Slide of the Seminar

Energy dissipation in rotating turbulence

Dr. Basile Gallet De Saint Aurin

ERC Advanced Grant (N. 339032) "NewTURB"
(P.I. Prof. Luca Biferale)

Università degli Studi di Roma Tor Vergata
C.F. n. 80213750583 – Partita IVA n. 02133971008 - Via della Ricerca Scientifica, I – 00133 ROMA

Energy dissipation in rotating turbulence

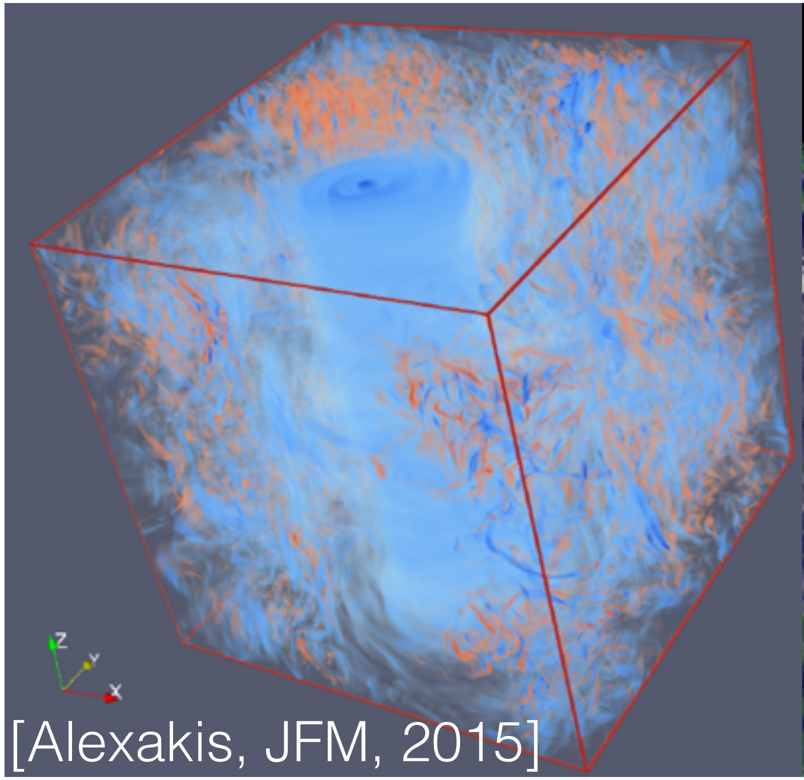
Basile Gallet, SPEC, CEA Saclay, France.

A. Campagne, N. Machicoane,
P.-P. Cortet, F. Moisy, FAST, Orsay

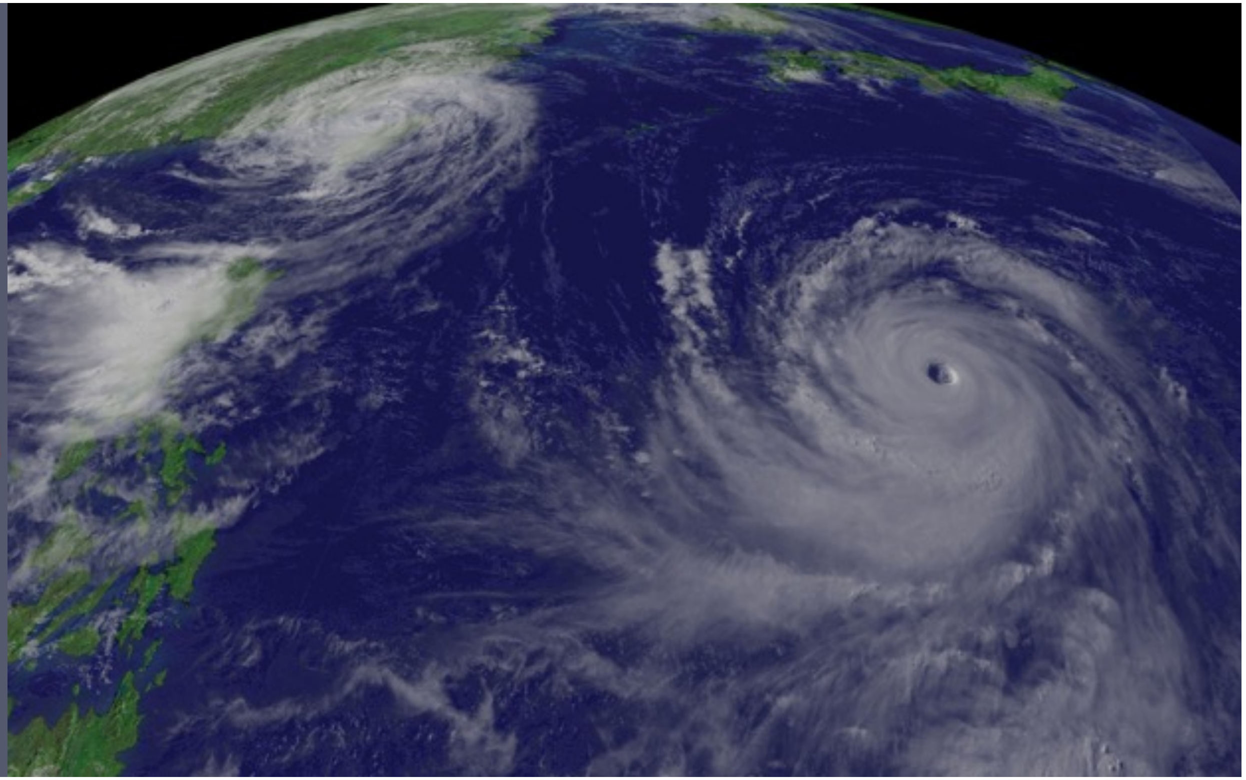
Rotating turbulence

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} + 2\Omega \mathbf{e}_z \times \mathbf{u} = -\boldsymbol{\nabla} p + \nu \Delta \mathbf{u}$$

- Two-dimensionalization
- Coherent structures
- Cyclone-anticyclone asymmetry



[Alexakis, JFM, 2015]



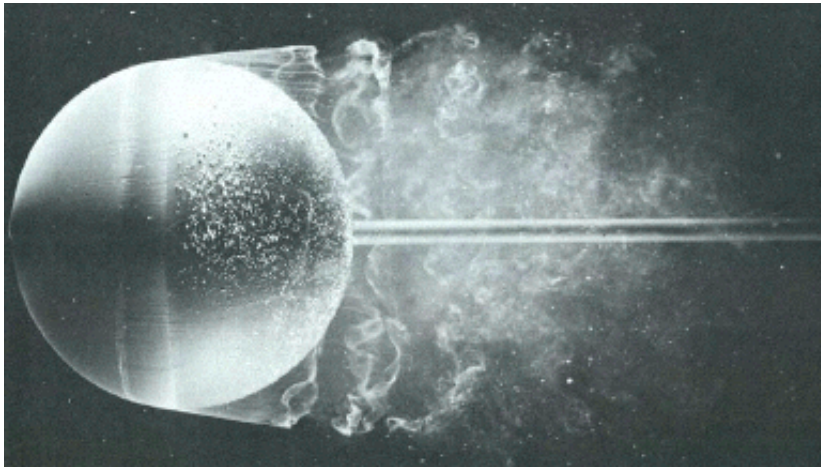
Turbulent energy dissipation

- Key quantity in Kolmogorov 41 theory
- Central question of turbulence for the engineers.

3D turbulence

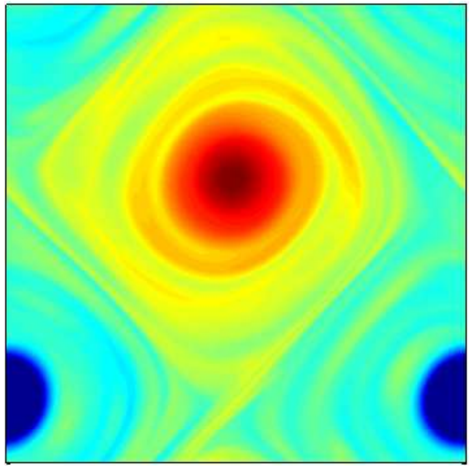
$$\underset{\nu \to 0}{\epsilon \sim} U^3/L \neq 0$$

dissipation anomaly



2D turbulence

$$\epsilon \sim \nu U^2/L^2 \underset{\nu \to 0}{\to} 0$$



Rotating turbulence is intermediate between 2D and 3D:
scaling for ϵ when Ω is large?

