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## Slide of the Seminar

# Learning to Flock, Flocking to Learn

***Dr. Mihir Durve***

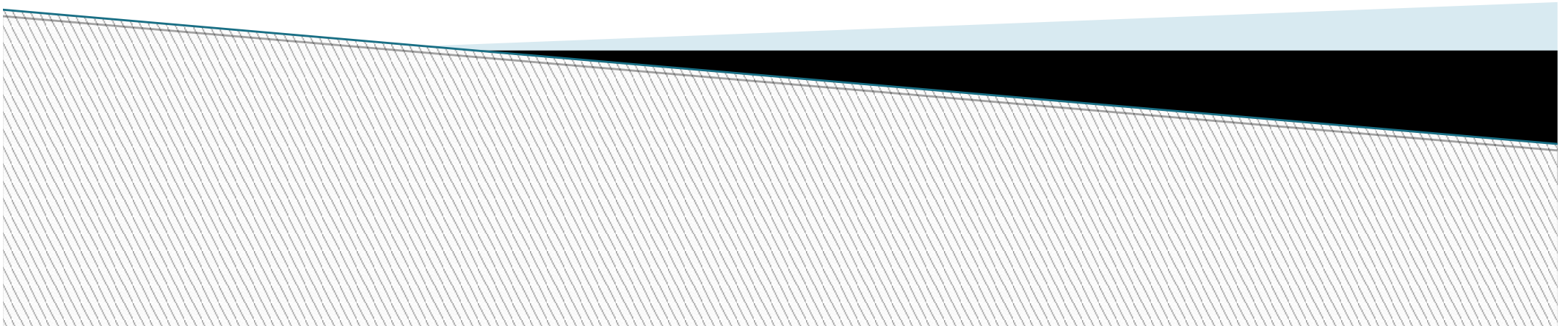
***ERC Advanced Grant (N. 339032) "NewTURB"  
(P.I. Prof. Luca Biferale)***

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# Learning to Flock, Flocking to Learn

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# Supervisors

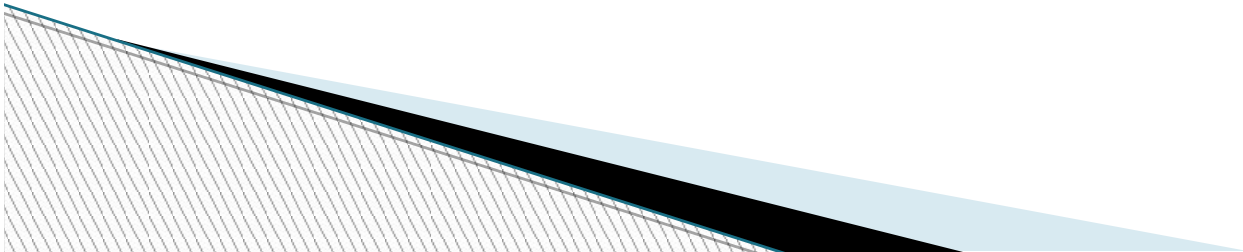
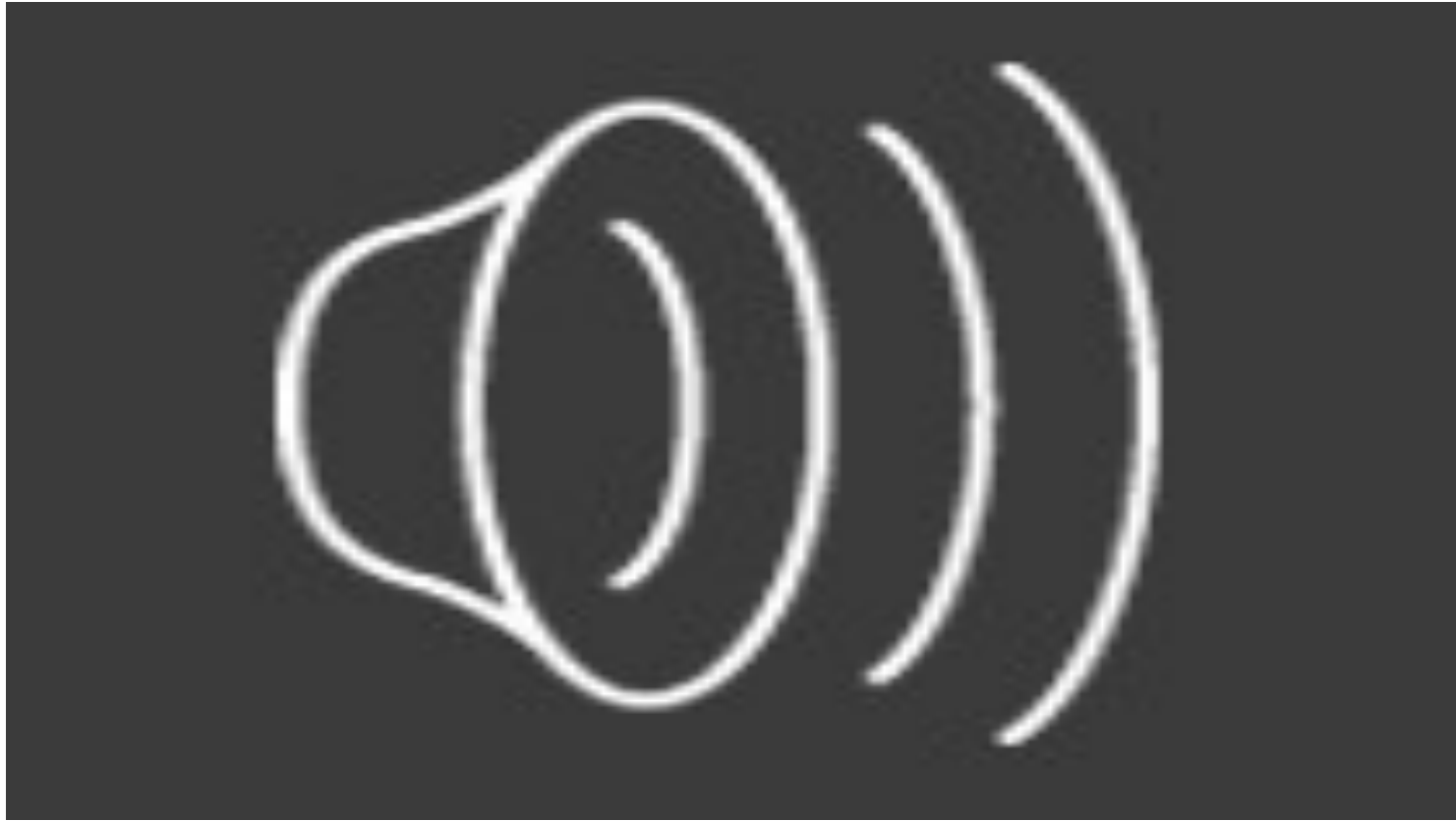


Prof. Antonio Celani



Prof. Edoardo Milotti

# Flock of Starlings



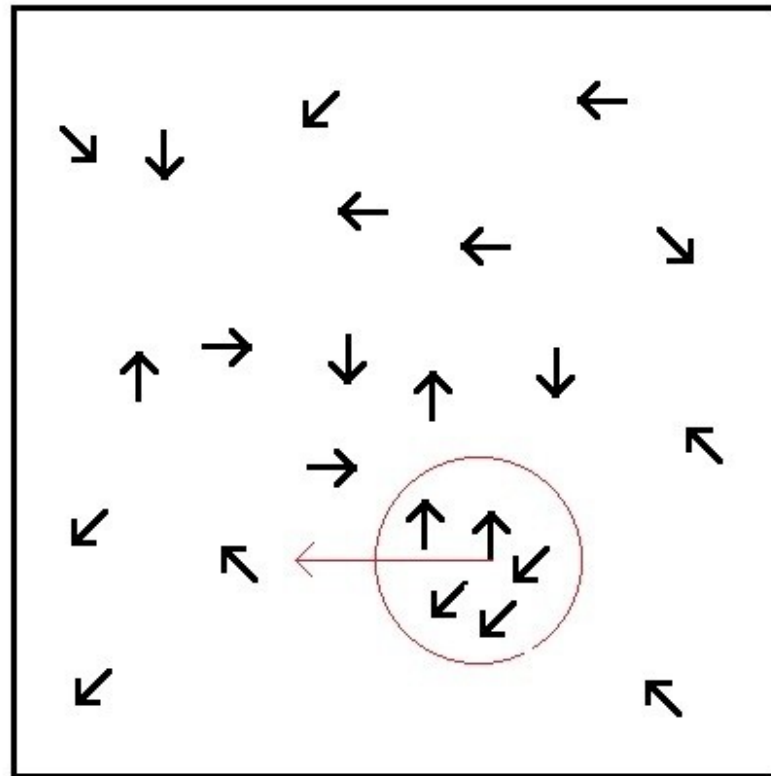
# Examples



Credits  
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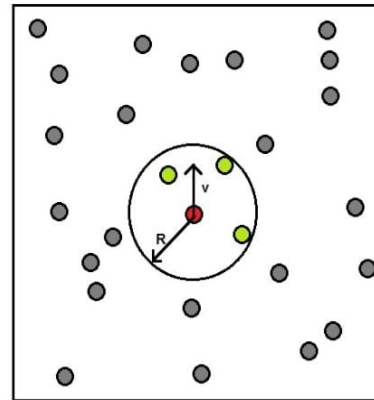
# Vicsek Model

- ▶ Do as your neighbours are doing



# Vicsek Model.

1. ' $N$ ' particles are placed randomly and uniformly in a box of  $L \times L$ .
2. All the particles initially have random velocity.
3. All the particles move with a constant speed  $v_0$
4. Neighborhood of interaction is a circle centered on the particle.



5. After every time interval all the particles adjust their direction to the average velocity of the particles in their neighborhood of interaction.
6. This adjustment is imperfect due to presence of noise.

# Vicsek Model

Continue...

