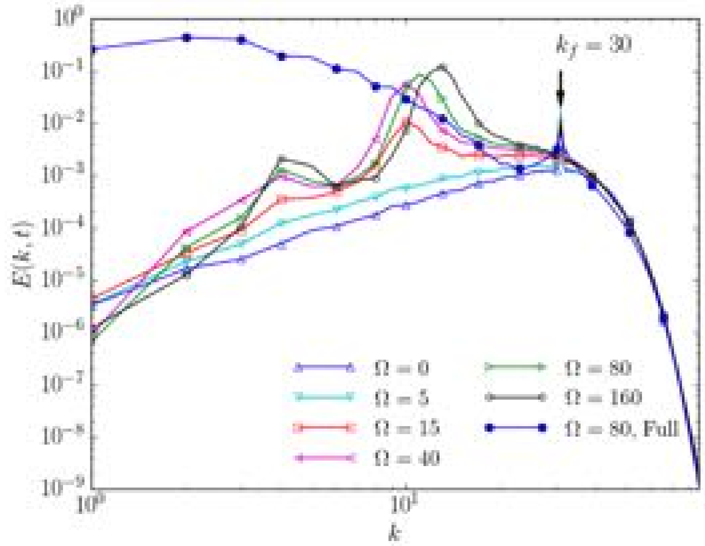


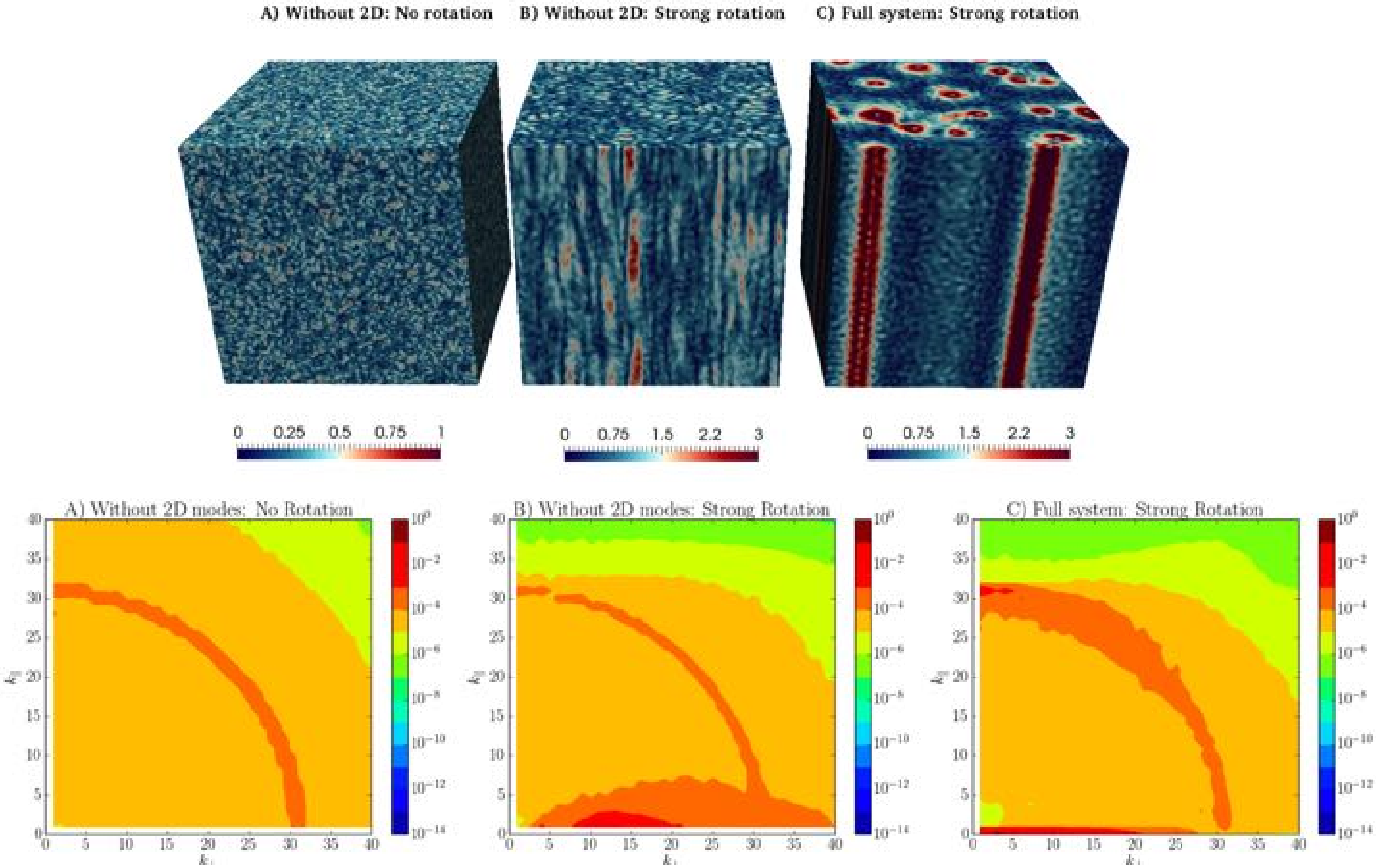
3D FAST-MANIFOLD ONLY

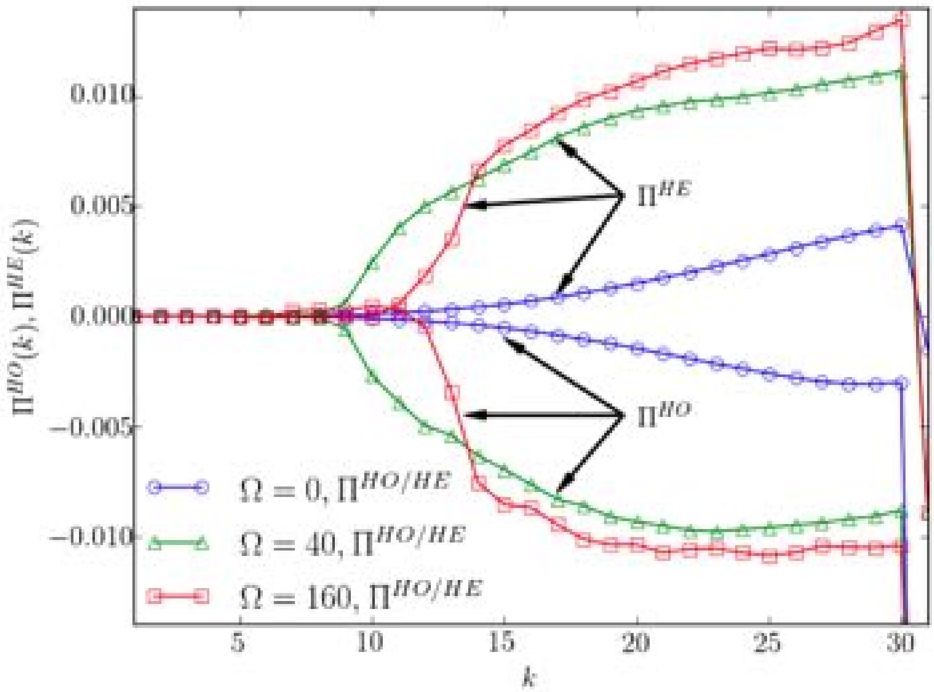


$$\begin{cases} \partial_t \boldsymbol{v} = \mathcal{P}[-\nabla p - (\boldsymbol{v} \cdot \nabla \boldsymbol{v})] - 2\boldsymbol{\Omega} \times \boldsymbol{v} + \nu(-1)^{\alpha+1}\Delta^{\alpha}\boldsymbol{v} \\ \nabla \cdot \boldsymbol{v} = 0. \end{cases}$$






A) Without 2D: No rotation
B) Without 2D: Strong rotation
C) Full system: Strong rotation
0 0.25 0.5 0.75 1
0 0.75 1.5 2.2 3
0 0.75 1.5 2.2 3
A) Without 2D modes: No Rotation
B) Without 2D modes: Strong Rotation
C) Full system: Strong Rotation
$k_{\parallel}$
$k_{\perp}$
0 5 10 15 20 25 30 35 40
10^{0} 10^{-2} 10^{-4} 10^{-6} 10^{-8} 10^{-10} 10^{-12} 10^{-14}