Taylor-Proudman theorem

Vorticity equation:

$$\partial_t \boldsymbol{\omega} - 2\Omega\partial_z \mathbf{u} = \boldsymbol{\nabla} \times (\boldsymbol{\omega} \times \mathbf{u}) + \nu\Delta\mathbf{u}$$

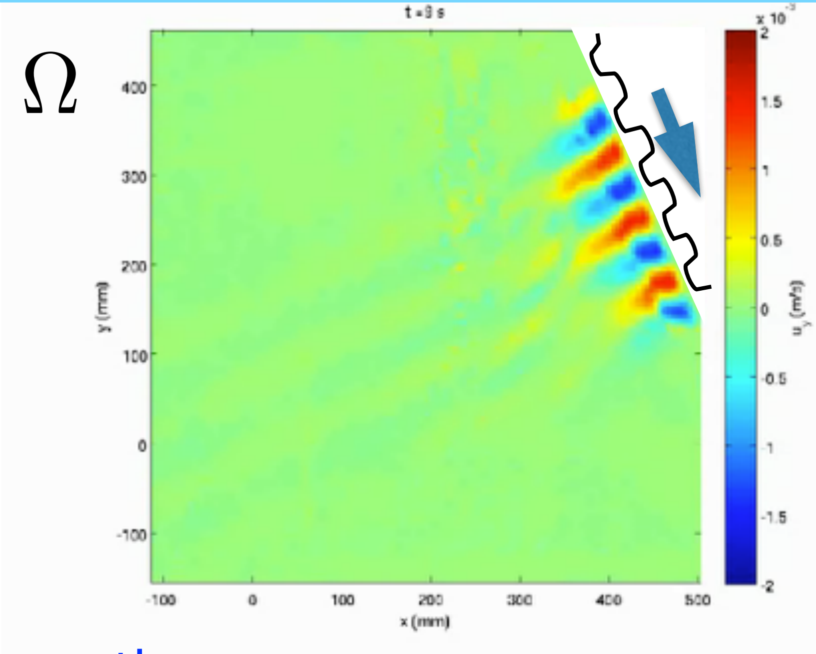
For slow, large-scale motion and large Ω, $\mathbf{u}$ becomes independent of z.

Two-dimensional three-component flow (2D3C)

Turbulent flows contain small-scale rapidly-evolving eddies: does Taylor-Proudman apply?

Wave turbulence

Linearized inviscid equation for large Ω

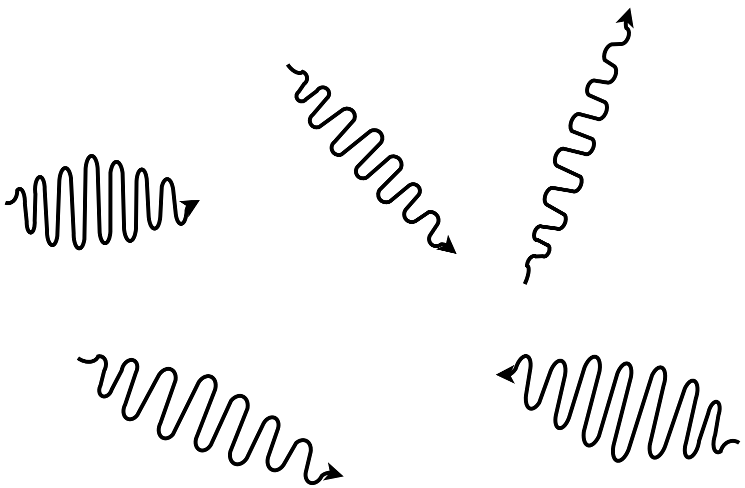


$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} + 2\Omega \mathbf{e}_z \times \mathbf{u} = -\boldsymbol{\nabla} p$$

$$\boxed{\sigma = \pm 2\Omega \frac{k_z}{k}}$$

Inertial waves

Weakly nonlinear regime: 3-wave interaction



Dimensional analysis: $\epsilon \sim \dfrac{U^4}{L^2 \Omega}$

[Iroshnikov, Kraichnan, Zhou]

Derived more rigorously in the framework of wave turbulence [Galtier, Cambon et al., etc.].

Incompatible theories?

Taylor-Proudman states that the flow becomes 2D, but considers only « slow enough » motion.

Wave-turbulence discards such 2D motion:

- 2D motion is not wave-like: no separation of time scales.
- Vanishing coupling coefficient between the 2D modes and two inertial waves.

→ The 2D modes remains zero through 3-wave interactions (if it is zero initially).

However, at the next order in the expansion, 4-wave resonances and quasi-resonant triads transfer energy to the 2D modes.

Outline

I   2D versus 3D flow structures in experimental rotating turbulence

Signature of inertial waves?

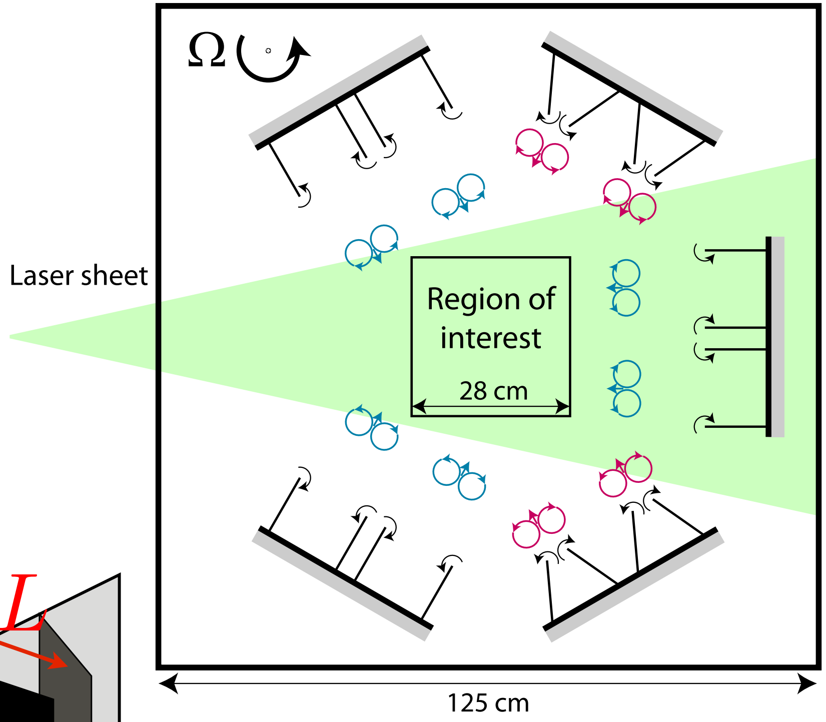
II   Exact two-dimensionalization of rapidly rotating flows

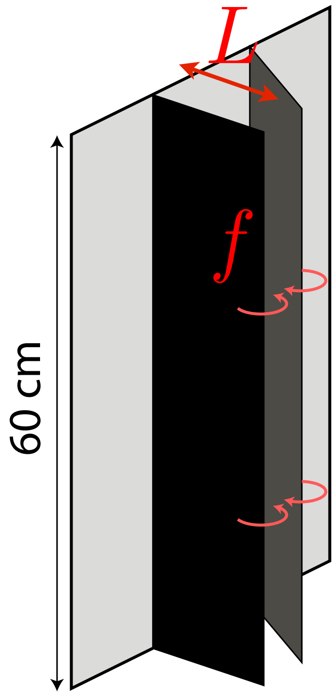
No dissipation anomaly at low Ro.

III Direct measurements of the dissipated power in rotating turbulence

Influence of the forcing geometry.