The noise is introduced in the model as;

$$\mathbf{v}'_i = v_0 \mathcal{R}(\theta) \hat{\mathbf{v}}(t) \quad (1)$$

here;

1.  $\hat{\mathbf{v}}(t)$  is the unit velocity in the direction of the mean velocity of the particles in the neighborhood.
2.  $\mathcal{R}(\theta)$  is the rotation operator which rotates the vector it acts upon ( i.e.,  $\hat{\mathbf{v}}(t)$  ) by an angle  $\theta$ . The angle  $\theta$  is a random variable uniformly distributed over the interval  $[-\eta\pi, \eta\pi]$ .
3.  $\eta$  is strength of the noise in the range [0 to 1]



# Vicsek Model

## Order Parameter

Order parameter  $\psi(t)$  is given by;

$$\psi(t) = \frac{1}{Nv_0} \left| \sum_{i=1}^N \mathbf{v}_i(t) \right| \quad (2)$$

$\psi(t) = 0$  : Disordered state

$\psi(t) > 0$  : Ordered state

# Vicsek Model

## Results

1. System undergoes second order phase transition as the noise is increased or particle density is decreased.

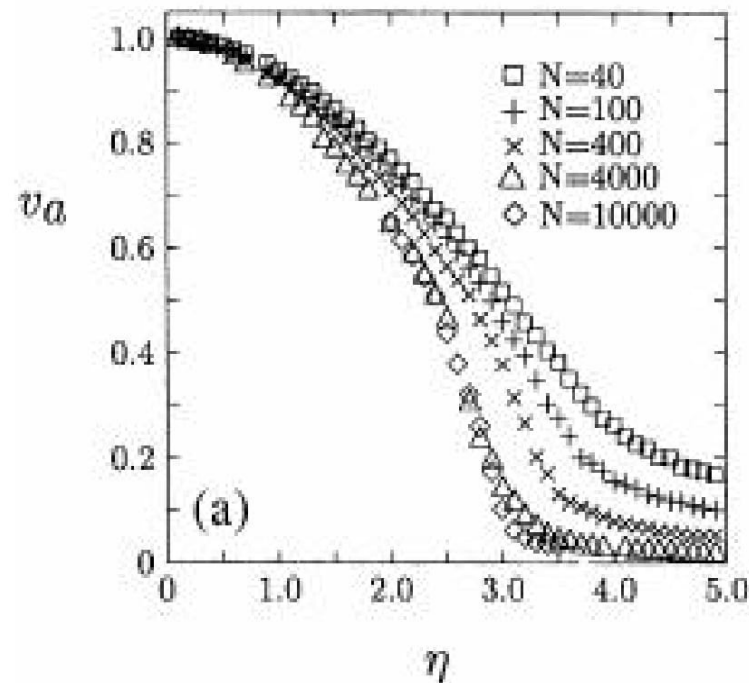


Figure :  $v_a$  Vs  $\eta$

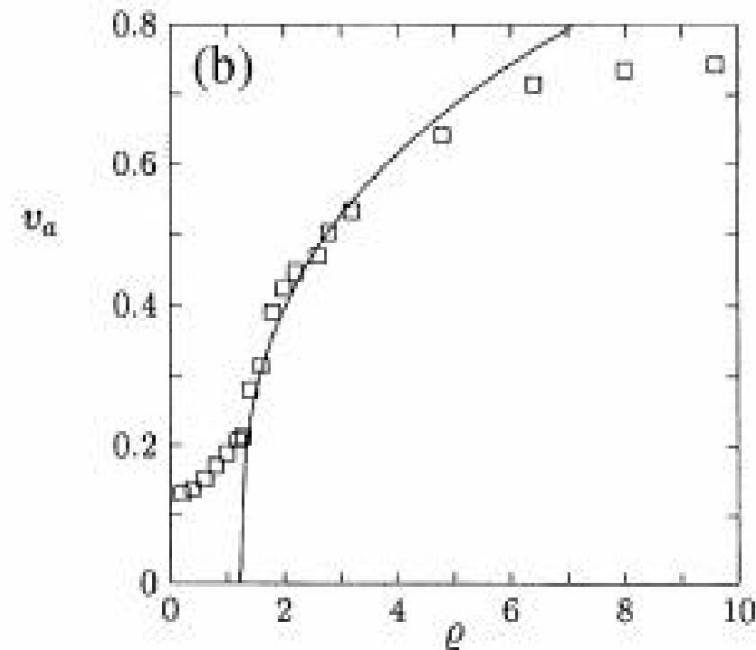


Figure :  $v_a$  Vs  $\rho$

Figure Credits : Vicsek et al. [PRL 1995]

# Vicsek Model



Random walkers

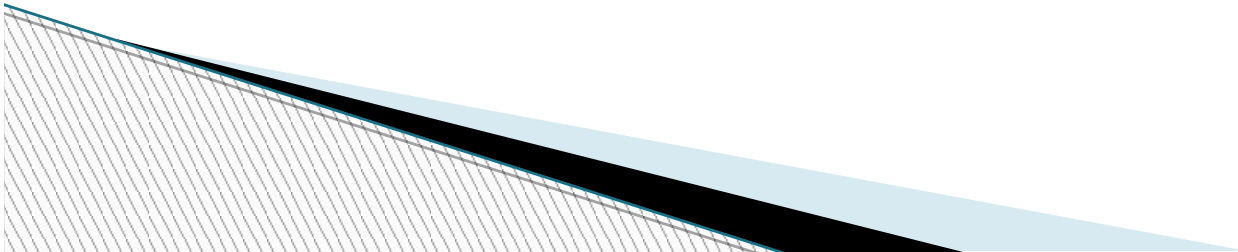


Flock

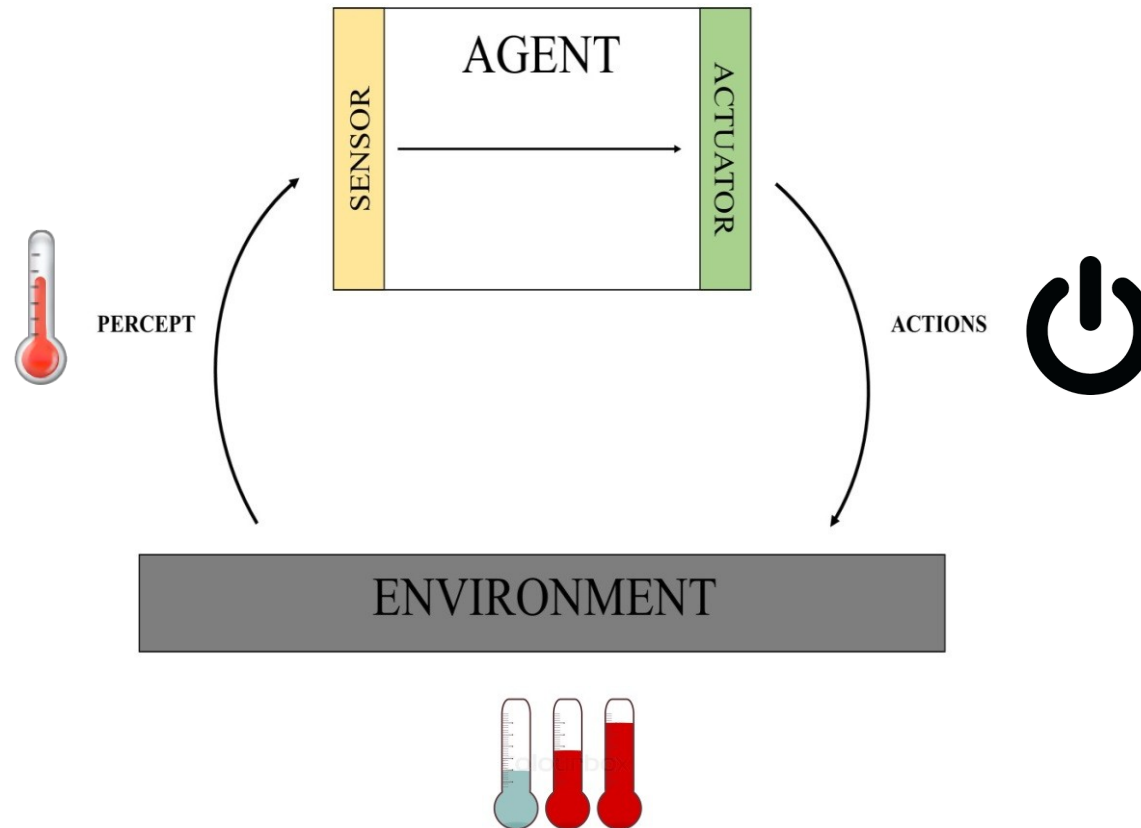
Vicsek et al. Phys Rev Lett (1995)

# Goal of the study

- ▶ For Flocking quantity to be optimized :  
Number of neighbours
- ▶ We use stochastic optimization techniques



