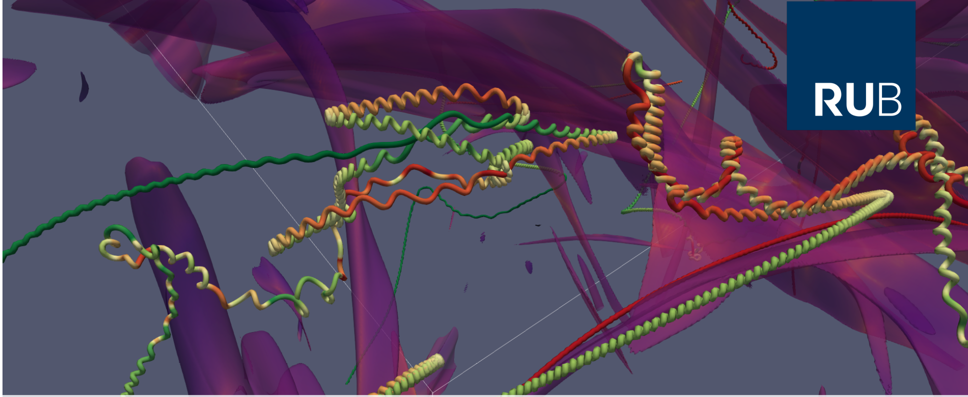

RUHR-UNIVERSITÄT BOCHUM

A Lagrangian perspective on magnetic Turbulence with energetic charged Tracer Particles

Physics Seminar at University of Rome, Tor Vergata

Ruhr University Bochum, Institute for Theoretical Physics I & IV
SFB-1491: Cosmic Interacting Matter, Project F1

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SFB1491

Outline

1 Plasma Parameters

2 MHD Turbulence

3 Motion of charged particles

4 Fieldline curvature

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Plasma Parameters

- Consider populations of fully ionized species with masses m_s and charges q_s, e.g. electrons $s = e$ and ions $s = i$
- Assume **quasi-neutrality** $n_e \simeq n_i$, i.e. comparable number densities
- The populations are characterized by their **kinetic temperature**

$$T_s = \frac{1}{3} m_s \langle v_s^2 \rangle$$

- Charges are sources to an electric potential via $\Delta\phi = q_s n_s$, but individual charges are shielded by the population of opposite charges
- The shielding scale is given by the **Debye length**

$$\lambda_D \sim \sqrt{\frac{T}{ne^2}}$$

- The typical number of particles in the Debye sphere is given by the **Plasma parameter**

$$\Lambda = \frac{4\pi}{3} n \lambda_D^3$$

- $\Lambda \ll 1$: sparse and cold, strongly coupled
- $\Lambda \gg 1$: dense and hot, weakly coupled

Plasma Parameters

- The most complete description is given by the Vlasov-Boltzmann transport equation of the distribution function $f_s(\mathbf{x}, \mathbf{v})$
$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{q_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f_s = C_s(f)$$
- This 6D problem can be simplified by computing the moments of f_s instead of evolving the entire distribution
  - **zeroth moment:** number density $n_s = \int f_s(\mathbf{x}, \mathbf{v})\,\mathrm{d}^3\mathbf{v}$
  - **first moment:** momentum density $n_s \mathbf{u}_s = \int (\mathbf{v} - \langle \mathbf{v} \rangle) f_s(\mathbf{x}, \mathbf{v})\,\mathrm{d}^3\mathbf{v}$
  - **second moment:** pressure tensor $P_s = \int m_s (\mathbf{v} - \langle \mathbf{v} \rangle)(\mathbf{v} - \langle \mathbf{v} \rangle) f_s(\mathbf{x}, \mathbf{v})\,\mathrm{d}^3\mathbf{v}$
- The evolution equations of the moments are known as the **multi-fluid description**
- On sufficiently large spatial and temporal scales, typical velocities much smaller than c_{sound}, and if $f(\mathbf{v})$ is close to equilibrium, one can further simplify to single-fluid incompressible **Magnetohydrodynamics (MHD)**

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MHD Turbulence

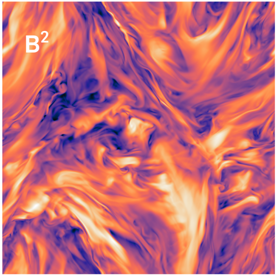
- A conducting magnetized fluid is governed by (in Alfvén units $[\mathbf{u}] = [\mathbf{B}]$)

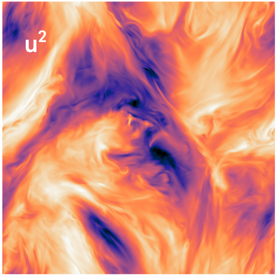
$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p$$
$$+\nu \Delta \mathbf{u} + \mathbf{f}$$
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \Delta \mathbf{B}$$

