Isotropic Helicoids in Complex Flows

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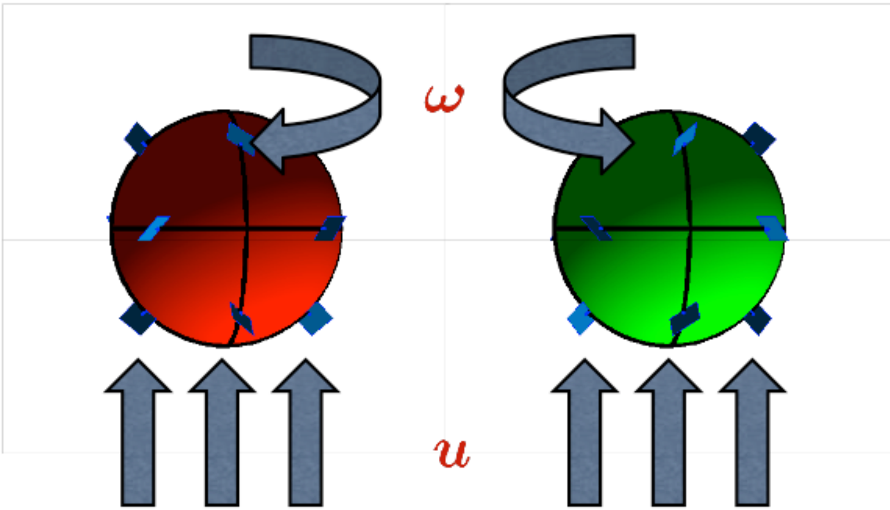
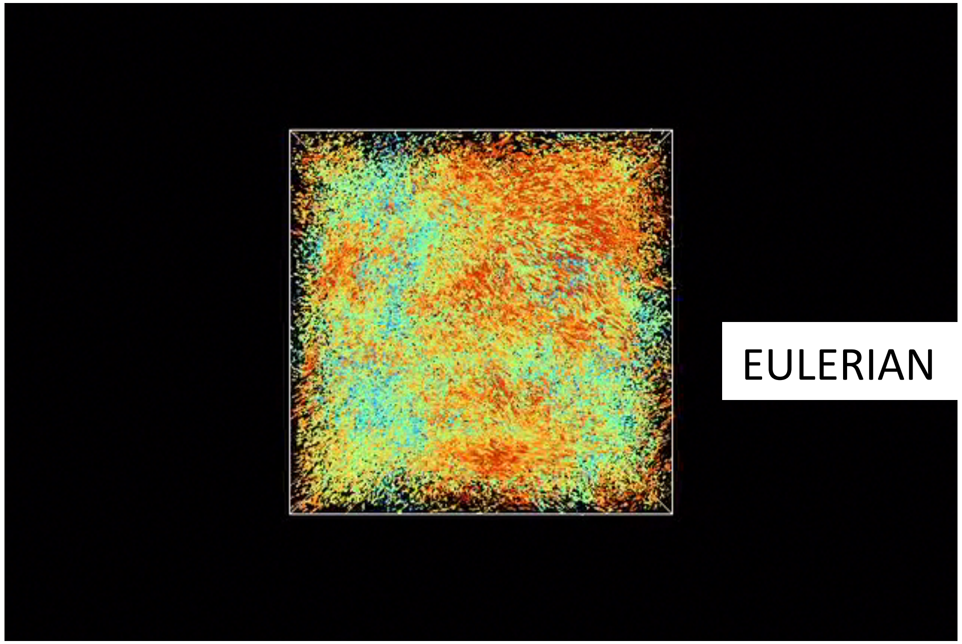
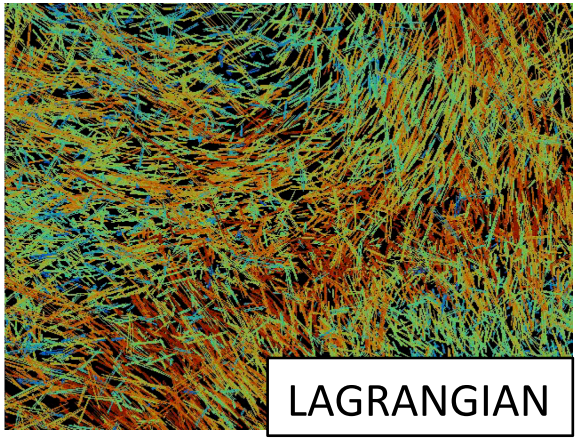
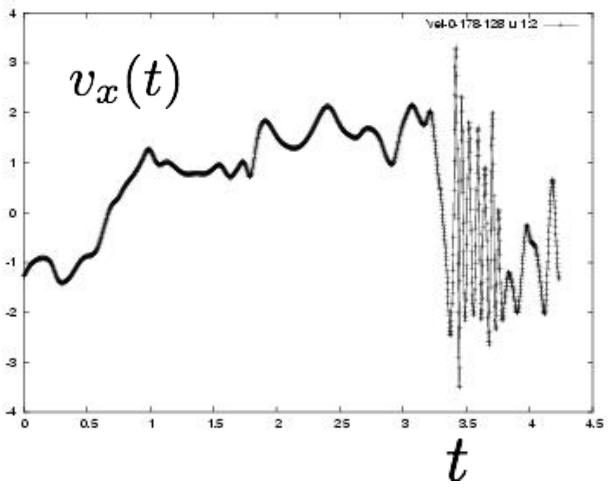


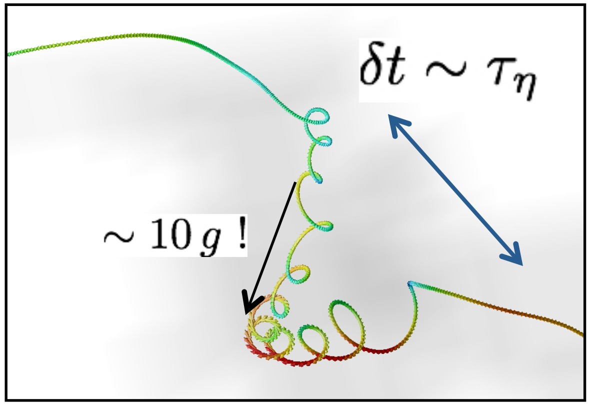
COMPLEX PARTICLES IN COMPLEX FLOWS:
HOW TO ESCAPE/FALL FROM/ON EULERIAN TURBULENT TRAPS?





Intermittency - Lagrangian





vortex trapping

PHYSICS OF FLUIDS 17, 021701 (2005)

Particle trapping in three-dimensional fully developed turbulence

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Lagrangian Properties of Particles in Turbulence

