On the direct and inverse energy transfer in 2-dimensional and 3-dimensional turbulent flows

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- Energy and enstrophy are conserved in 2D Navier-Stokes equations.
- Forward cascade of energy is blocked, since enstrophy is positive and definite. (Boffetta Ann. Rev. Fluid Mech 2012)
- Energy and Helicity are invariants of 3D Navier-Stokes equations.
- Both cascades forward, from large scales to small scales. (Chen, Phys. Fluids 2003)
- Helicity could be positive or negative.

What happens when we change the relative weight between positive and negative helicity modes?

Following Waleffe Phys. Fluids (1992)

$$\mathbf{u}(\mathbf{k},t) = a^{+}(\mathbf{k},t)\mathbf{h}^{+}(\mathbf{k}) + a^{-}(\mathbf{k},t)\mathbf{h}^{-}(\mathbf{k})$$

where $\mathbf{h}^{\pm}(\mathbf{k})$ are the eigenvectors of the curl operator $i\mathbf{k} \times \mathbf{h}^{\pm}(\mathbf{k}) = \pm k\mathbf{h}^{\pm}(\mathbf{k})$.
Projection operator:

$$\mathcal{P}^{\pm}(\mathbf{k}) \equiv \frac{\mathbf{h}^{\pm}(\mathbf{k}) \otimes \mathbf{h}^{\pm}(\mathbf{k})^{*}}{\mathbf{h}^{\pm}(\mathbf{k})^{*} \cdot \mathbf{h}^{\pm}(\mathbf{k})}$$

$$\mathbf{u}^{\pm}(\mathbf{k},t) = \mathcal{P}^{\pm}(\mathbf{k})\mathbf{u}(\mathbf{k},t)$$

$$\mathbf{u}(\mathbf{k},t) = \mathbf{u}^{+}(\mathbf{k},t) + \mathbf{u}^{-}(\mathbf{k},t)$$

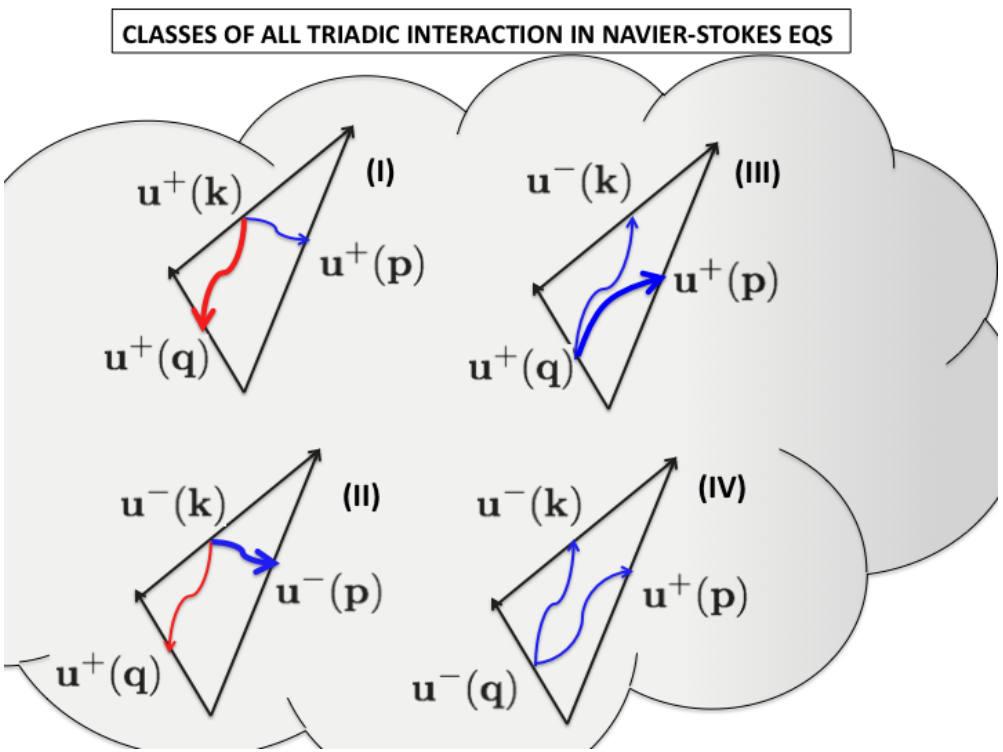
Decimated Navier-Stokes equations in Fourier space:

