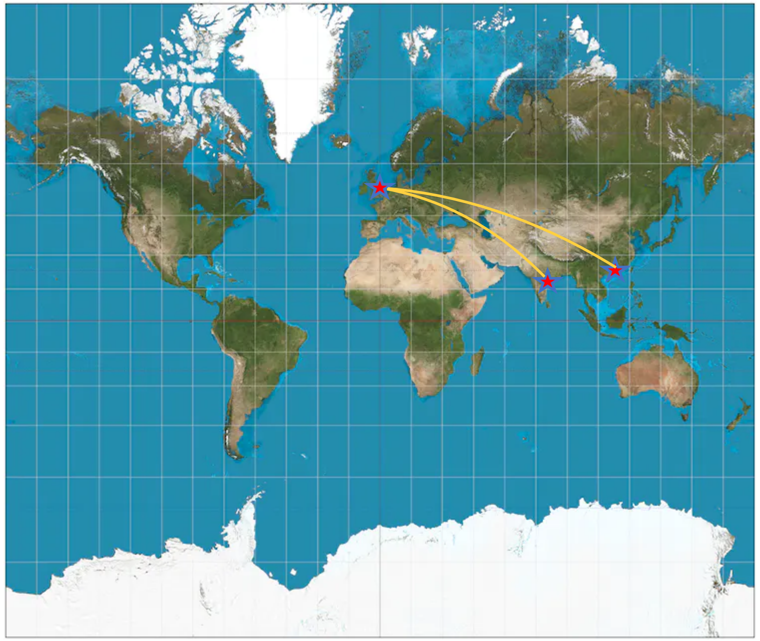
Prediction of turbulent systems from limited measurements: classical methods to machine learning

Vikrant Gupta

Southern University of Science & Technology

Collaborators: **Prof. Minping Wan**, SUSTech; Prof. Shiyi Chen, SUSTech; Prof. Larry K.B. Li, HKUST; Prof. Simon J. Illingworth, U. Melbourne; Prof. Matthew P. Juniper, U. Cambridge; Dr Dachuan Feng, TU Delft; Dr Wen Zhang, SUSTech; Dr Anagha Madhusudanan, Caltech.



Bachelor's and Master's degrees
Indian Institute of Technology Madras, India

Thermoacoustic instabilities in gas-turbine engines

Doctoral studies and postdoctoral research
University of Cambridge, UK

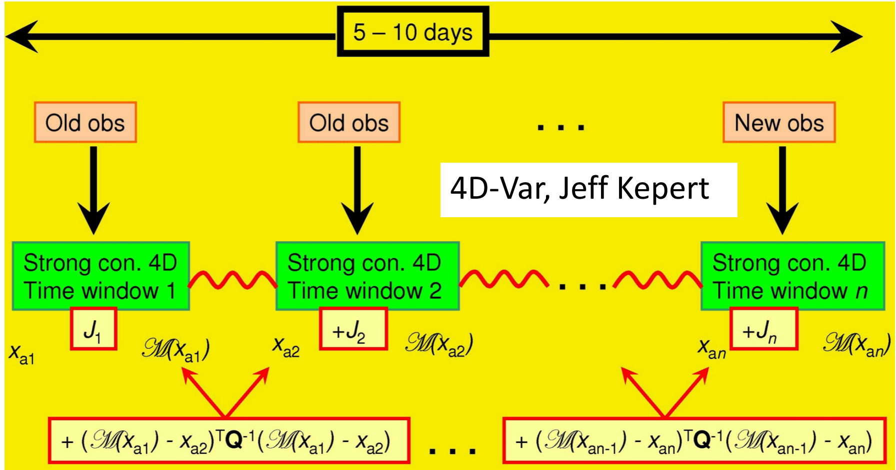
Flow instabilities in channel flows and gas-turbine engines; Tidal energy

Since 2016
Southern University of Science & Technology, China

Wind energy; Wall-bounded turbulence; Wake and jet flow instabilities; Aeroacoustics.