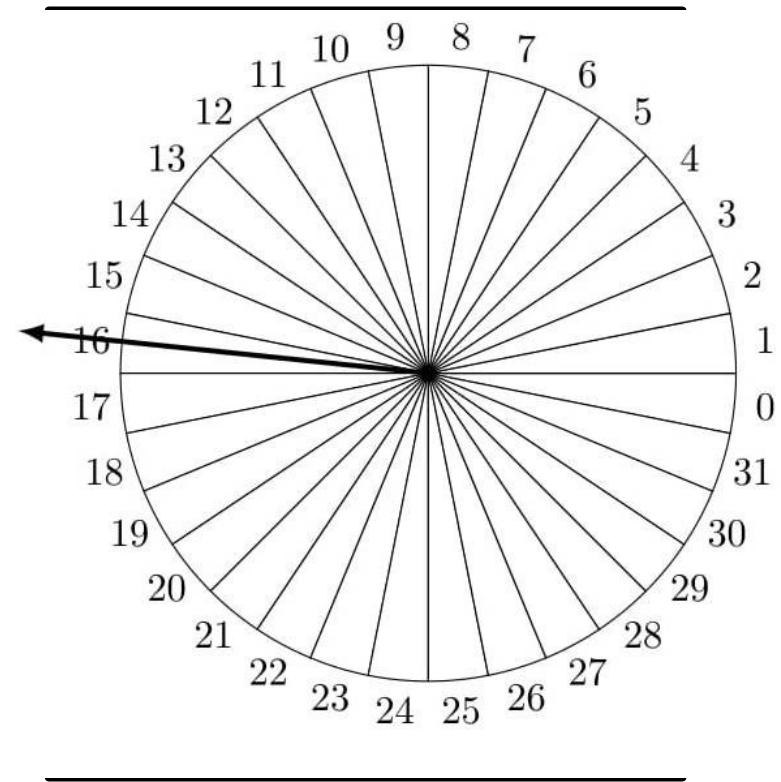
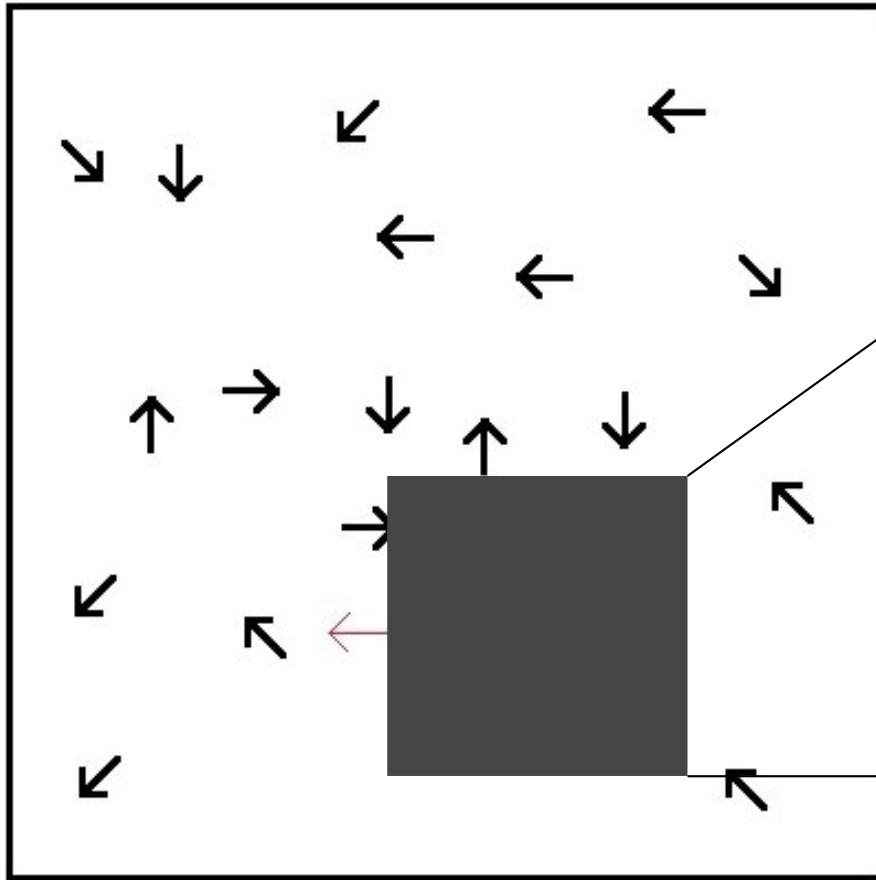
# Reinforcement Learning



Sutton and Barto (1998)

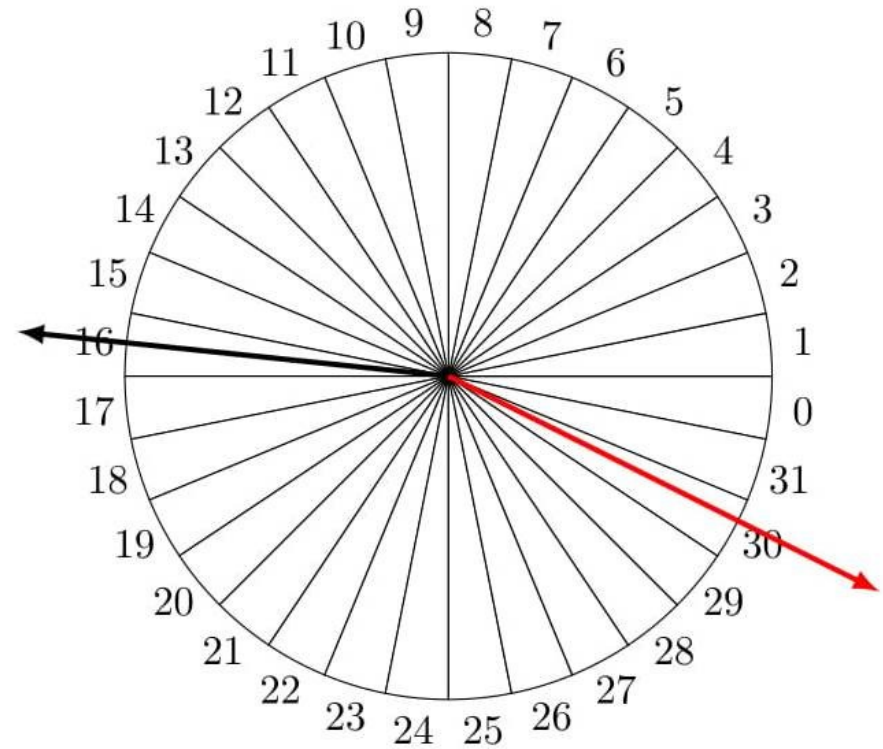
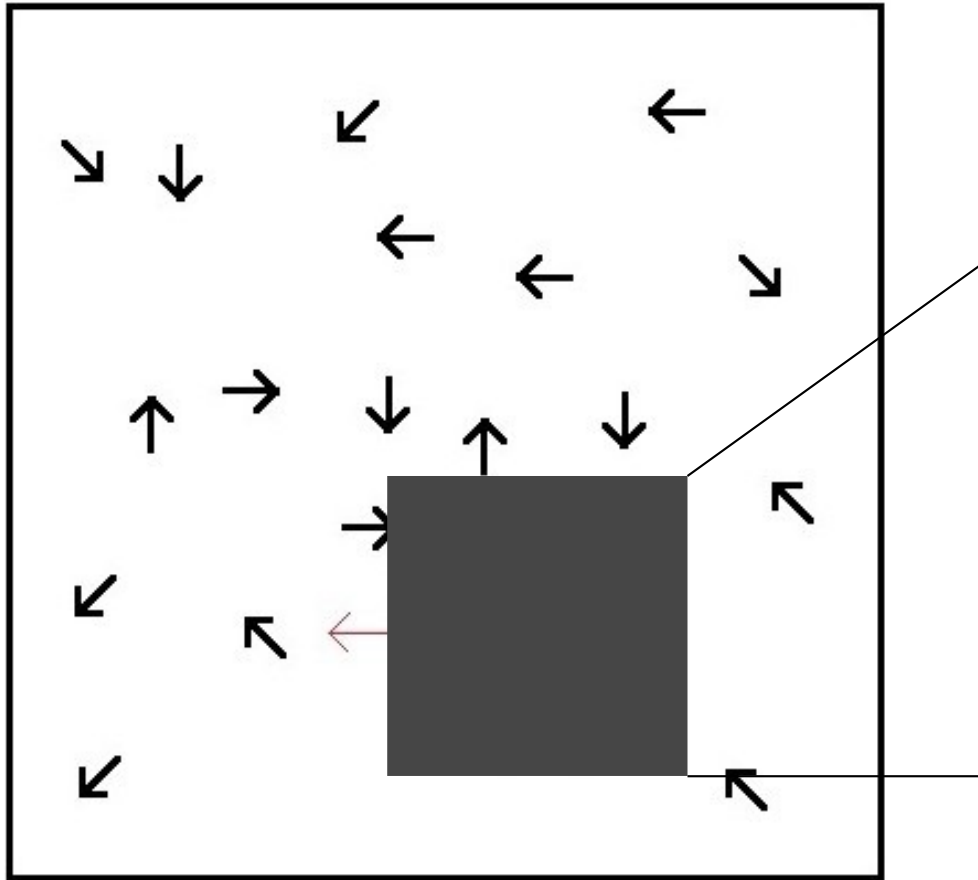
# RL in multi agent system :

## States, Actions



# RL in multi agent system :

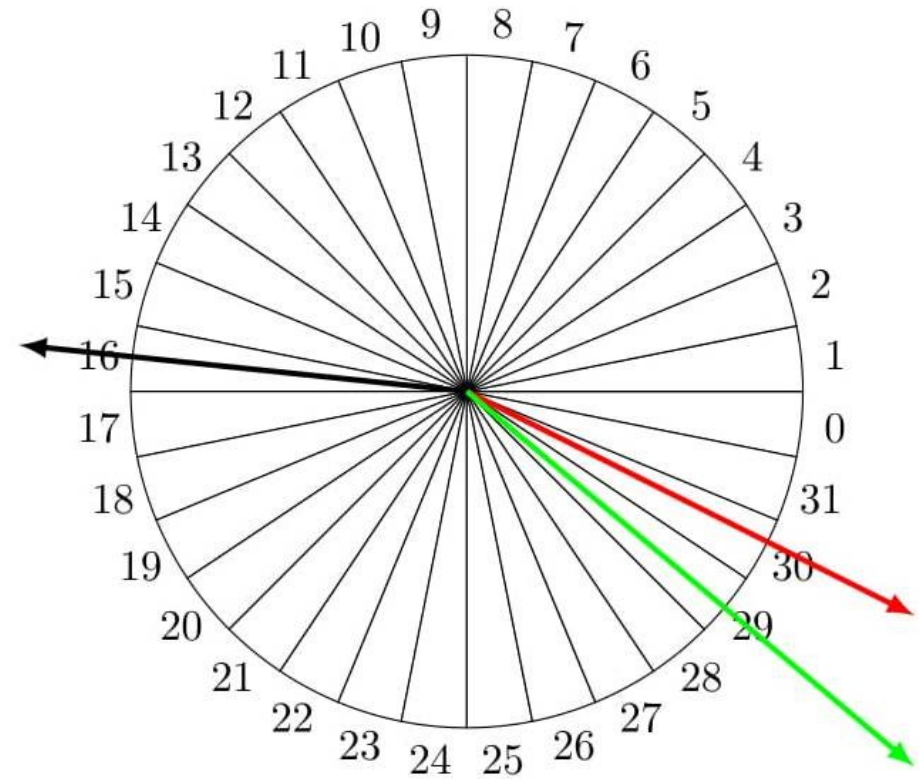
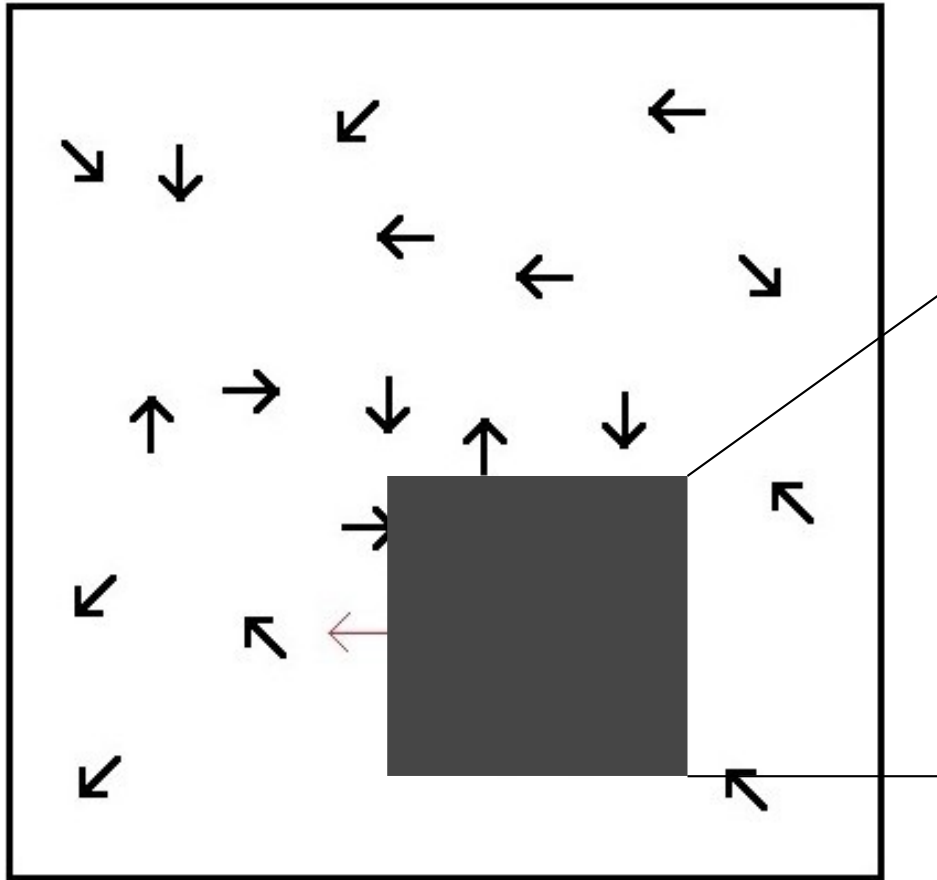
## States, Actions