$$\partial_t \mathbf{u}^{\pm}(\mathbf{k},t) = \mathcal{P}^{\pm}(\mathbf{k})\mathbf{N}_{u^{\pm}}(\mathbf{k},t) + \nu k^2 \mathbf{u}^{\pm}(\mathbf{k},t) + \mathbf{f}^{\pm}(\mathbf{k},t)$$

where ν is kinematic viscosity and $\mathbf{f}$ is external forcing.

Helical-decimated Navier-Stokes equations

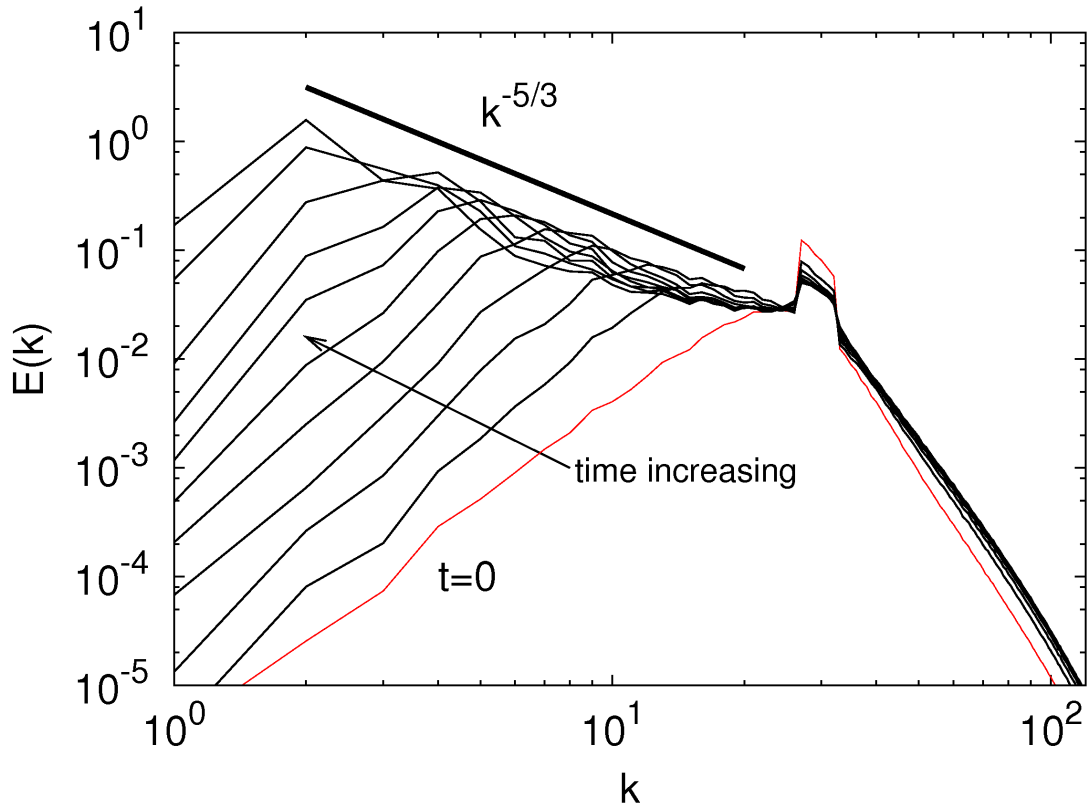
$$\mathbf{N}_{u^\pm}(\mathbf{k}, t) = \mathcal{F}\mathcal{T}(\mathbf{u}^\pm \cdot \boldsymbol{\nabla}\mathbf{u}^\pm - \boldsymbol{\nabla} p)$$



- Four classes of nonlinear triadic interactions with definite helicity signs under helical decomposition of NS equations.
- Energy and helicity are conserved for each triads.