Forced rotating turbulence





$$Re = \frac{L^2 f}{\nu} \simeq 1000$$

$$Ro = \frac{f}{\Omega} \in [0.1\,, \infty]$$

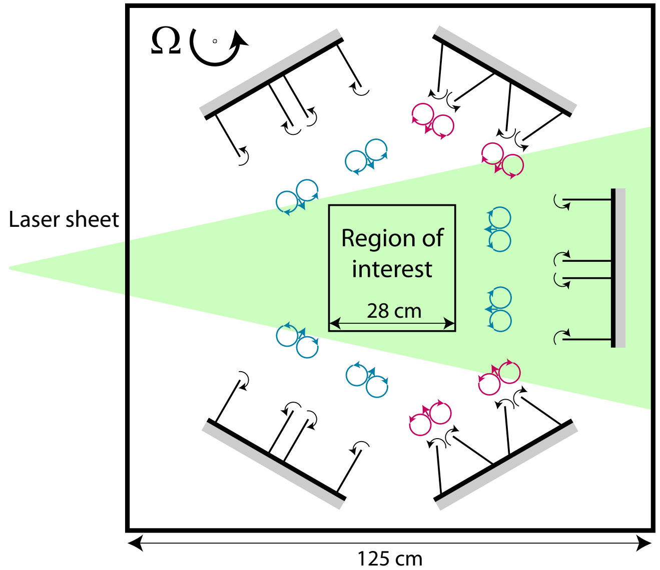
Turbulence in a rotating frame :

An arena of vortex dipole generators
on the Gyroflow rotating platform

Laboratory FAST, University Paris Sud and CNRS

in collaboration with Laboratory LadHyx, Ecole Polytechnique

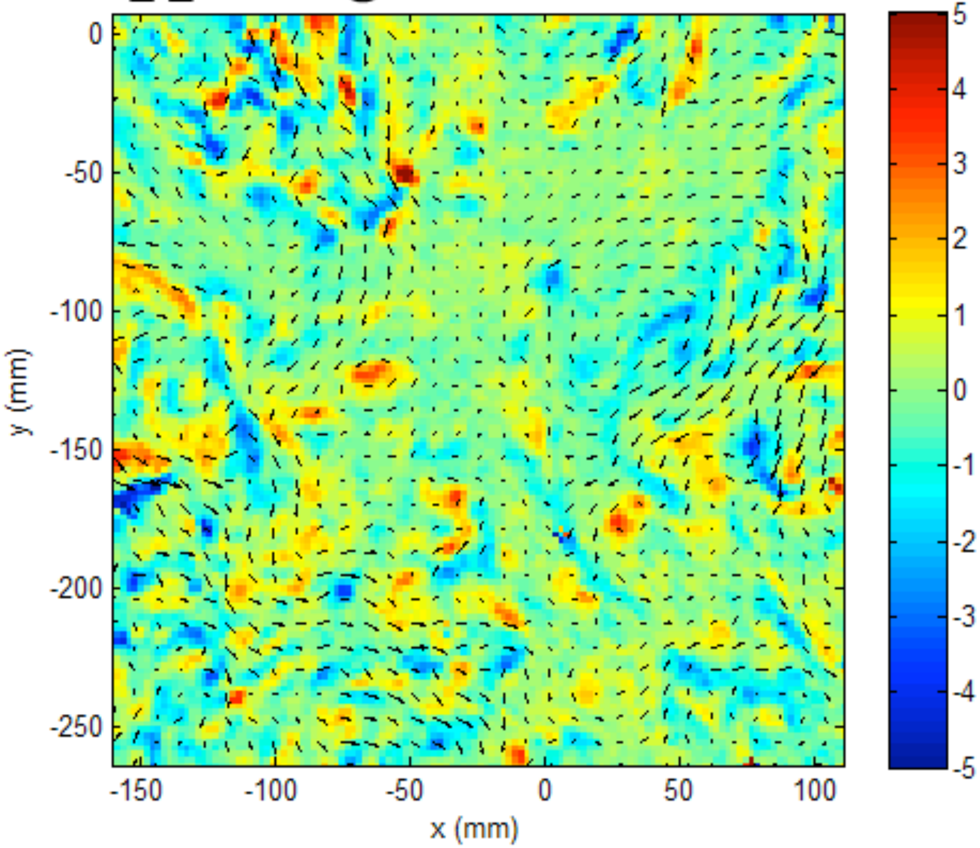
Predominance of cyclones



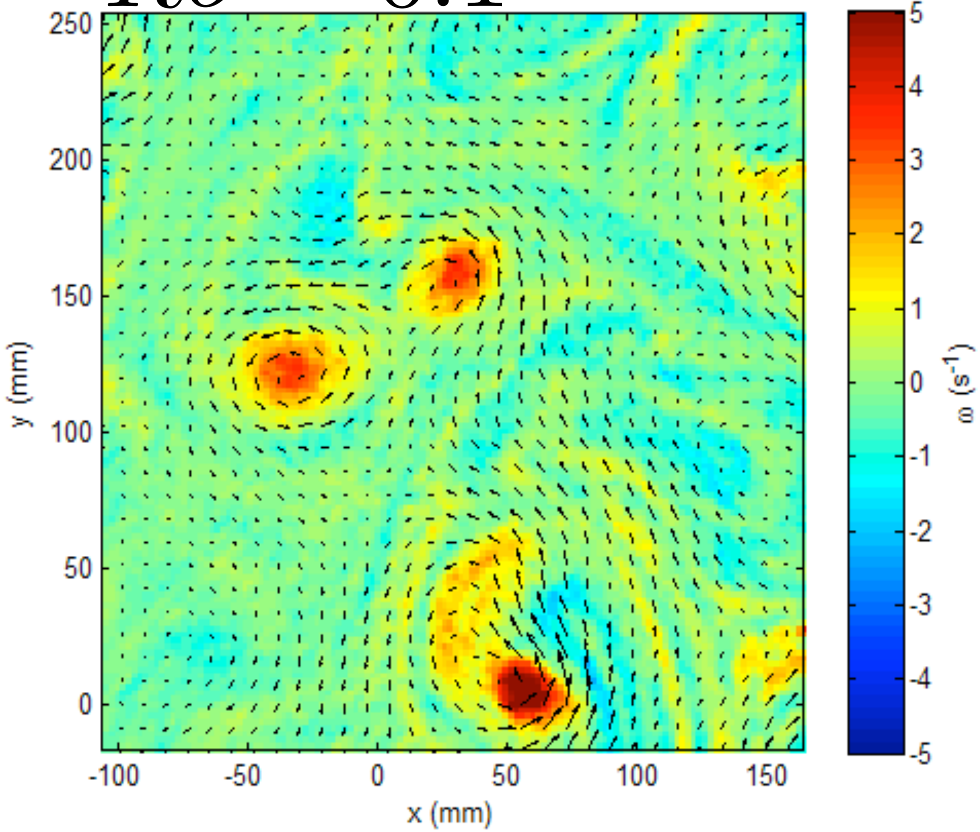
Vertical PIV vorticity, top view.

[A. Campagne, B. Gallet, P.-P. Cortet, F. Moisy, Phys. Fluids, 2014]

