TURBULENT AT HIGH AND LOW ROTATION RATES: EULERIAN AND LAGRANGIAN STATISTICS

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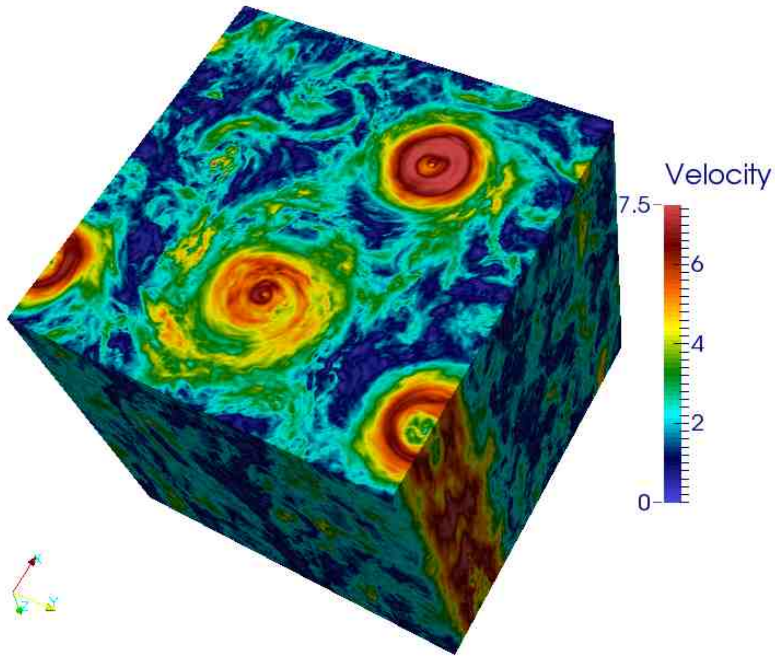
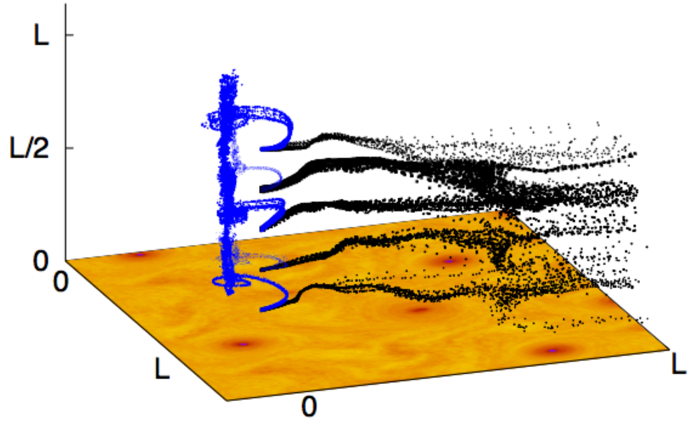
ICTAM2016





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PRACE 09_2256
ROTATING TURBULENCE
2015 – 55MH

NAVIER_STOKES EQS IN A ROTATING FRAME (NO BOUNDARIES)

DNS: A. Pouquet, P. Mininni, A. Alexakis, S. Chen, G. Eyink ....

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{\Omega} \times \mathbf{v} = \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{F} - \alpha \mathbf{v}$$