State label : 30

# RL in multi agent system :

## States, Actions

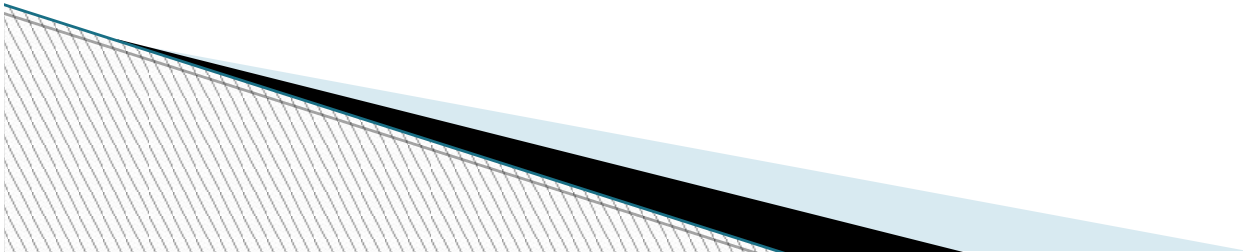


State label : 30  
Possible Actions : 0-31



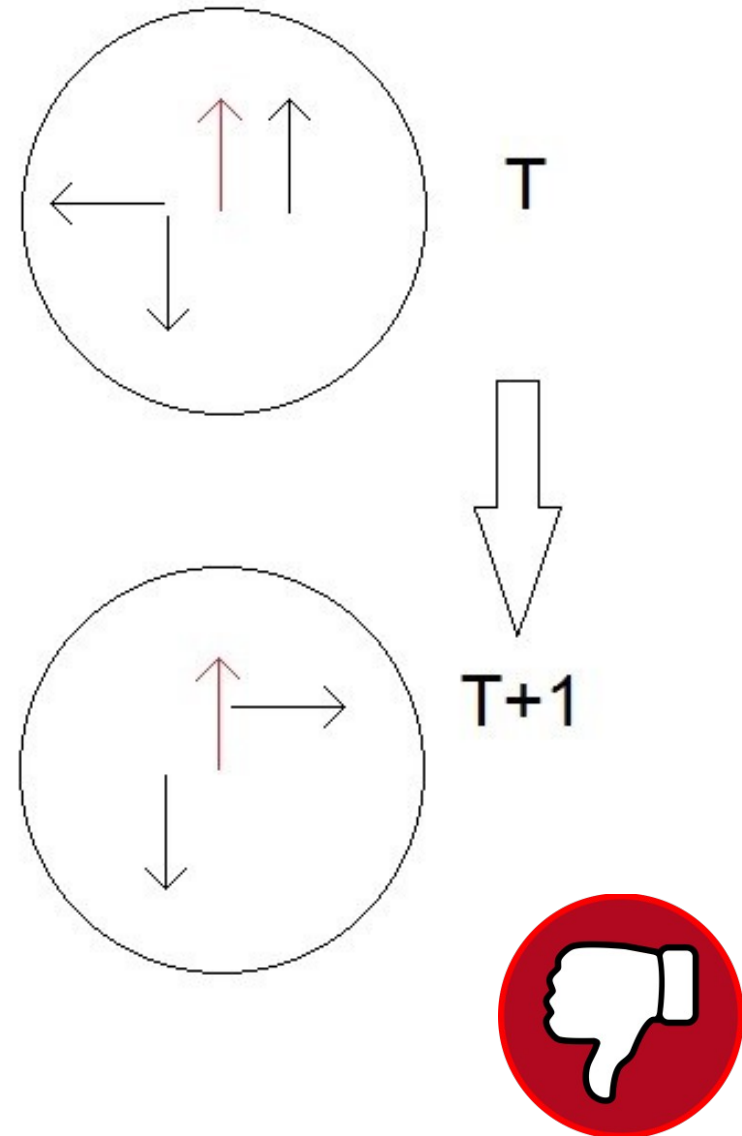
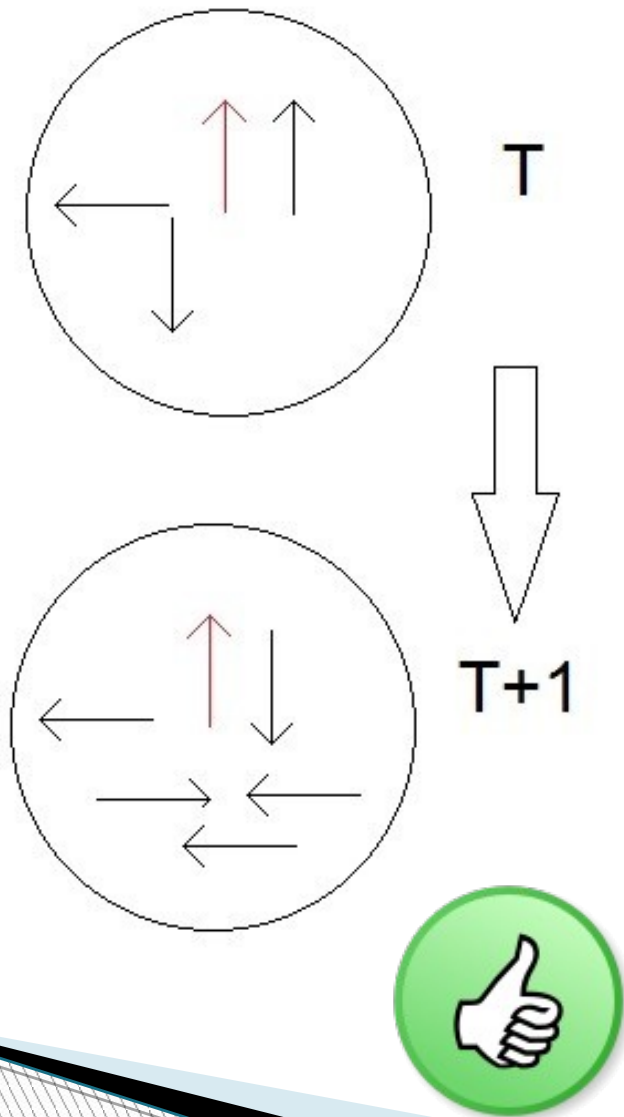
# RL in multi agent system :

$$x_i(t + 1) = x_i(t) + v_a(i) \times \Delta T$$



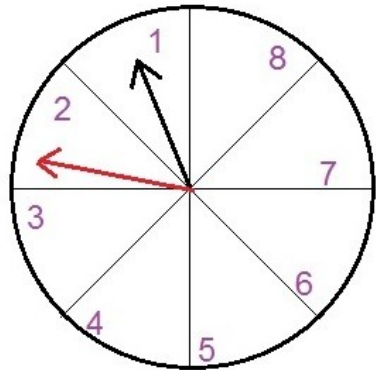
# RL in multi agent system :

## Reward for individual agents



# Q-matrix

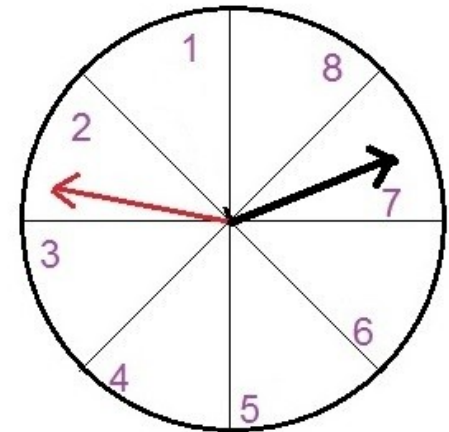
S	A	1	2	3	4	5	6	7	8
1		Q(1,1) )	Q(1,2) )	...					
2		Q(2,1) )	...						
3		...							
4									
5									
6									
7									
8									



$$\max_{a'} Q(s, a') p(1 - \epsilon)$$



Random  $a$   $p(\epsilon)$



# RL in multi agent system

## Q-update rule

$$Q(s_n, a_n) \leftarrow Q(s_n, a_n) + \alpha[r_n - Q(s_n, a_n)]$$



$$r_n = +R_f$$



$$r_n = -R_f$$

S	A	1	2	3
1		Q(1,1)	Q(1,2)	...
2		Q(2,1)	...	
3		...		