$\Omega = 0$

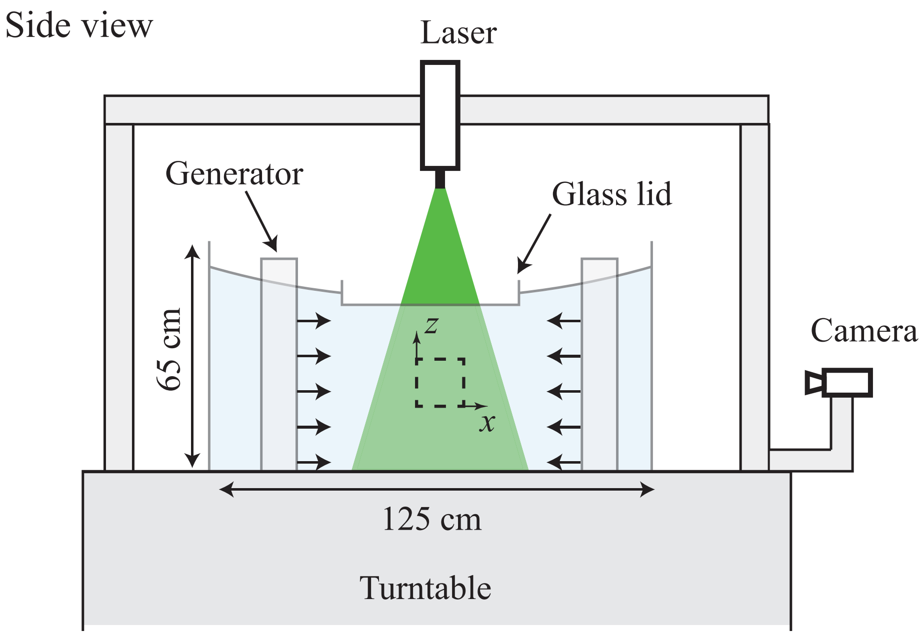


$Ro = 0.1$

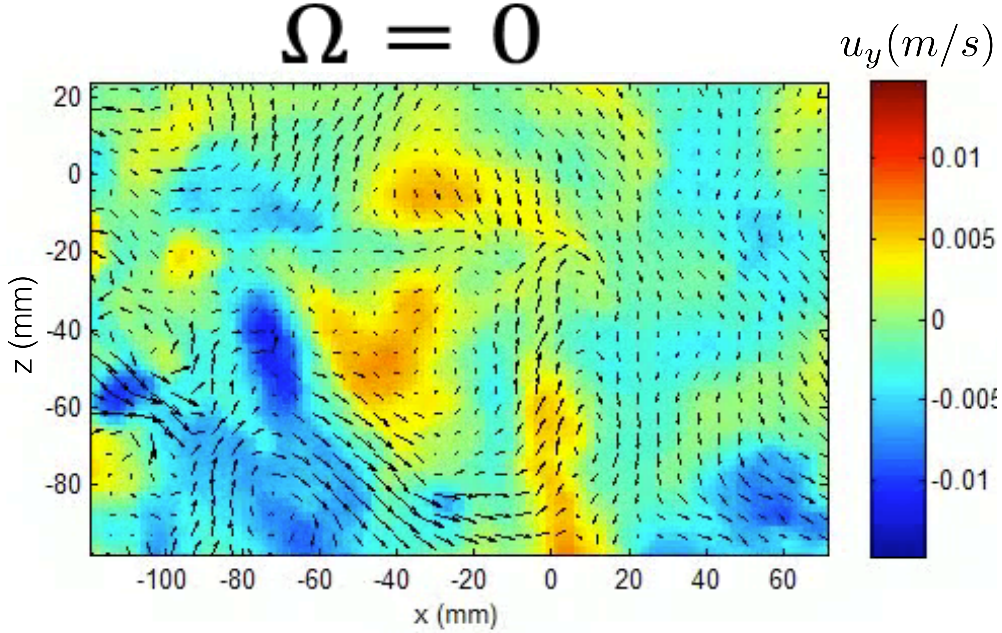


Two-dimensionalization

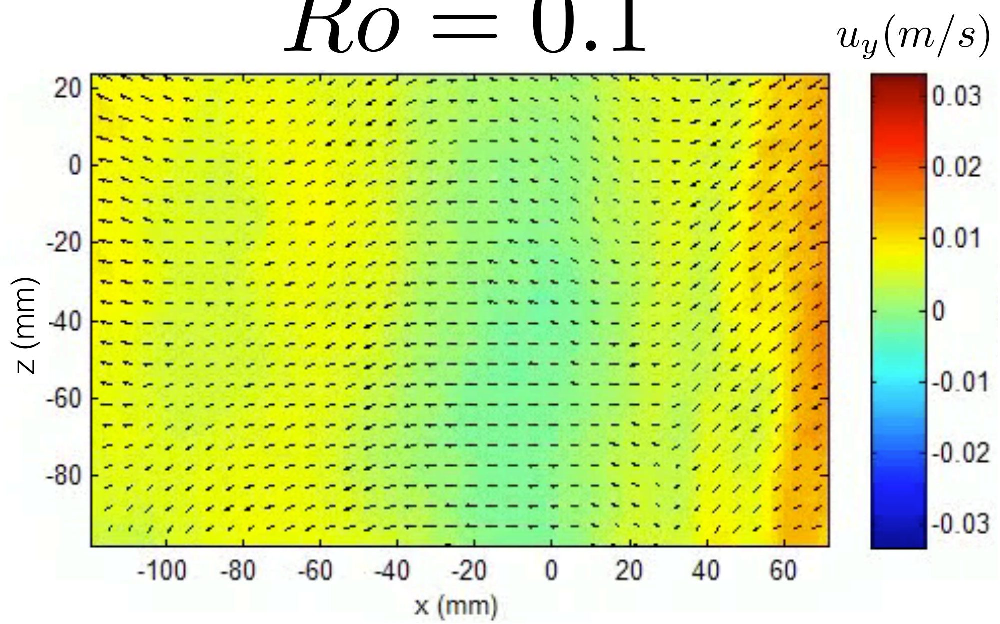
PIV in a vertical plane



$\Omega = 0$



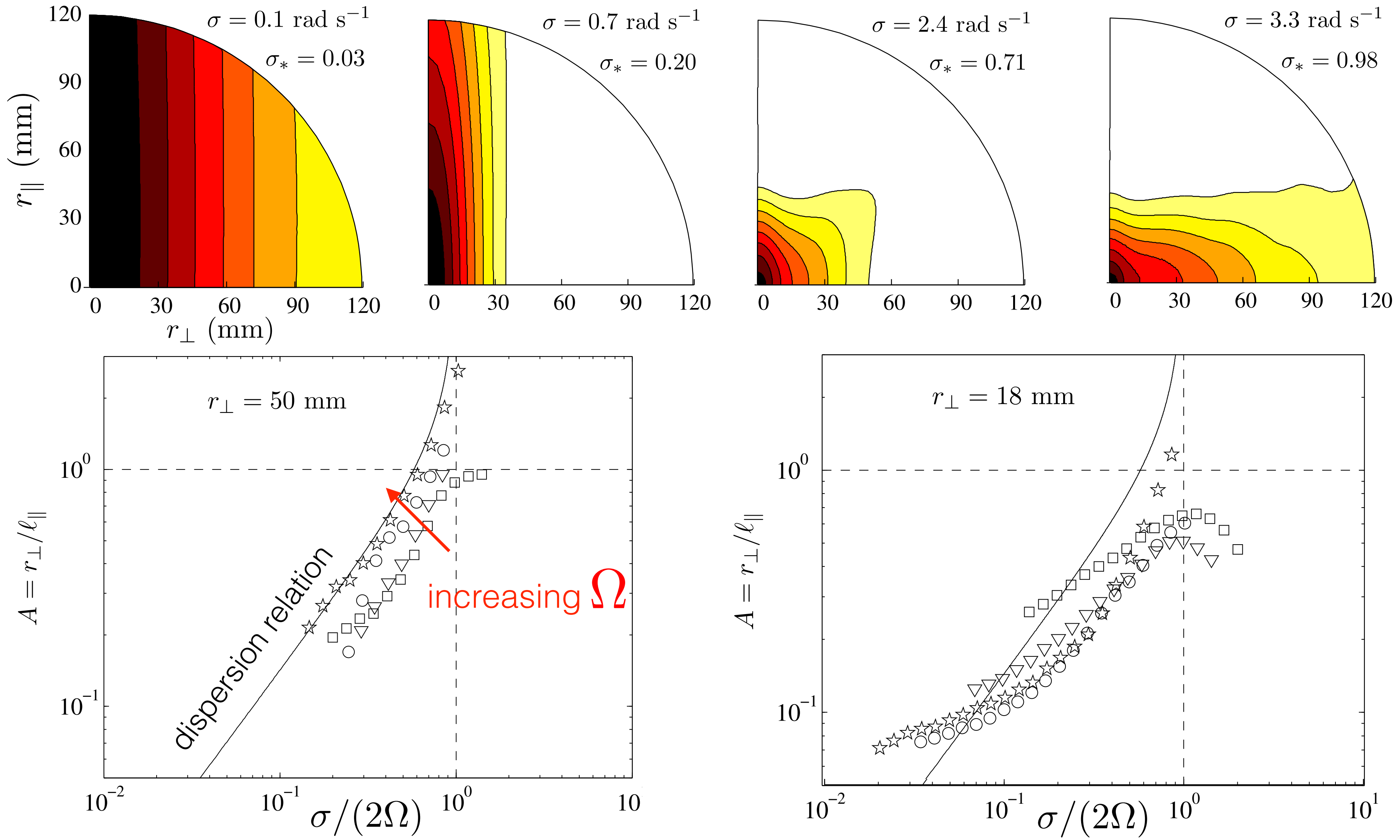
$Ro = 0.1$



Only a few percent of the kinetic energy contained in z-dependent structures (inside the PIV field).