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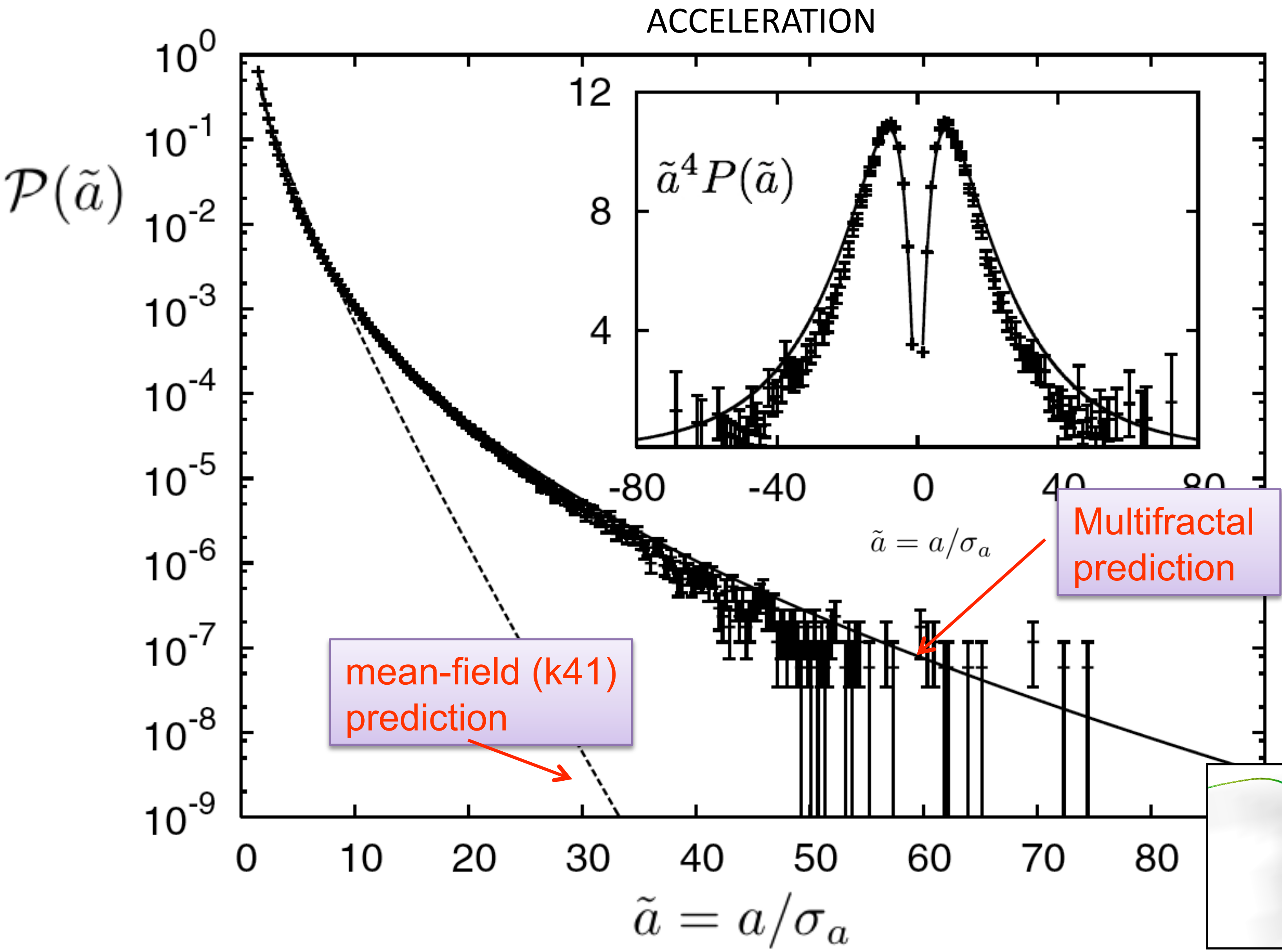
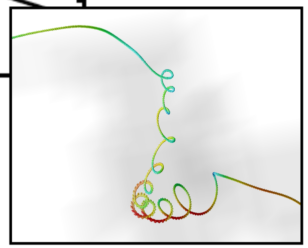
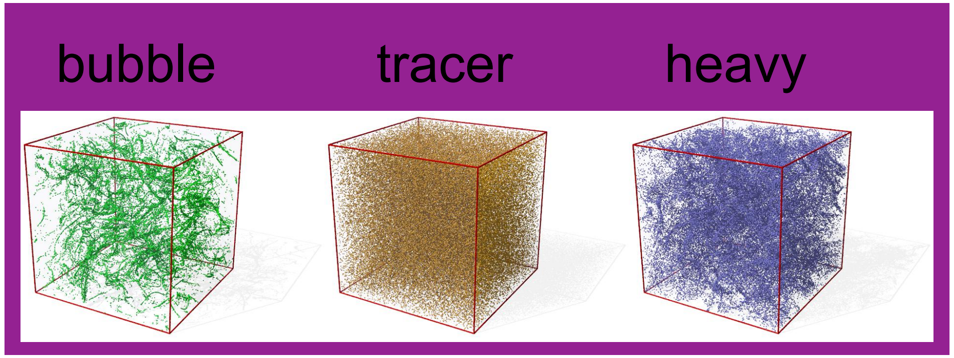
L. B., G. Boffetta, A. Celani, B. Devenish, A. Lanotte and F. Toschi *Phys. Rev. Lett.* **93**, 064502, 2004

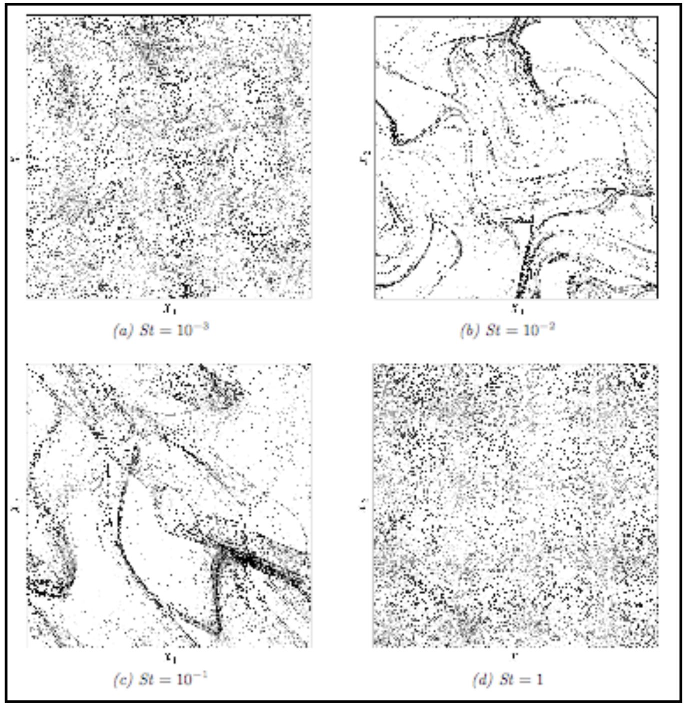




$$\mathcal{P}(a) \sim \int_{h\in I} \mathrm{d}h\, a^{\frac{h-5+D(h)}{3}} \nu^{\frac{7-2h-2D(h)}{3}} L_0^{D(h)+h-3} \sigma_v^{-1} \times \quad \exp\left(-\frac{a^{\frac{2(1+h)}{3}} \nu^{\frac{2(1-2h)}{3}} L_0^{2h}}{2\sigma_v^2}\right)$$

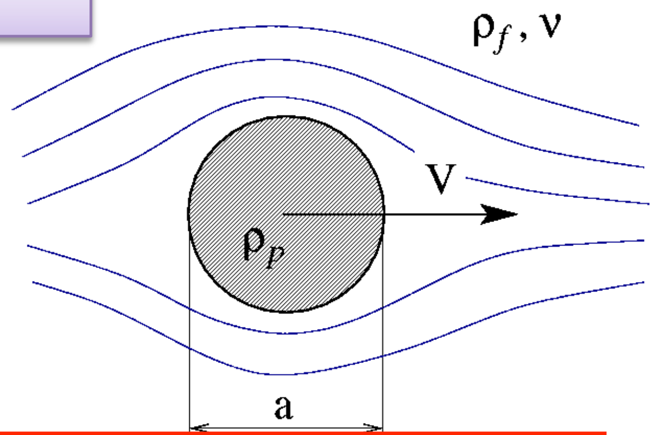
- -LAGRANGIAN TURBULENCE
- -INERTIAL PARTICLES
- -EFFECTS OF PREFERENTIAL CONCENTRANTIONS
- -EFFECTS OF CAUSTICS
- -COMPLEX PARTICLES (TODAY)
- -SMART PARTICLES