# Q-learning

## Episode

- ▶ Each agent begins in a box each with its own Q-matrix ( initially flat)
- ▶ DO  $T=0, T=T_{\max}$
- ▶ Observes the state  $s$
- ▶ Chooses action  $a$
- ▶ Updates position and orientation
- ▶ Receives reward
- ▶ Update Q-matrix
- ▶ End DO

S	A	1	2	3
1		$Q^*(1,1)$	$Q^*(1,2)$	...
2		$Q^*(2,1)$	...	
3		...		

# Preliminary Results



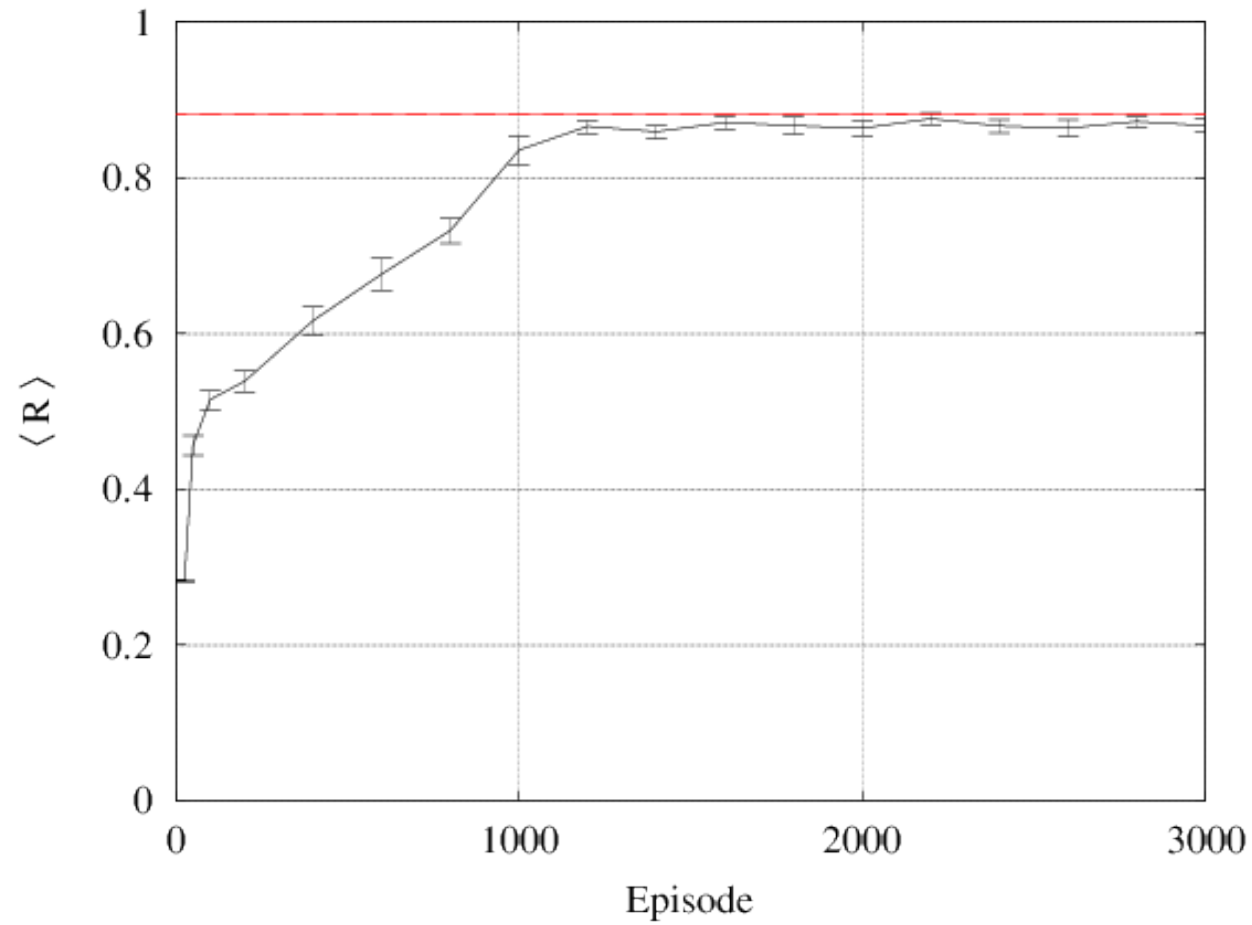
In the beginning



In the end