Inertial waves?

Spatio-temporal analysis: 2-point correlation of the temporal Fourier transform. [A. Campagne, B. Gallet, F. Moisy, P.-P. Cortet, PRE, 2015]



Small-scale waves are strongly swept by the intense 2D flow.

Conclusions of part 1

- Global rotation induces strong two-dimensionalization and cyclone-anticyclone asymmetry.
- Large-scale 3D structures follow the inertial-wave dispersion relation.
- Small-scale 3D structures undergo intense sweeping by the 2D flow.

→ Very small fraction of the kinetic energy on the inertial-wave dispersion relation
(a prerequisite for wave turbulence theories).

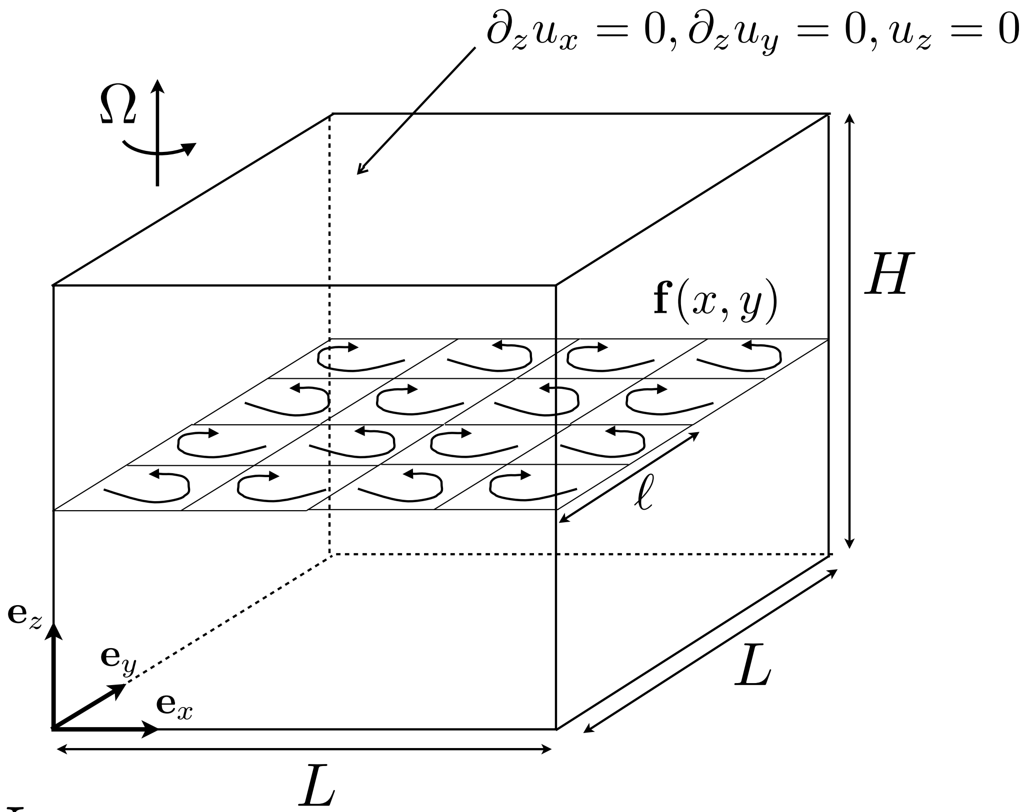
Part II:
Exact two-dimensionalization of rapidly rotating flows.

What happens at even lower Ro?

- How far does two-dimensionalization proceed? Are there still inertial waves?
- Does cyclone-anticyclone asymmetry remain at small Ro?
- Is there a dissipation anomaly?

Theoretical setup

- z-invariant forcing
- Stress-free top and bottom boundaries, or 3D periodic domain.
- Solutions to 2D Navier-Stokes: are they stable to 3D perturbations?



$$Re = \frac{U\ell}{\nu} \qquad Ro = \frac{U}{\ell\Omega}$$

The challenge is to derive stability criteria for a 2D turbulent base flow.

Linear two-dimensionalization

Linear stability of the solution V of 2D Navier-Stokes:

Proof of the existence of a critical Rossby number $Ro_c(Re)$ under which the 2D flow is stable to 3D perturbations.

Lower-bound $Ro_<(Re)$ on $Ro_c(Re)$:

$$Ro_< = c \, Re^{-6} \ln^{-2}\left(Re\frac{L}{\ell}\right) \frac{\ell^{10}}{L^6 H^3 (H+\ell)}$$

For any Re and strong enough global rotation, the flow becomes **exactly** 2D in the long-time limit.

[B. Gallet, JFM, 2015]

Sketch of the proof

- Linearize the rotating Navier-Stokes equation about the 2D turbulent base-flow V(x,y,t).

$$\partial_t \mathbf{v} + (\mathbf{V}\cdot\boldsymbol{\nabla})\mathbf{v} + (\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{V} + 2\Omega\mathbf{e}_z \times \mathbf{v} = -\boldsymbol{\nabla}p' + \nu\Delta\mathbf{v}$$

- Write the evolution equation for the energy in the 3D perturbation v(x,y,z,t)

$$\mathrm{d}_t\left(\int_{\mathcal{D}} \frac{|\mathbf{v}|^2}{2}\mathrm{d}^3\mathbf{x}\right) = -\int_{\mathcal{D}} \mathbf{v}\cdot(\boldsymbol{\nabla}\mathbf{V})\cdot\mathbf{v}\,\mathrm{d}^3\mathbf{x} - \nu\int_{\mathcal{D}} |\boldsymbol{\nabla}\mathbf{v}|^2\mathrm{d}^3\mathbf{x}$$

2D→3D transfers　　viscous dissipation

- Greenspan, Waleffe: the energy transfers between the waves and the 2D modes are very weak for large Ω.
  ➡ I prove that they decrease as $1/\Omega$.

- For large enough Ω, they cannot overcome viscous dissipation. ➡ The 3D perturbation decays.

[B. Gallet, JFM, 2015]

Physical consequences

I can extend this result to perturbations of arbitrary amplitude: above a threshold value of the rotation rate, the flow becomes 2D regardless of the initial condition.

- The flow becomes exactly 2D, with no inertial waves at all.
- Ω is absent from the 2D Navier-Stokes equation, so cyclone-anticyclone asymmetry disappears.
- No dissipation anomaly for such 2D flows.

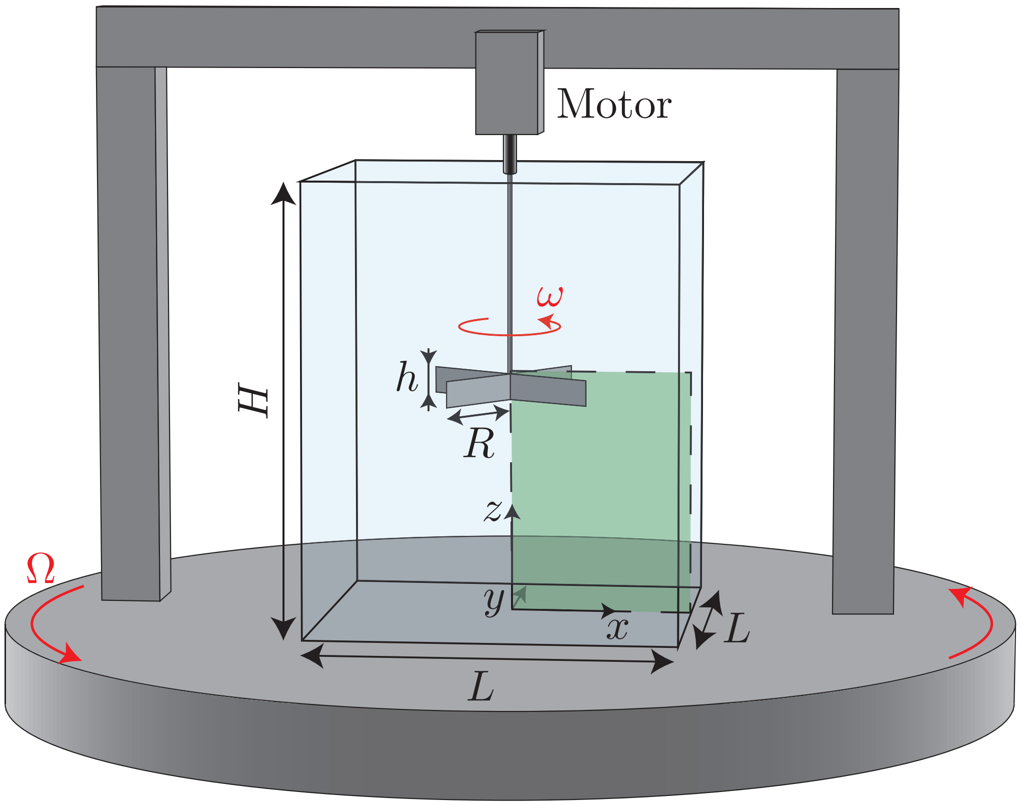
[B. Gallet, JFM, 2015]