Inverse energy cascade

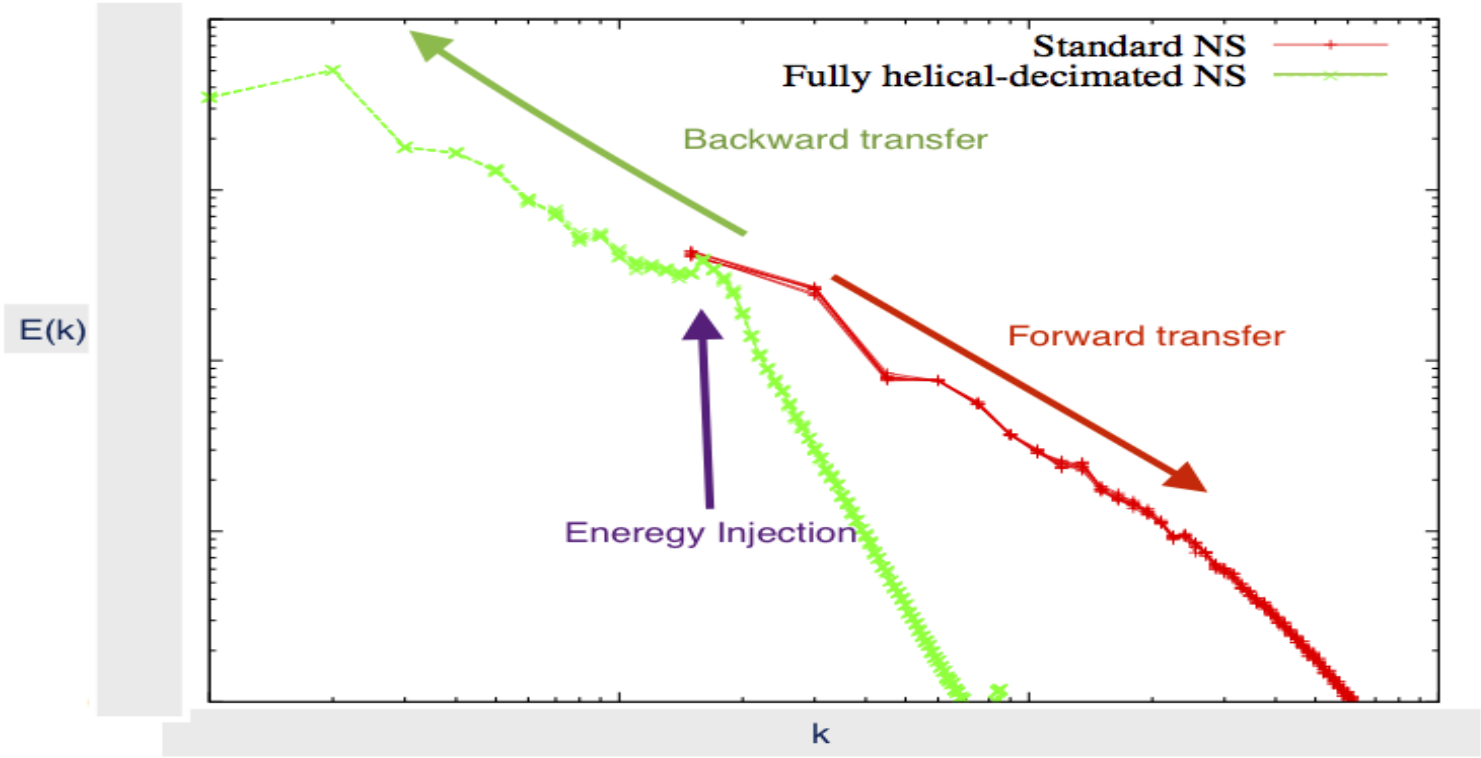
Triads with only $\mathbf{u}^+$ lead to reversal of energy cascade.



Energy spectra in the inverse cascade regime shows $k^{-5/3}$ slope.