$$\Pi(k) = \Pi^{HE}(k) + \Pi^{HO}(k) = 0$$

$$\Pi^{HE}(k) = -\Pi^{HO}(k) \neq 0$$

Out-of-equilibrium flux-loop cascade

M. Buzzicotti, P. Clark di Leoni, L.B. Eur. Phys. J. E 41: 131 (2018)

2D3C SLOW-MANIFOLD ONLY

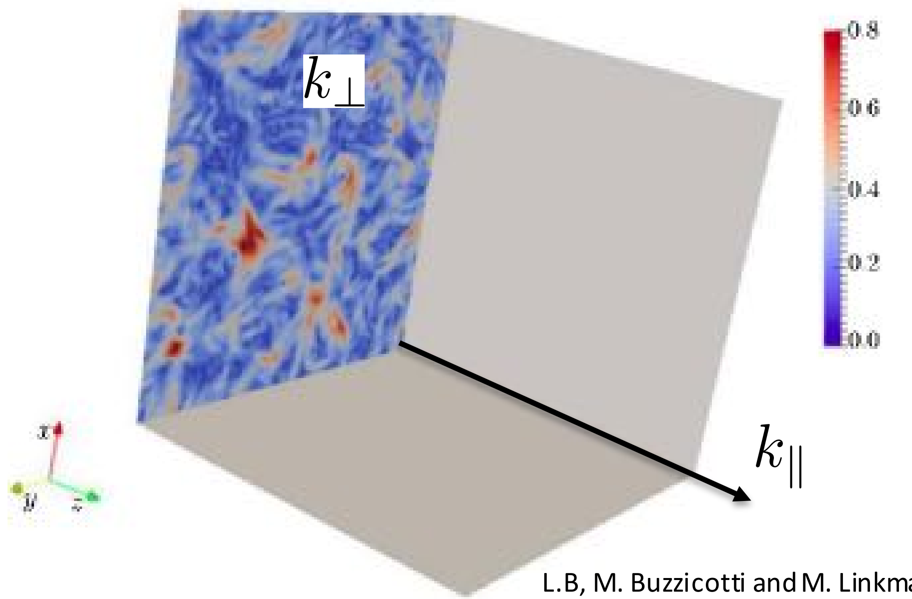
$\boldsymbol{u}(x,y,t)$ → **three** components; → **constant** in the z-direction

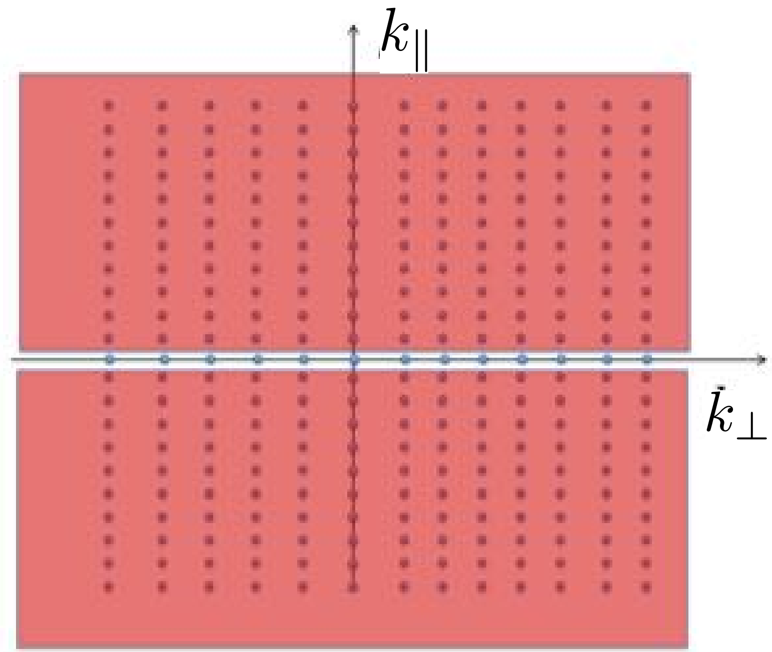
$$\boldsymbol{u} = \boldsymbol{u}^{2\mathrm{D}} + \theta$$

$$\boldsymbol{u}^{2\mathrm{D}} = \begin{pmatrix} u_x \\ u_y \\ 0 \end{pmatrix}; \qquad \theta = \begin{pmatrix} 0 \\ 0 \\ u_z \end{pmatrix}$$

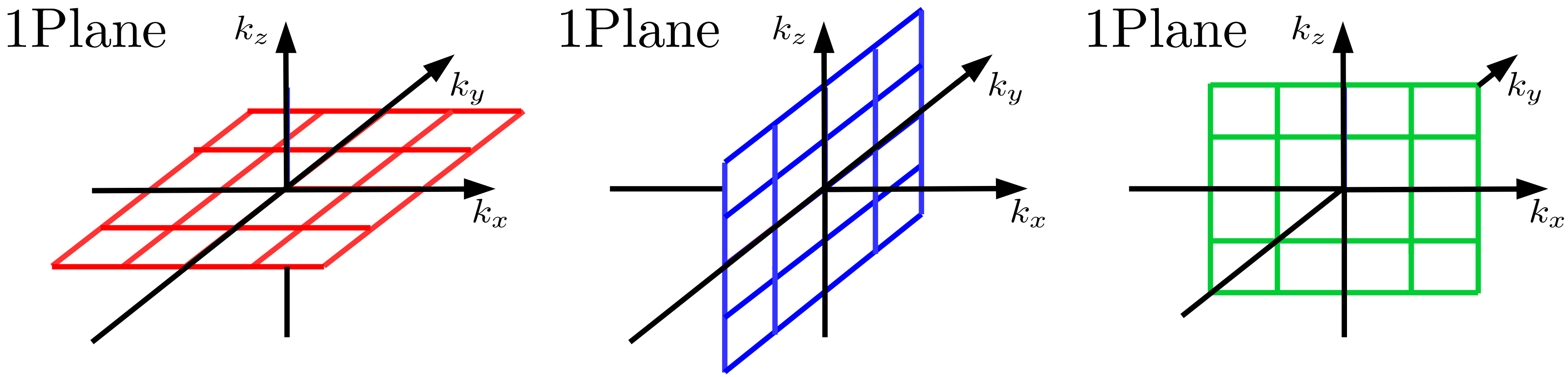
$$\begin{cases} \partial_t \boldsymbol{u}^{2\mathrm{D}} &= -(\boldsymbol{u}^{2\mathrm{D}} \cdot \nabla)\boldsymbol{u}^{2\mathrm{D}} - \nabla P + \nu \Delta \boldsymbol{u}^{2\mathrm{D}} \\ \partial_t \theta &= -(\boldsymbol{u}^{2\mathrm{D}} \cdot \nabla)\theta + \nu \Delta \theta \\ \nabla \cdot \boldsymbol{u} &= 0 \end{cases}$$