- Fluid and magnetic Reynolds numbers

$$\mathcal{R} = \frac{(\mathbf{u}\cdot\nabla)\mathbf{u} \sim U^2/L}{\nu \Delta \mathbf{u} \sim \nu U/L^2} \sim \frac{UL}{\nu}$$
$$\mathcal{R}_m = \frac{\nabla \times (\mathbf{u} \times \mathbf{B}) \sim UB/L}{\eta \Delta \mathbf{B} \sim \nu B/L^2} \sim \frac{UL}{\eta}$$

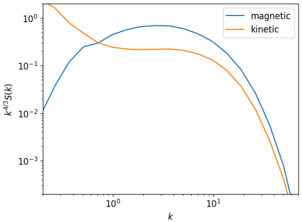
- If $\eta \to 0$, flow and magnetic field are frozen into each other (flow advects, magnetic field exerts tension)



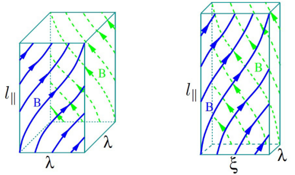


MHD Turbulence

- **Kolmogorov (1941):** constant energy flux from scale to scale with rate $\varepsilon \Rightarrow E(k) \sim \varepsilon^{2/3} k^{-5/3}$, $\delta v_\lambda \sim (\varepsilon\lambda)^{1/3}$
- **Kraichnan&Iroshnikov (1965/63):** collisions of counter-traveling Alfvén waves with $v_A = B_0$ $\Rightarrow E(k) \sim (\varepsilon v_A)^{1/2} k^{-3/2}$, $\delta v_\lambda \sim (\varepsilon v_A \lambda)^{1/4}$
- **Goldreich&Sridhar (1995):** MHD turbulence is anisotropic on all scales due to $\mathbf{B}_0$. Critical balance $l_\parallel / v_A \sim \lambda/\delta v_\lambda \quad (\lambda \sim k_\parallel)$ $\Rightarrow E(k_\perp) \sim \varepsilon^{2/3} k_\perp^{-5/3}$, $l_\parallel \sim v_A \varepsilon^{-1/3} \lambda^{2/3}$
- **Boldyrev (2006):** $\delta \mathbf{v}_\perp$ and $\delta \mathbf{B}_\perp$ tend to align with each other $\Rightarrow$ sheet-like structures with aspect ratio $\lambda/\xi \sim \sin\theta_\lambda$ (with $\boldsymbol{\xi} \parallel \delta\mathbf{B}_\perp$)



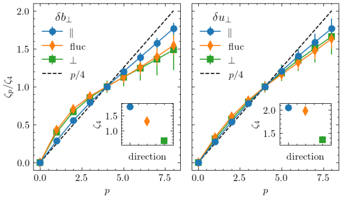
Spectra of magnetic and kinetic energy, with $k^{4/3}$ (Grete+ 2021)



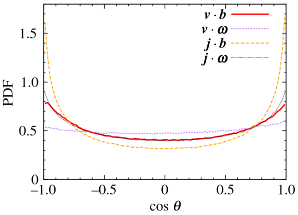
Sketch of critical balance and aligned eddy (Boldyrev 2006)

MHD Turbulence

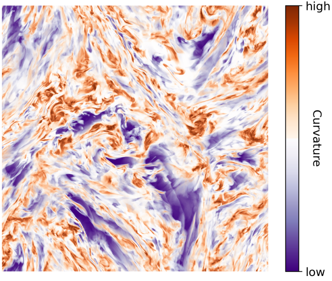
- **Boldyrev (cont.):** $\Rightarrow E(k_\perp) \sim k_\perp^{-3/2}$, $l_\parallel \sim \lambda^{1/2}$, $\sin\theta_\lambda \sim \lambda^{1/4}$
- Scale-invariance is broken, λ is more intermittent than $l_\parallel$, **B** is more intermittent than **v**
- strong alignment reduces non-linearity, MHD organizes in **coherent structures** of aligned fields, separated by highly non-linear regions



Anisotropic structure function scaling



Distributions of alignment angles (Matthaeus+ 2015)



Non-linearity in the B-field

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Motion of charged particles

- Equation of motion due to the **Lorentz force**

$$\ddot{\mathbf{x}} = q/m\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})\,, \quad \mathbf{v} = \dot{\mathbf{x}}$$