Biferale PRL (2012)

- Full decimation of $\mathbf{u}^+$ or $\mathbf{u}^-$ $\rightarrow$ inverse cascade of energy.
  No decimation $\rightarrow$ forward cascade of energy



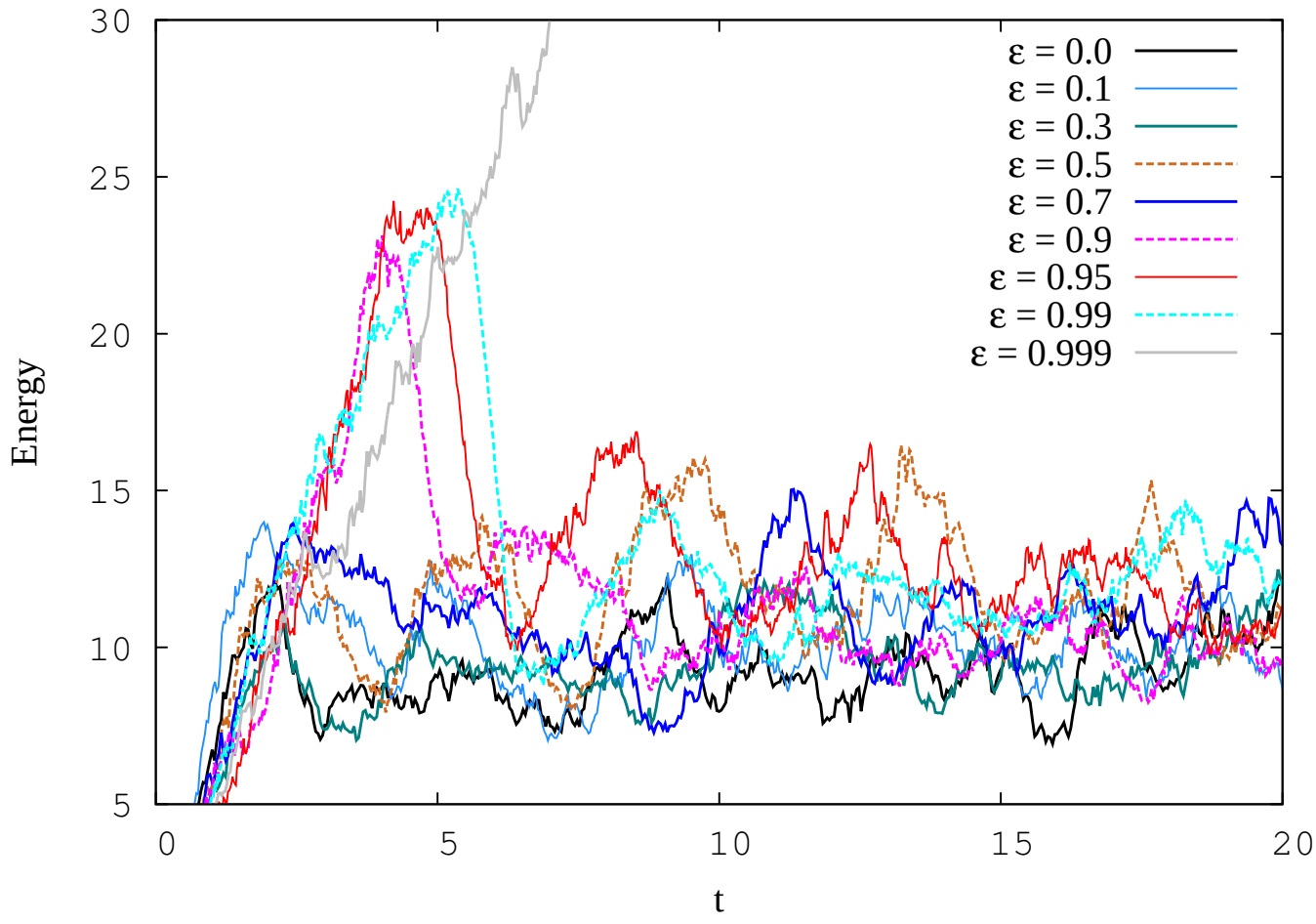
What happens in between??