$\mathbf{\Omega}$ =rotation

$$P = P_0 + \frac{1}{2} |\mathbf{\Omega} \times \mathbf{r}|^2$$

$\mathbf{F}$ =large scale Forcing

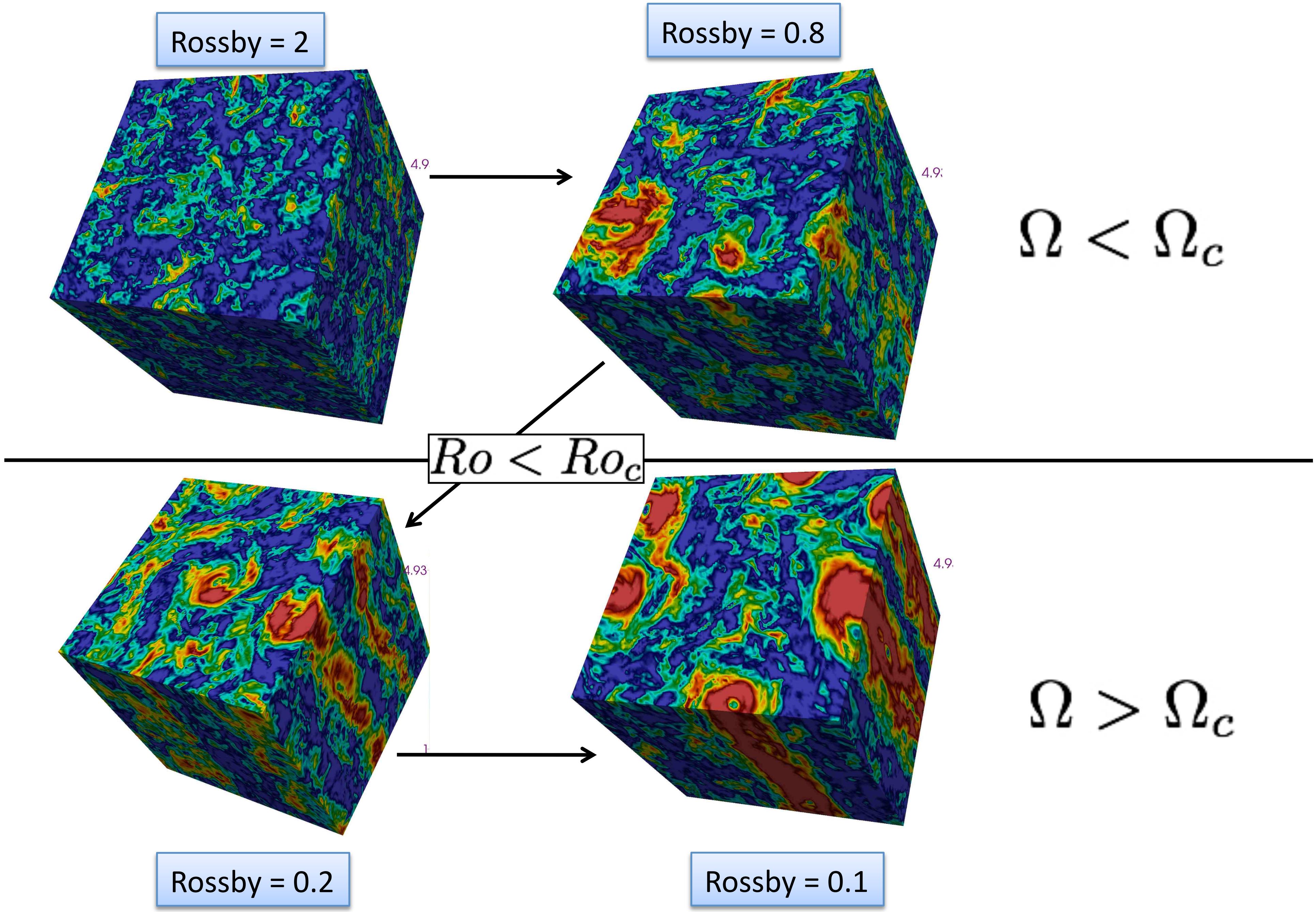
$\alpha =$ large scale energy sink

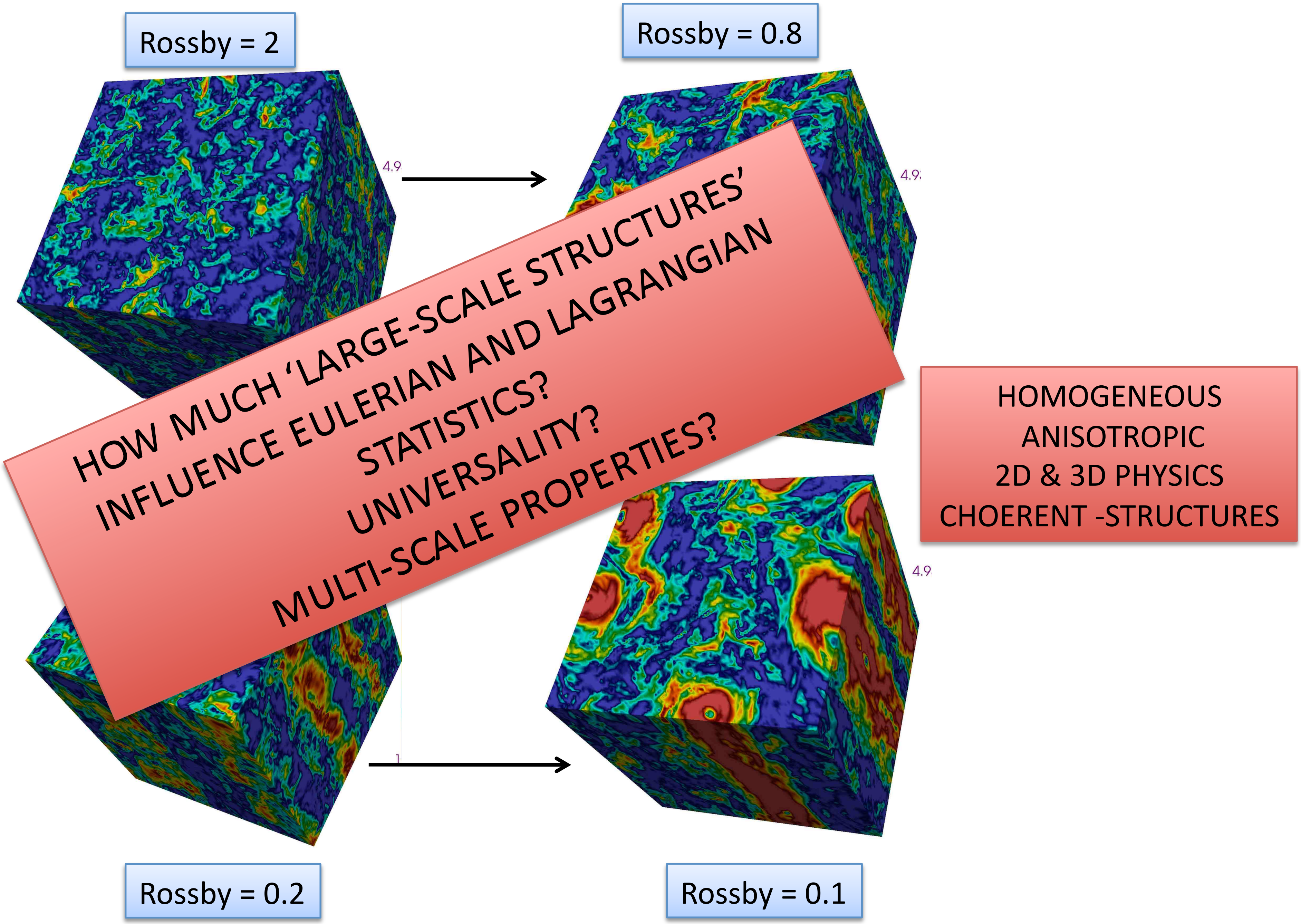
ROSSBY NUMBER ~ NON-LINEAR/ROTATION

$$\mathrm{Ro} \sim \frac{v_0}{\Omega L_0}$$

$\mathrm{Ro} \geq Ro_c \rightarrow$ FORWARD ENERGY TRANSFER

$\mathrm{Ro} \leq Ro_c \rightarrow$ FORWARD & BACKWARD ENERGY TRANSFER


Rossby = 2
Rossby = 0.8
$\Omega < \Omega_c$
$Ro < Ro_c$
$\Omega > \Omega_c$
Rossby = 0.2
Rossby = 0.1