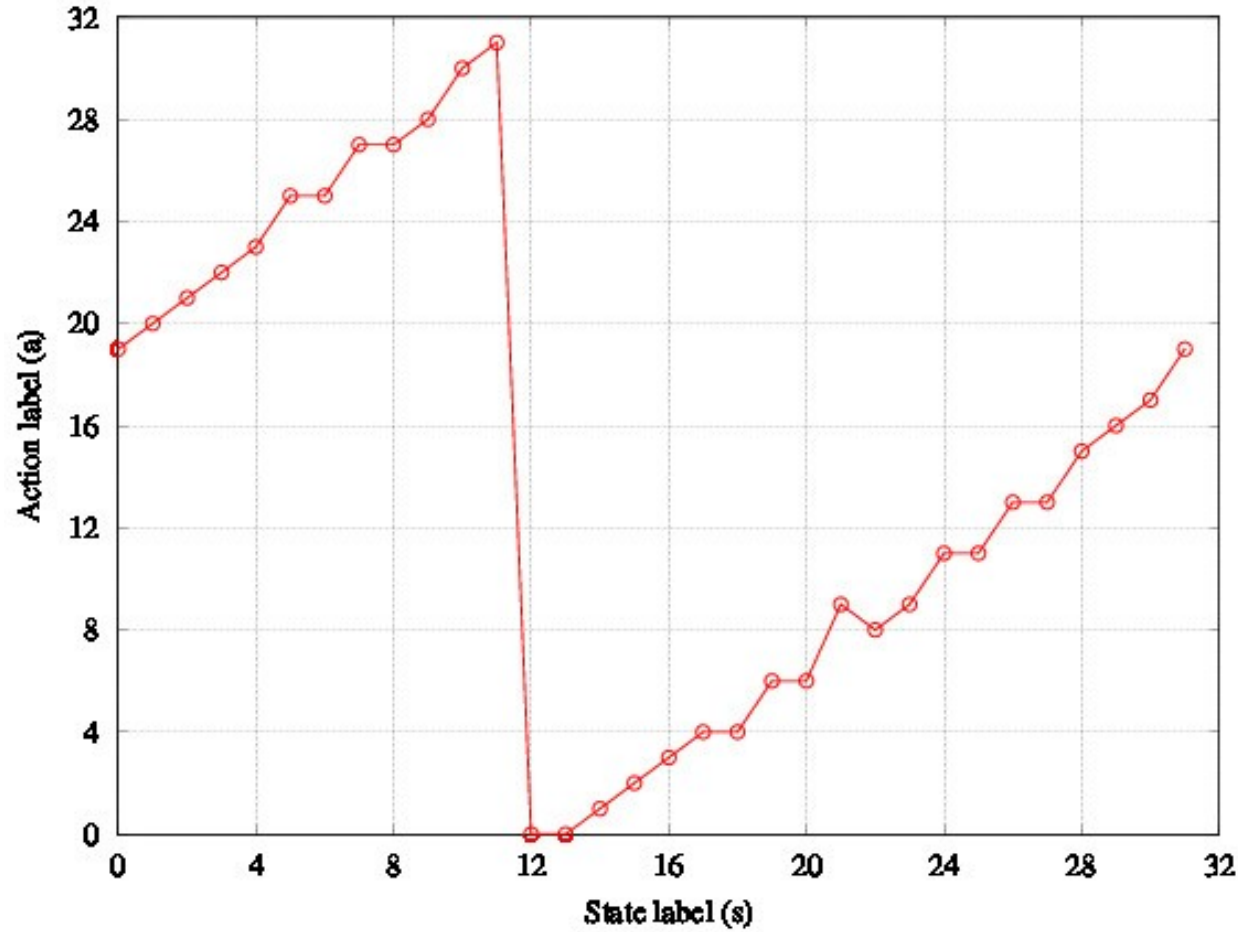
# Preliminary Result

## Average Reward with episodes



# Preliminary Results

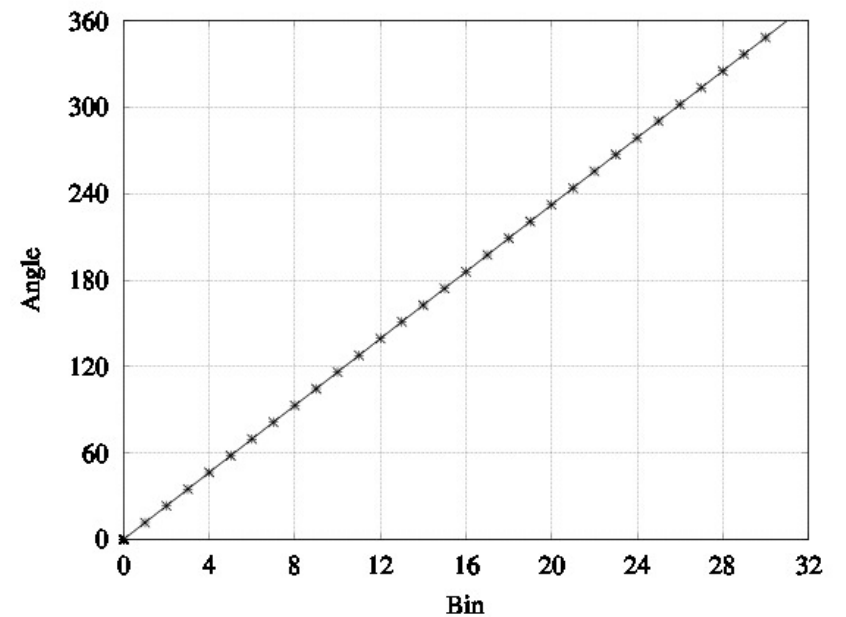
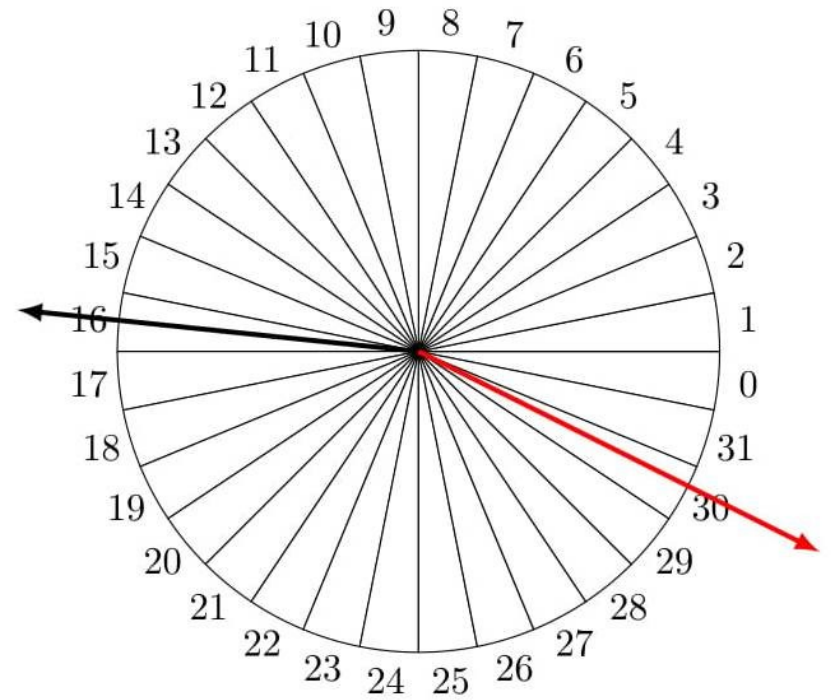
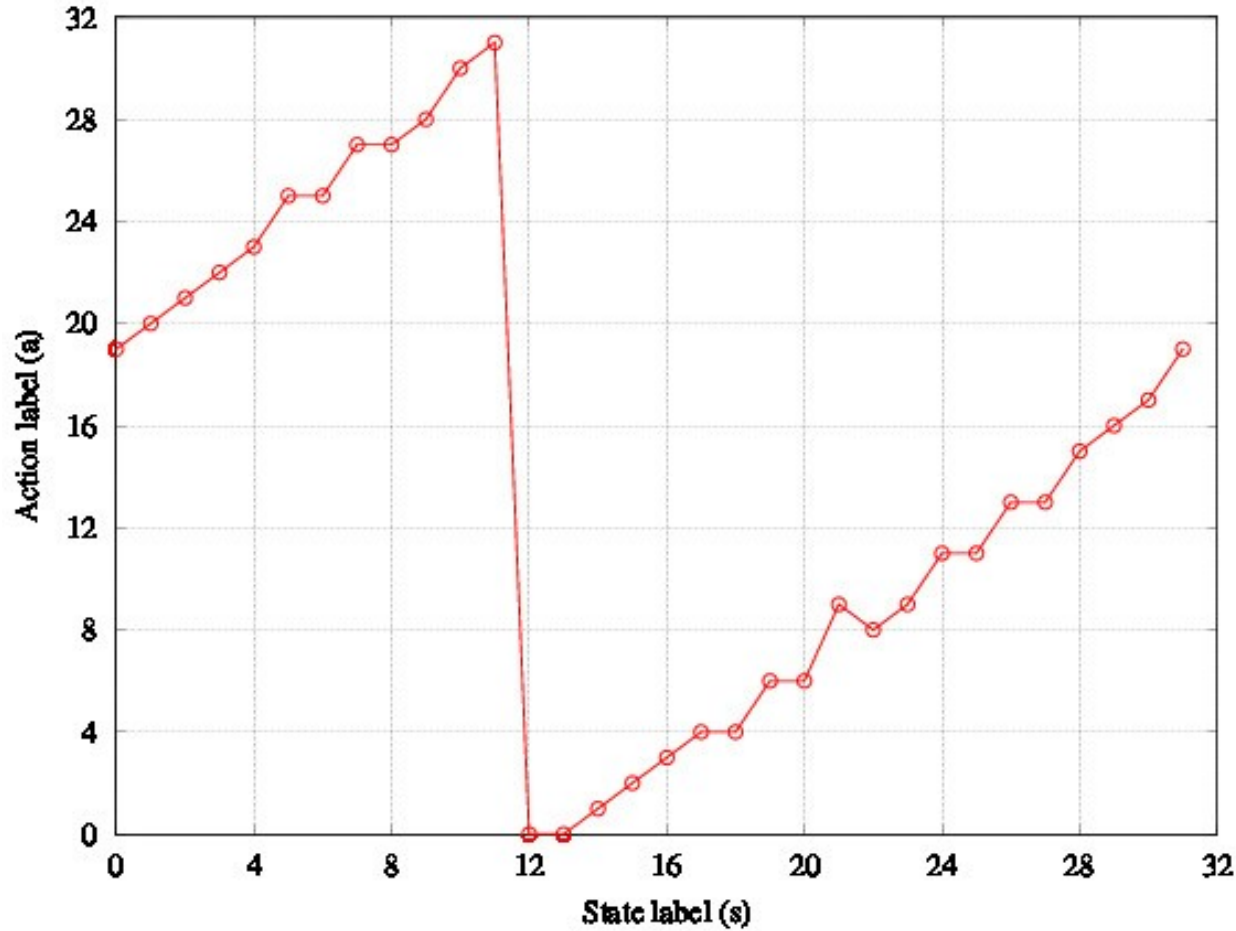
## Best $a$ for $s$ in Q-matrix





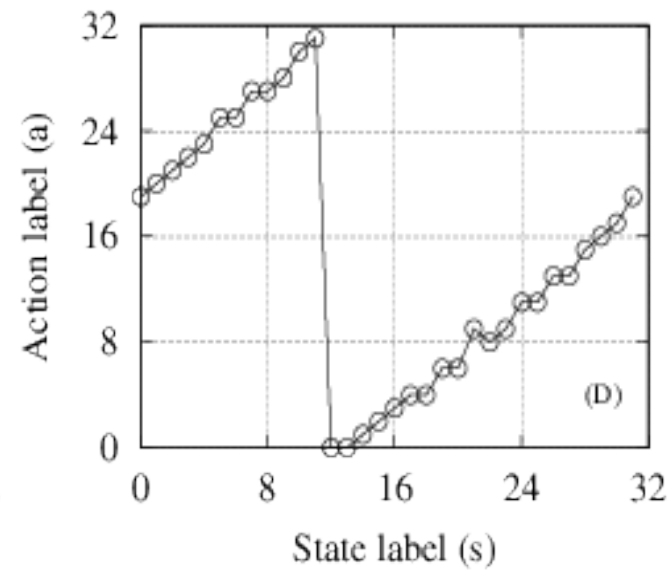
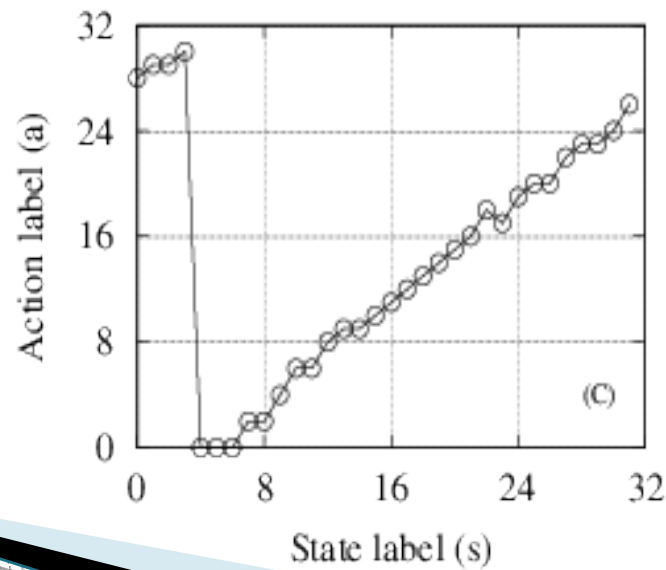
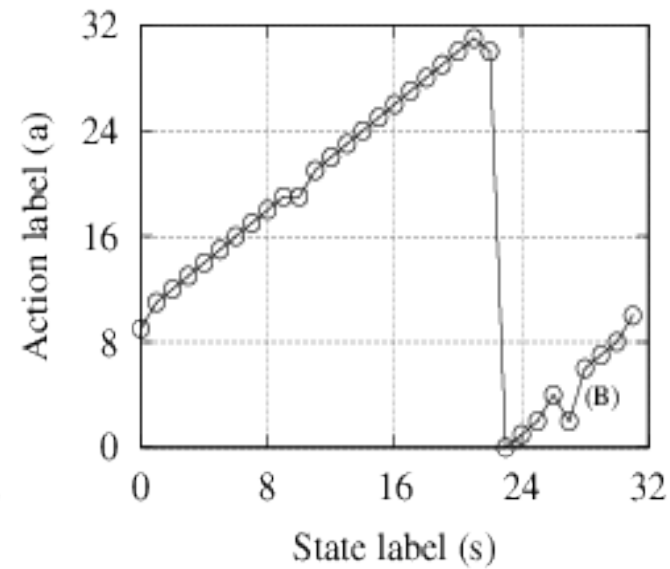
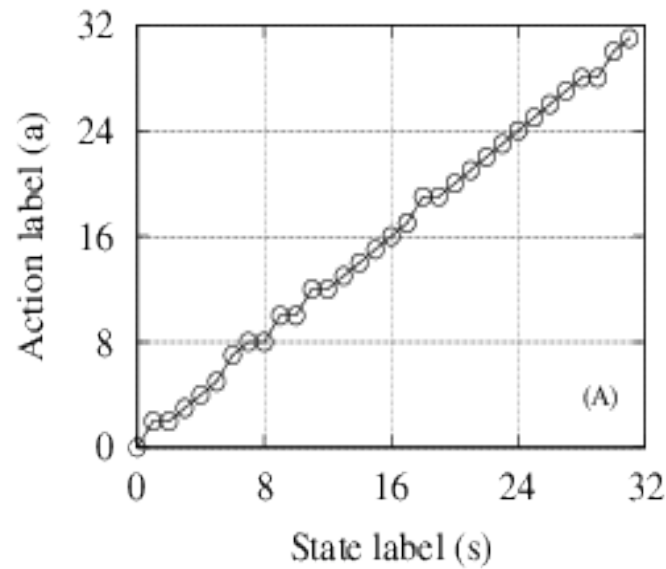
# Preliminary Results