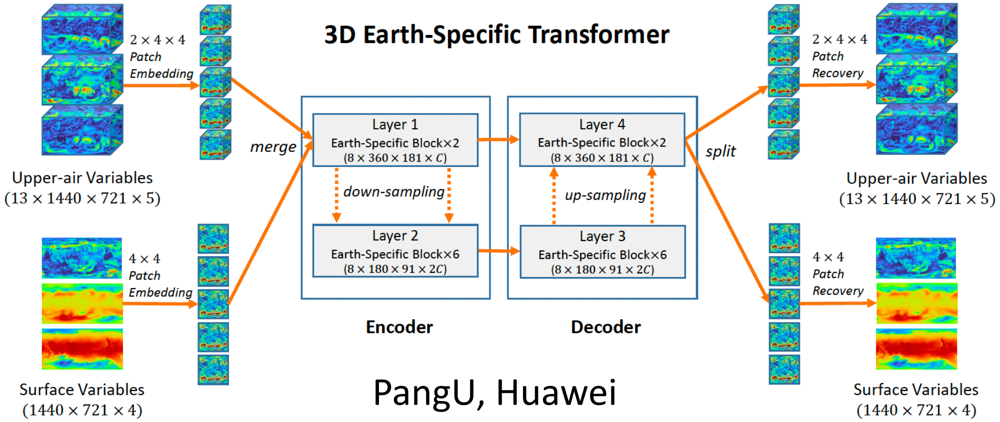
Motivation: rapid development of data-driven predictive methods

Model-based methods (4D-Var, Kalman filter, PINN)



Data assimilation: The dynamics is known and used as constrained

Model-free methods (Neural networks, DMD, POD)



Purely data-driven: The dynamics is learned

Vikrant Gupta (vikrant@sustech.edu.cn)

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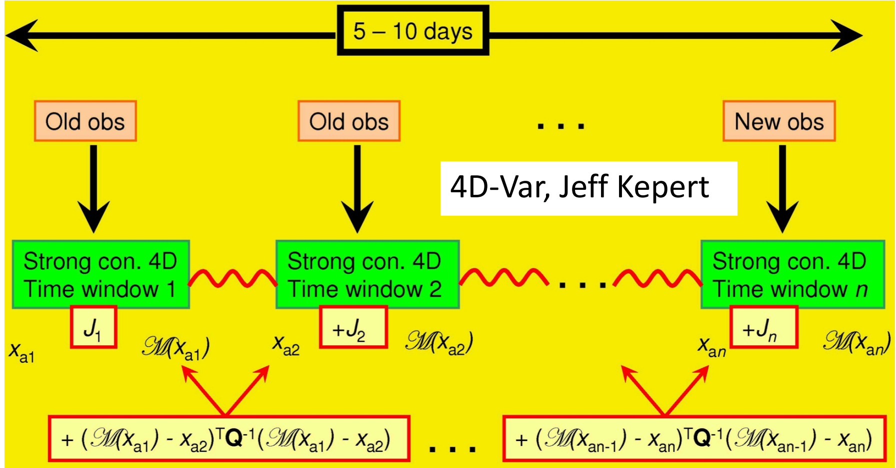