Eqs of motion for a single particle

Maxey & Riley Phys. Fluids 26, 883 (1983)



Small particles
Small Reynolds numbers (on the particle radiud)
Undeformable
Small volume fraction
collisionless

$$\frac{a(u-V)}{\nu} \ll 1 \qquad a \ll \eta$$

$$m_p \frac{dV_i}{dt} = (m_p - m_f) g_i + m_f \left.\frac{Du_i}{Dt}\right|_{\boldsymbol{X}(t)}$$

Buoyoancy + fluid acceleration

$$-6\pi a \mu \left[V_i(t) - u_i(\boldsymbol{X}(t), t) - \frac{1}{6} a^2 \left. \nabla^2 u_i \right|_{\boldsymbol{X}(t)} \right]$$

Stokes drag

$$-\frac{m_f}{2} \frac{d}{dt} \left[V_i(t) - u_i(\boldsymbol{X}(t), t) - \frac{1}{10} a^2 \left. \nabla^2 u_i \right|_{\boldsymbol{X}(t)} \right]$$

Added mass

$$-6\pi a \mu \int_0^t ds \left(\frac{d/ds \left[V_i(s) - u_i(\boldsymbol{X}(s), s) - \frac{1}{6} a^2 \left. \nabla^2 u_i \right|_{\boldsymbol{X}(s)} \right]}{\sqrt{\pi \nu (t-s)}} \right)$$

Basset-history terms

Simplified limit

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{V}$$

$$\tau_p = \frac{a^2}{3\nu\beta}$$

$$\frac{d\boldsymbol{V}}{dt} = \beta\frac{D\boldsymbol{u}(\boldsymbol{X},t)}{Dt} + \frac{\boldsymbol{u}(\boldsymbol{X},t)-\boldsymbol{V}}{\tau_p} + (1-\beta)\boldsymbol{g}$$

Three-parameters problem

τ_f Fluid characteristic time

τ_p Particle's characteristic time

$\rho_p \gg \rho_f \rightarrow \beta = 0$ HEAVY

$\rho_f = \rho_p \rightarrow \beta = 1$ TRACERS

$\rho_f \gg \rho_p \rightarrow \beta = 3$ LIGHT

Stokes number

$$St = \frac{\tau_p}{\tau_f}$$

Density contrast

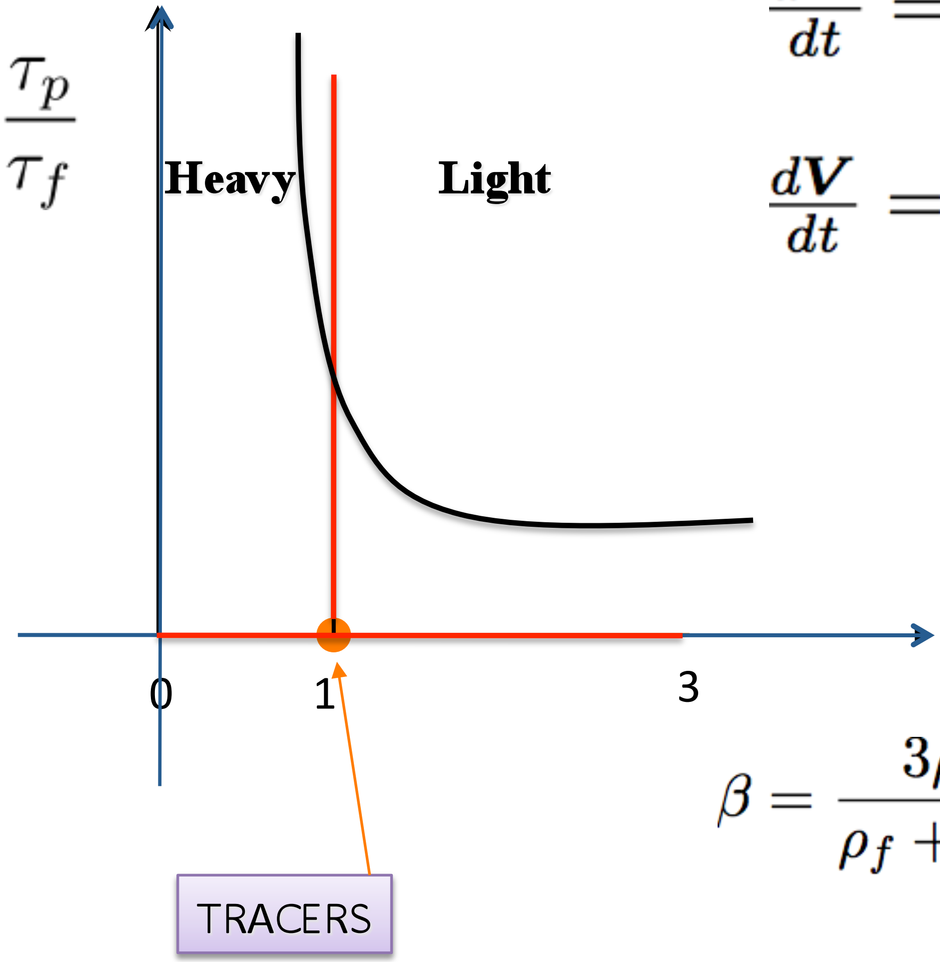
$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$

Reynolds

$$Re = \frac{UL}{\nu}$$

Validity of assumption $\boxed{a/\eta < 1}$

$$St = \frac{\tau_p}{\tau_f}$$



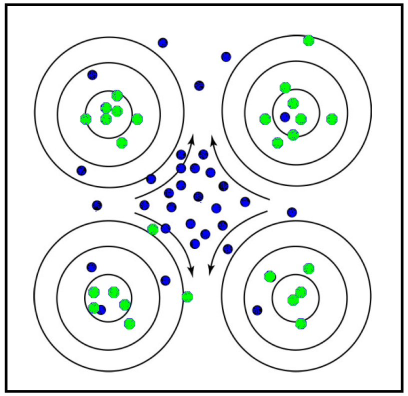
$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{V}$$

$$\frac{d\boldsymbol{V}}{dt} = \beta \frac{D\boldsymbol{u}(\boldsymbol{X},t)}{Dt} + \frac{\boldsymbol{u}(\boldsymbol{X},t) - \boldsymbol{V}}{St}$$

$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{V}$$

$$\frac{d\boldsymbol{V}}{dt} = \beta\frac{D\boldsymbol{u}(\boldsymbol{X},t)}{Dt} + \frac{\boldsymbol{u}(\boldsymbol{X},t)-\boldsymbol{V}}{St}$$

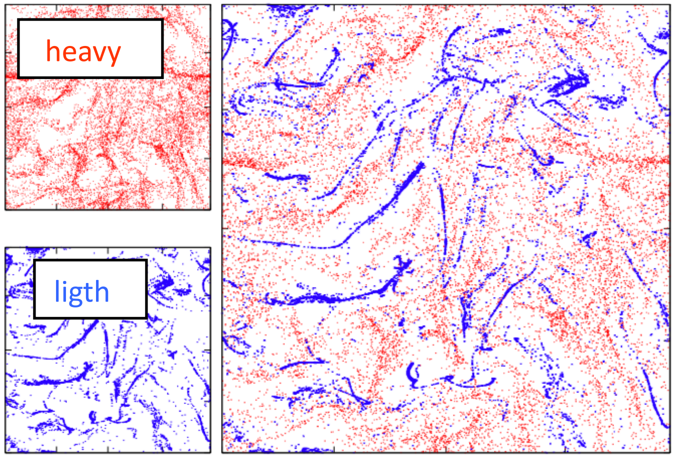


$$\boldsymbol{V}(\boldsymbol{x},t) \approx \boldsymbol{u}(\boldsymbol{x},t) + St(\beta-1)[\partial_t\boldsymbol{u}(\boldsymbol{x},t) + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}]$$