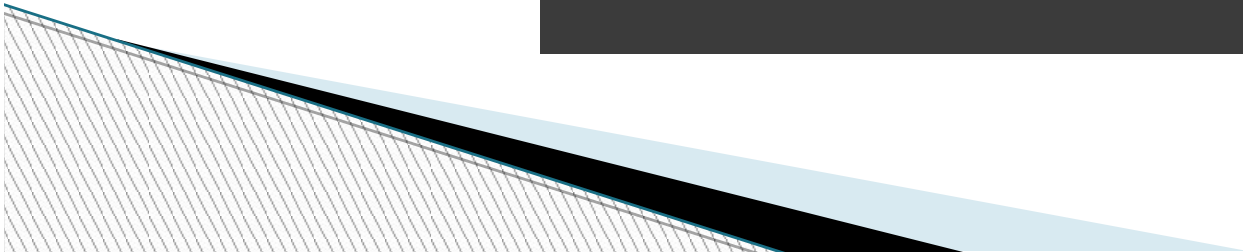
## Best $a$ for $s$ in Q-matrix



# Preliminary Results

## Max (Q(s,a))

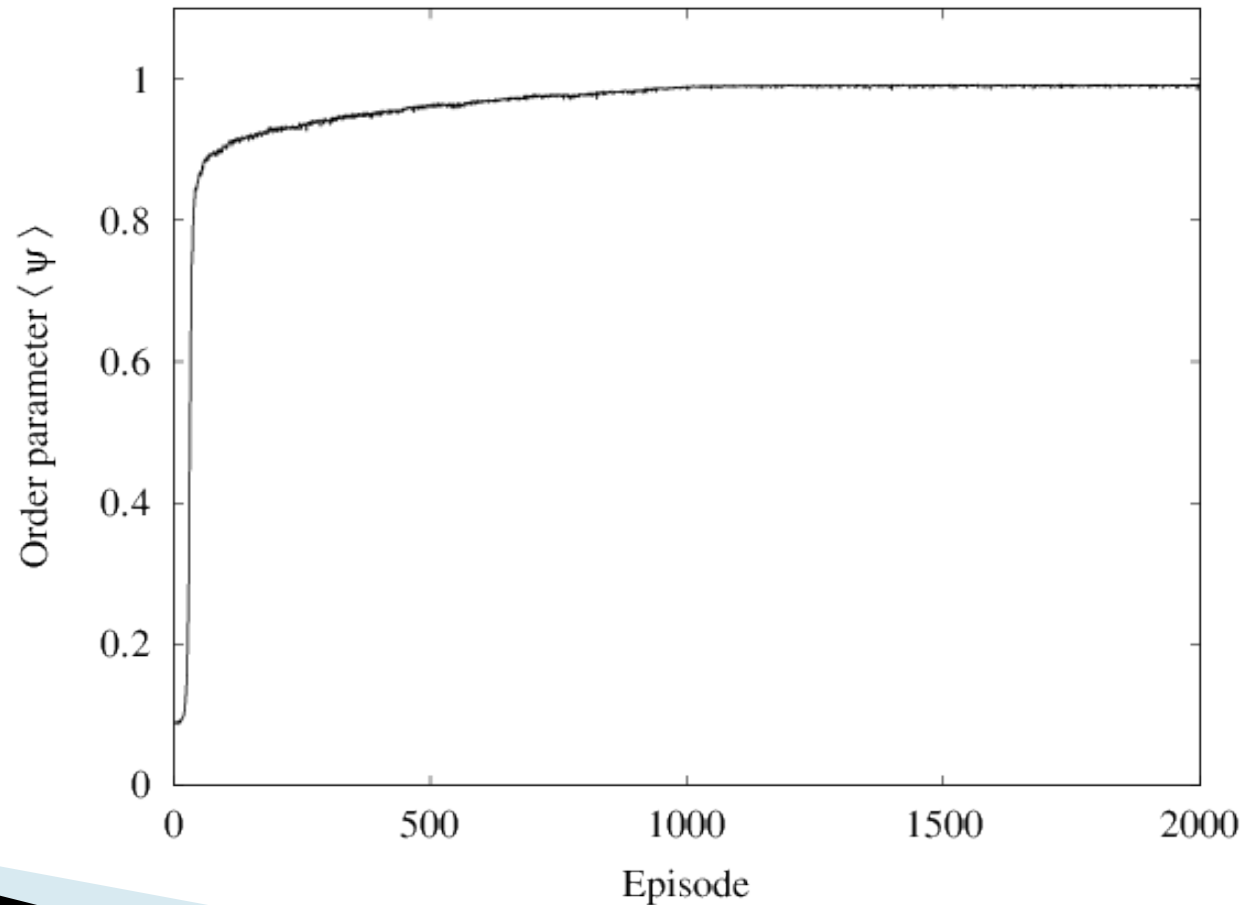




# Preliminary Results

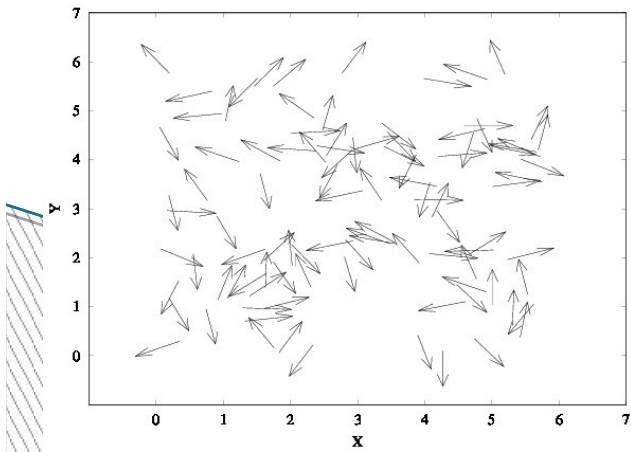
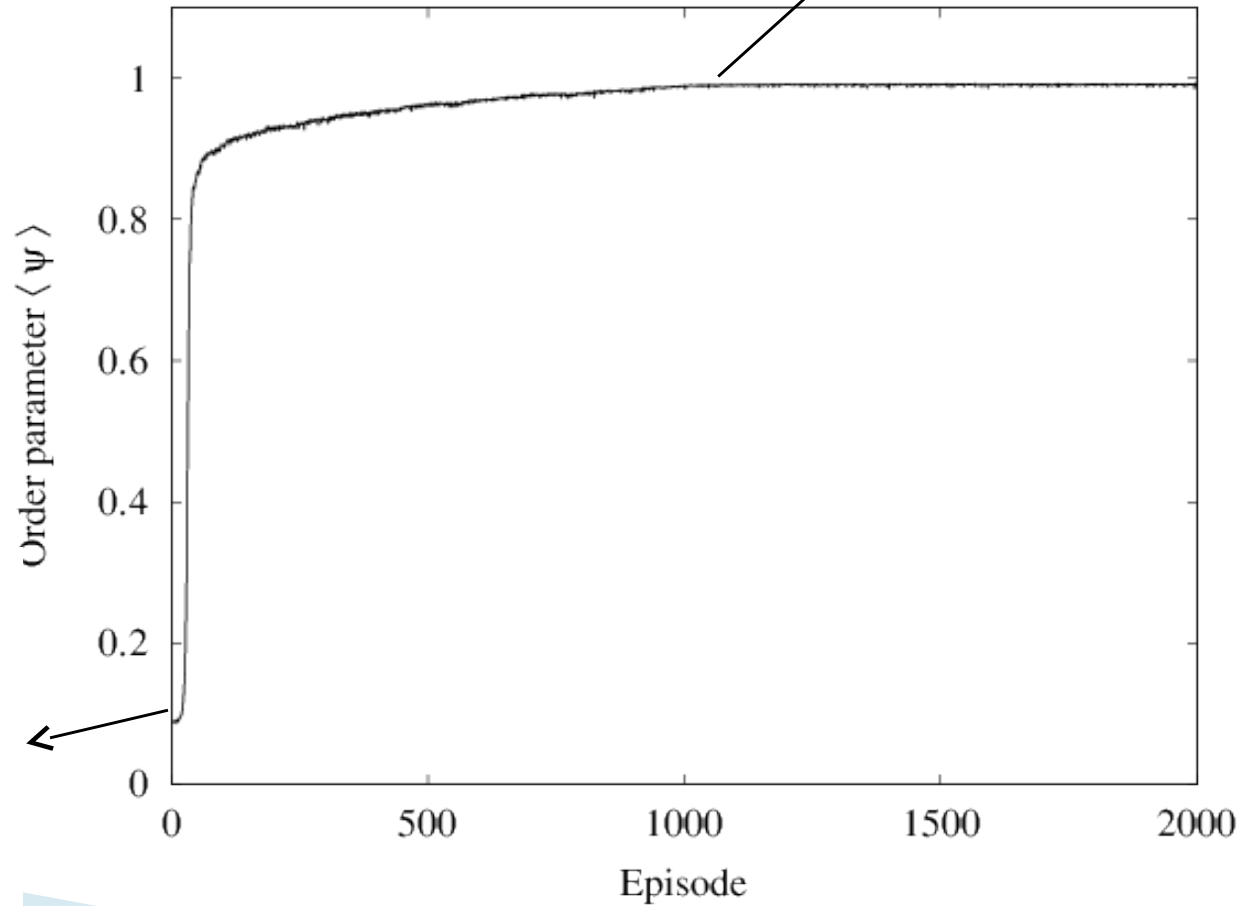
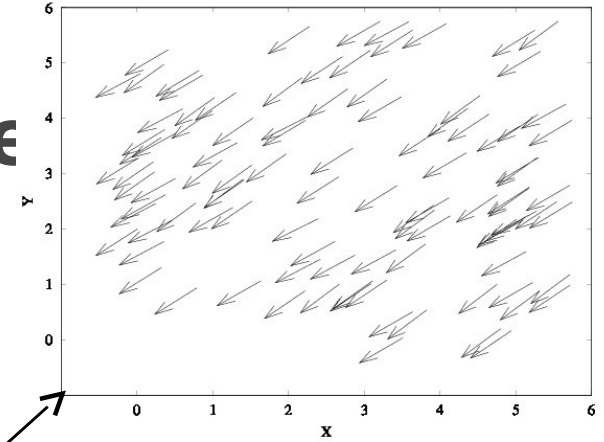
## Order parameter with Episodes