physical space:

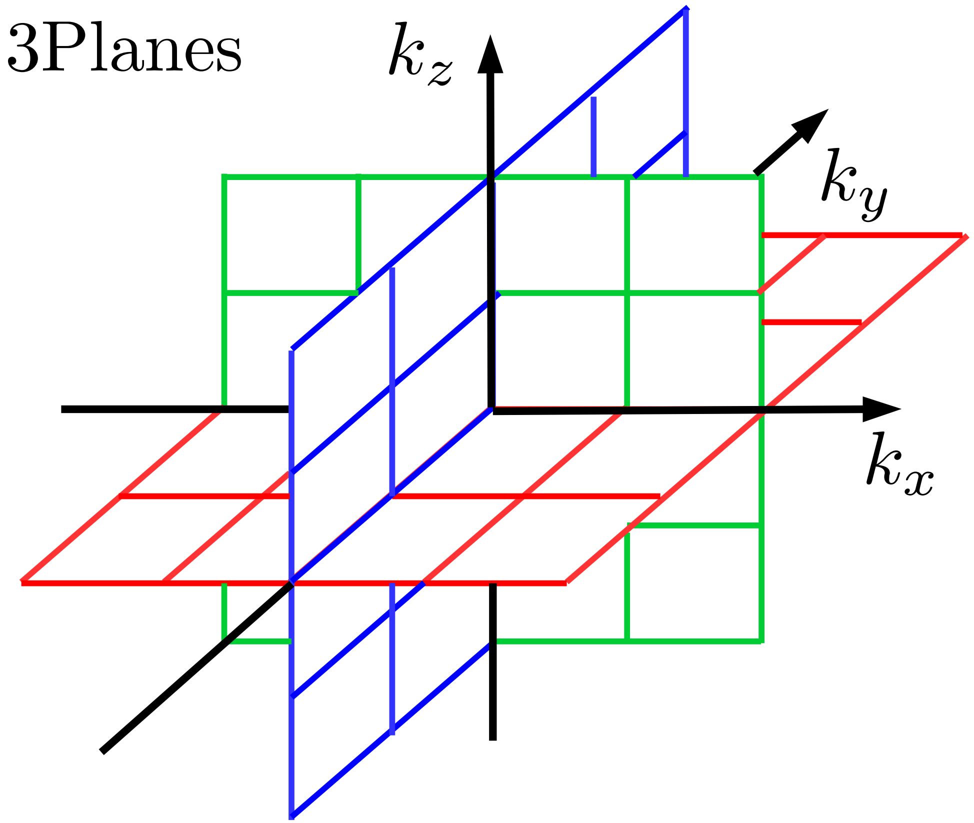




L.B, M. Buzzicotti and M. Linkmann PoF 29, 111101 (2017)



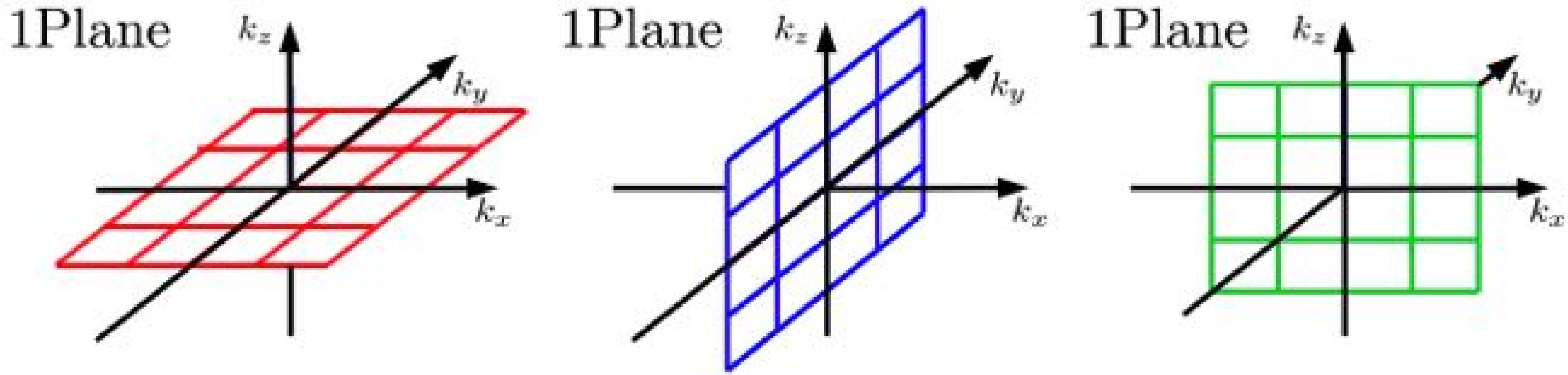
FROM 2D TO 3D BY SUPERPOSING 2D3C PLANES