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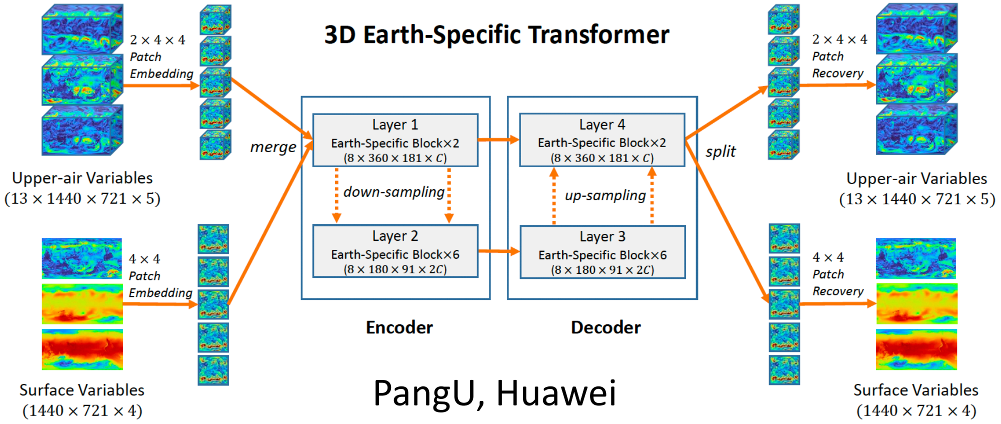
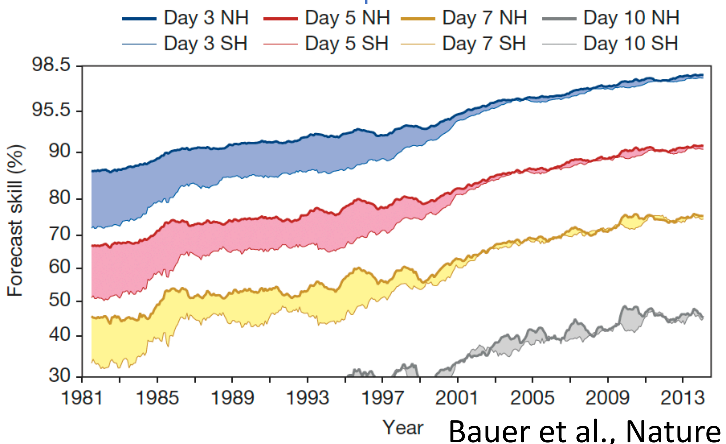


Purely data-driven: The dynamics is learned

Improvements in data assimilation, models and measurements have gradually improved NWP over the past three decades.



Bauer et al., Nature 2015

PangU (Huawei) and FourCastNet (Nvidia and Lawrence Berkeley NL) are recently developed and already claim better performance than existing NWP.

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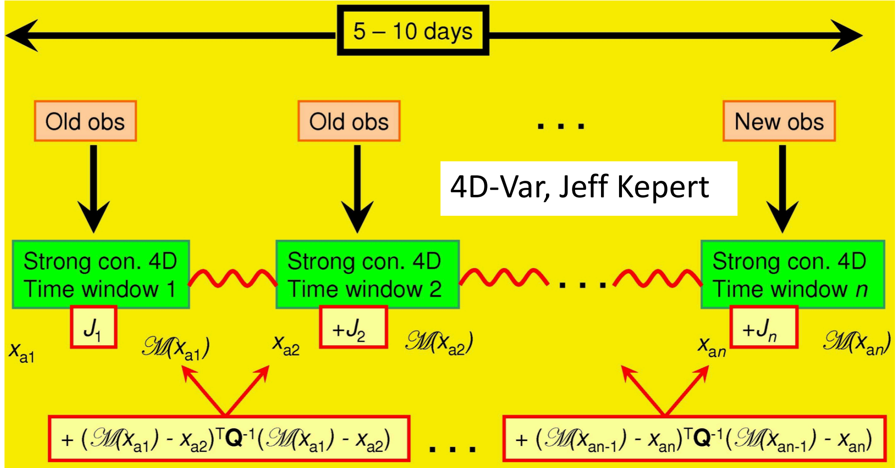