Order parameter :  $\psi(t) = \frac{1}{Nv_0} \left| \sum_{i=1}^N \mathbf{v}_i(t) \right|$

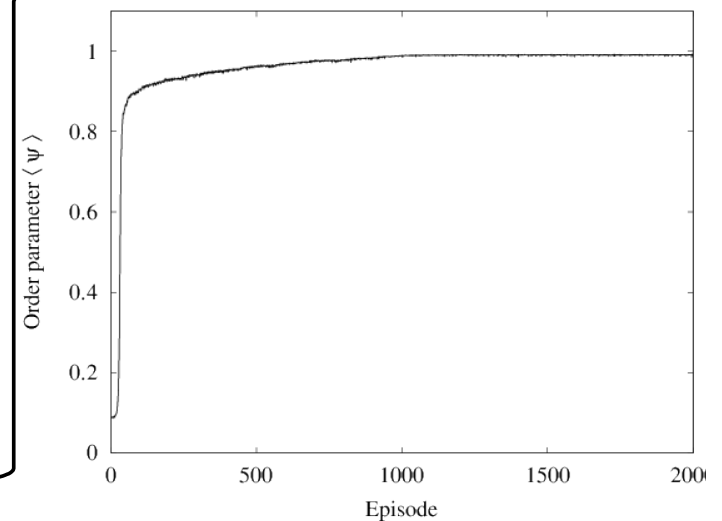
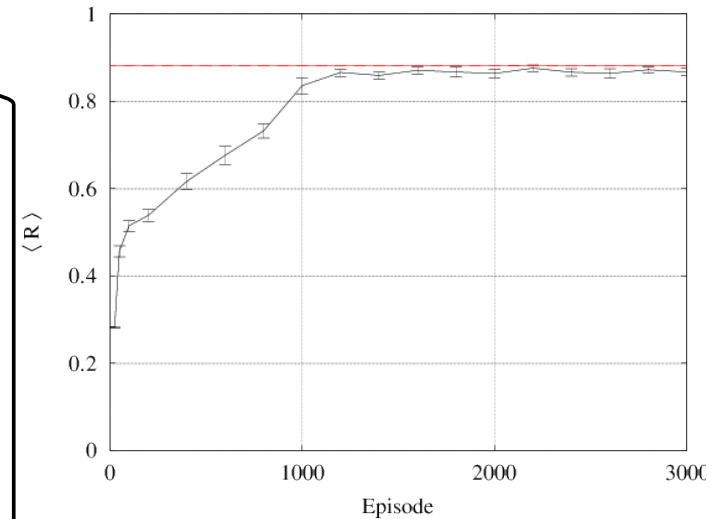
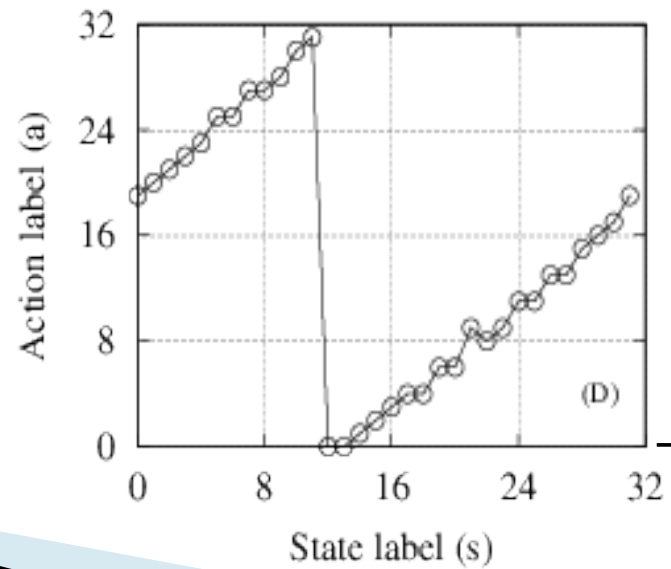
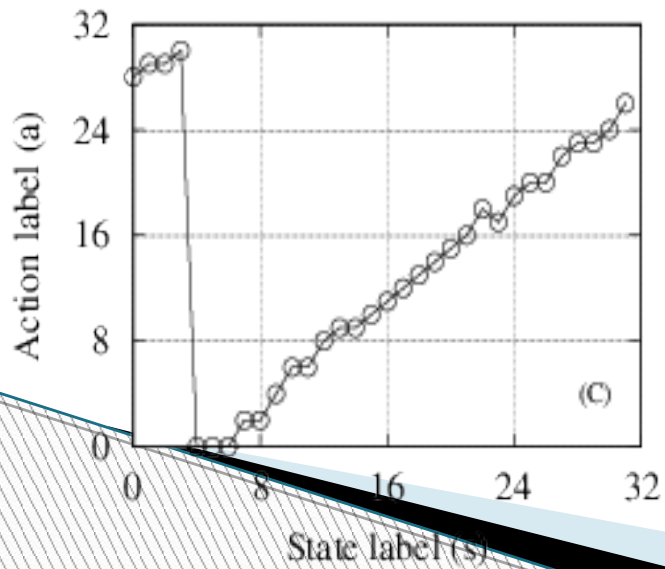
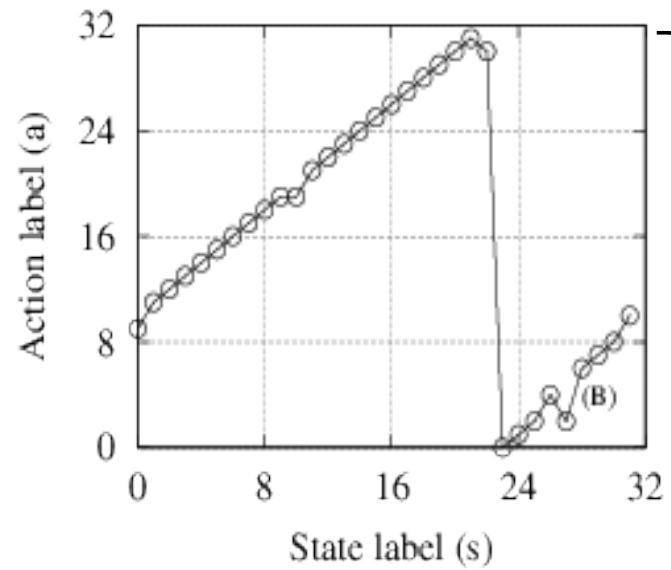
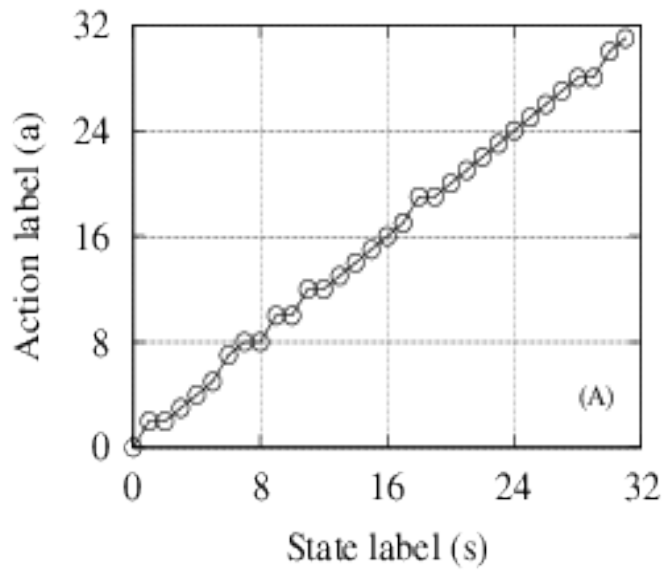


# Preliminary Results

## Order parameter with Episode

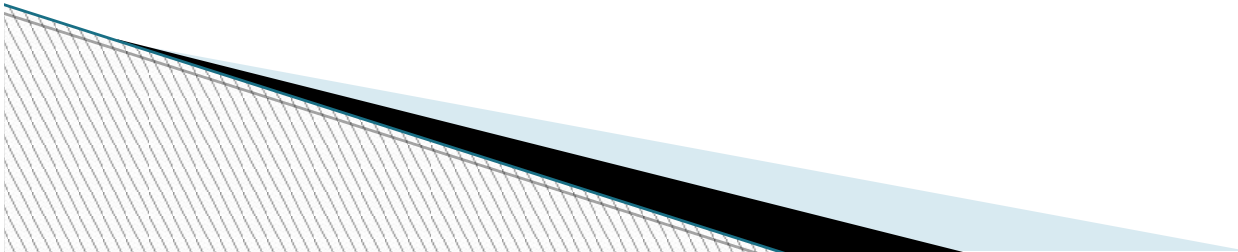


# Preliminary Results Policies



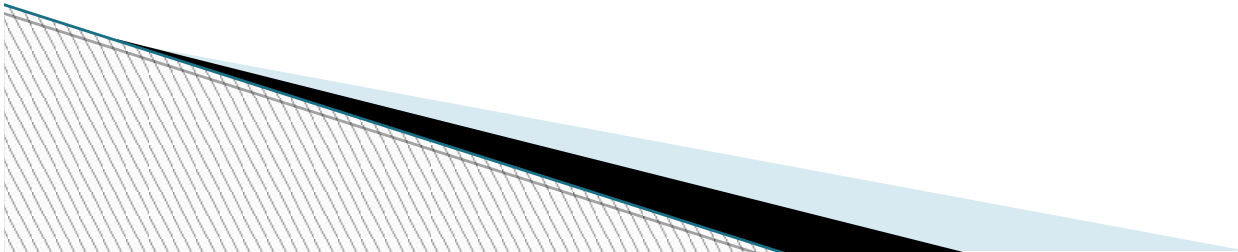
# Conclusion

- ▶ Multi-agent system optimizing aggregation formed highly polar ordered state.



# Next plans

- ▶ Restricting the set of actions
- ▶ Changing the reward schemes
- ▶ Changing the percept







THANK  
YOU

$$Q(s_n, a_n) = Q(s_n, a_n) + \alpha(r_n + \gamma \max_{a'} Q(s_{n+1}, a') - Q(s_n, a_n))$$

$$Q_{\pi}^*(s_n, a_n) = \langle r_{n+1} + \gamma \max_a Q_{\pi}(s_{n+1}, a) \rangle$$