- Modified projection operator:

$$\mathcal{P}_\epsilon^+(\mathbf{k})\mathbf{u}(\mathbf{k},t) = \mathbf{u}^+(\mathbf{k},t) + \theta_\epsilon(\mathbf{k})\mathbf{u}^-(\mathbf{k},t)$$

  where $\theta_\epsilon(\mathbf{k})$ is 0 or 1 with probability ϵ and $1-\epsilon$, respectively.
- $\epsilon = 0 \rightarrow$ Standard Navier-Stokes.
  $\epsilon = 1 \rightarrow$ Fully helical-decimated Navier-Stokes.
- Critical value of ϵ at which forward cascade of energy stops? alternatively, inverse cascade of energy stops if forced at small scales.
- Pseudo-spectral DNS on a triply periodic cubic domain of size $L = 2\pi$ with resolutions upto 512^3 collocation points.

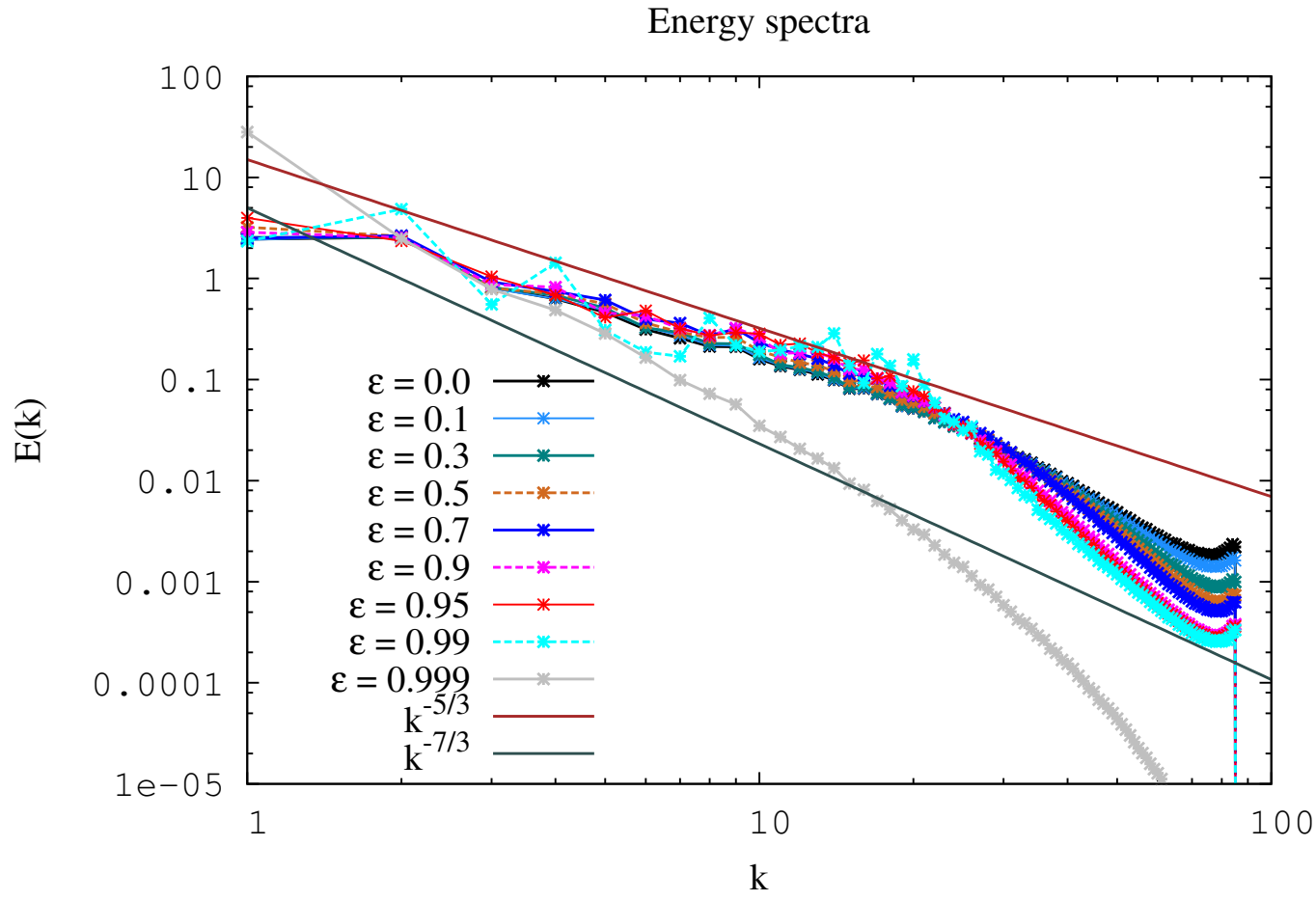
Evolution of energy



Energy vs time shows a initial block at the large scales before reaching a steady state.