Main properties:

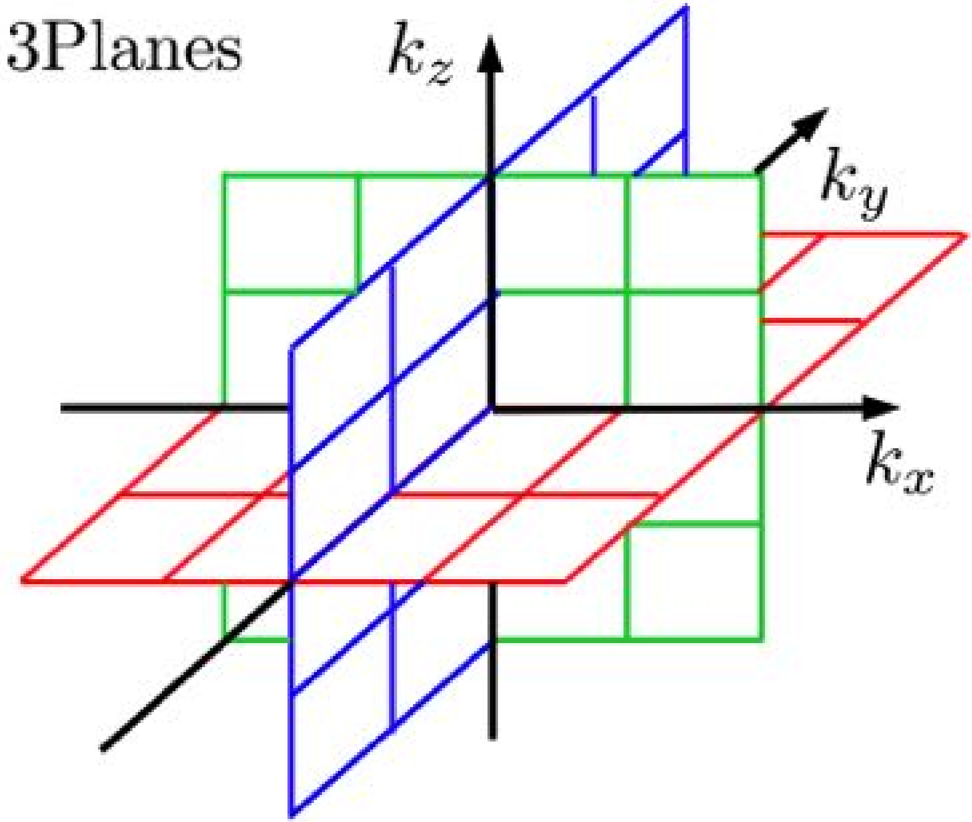
three-dimensionality

isotropy
under discrete rotations