- Particle energy is conserved in static B-fields, i.e. $\|\dot{\mathbf{x}}\| = \text{const.}$
- Uniform **B** and vanishing **E** results in **gyro motion** perpendicular to **B** with frequency and radius

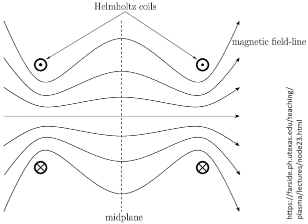
$$\omega_g = |q|B/m, \quad r_g = v_\perp/\omega_g$$

- If $r_g \ll B/\nabla B$, split dynamics in fast oscillation about **guiding center** and slow drift, such as gradient and curvature drifts

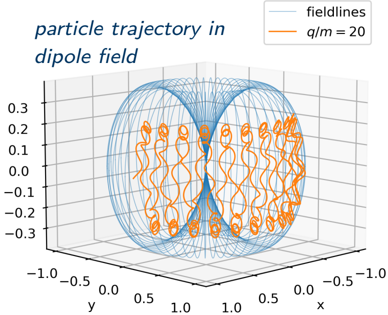
$$\mathbf{v}_{\nabla B} = \bar{\mu}/m\omega_g\,\mathbf{B} \times \nabla B/B^2, \quad \mathbf{v}_{curv} = v_\parallel^2/\omega_g\,\hat{\mathbf{B}} \times (\mathbf{B} \cdot \nabla \mathbf{B})/B^2$$

- Dynamics are typically very complicated, only simple setups admit analytical treatment

Motion of charged particles



- The **magnetic moment** $\bar{\mu} = mv_\perp^2/2B$ is conserved in the guiding center approximation
- Particle energy is conserved in static B-fields, i.e. $\mathcal{E} = mv_\parallel^2/2 + \bar{\mu}B = \text{const.}$
- A particle moving along an increasing B-field has to decrease its $v_\parallel$ until it reverses its direction
- The particle energy at this bounce point is $\mathcal{E} = \bar{\mu}B_{max}$ and the trapping condition is
$$\left|v_\parallel\right|/\left|v_\perp\right| < \sqrt{B_{max}/B_{min} - 1}$$



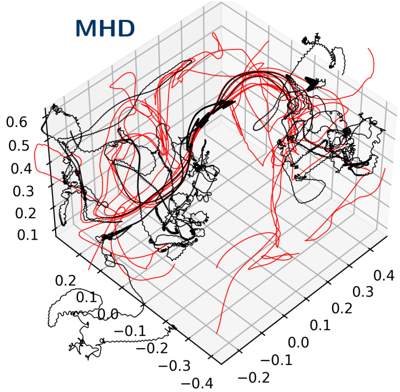
Motion of charged particles

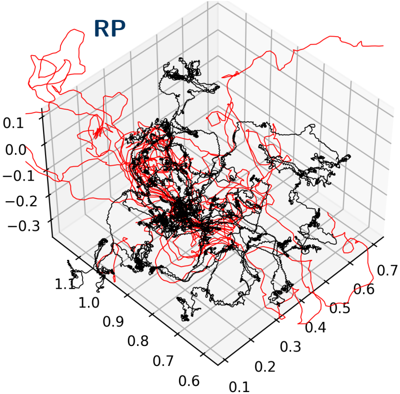
- In magnetized turbulence, particle motion becomes highly chaotic
- Quasi-linear theory assumes a strong regular magnetic field and weak fluctuations $\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$, $\|\delta\mathbf{B}\| \ll \|\mathbf{B}_0\|$
- Particles are said to **interact resonantly** with magnetic fluctuations with wavenumbers k_m such that $k_m r_g \mu \sim 1$
- Particles undergo scattering of pitch angle $\mu = \hat{\mathbf{v}} \cdot \hat{\mathbf{B}}$, resulting in diffusive behaviour, described by a distribution $f(t, z, \mu)$ with a transport equation
$$\frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu}\left(D_{\mu\mu} \frac{\partial f}{\partial \mu}\right),$$
where $D_{\mu\mu} = \langle \Delta\mu^2 \rangle / 2\Delta t$ is the pitch angle diffusion coefficient

Motion of charged particles

- QLT models the magnetic fluctuations $\delta\mathbf{B}$ as a superposition of waves with random phases and a prescribed spectrum

$$\delta\mathbf{B}(\mathbf{r}) = \sum_{n=0}^{N-1} A(k_n)\hat{\boldsymbol{\xi}} \cos\left(\mathbf{k}_n \cdot \mathbf{r} + \phi_R\right), \quad A(k)^2 \sim k^{-5/3}, \quad \hat{\boldsymbol{\xi}} \cdot \hat{\mathbf{k}} = 0.$$