With increase in ϵ the peak grows, a signature of inverse cascade.

Energy spectra

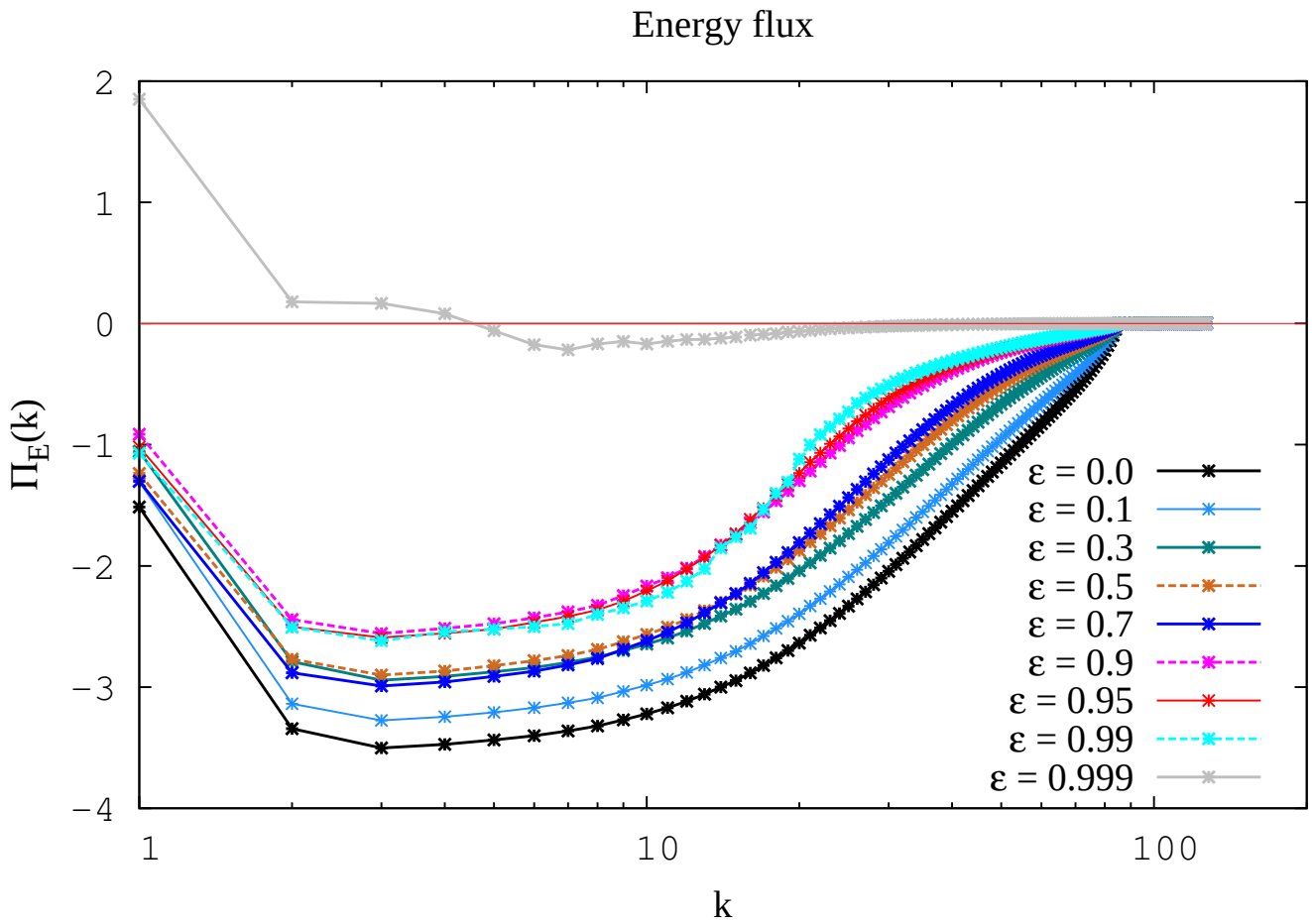


At $\epsilon = 0.99$ the spectrum shows large fluctuations. (Critical Value!)