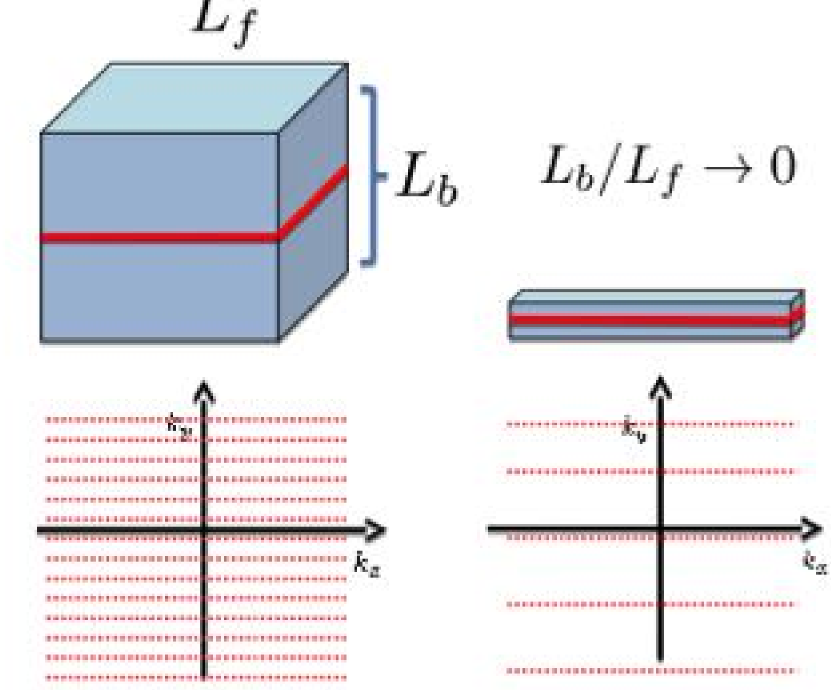


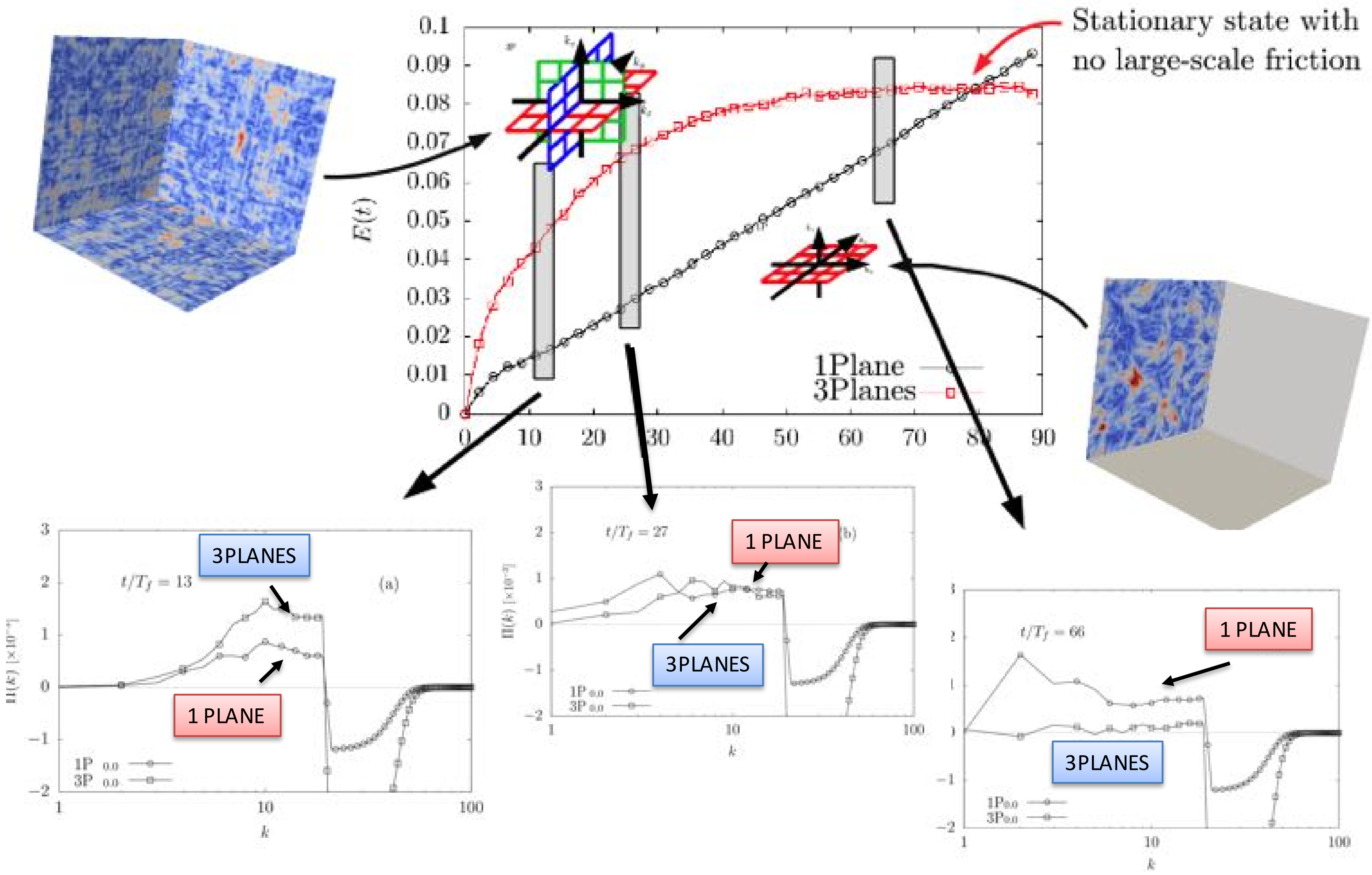
FROM 2D TO 3D BY SUPERPOSING 2D3C PLANES