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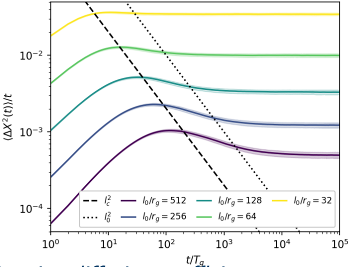
Fieldline curvature

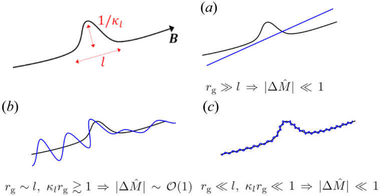
- No definite transport theory of charged particles in strong MHD turbulence exists
- Particles experience **fast super-diffusion along coherent structures** characterized by small curvatures and lengths $\sim l_0$
- Consider the fieldline curvature as a scattering mechanism *(Kempski+ 2023, Lemoine 2023)*

$$\kappa = \left\| \hat{\mathbf{B}} \times (\mathbf{B} \cdot \mathbf{B}) \right\| / B^2$$

- Conservation of the particle's magnetic moment $\bar{\mu}$ is violated through interactions with resonant fieldline curvature $\langle \kappa \rangle_{r_g} r_g \sim 1$



Running diffusion coefficients



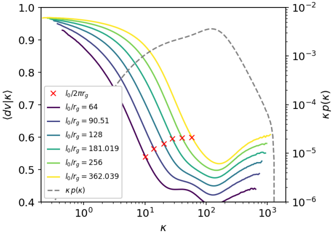
Interaction of a particle with a localized fieldline bend (Lemonie 2023)

Fieldline curvature

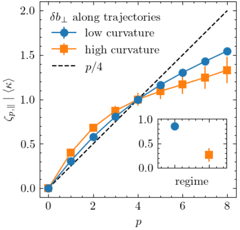
- Consider **local guiding center average** of some quantity X along particle trajectory $\mathbf{x}(t)$

$$X_{T_g}(\mathbf{x}(t)) = \frac{1}{T_g(\mathbf{x}(t))} \int_0^{T_g(\mathbf{x}(t))} X(\mathbf{x}(t-\tau))\,\mathrm{d}\tau$$

- Record joint distribution $p(\delta v_{T_g}, \kappa_{T_g})$ of scattering angle cosine $\delta v_{T_g} = \hat{\mathbf{v}}_{T_g}(t_i) \cdot \hat{\mathbf{v}}_{T_g}(t_{i-1})$ and fieldline curvature κ_{T_g}
- Conditional average $\langle \delta v_{T_g} | \kappa_{T_g} \rangle$ reveals two distinct transport regimes with resonant threshold $\kappa_{thres} = l_0/2\pi r_g$
- coherent geometry ⇒ fast transport, non-linear geometry ⇒ slow transport



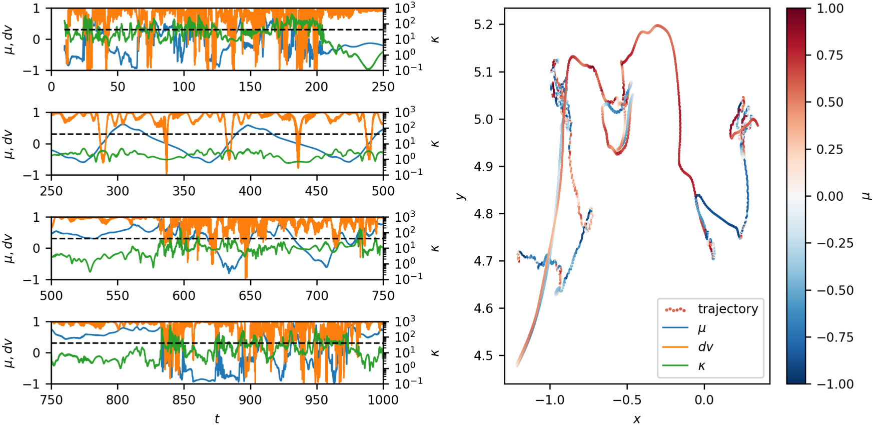
conditional scattering angle cosine average for various particle energies



roughness and intermittency along particle trajectories conditional on avg. fieldline curvature