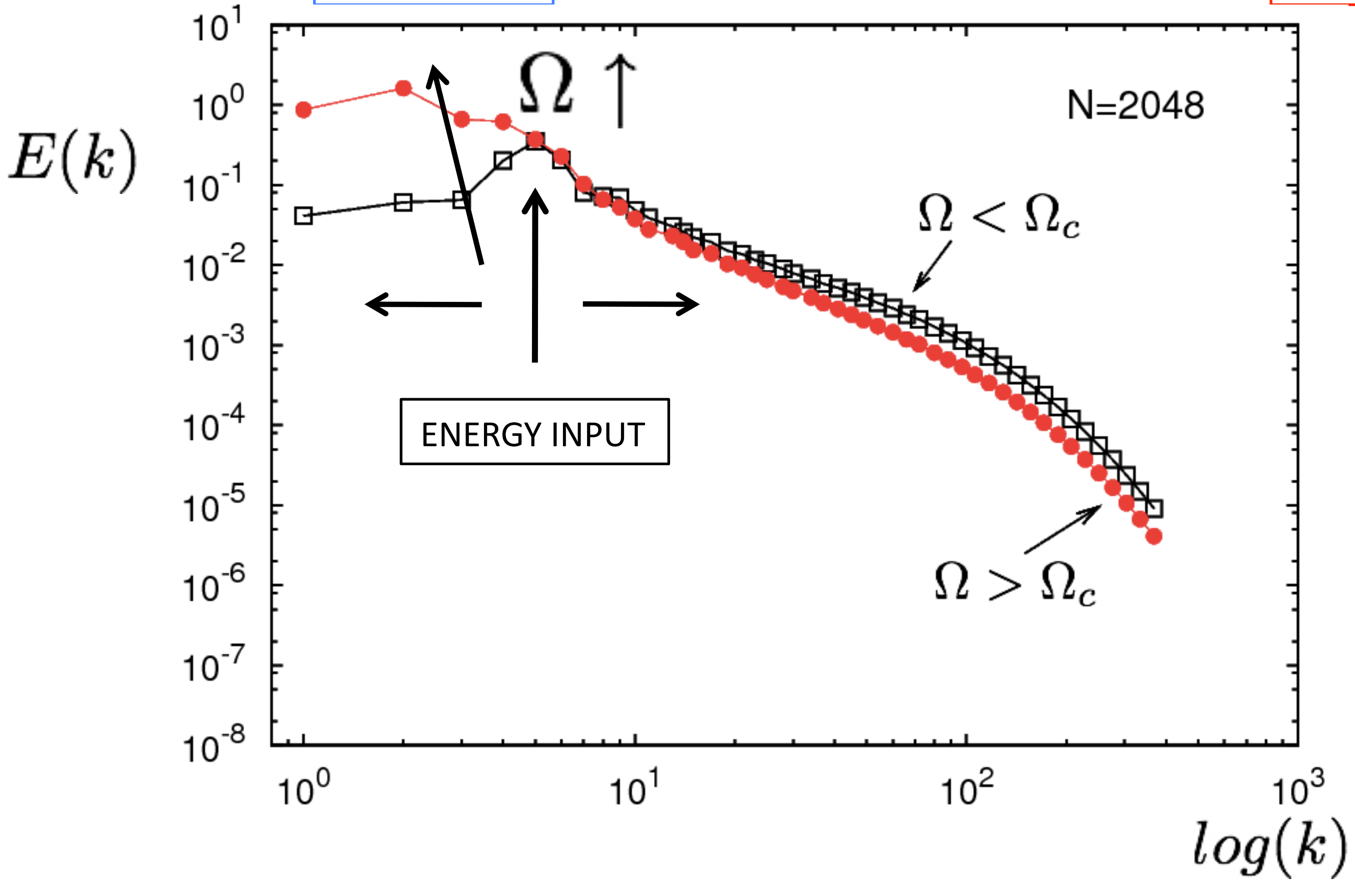
Rossby = 2
Rossby = 0.8
Rossby = 0.2
Rossby = 0.1
HOW MUCH ‘LARGE-SCALE STRUCTURES’ INFLUENCE EULERIAN AND LAGRANGIAN STATISTICS?
UNIVERSALITY?
MULTI-SCALE PROPERTIES?
HOMOGENEOUS
ANISOTROPIC
2D & 3D PHYSICS
CHOERENT -STRUCTURES

HOMOGENEOUS-ANISOTROPIC

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{\Omega} \times \mathbf{v} = \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{F} - \alpha \mathbf{v}$$

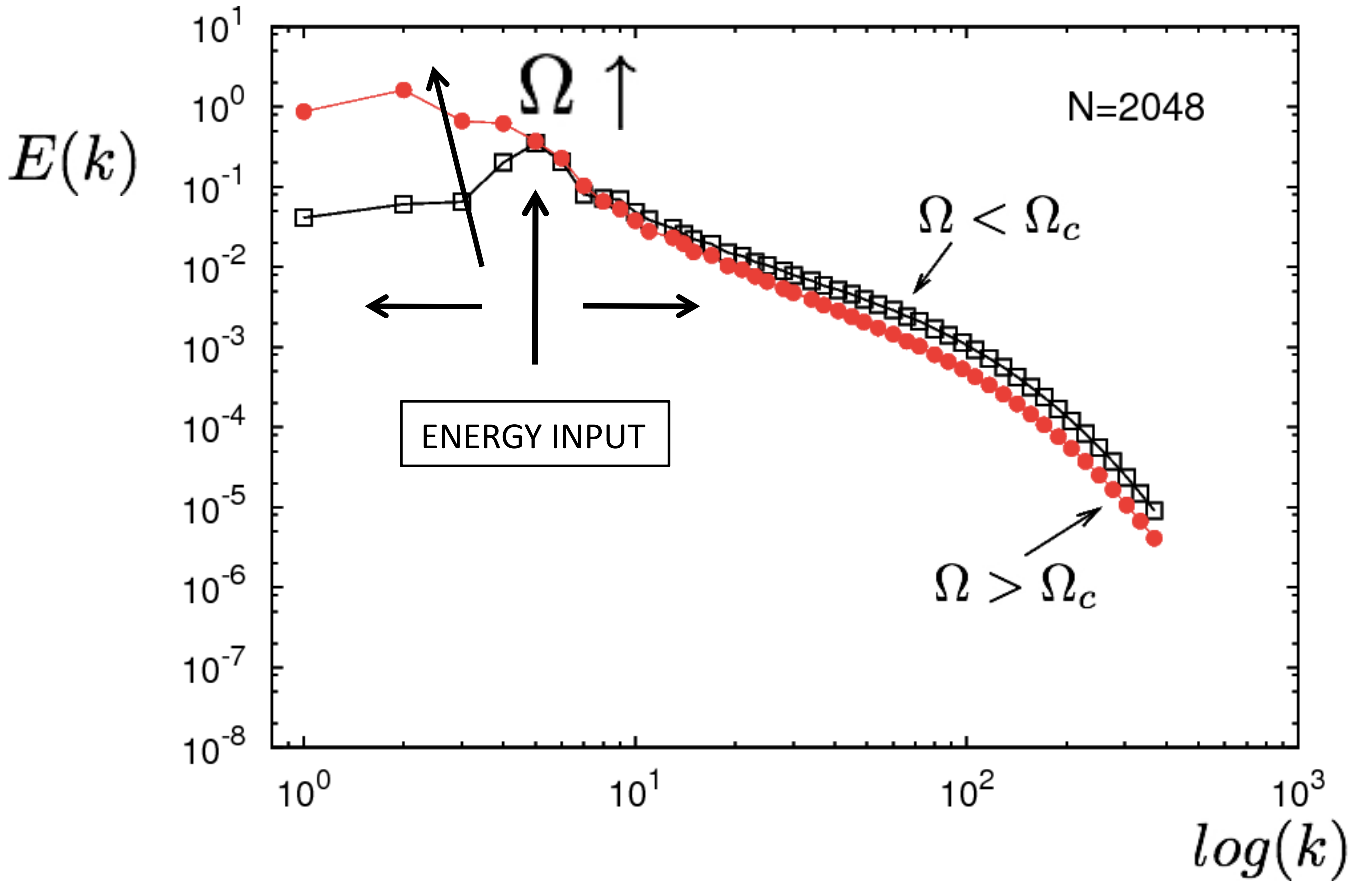
ROTATION

DAMPING:



FORCING: 2°-order OU-PROCESSS: ISOTROPIC, HOMOGENEOUS **NOT** DELTA-CORRELATED

WHAT ABOUT THE EFFECTS OF THE LARGE-SCALE COHERENT STRUCTURES ON THE EULERIAN AND LAGRANGIAN STATISTICS?