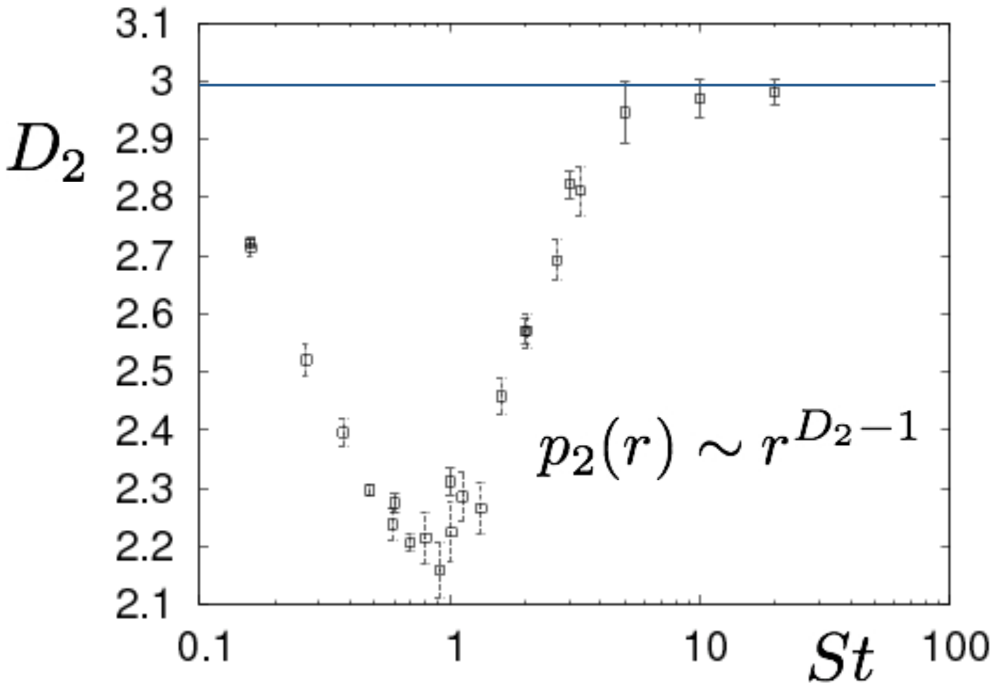
$$\boldsymbol{\nabla}\cdot\boldsymbol{V}(\boldsymbol{x},t) = St(\beta-1)\boldsymbol{\nabla}\cdot[\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}] = St(\beta-1)\sum_{ij}\left(\frac{\partial u_j}{\partial x_i}\right)\left(\frac{\partial u_i}{\partial x_j}\right)$$

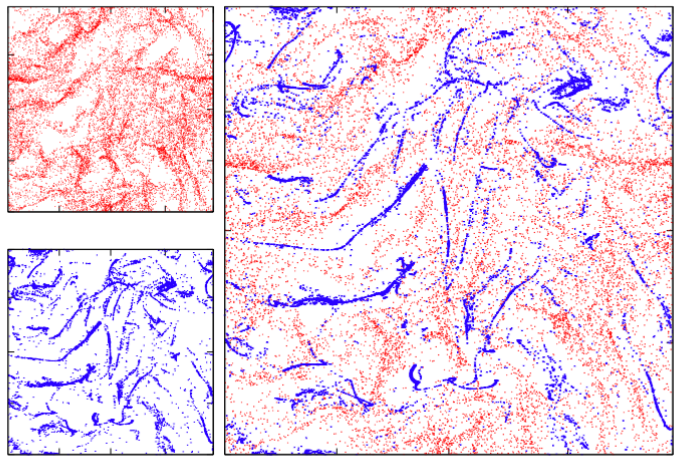
$$\beta < 1 \quad S^2 > \Omega^2 \Longrightarrow \boldsymbol{\nabla}\cdot\boldsymbol{V} < 0$$

$$\beta > 1 \quad \Omega^2 > S^2 \Longrightarrow \boldsymbol{\nabla}\cdot\boldsymbol{V} < 0$$



Preferential Concentration





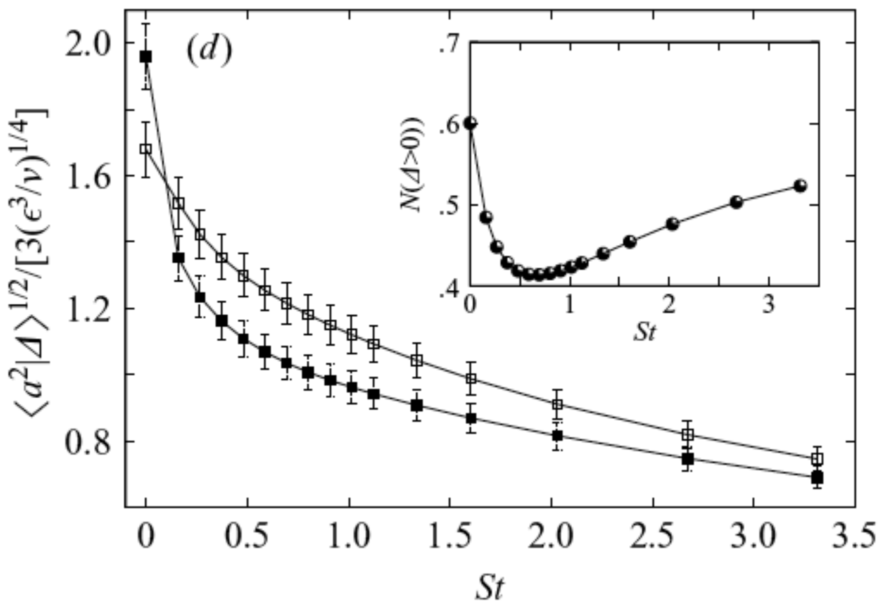
$$\Delta = \left(\frac{\det[\hat{\boldsymbol{\sigma}}]}{2}\right)^2 - \left(\frac{\mathrm{Tr}[\hat{\boldsymbol{\sigma}}^2]}{6}\right)^3$$

$\Delta \leq 0$

$\Delta > 0$

Okubo–Weiss parameter Q
is the determinant of the strain matrix

$$\sigma_{ij} = \frac{\partial u_i}{\partial x_j}$$



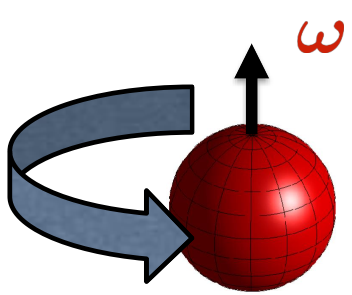
J. Bec et al. Phys. Rev. Lett. 98, 084502 (2007)

Spherical particle

Equations for velocity $\boldsymbol{v}$ and angular velocity $\boldsymbol{\omega}$ for small spherical particle at position $\boldsymbol{r}$: Happel & Brenner, Low Reynolds number hydrodynamics (1963)

$$\dot{\boldsymbol{v}} = \frac{1}{\tau_{\mathrm{p}}}\left[\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v}\right]$$