At $\epsilon = 0.999$ forward cascade of energy subsides.

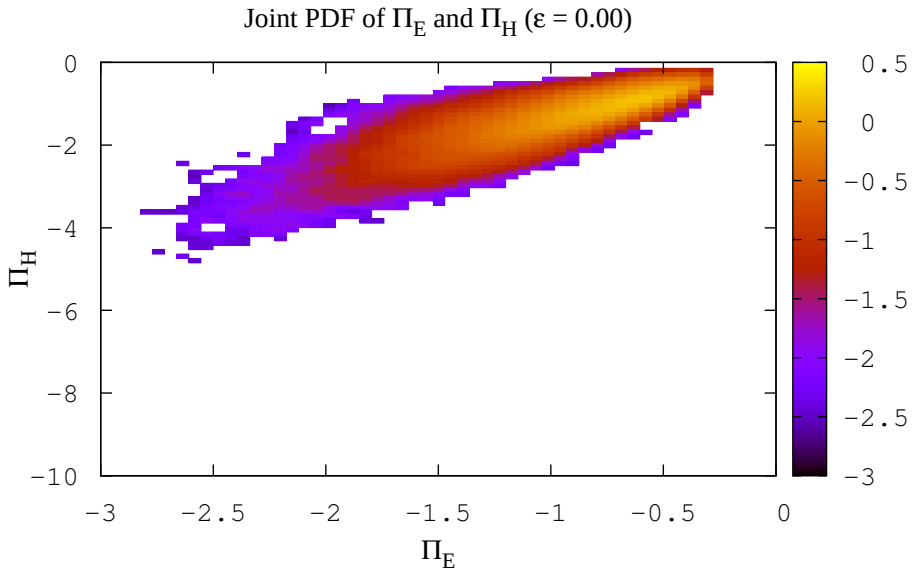
and Energy spectra shows a $k^{-7/3}$ spectrum due to forward helicity