OUR DNS DATA-BASE (EULERIAN + LAGRANGIAN)

NEW FEATURES:

1) IDEAL FORCING MECHANISM (AS NEUTRAL AS POSSIBLE: ISOTROPIC; NON HELICAL, TIME-COLORED) + LARGE SCALE FRICTION
2) UNPRECEDENTED NUMERICAL RESOLUTION/SCALE SEPARATION (**UP TO 4096^3**)
3) LAGRANGIAN STATISTICS (MILLIONS OF TRACERS AND INERTIAL PARTICLES)

N	Ω	ν	ϵ	ϵ_f	u_0	η/dx	τ_η/dt	Re_λ	Ro	f_0	τ_f	T_0	α
1024	4	7×10^{-4}	1.2	1.2	1.05	0.67	120	150	0.78	0.02	0.023	0.17	0.0
1024	10	6×10^{-4}	0.46	0.59	1.6	0.76	294	580	0.24	0.02	0.023	0.25	0.1
2048	4	2.8×10^{-4}	1.2	1.2	1.05	0.67	380	230	0.76	0.02	0.023	0.17	0.0
2048	10	2.2×10^{-4}	0.45	0.64	1.7	0.72	550	1170	0.25	0.02	0.023	0.3	0.1
4096	10	1×10^{-4}	0.46	0.65	1.7	0.78	1010	1600	0.25	0.02	0.023	0.3	0.1

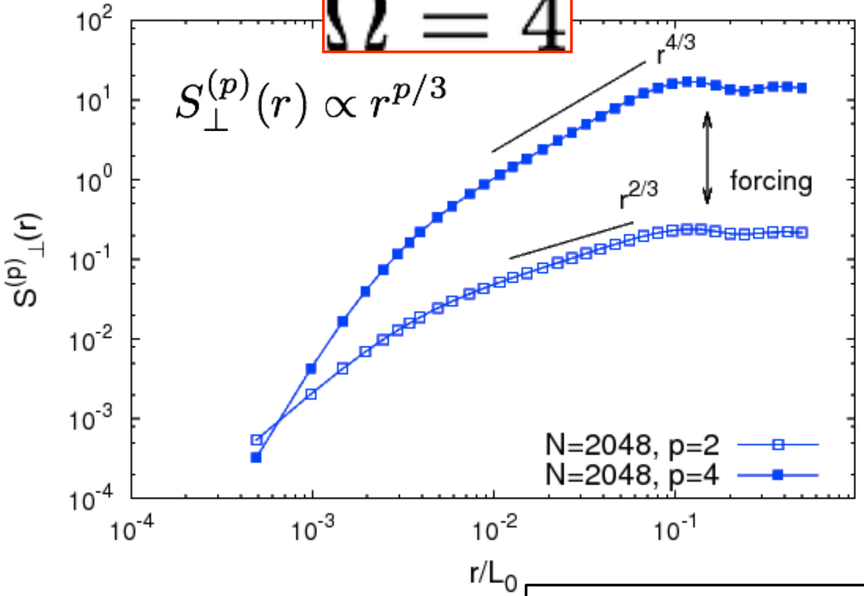
TABLE I: Eulerian dynamics parameters. N: number of collocation points per spatial direction; Ω: rotation rate; ν: kinematic viscosity; $\epsilon = \nu \int d^3x \sum_{ij} (\nabla_i u_j)^2$: viscous energy dissipation; $\epsilon_f = \int d^3x \sum_i f_i u_i$: energy injection; $u_0 = 1/3 \int d^3x \sum_i u_i^2$: mean kinetic energy; $\eta = (\nu^3/\epsilon)^{1/4}$: Kolmogorov dissipative scale; $dx = L_0/N$: numerical grid spacing; $L_0 = 2\pi$: box size; $\tau_\eta = (\nu/\epsilon)^{1/2}$: Kolmogorov dissipative time; $Re_\lambda = (u_0 \lambda)/\nu$: Reynolds number based on the Taylor micro-scale; $\lambda = (15 \nu u_0^2/\epsilon)^{1/2}$: Taylor micro-scale; $Ro = (\epsilon_f k_f)^{1/3}/\Omega$: Rossby number defined in terms of the energy injection properties, where $k_f = 5$ is the wavenumber where the forcing is acting; f_0: intensity of the Ornstein-Uhlenbeck forcing; τ_f: decorrelation time of the forcing; $T_0 = u_0/L_0$: Eulerian large-scale eddy turn over time; α: coefficient of the damping term $\alpha \Delta^{-1} \boldsymbol{u}$.

MAX RESOLUTION

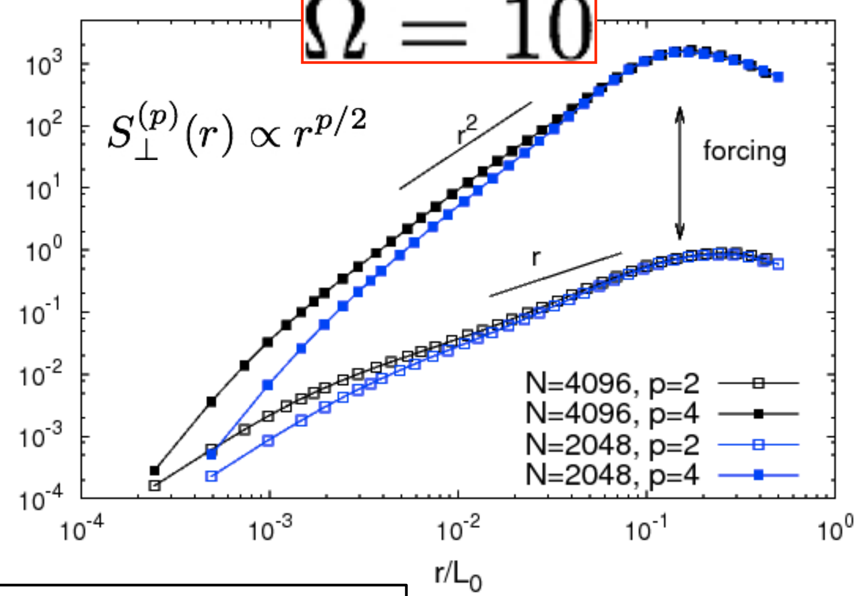
$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{\Omega} \times \mathbf{v} = \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{F} - \alpha \mathbf{v}$$