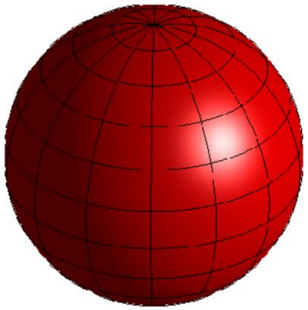
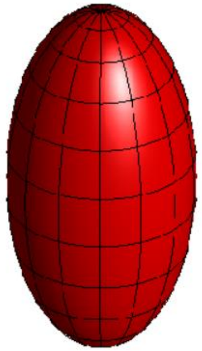
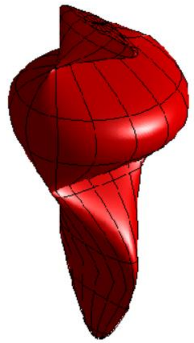
$$\dot{\boldsymbol{\omega}} = \frac{1}{\tau_{\mathrm{p}}}\left[\frac{10}{3}(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega})\right]$$



- $\boldsymbol{u}$ Fluid velocity
- $\boldsymbol{\Omega}$ Half fluid vorticity
- τ_{p} Particle relaxation time

Dynamics statistically invariant under rotations and reflections if $\boldsymbol{u}$ statistically invariant under rotations and reflections

Particle symmetries

Rotation invariance / Reflection invariance	✓	✗
✓		
✗	**'Isotropic helicoid'** (this talk)	

Example of an isotropic helicoid

Recipe from Lord Kelvin:

"An isotropic helicoid can be made by attaching projecting vanes to the surface of a globe in proper positions; for instance cutting at 45º each, at the middles of the twelve quadrants of any three great circles dividing the globe into eight quadrantal triangles."

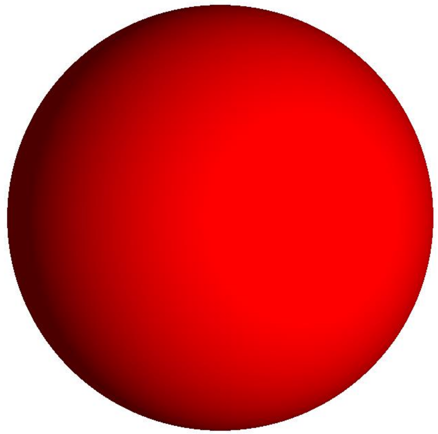
Kelvin, Phil. Mag. **42** (1871)

THES SIMPLEST (BUT NOT SIMPLER) GENERALISATION OF SPHERICAL HEAVY PARTCILES

Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

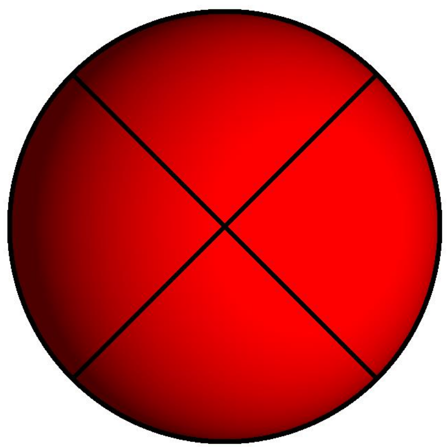
Start with a sphere



Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

✓ Start with a sphere
Draw 3 great circles

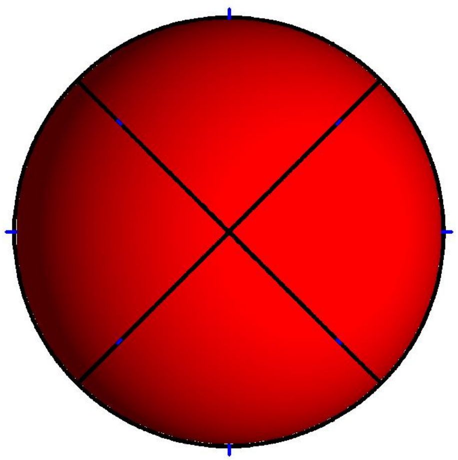


Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

- ✓ Start with a sphere
- ✓ Draw 3 great circles

Identify 12 vane positions at midpoints of quarter-arcs

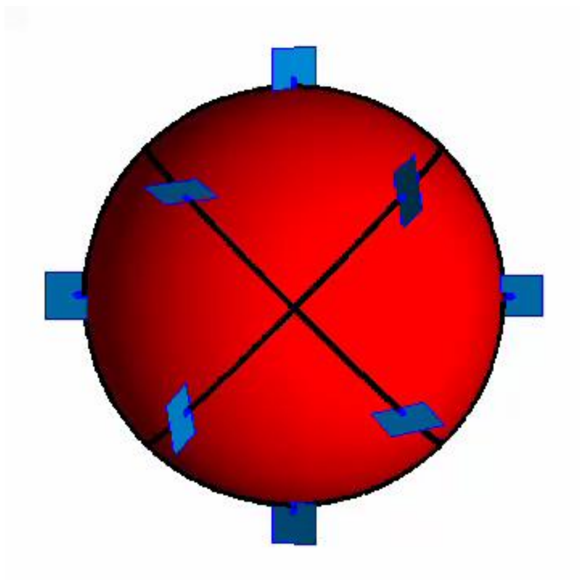


Example of an isotropic helicoid

Recipe from Lord Kelvin (1884)

- ✓ Start with a sphere
- ✓ Draw 3 great circles
- ✓ Identify 12 vane positions at midpoints of quarter-arcs

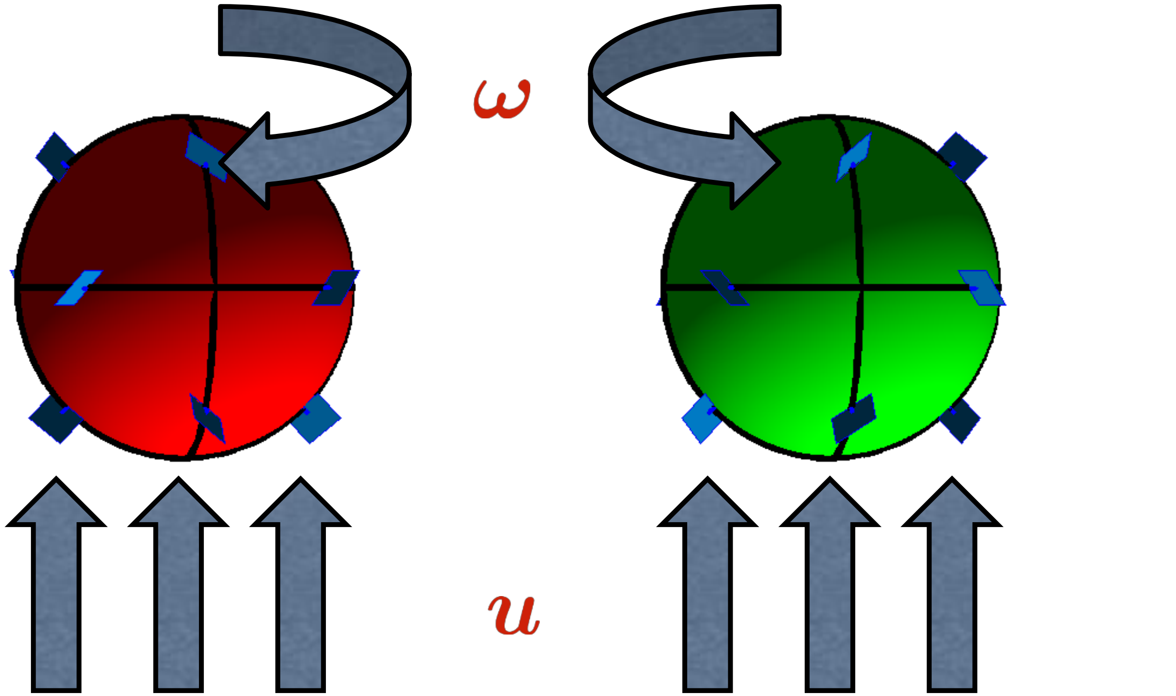
Put a vane on each vane position (45° to arc line)



Chirality

In a constant flow $\boldsymbol{u}$, the isotropic helicoid starts spinning around the flow direction with angular velocity $\boldsymbol{\omega}$.