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ECMWF

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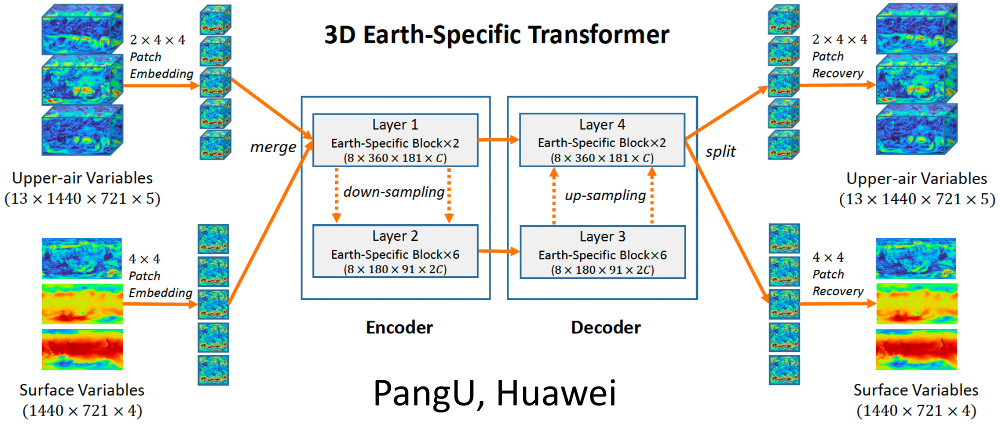

PangU, Huawei

Purely data-driven: The dynamics is learned

Hesitancy to use deep learning for end-to-end use limit their practical implementation.

There is little to no understanding on under what conditions can a purely data-driven model-free approach perform better than a model-based approach.

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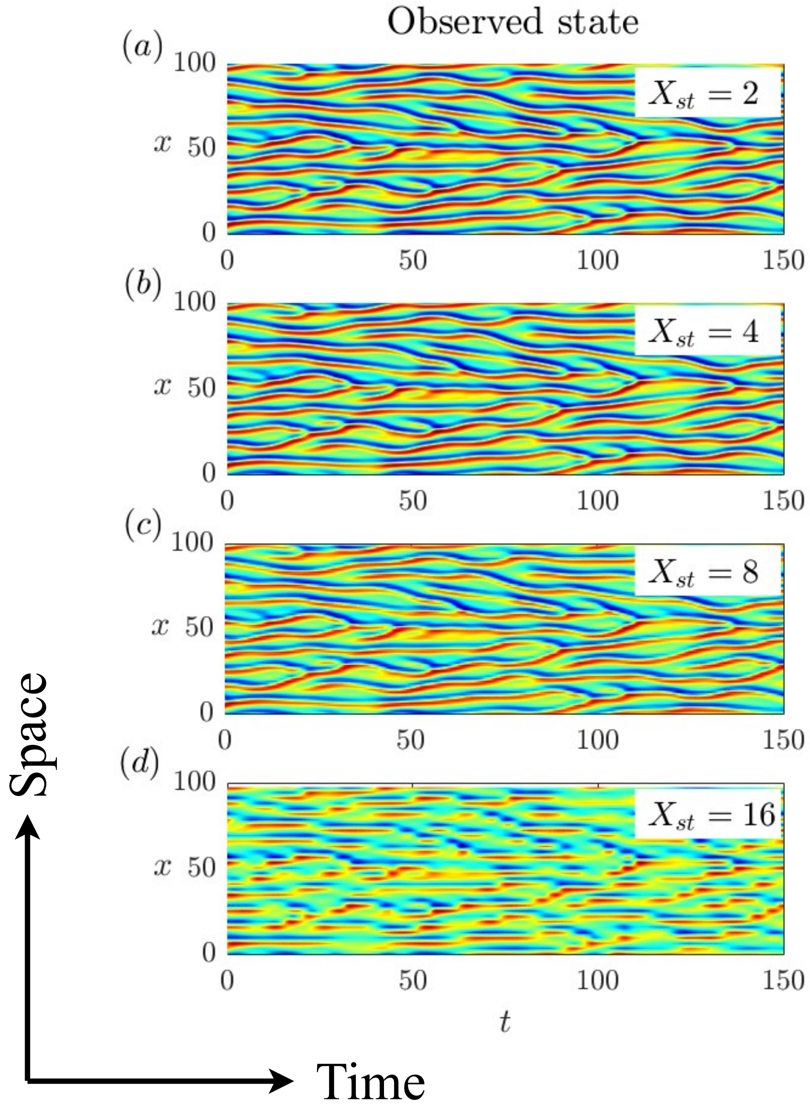

- Is there a limit of spatial resolution beyond which data assimilation methods cannot estimate/predict?
- Do model-free machine learning methods need higher or lower resolution?
- Can classical methods still be useful when data-driven methods fail?