$$S_{\perp}^{(p)}(r) = \langle (\delta u(r)_{\perp})^p \rangle$$

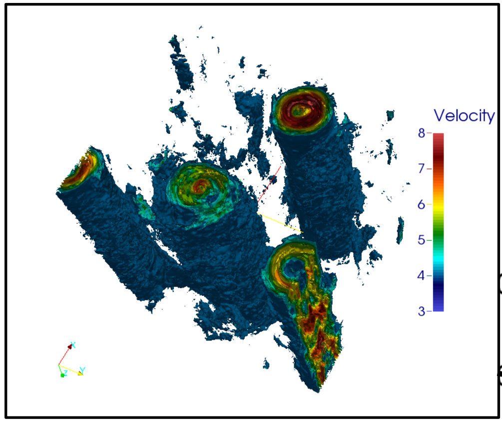
$\Omega = 4$

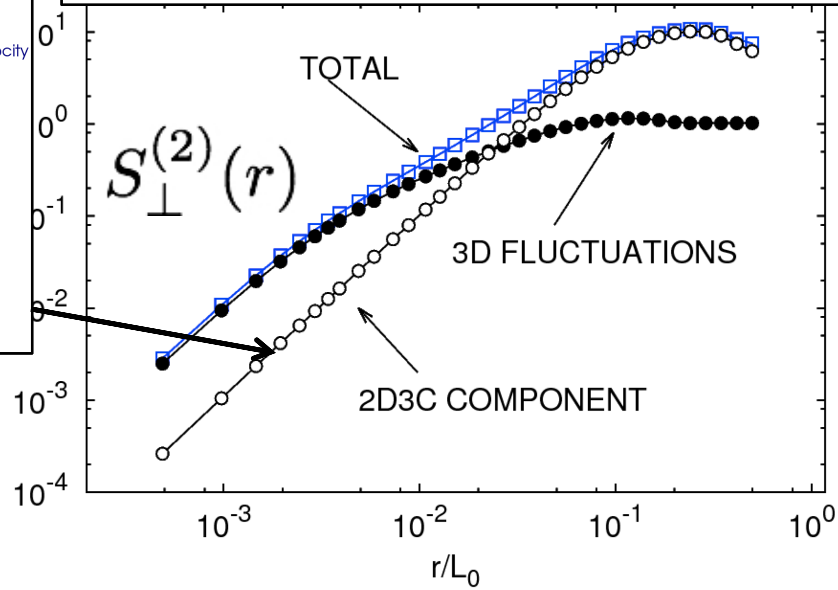


$\Omega = 10$



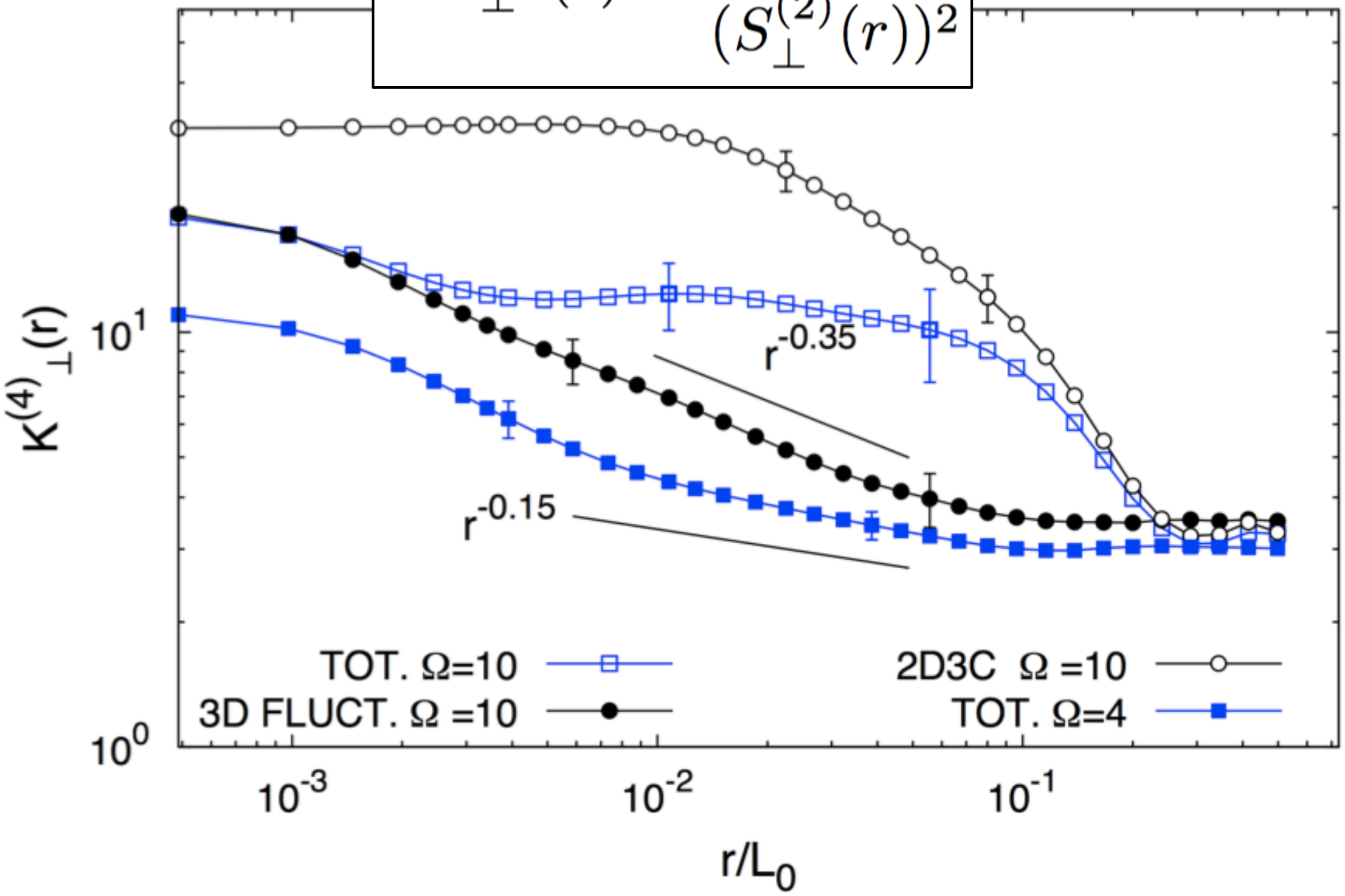
$$\boldsymbol{u}(x, y, z|t) = \boldsymbol{u}_{2D}(y, z|t) + \boldsymbol{u}'(x, y, z|t)$$