The spinning direction depends on the chirality of the vanes.



Left handed $C_0 < 0$ *Right handed* $C_0 > 0$

Motion of an ‘isotropic helicoid’

Equations for velocity $\boldsymbol{v}$ and angular velocity $\boldsymbol{\omega}$ for small isotropic helicoid:

Happel & Brenner, Low Reynolds number hydrodynamics (1963)

$$\dot{\boldsymbol{v}} = \frac{1}{\tau_{\mathrm{p}}}\left[\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v} + \frac{2a}{9}C_0(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega})\right]$$

$$\dot{\boldsymbol{\omega}} = \frac{1}{\tau_{\mathrm{p}}}\left[\frac{10}{3}(\boldsymbol{\Omega}(\boldsymbol{r},t) - \boldsymbol{\omega}) + \frac{5}{9a}C_0(\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{v})\right]$$

Stokes’ law translation – rotation coupling (scalar)

$a = \sqrt{5I_0/(2m)}$ Particle ‘size’ (defined by mass m and moment of inertia I_0)

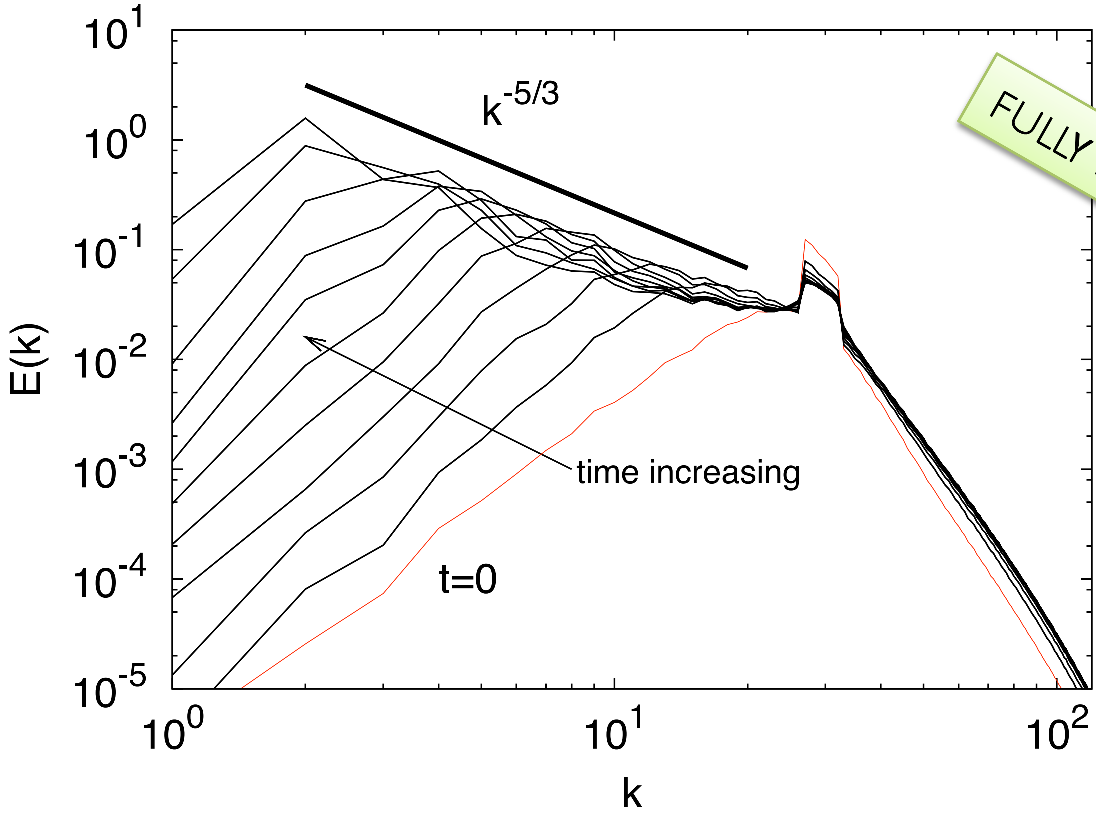
C_0 Helicoidality

Ratio of rotational and translational inertia fixed to that of sphere

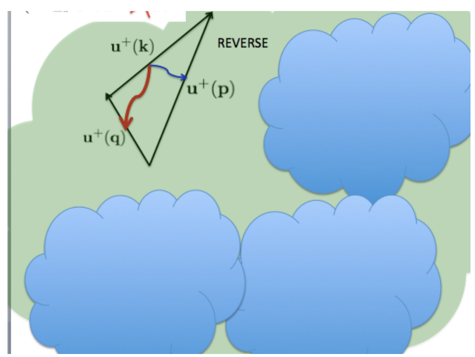
Equations break spatial reflection symmetry (ω pseudovector)

REGIONS WITH STRONGLY MULTI-SCALE HELICAL CHOERENCY -> REVERT ENERGY CASCADE

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$



FULLY 3D AND ISOTORPIC



L.B., S. MUSACCHIO & F. TOSCHI Phys. Rev. Lett. 108 164501, 2012.

Dimensionless parameters

Stokes number $\mathrm{St} \equiv \frac{\tau_{\mathrm{p}}}{\tau_\eta}$ Size $\overline{a} \equiv \frac{a}{\eta}$ Helicoidality C_0

with τ_η and η smallest time- and length scales of flow.

Dynamics may grow indefinitely unless $-\sqrt{27} < C_0 < \sqrt{27}$.

St and $\overline{a}$ constrained by particle density higher than that of the fluid and geometrical size must be smaller than η .

Simulations and theory is done using a random single-scale flow characterised by the Kubo number

$$\mathrm{Ku} \equiv \frac{u_0 \tau_\eta}{\eta}$$

with u_0 typical speed of flow.

Clustering at small St

Expand compressibility of particle-velocity field $\boldsymbol{\nabla} \cdot \boldsymbol{v}$ in small $\mathrm{St} \sim \tau_{\mathrm{p}}$

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = -\frac{27}{27 - C_0^2} \tau_{\mathrm{p}} \left[\mathrm{Tr}(\boldsymbol{\nabla u}^{\mathrm{T}} \boldsymbol{\nabla u}^{\mathrm{T}}) - \frac{1}{15} \mathrm{aC_0 Tr}(\boldsymbol{\nabla u}^{\mathrm{T}} \boldsymbol{\nabla \Omega}^{\mathrm{T}}) \right]$$

Centrifuge effect with modified amplitude
Maxey, J. Fluid Mech. **174** (1987)