Model-based methods

* Data assimilation methods
  * 4D-Var (variational method)
  * Ensemble Kalman filter (sequential method)
* Kuramoto-Sivashinsky system
  * Measurement conditions
* Fully developed turbulence (criterion)

Model-free methods

* Recurrent neural networks
  * Reservoir computing
  * LSTM
* Kuramoto-Sivashinsky system
  * Measurement conditions
* Data assimilation (criterion)

Linearised (low-rank approximation) models

* Model deduction for wall turbulence

Methods: 4DVar, EnKF and Interpolation

4D-Var (variational method)

$$J = \frac{1}{2} \sum_{k=-T_d}^{k=0} \left(\mathbf{v}_k - h\left(\mathbf{u}_k\right)\right)^T \left(\mathbf{v}_k - h\left(\mathbf{u}_k\right)\right)$$

- Most common in numerical weather prediction
- Use variational calculus and adjoint equations
- Iterative calculations for optimal solution

EnKF (sequential method)

$$\mathbf{u}_k^a = \mathbf{u}_k^f + \mathbf{P}_k^f \mathbf{H}_k^T \left(\mathbf{H}_k \mathbf{P}_k^f \mathbf{H}_k^T + \mathbf{R}\right)^{-1} \left(\mathbf{v}_k - \mathbf{H}_k \mathbf{u}_k^f\right)$$

- Cost effective, common for turbulent flows
- Use Kalman filter and Monte Carlo sampling
- Sequential calculations for optimal solution

Interpolation (no data assimilation)

- Predictions are obtained by simply time-marching the interpolated initial conditions
- Data assimilation methods must be significantly better than interpolation

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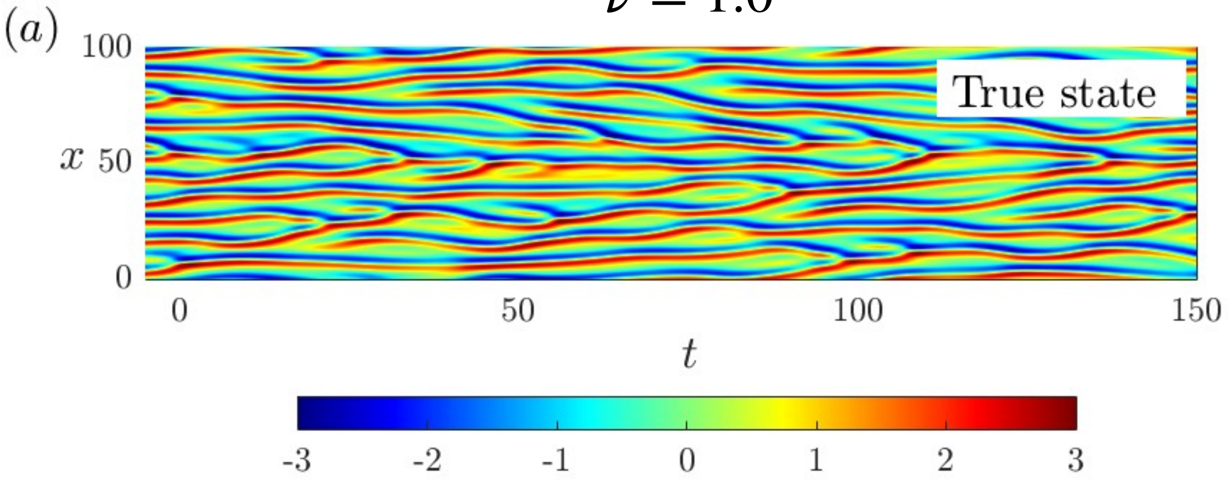
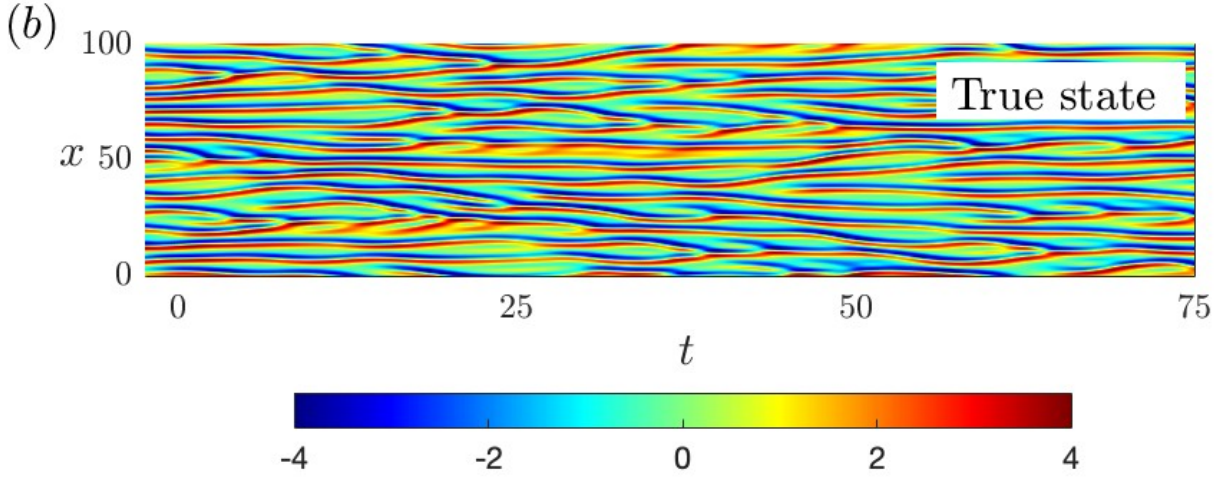
Kuramoto-Sivashinsky system: a model for weak turbulence (spatiotemporal chaos)