FLUCTUATIONS: FLATNESS

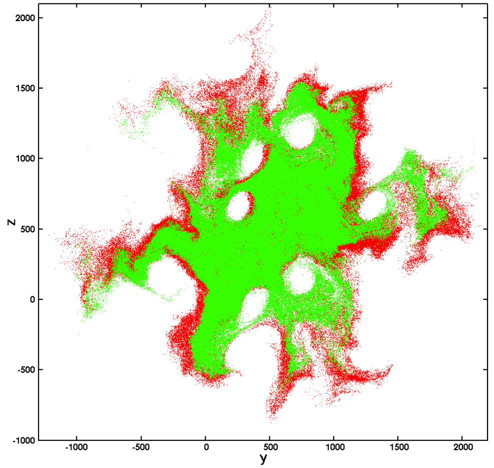
$$K_{\perp}^{(4)}(r) \equiv \frac{S_{\perp}^{(4)}(r)}{(S_{\perp}^{(2)}(r))^2}$$



- NON-GAUSSIAN PROPERTIES DEPEND ON THE WAY YOU DECOMPOSE THE FIELD
- AFTER FILTERING THE 2D3C COMPONENT: SCALING PROPERTIES ARE BACK (BUT NOT HIT!)

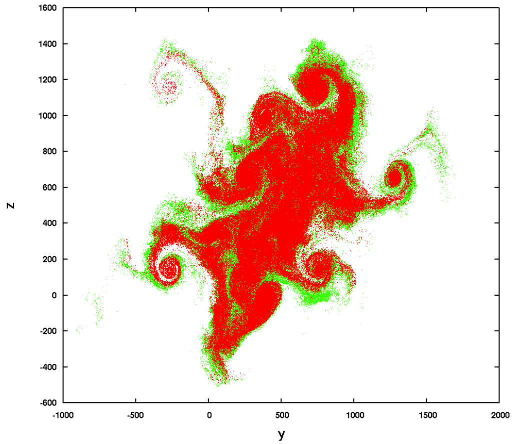
RMS FORCES ALONG TRAJECTORIES

$$\frac{d\mathbf{v}}{dt} = \beta\frac{D\mathbf{u}}{Dt} - \frac{1}{\tau_p}(\mathbf{v}-\mathbf{u}) + 2(\mathbf{v}-\beta\mathbf{u})\times\boldsymbol{\Omega} - (1-\beta)\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})$$



HEAVY

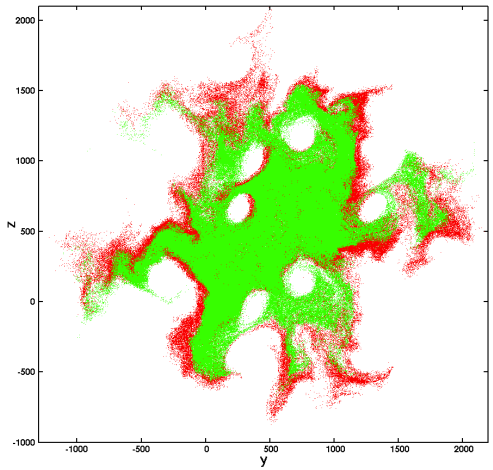
$$\begin{cases} a_{rms}^{tot}(t) = \langle \dot{v}^2 \rangle; & \text{total} \\ a_{rms}^{am}(t) = \beta^2 \langle (D_t\mathbf{u})^2 \rangle; & \text{added mass} \\ a_{rms}^{St}(t) = 1/\tau_p^2 \langle (\mathbf{v}-\mathbf{u})^2 \rangle; & \text{Stokes drag} \\ a_{rms}^{Co}(t) = 4\langle [\boldsymbol{\Omega}\times(\mathbf{v}-\beta\mathbf{u})]^2 \rangle; & \text{Coriolis} \\ a_{rms}^{Cp}(t) = (1-\beta)^2 \langle [\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times(\mathbf{r}_t-\mathbf{r}_0)]^2 \rangle; & \text{centripetal.} \end{cases}$$



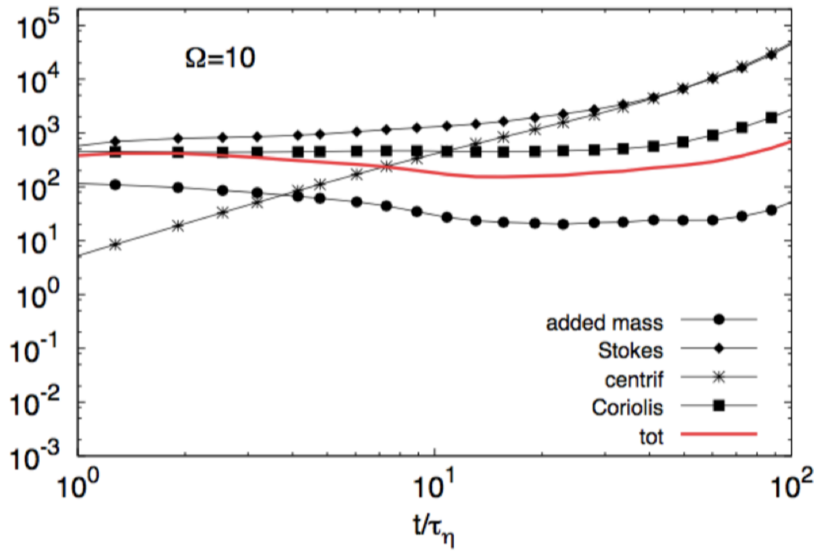
LIGHT

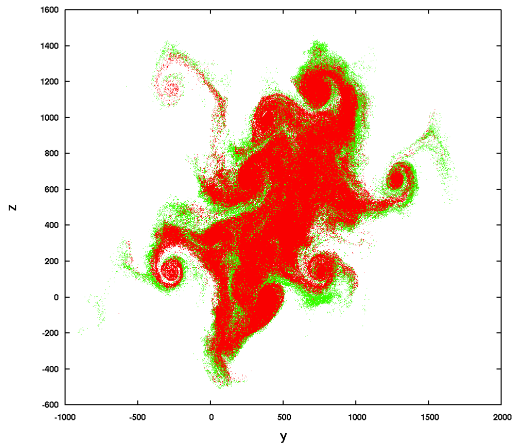
RMS FORCES ALONG TRAJECTORIES

$$\frac{d\mathbf{v}}{dt} = \beta\frac{D\mathbf{u}}{Dt} - \frac{1}{\tau_p}(\mathbf{v}-\mathbf{u}) + 2(\mathbf{v}-\beta\mathbf{u})\times\mathbf{\Omega} - (1-\beta)\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})$$