Term due to parity breaking of system

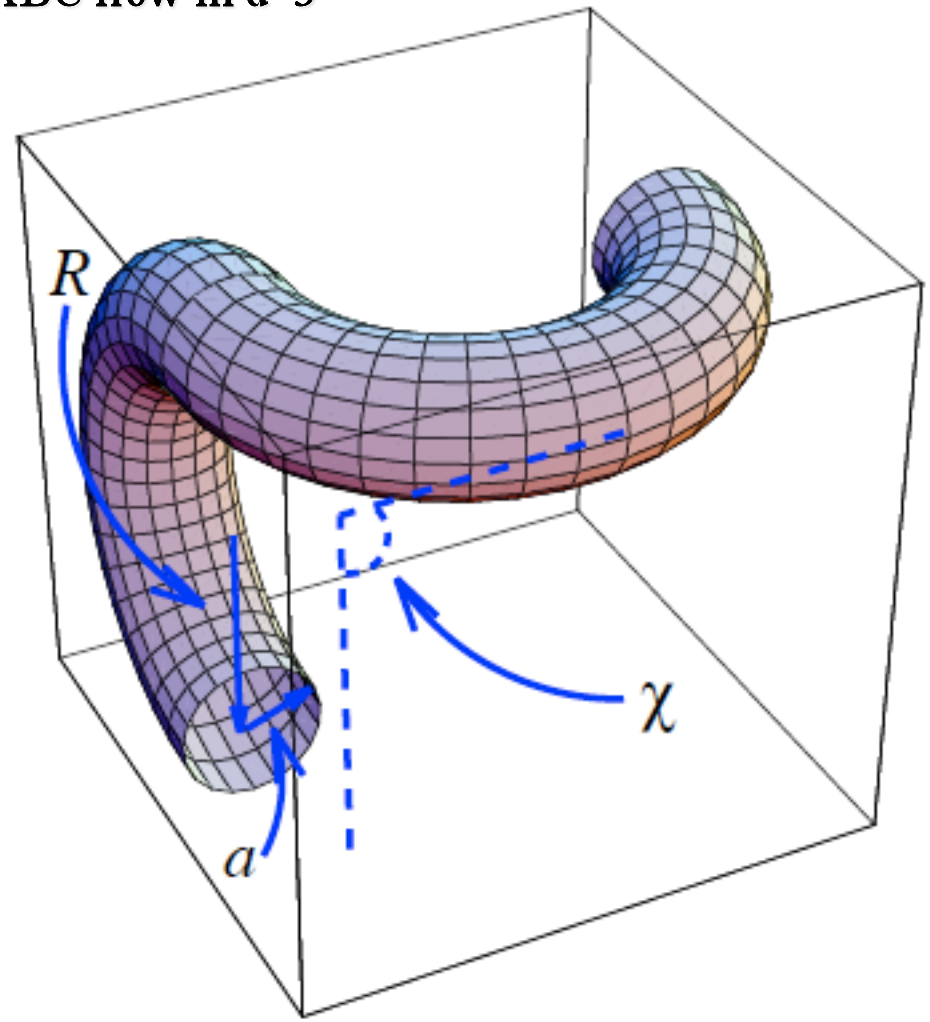
Reflection-invariant systems have $\langle \mathrm{Tr}(\boldsymbol{\nabla u}^{\mathrm{T}} \boldsymbol{\nabla \Omega}^{\mathrm{T}}) \rangle = 0$

Isotropic helicoids violate that relation $\langle \mathrm{Tr}(\boldsymbol{\nabla u}^{\mathrm{T}} \boldsymbol{\nabla \Omega}^{\mathrm{T}}) \rangle \propto \tau_{\mathrm{p}} \mathrm{C_0}$

$\Rightarrow$ In a parity-invariant isotropic flow clustering does not depend on sign of C_0

Eulerian smooth but Lagrangian non-trivial

ABC flow in d=3



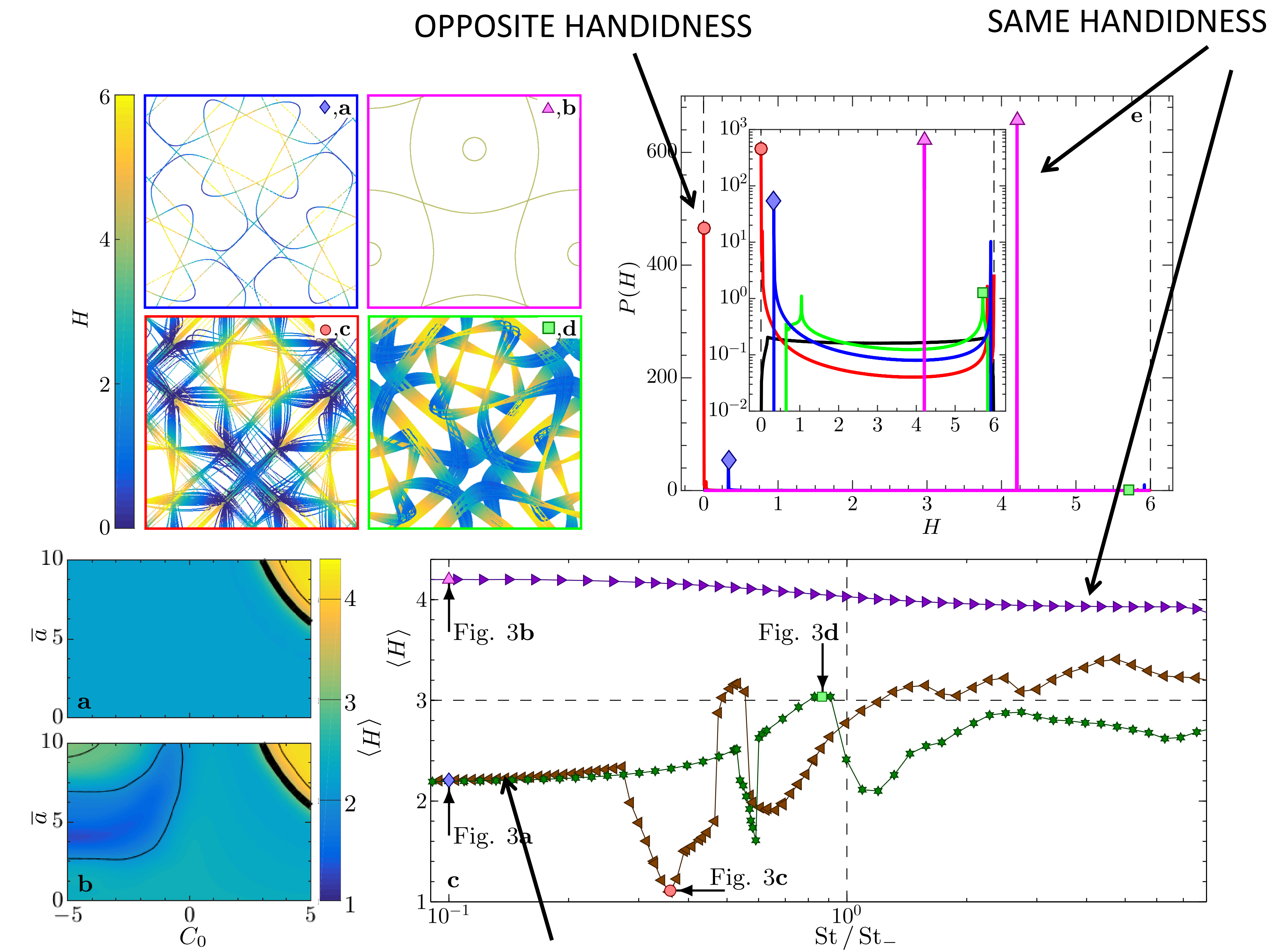
$$\begin{aligned} \dot{x} &= A \sin z + C \cos y, \\ \dot{y} &= B \sin x + A \cos z, \\ \dot{z} &= C \sin y + B \cos x. \end{aligned}$$

$$\mathbf{v} \parallel \omega$$

Exact stationaty solution of Euler equation

$$\partial_i v_i \propto -\mathrm{Tr}[\mathbb{A}^2](27 - \frac{9\bar{a}C_0}{10})\,.$$