Part III: Turbulent drag in a rotating frame

Motivations

- Study the influence of global rotation on the dissipated power in a simple experiment.
- Direct measurement of the dissipated power.
- Study its behavior in the regime of moderately low Ro and large Re.
- Study the influence of the forcing geometry (not necessarily invariant along z!).

Drag in a rotating frame



- Motor in the rotating frame: propeller rotating at ω with respect to the platform.

$$Re = \frac{R^2\omega}{\nu} \qquad Ro = \frac{\omega}{\Omega}$$

- Measurement of the time-averaged torque Γ.

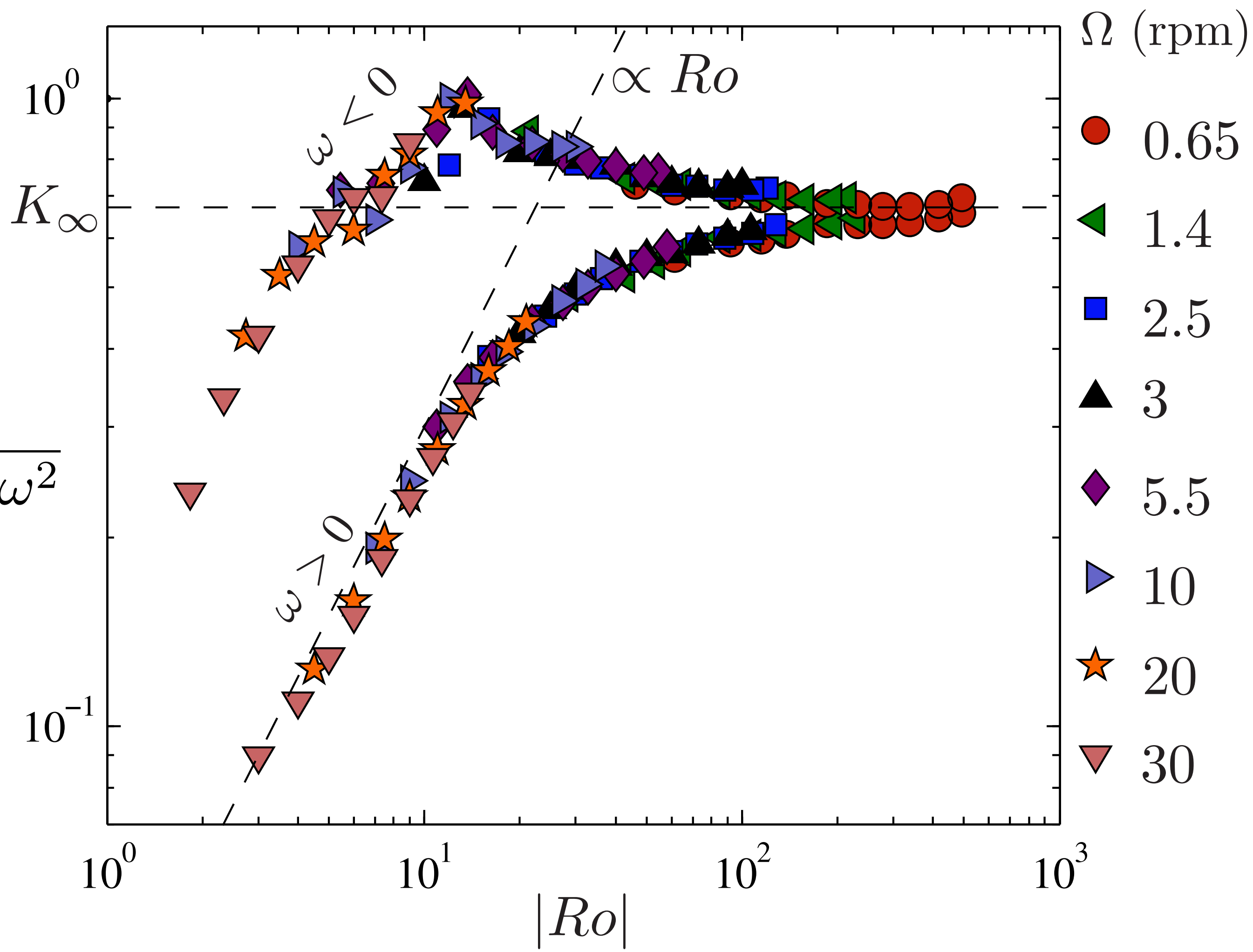
$$K = \frac{\Gamma}{\rho R^4 h \omega^2}$$ drag coefficient

- PIV measurements.

Turbulent drag

High-Re data collapse onto two branches:

$$K = \frac{\Gamma}{\rho R^4 h \omega^2}$$



Strong decrease of K for rapid rotation, approx. $K \sim Ro$

2 possible explanations

Explanation 1: the energy dissipation rate decreases because of inertial-wave dynamics. Following wave turbulence type of arguments:

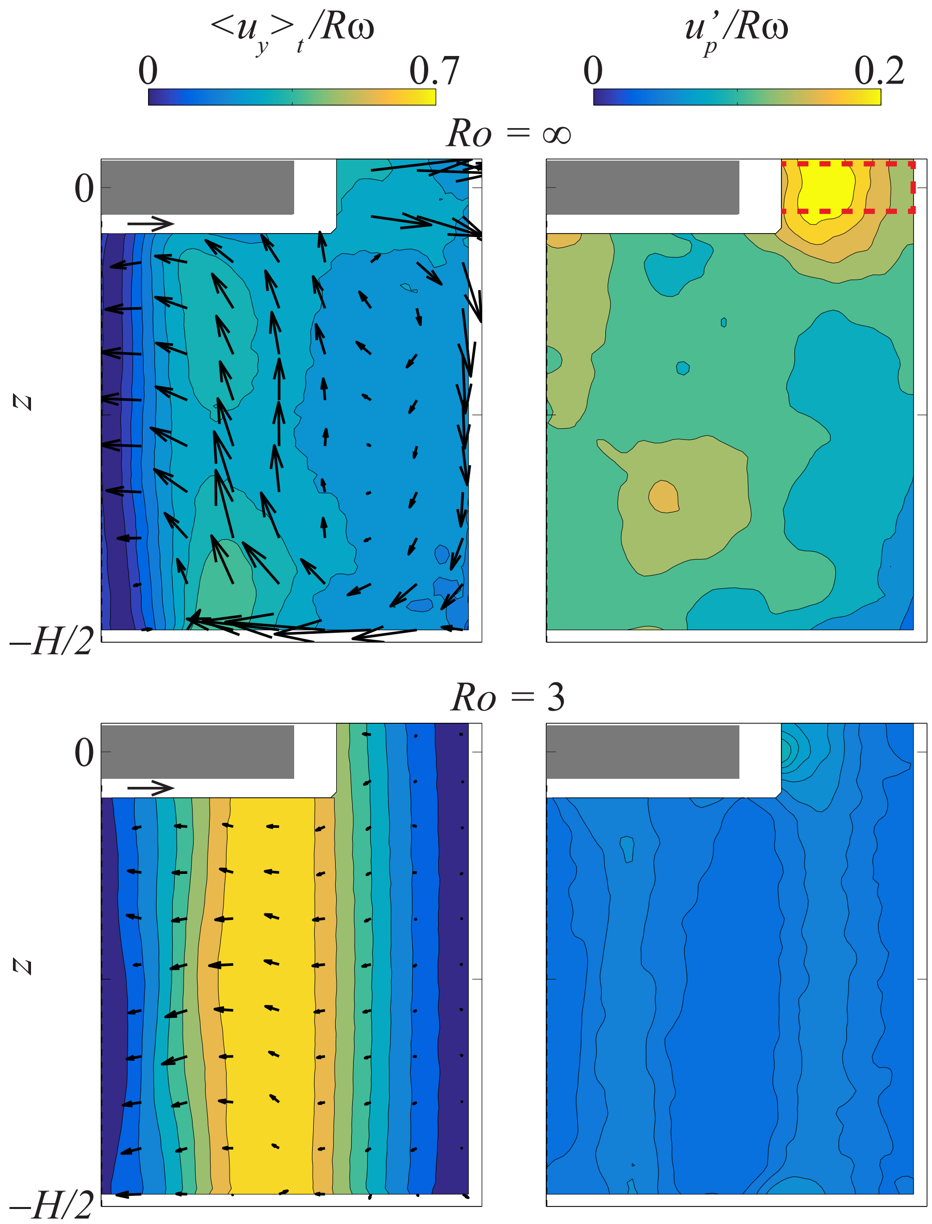
$$\epsilon \sim Ro \frac{U^3}{L} \Rightarrow K \sim Ro$$