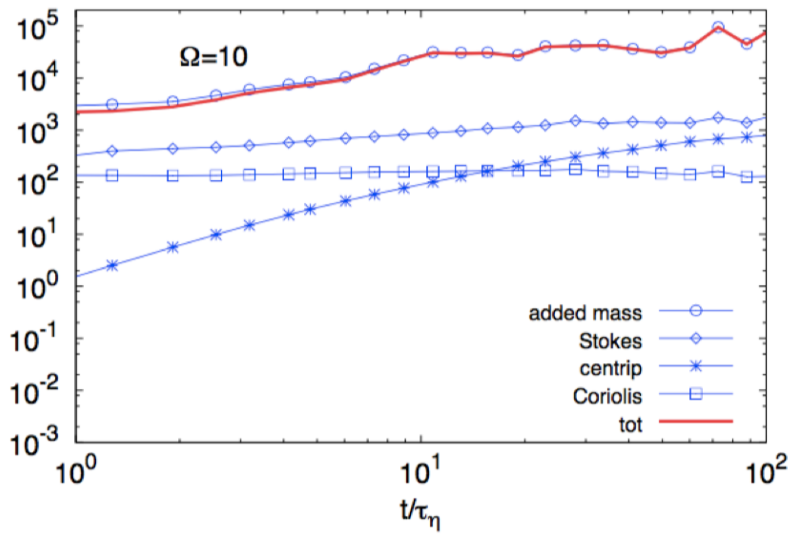


HEAVY

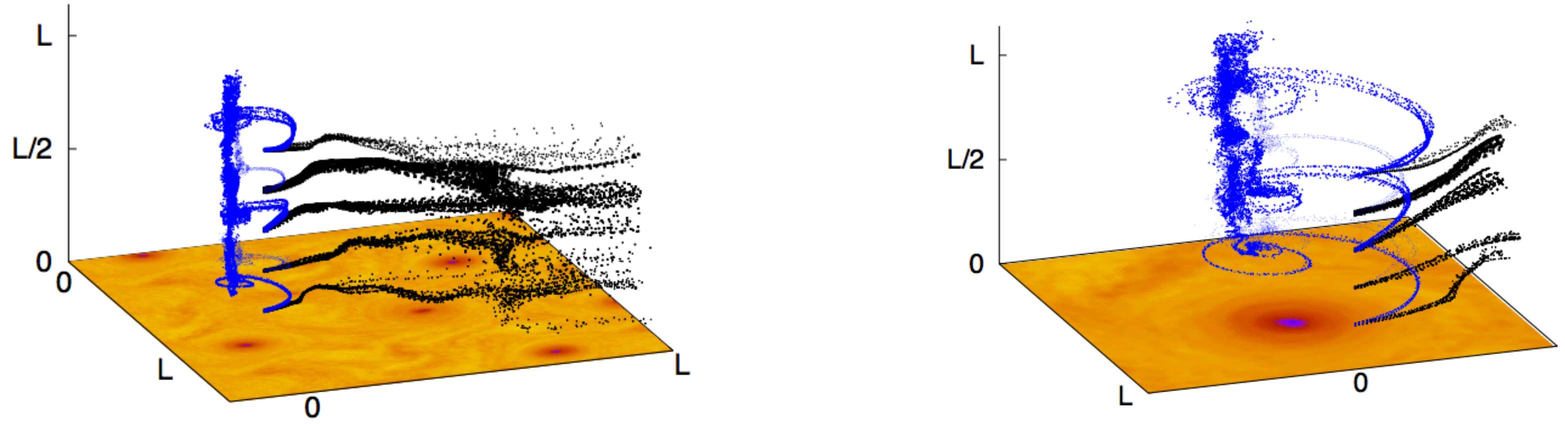




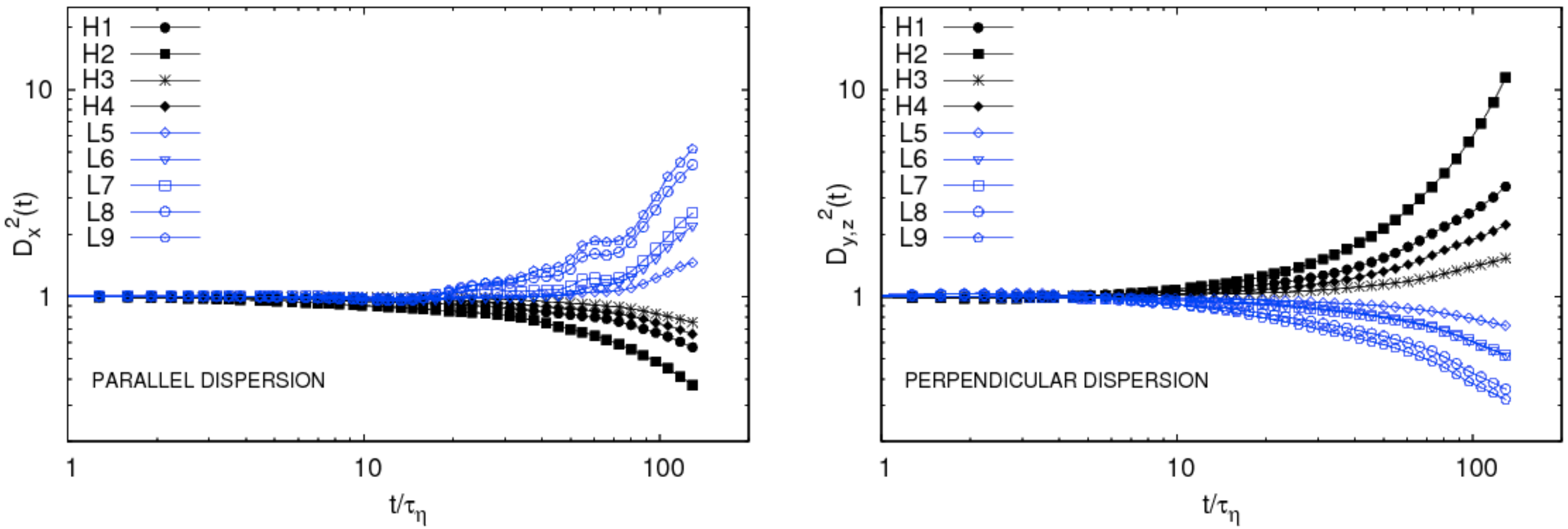
LIGHT



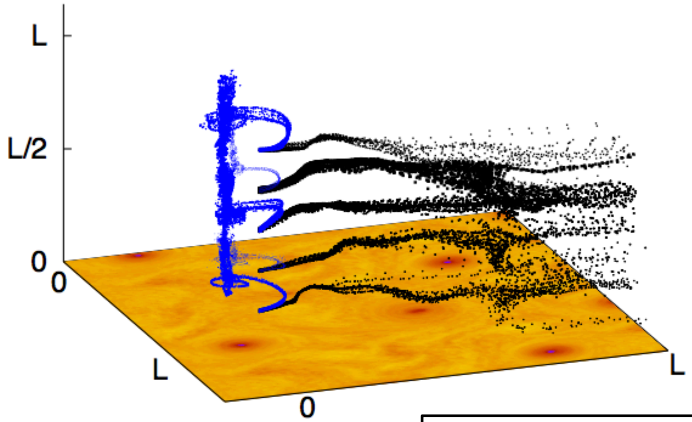
INERTIA: SINGULAR EFFECT ON SINGLE PARTICLE DISPERSION

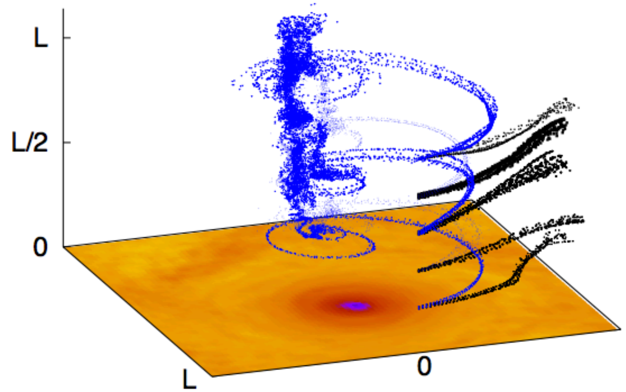


$$D^{i}_{St,\beta}(t) = \frac{\langle (r^i_t - r^i_0)^2 \rangle_{St,\beta}}{\langle (r^i_t - r^i_0)^2 \rangle_{tracer}}$$

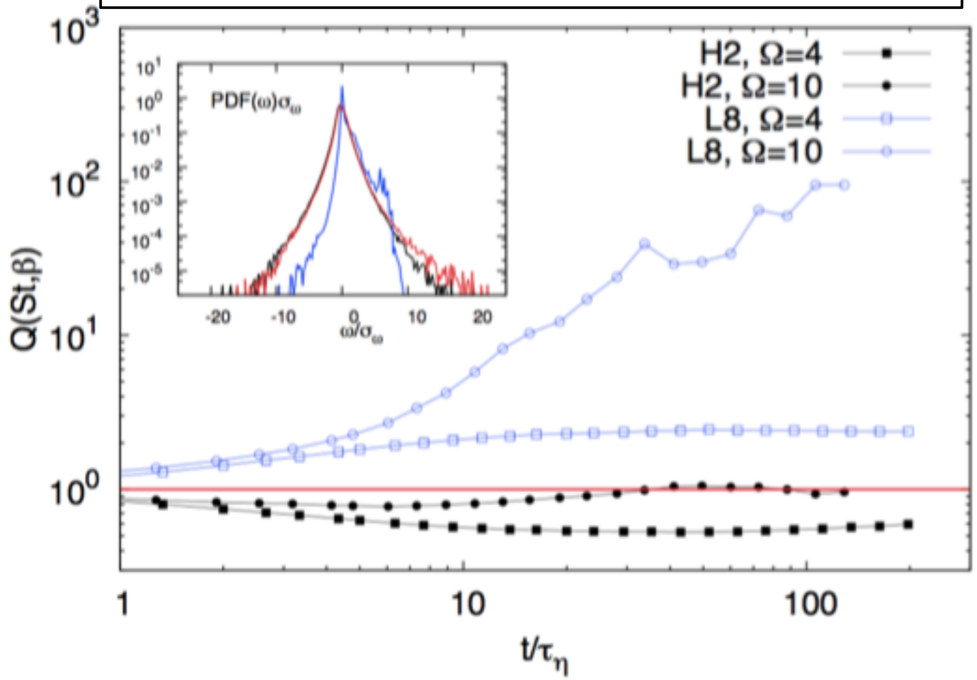


INERTIA: SINGULAR EFFECT ON SINGLE PARTICLE DISPERSION





$$Q_{St,\beta}(|t) = \frac{\langle [w_x(\mathbf{r}_t, t)]^2 \rangle_{\beta,St}}{\langle [w_x(\mathbf{r}_t, t)]^2 \rangle_{tracer}} .$$



CONCLUSIONS

-HIGH RESOLUTION ROTATING TURBULENCE: FIRST ATTEMPT TO CONTROL SIMULTANEOUSLY EULERIAN & LAGRANGIAN STATISTICS

-IDEAL SET-UP (1): HOMOGENEOUS AND ISOTROPIC TIME-COLORED FORCING

-IDEAL SET-UP (2): SCALE-SEPARATION

-STRONG INFLUENCE OF LARGE-SCALE (NON-UNIVERSAL?) VORTICAL STRUCTURES

-DEPARTURE FROM GAUSSIANITY (DEPENDING ON HOW YOU MEASURE IT: 2D3C-3D3D)

-EFFECTS OF LARGE-SCALE STRUCTURES ON PARTICLES' DISPERSION

-DISENTANGLING INVERSE CASCADES IN TERMS OF HELICAL-FOURIER INTERACTIONS

L.B., F. Bonaccorso, I. Mazzitelli, A. Lanotte, S. Musacchio, P. Perlekar, F. Toschi and M. Hinsberg, PRX submitted 2016