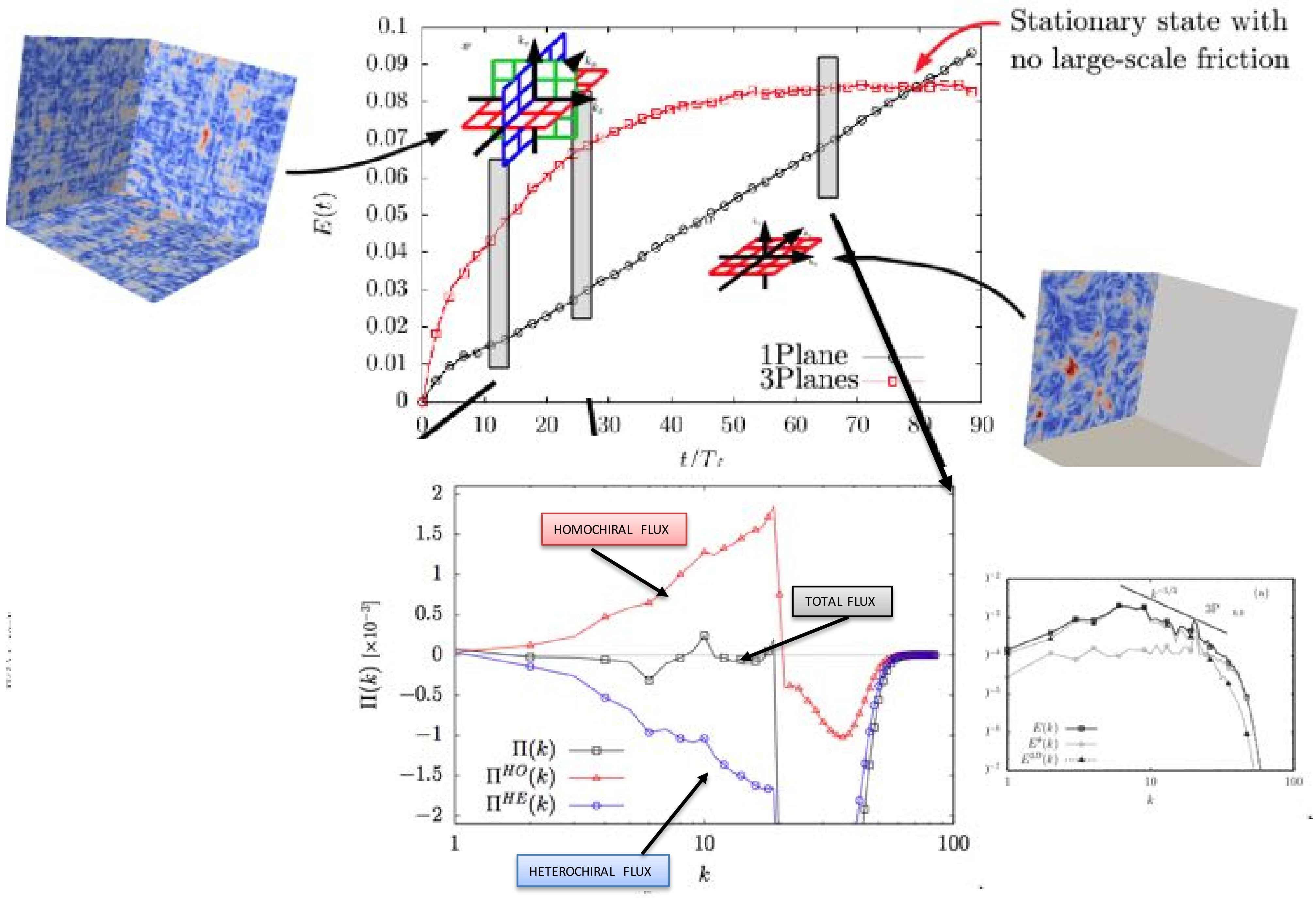
UV 2D AND IR 3D !
OPPOSITE OF A THIN LAYER!



$\neq$

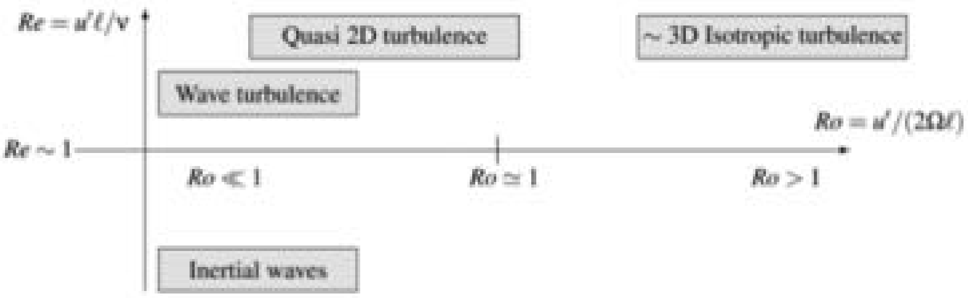



Stationary state with
no large-scale friction
E(t)
1Plane
3Planes
3PLANES
1 PLANE
t/T_f = 13
(a)
1P 0.0
3P 0.0
k
t/T_f = 27
(b)
1 PLANE
3PLANES
1P 0.0
3P 0.0
k
t/T_f = 66
1 PLANE
3PLANES
1P0.0
3P0.0
k