Explanation 2: forcing compatible with Taylor-Proudman. Because of two-dimensionalization, the rapidly rotating flow ressembles solid-body rotation, with weak 3D poloidal recirculation.

⇒ Dissipation is due to the weak 3D recirculation only.

PIV measurements

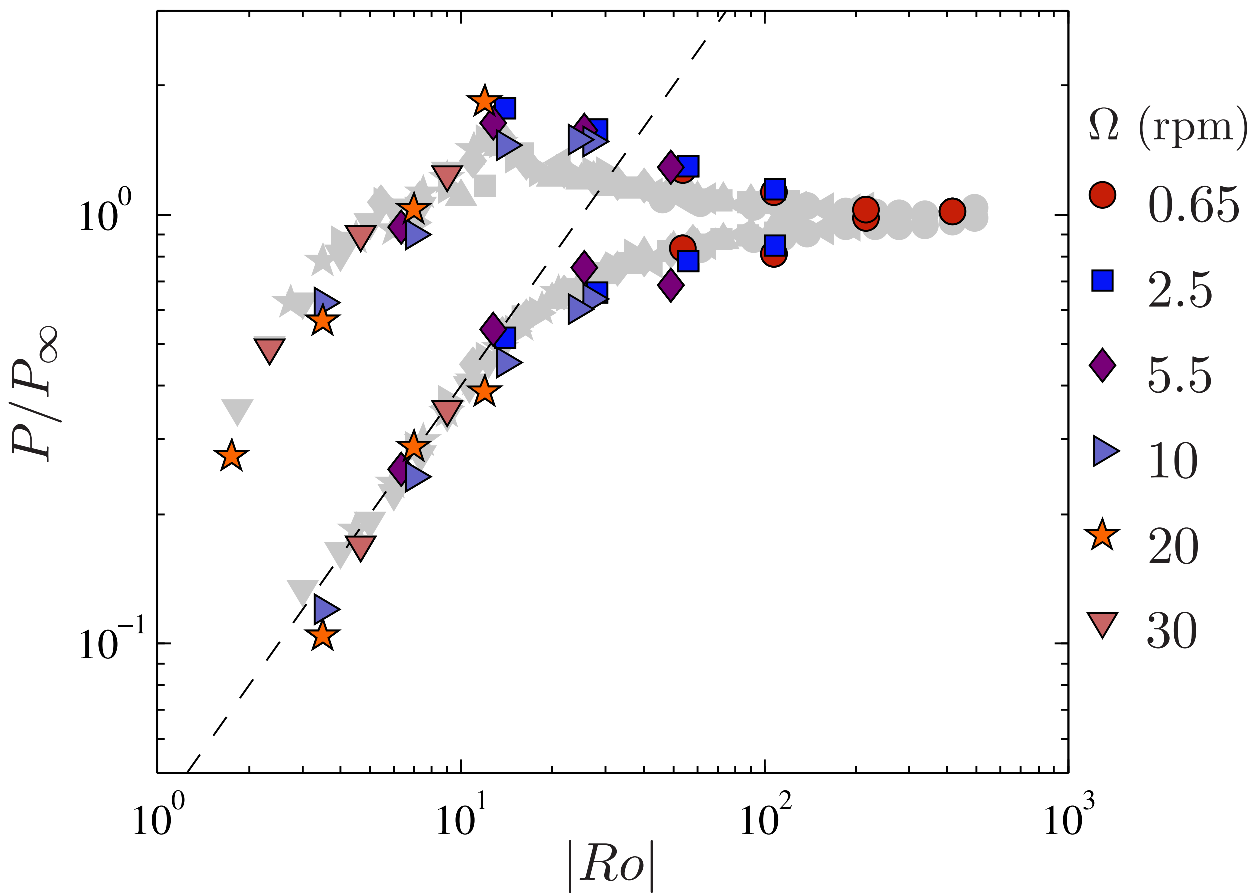
- No signature of inertial waves could be identified.
- Mean flow dominated by toroidal solid-body rotation
- Turbulent poloidal velocity fluctuations decrease with increasing Ω.



supports the two-dimensionalization scenario.

PIV measurements

Dissipation due to the poloidal recirculation evaluated using the 3D non-rotating estimate $\epsilon \sim u_p'^3/h$

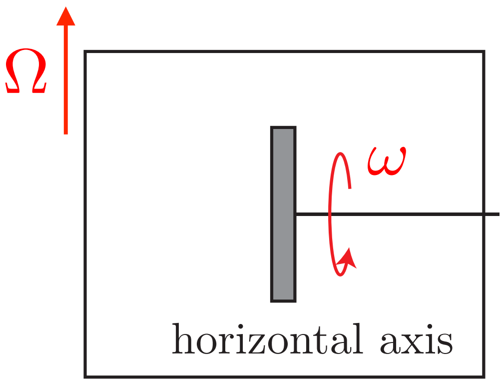


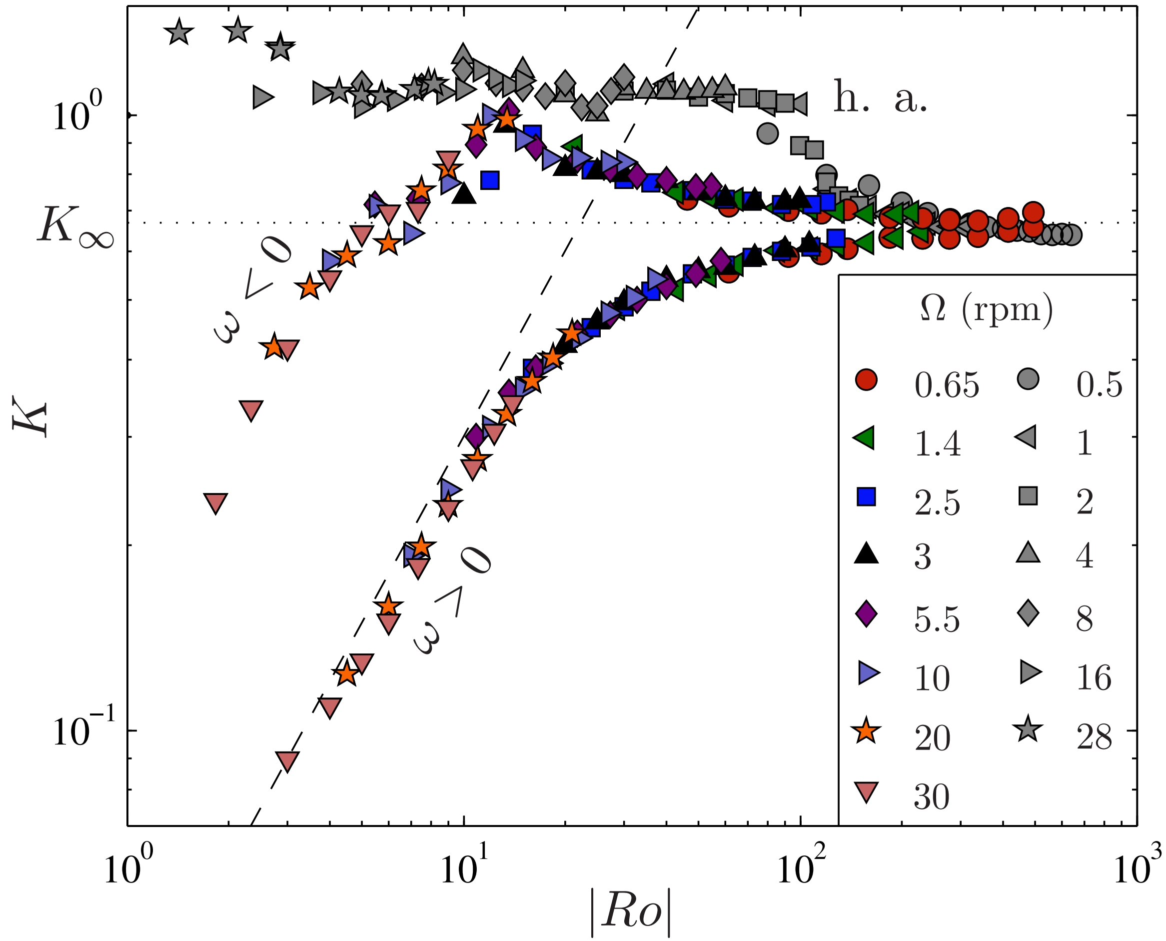
ϵ is not modified by some inertial-wave dynamics.