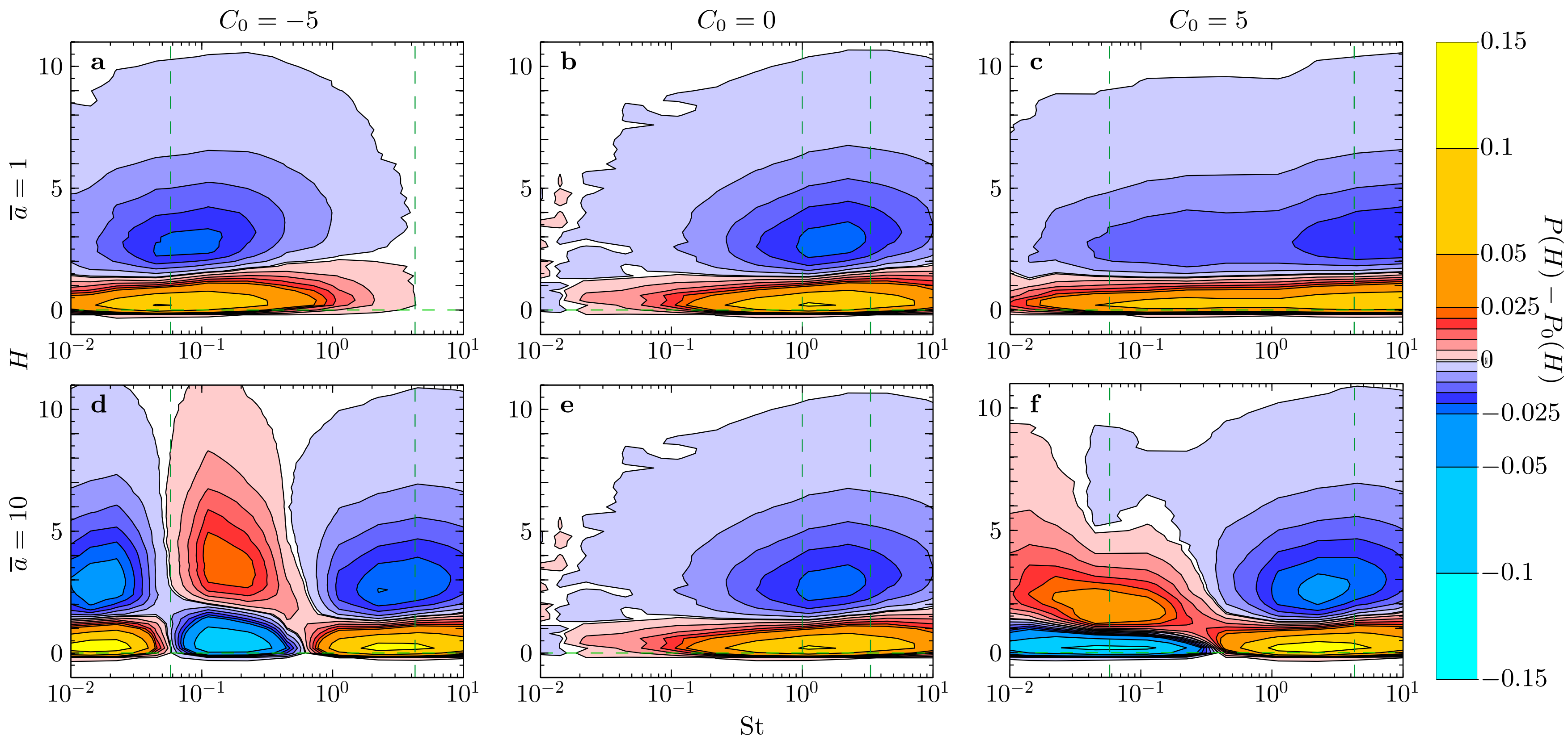
HELICOIDS MIGHT BEHAVE AS LIGHT OR HEAVY PARTICLES !!!

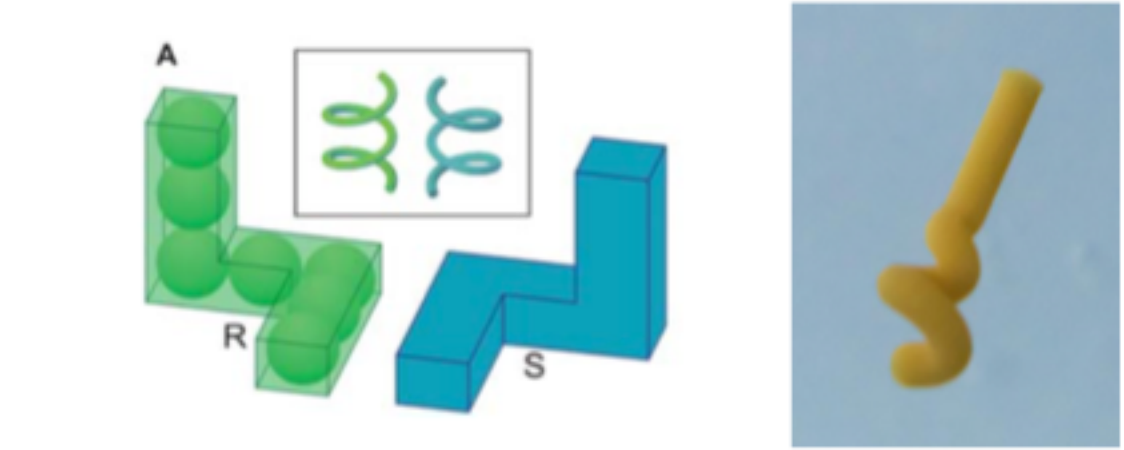


$$\partial_i v_i \propto -\mathrm{Tr}[\mathbb{A}^2](27 - \frac{9\overline{a}C_0}{10})\,.$$

STOCHASTIC HELICAL FLOW

$$P_{\mathrm{M}_H}(H) = \frac{|H| \exp\left[\frac{\alpha H \, \mathrm{M}_H}{\beta + \gamma \, \mathrm{M}_H^2}\right] K_1 \left[\frac{\delta |H|}{\beta + \gamma \, \mathrm{M}_H^2}\right]}{\sqrt{\beta + \gamma \, \mathrm{M}_H^2}}$$





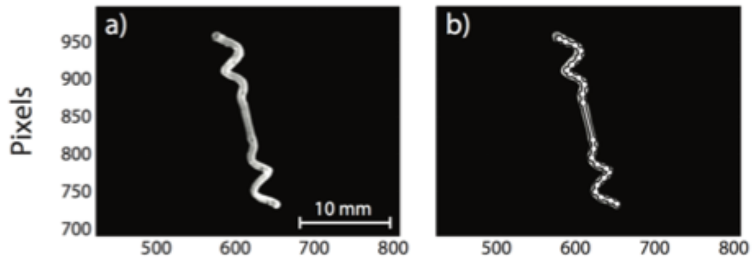
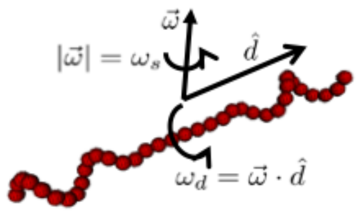


Figure 5: An example of a complex particle, a "strain probe" is basically a chiral-dipole and is sensitive to the local strain level in turbulence. This type of 3D printed particles have been designed and tracked for both position and orientation in turbulent flows by means of optical techniques [20], [24]. Similarly shaped particles were studied numerically by means of Stokesian dynamics simulations [25] (see Figure 6).



S. Kramel, S. Tympel, F. Toschi, and G. A. Voth, "Preferential rotation of chiral dipoles in isotropic turbulence."

Conclusions

Isotropic helicoids are rotation invariant particles which break reflection invariance (two chiralities)

Coupling between translational and rotational degrees of freedom changes dynamics compared to spherical particles (modified clustering, preferential sampling etc.)

The two chiralities may show different dynamics if the particle size is not too small and flow is persistent

Flows with broken parity invariance increase the differences in the dynamics of the two particles

K. GUSTAFSSON AND L.B. PHYS. REV. FLUIDS (IN PRESS) 2016.