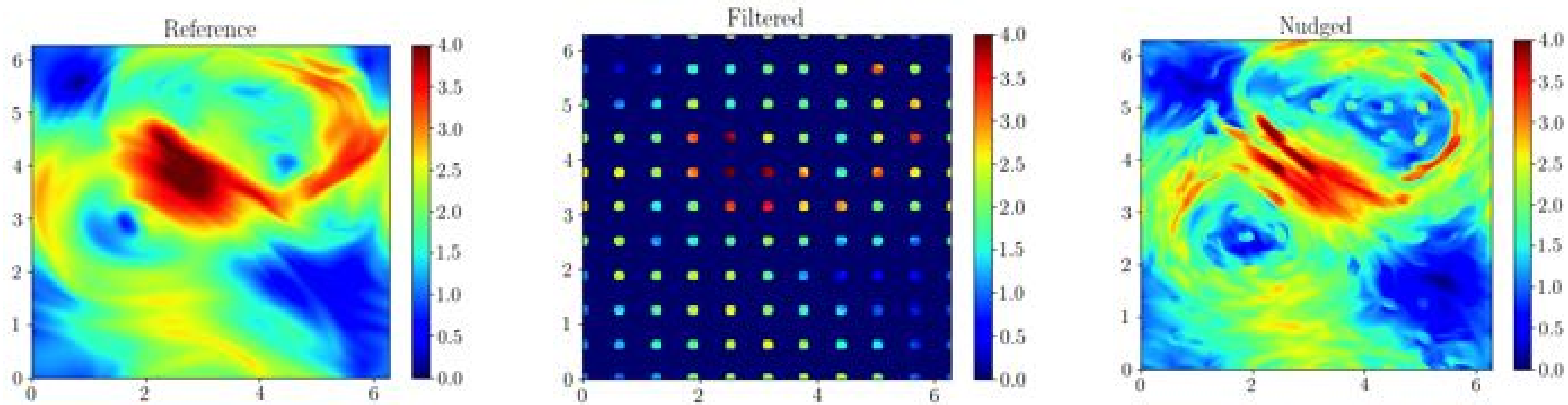
L. B., M. Buzzicotti and M. Linkmann PoF 29, 111101 (2017); M. Linkmann, M. Buzzicotti and L. B., EPJE (2018)

CONCLUSIONS



Godeferd & Moisy App. Mech. Rev. 2015

- PHASE-SPACE OF ROTATING TURBULENCE IS HIGHLY NON-TRIVIAL (Ro, Re, H, $K_{forcing}$, etc…)
- MORE THAN ONE ENERGY-TRANSFER MECHANISM ACTING IN THE SYSTEM (FLUX LOOP)
- FOURIER vs CONFIGURATIONAL SPACE DICHOTOMY: CAN WE IDENTIFY THE DEGREES-OF-FREEDOM THAT DEFINE THE TURBULENT BACKBONE?



P. Clark Di Leoni, A. Mazzino, L.B. Inferring flow parameters and turbulent configuration with physics-informed data-assimilation and spectral nudging. Physical Review Fluids 3 (10), 104604 (2018)

Physics Reports 767–769 (2018) 1–101

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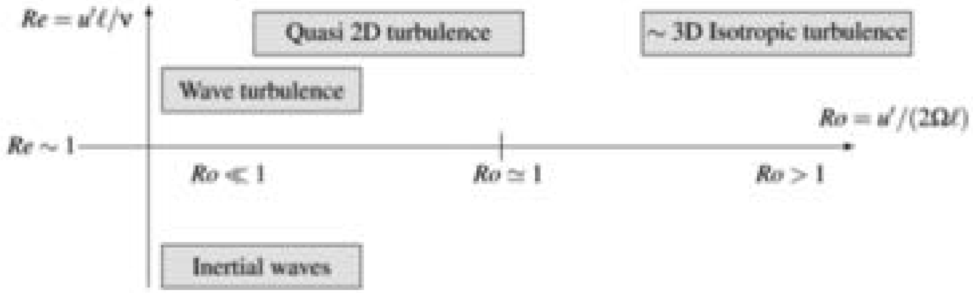
journal homepage: www.elsevier.com/locate/physrep

Cascades and transitions in turbulent flows

A. Alexakis [a], L. Biferale [b,*]



Godeferd & Moisy App. Mech. Rev. 2015