➡ supports the two-dimensionalization scenario.

Horizontal axis

A forcing configuration that is incompatible with Taylor-Proudman





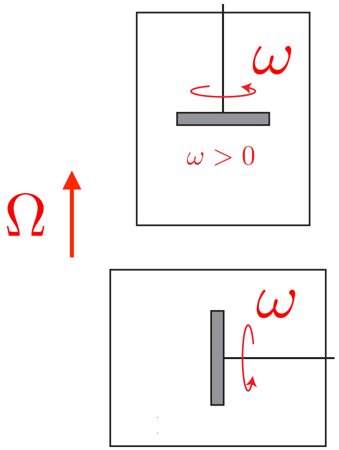
K does not decrease for increasing Ω!

The decrease in K takes place only when the forcing is compatible with TP, and allows for two-dimensionalization.

Conclusions of part III

A strong decrease of the drag coefficient for rapid global rotation [A. Campagne, N. Machicoane, B. Gallet, P.-P. Cortet, F. Moisy, JFM, 2016]

- Due to the two-dimensionalization of the velocity field, and not to IW dynamics.
- Takes place only when the forcing is compatible with Taylor-Proudman.
- Dissipation well-estimated by the non-rotating estimate u'^3/L using the turbulent 3D recirculation.

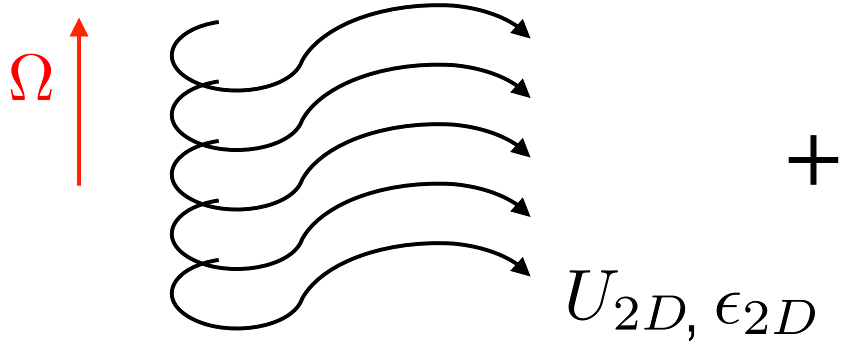


K strongly decreases with Ω for motion $\perp$ to Ω.

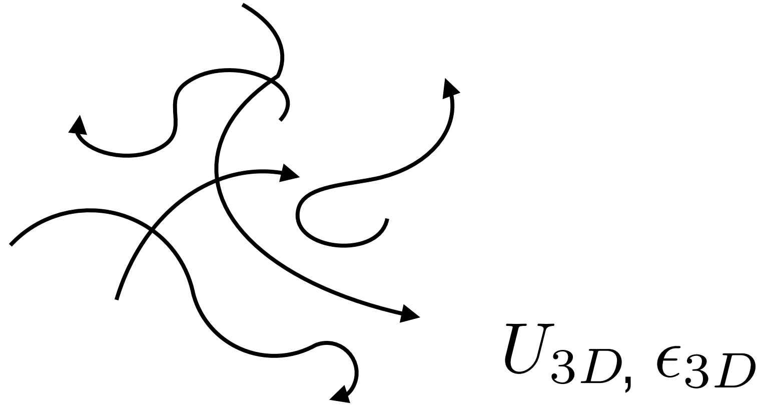
K is weakly affected by Ω for motion $\parallel$ to Ω.

General conclusion

Structure of high-Re low-Ro flows:



+



z-invariant 2D flow

- laminar dissipation rate

$$\epsilon_{2D} \sim \nu \frac{U_{2D}^2}{L^2}$$

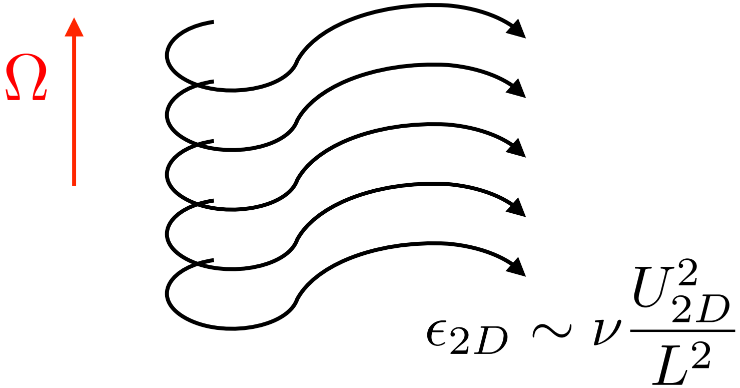
3D fluctuations

- strongly swept by the 2D flow: not described by the IR dispersion relation.
- turbulent dissipation $\epsilon_{3D} \sim \frac{U_{3D}^3}{L}$

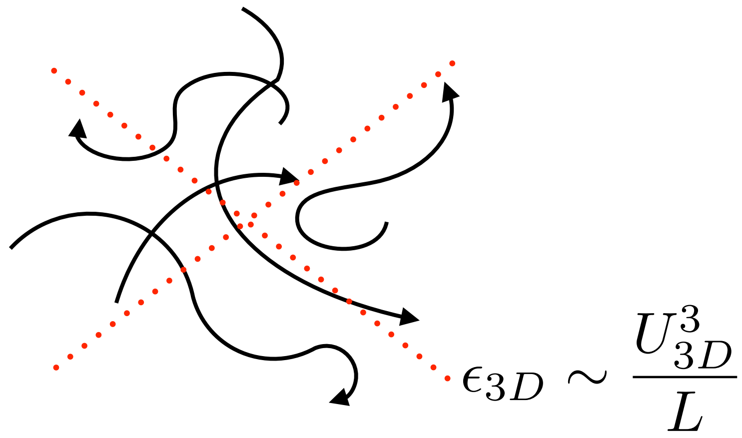
  (at intermediate Ro, may be lower for Ro$\ll$1).

General conclusion

Case 1: forcing compatible with Taylor-Proudman:



+



- U_{3D} strongly decreases for decreasing Ro.
  ➡ $\epsilon = \epsilon_{2D} + \epsilon_{3D}$ decreases.
- Theoretically, exact two-dimensionalization for $Ro < Ro_c(Re)$
  ➡ laminar dissipation $\epsilon = \epsilon_{2D} \sim \nu U_{2D}^2/L^2$

Case 2: forcing incompatible with Taylor-Proudman:

Imposed U_{3D} ➡ $\epsilon \simeq \epsilon_{3D} \sim U_{3D}^3/L$