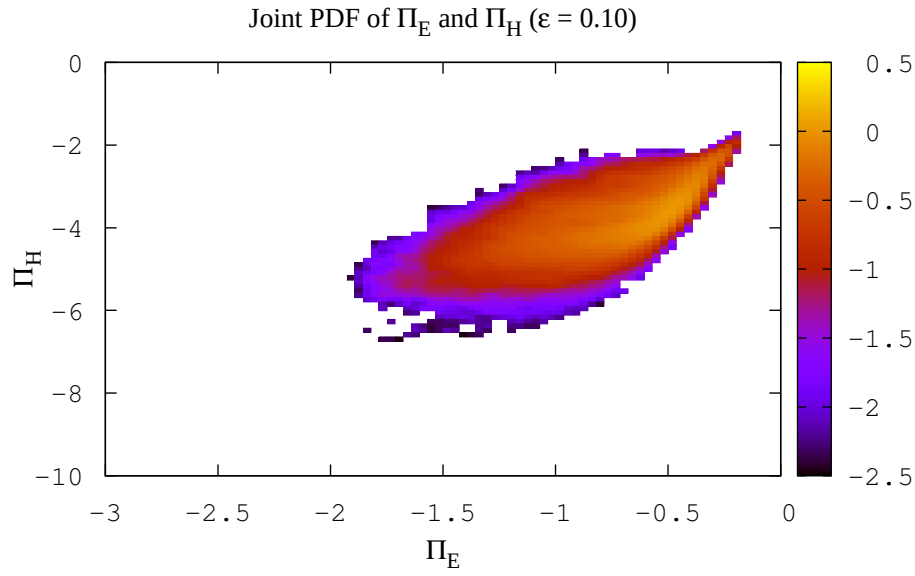
Energy flux

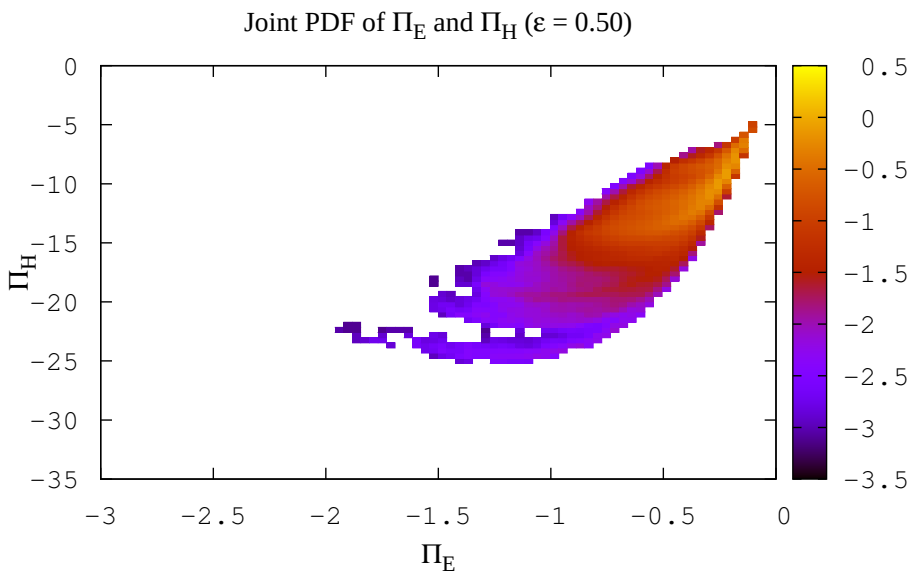


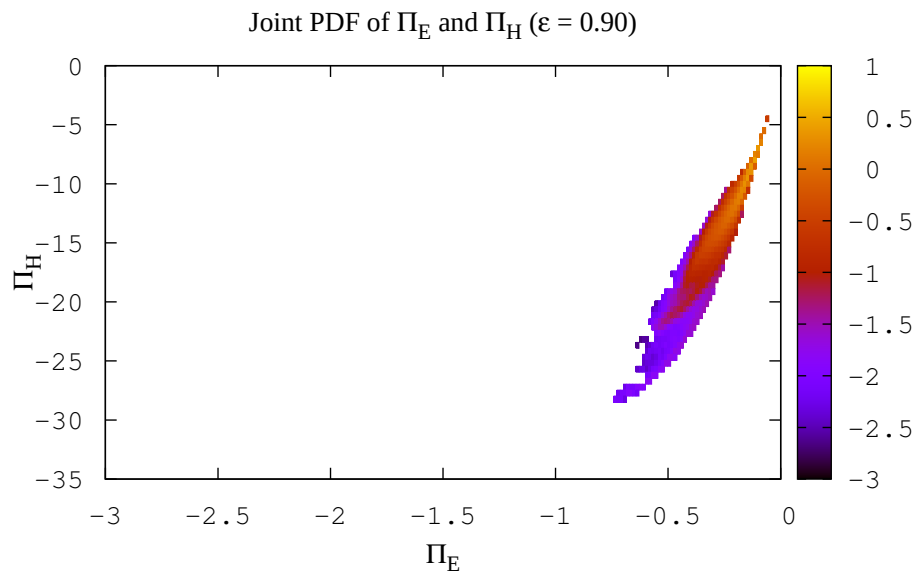
Energy flux gets depleted in the small scales with increasing ϵ.
There is sudden reversal of flux as we change ϵ from 0.99 to 0.999.

Joint PDF of helicity and energy fluxes









The helicity flux attains higher values whereas the energy flux depletes with increasing ϵ.

Thank you!

- As we increase ϵ, the contribution of triads leading to inverse energy cascade grows.
- Only when ϵ is very close to 1 inverse energy cascade takes over the forward cascade.
- Critical value of ϵ may have Reynolds number dependence!
- Can both forward and inverse cascade co-exist?
- What about intermittency in the forward cascade regime at changing ϵ.