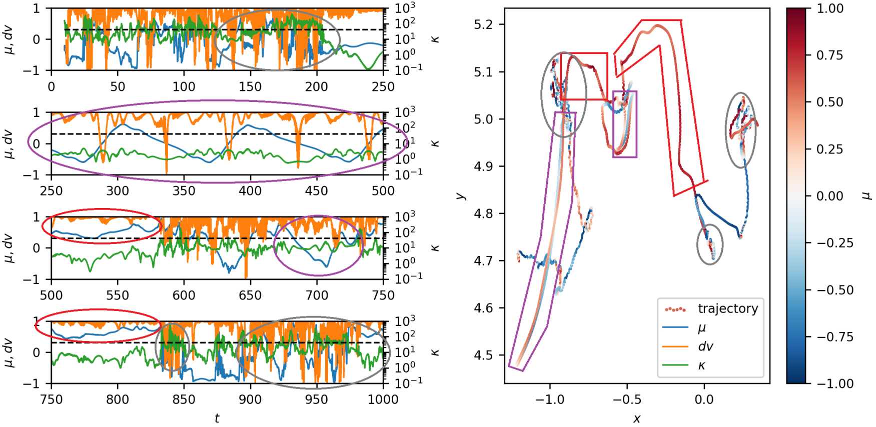
Fieldline curvature

Example of a particle trajectory with $l_0/r_g = 256$.
Shown are the pitch angle cosine, scattering angle cosine and fieldline curvature averaged along the particle trajectory.



Fieldline curvature

Example of a particle trajectory with $l_0/r_g = 256$.
Highlighted are free streaming, mirror confinement and chaotic confinement.



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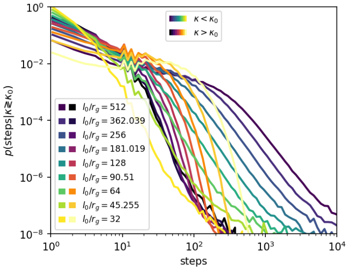
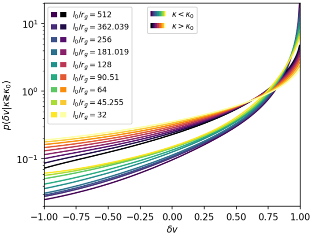
Modeling

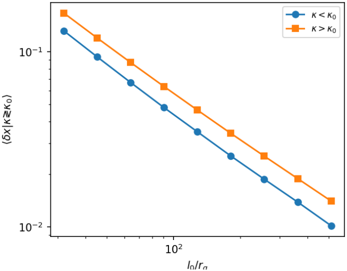
Correlated Random Walk

- sample direction $\hat{\mathbf{v}} \in S^2$, regime $\hat{\kappa} \in \{0, 1\}$
- while $\texttt{step} < \texttt{max\_steps}$ do
  - sample $\texttt{substeps} \in p(\texttt{steps}|\hat{\kappa})$
  - while $\texttt{substep} < \texttt{substeps}$ do
    - sample $\delta v \in p(\delta v|\kappa)$
    - $\hat{\mathbf{v}} \leftarrow$ rotate $\hat{\mathbf{v}}$ by δv
    - $\mathbf{x} \leftarrow \mathbf{x} + \hat{\mathbf{v}} \cdot \delta x(\kappa)$
    - $\texttt{substep}{+}{+}$, $\texttt{step}{+}{+}$





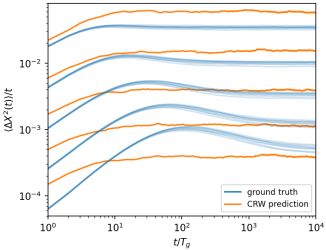
Distributions of deflection angle cosines (left), regime escape times (upper right) and average guiding center step width (lower right), conditional on fast and slow curvature regime



Modeling

CRW Results, preliminary

- Diffusion coefficients are only roughly reproduced
- Initial ballistic and intermediate subdiffusive transport are insufficiently reproduced
- Magnetic mirroring is not explicitly modeled



Generative Diffusion Models

- Generative diffusion models (GDM) are able to learn complicated, high-dimensional probability distributions
- See synthetic Lagrangian trajectories by *Li+ 2023*
- **Aim:** reproduce trajectory features over all relevant scales (small: intermittency, large: confinement and free streaming)

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MHD Turbulence

- intrinsically anisotropic, (local) $\mathbf{B}_0$ persists on all scales
- forms large high-aspect ration coherent structures with reduced non-linearity, interleaved with highly non-linear chaotic regions

Motion of charged particles

- Gyro motion due to the Lorentz force
- Confinement due to magnetic mirror configurations
- no transport theory for strong turbulence yet

Fieldline curvature

- Particles exhibit three distinct transport regimes (free streaming, mirror confinement, chaotic confinement)
- Free streaming and chaotic confinement is distinguished by the fieldline curvature

Modeling

- CRW appears natural on the first glance, but implementation is not trivial
- Challenge: reproduce small-scale intermittency features and large-scale transport features

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