$$u_t = -uu_x - u_{xx} - \nu u_{xxxx}$$

ν is the eddy viscosity and $L = 32\pi$ is the system size

$\nu = 1.0$





- Introduced to describe turbulence in magnetised plasma, flame front propagation and chemical reaction diffusion process
- More complex than Lorenz system but much simpler than the Navier—Stokes equations

Kuramoto-Sivashinsky system: a model for weak turbulence (spatiotemporal chaos)

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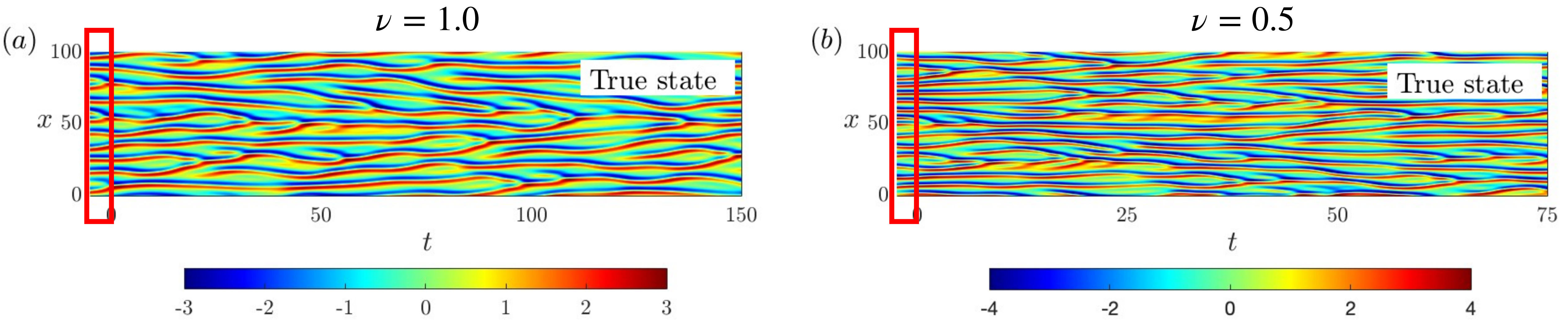
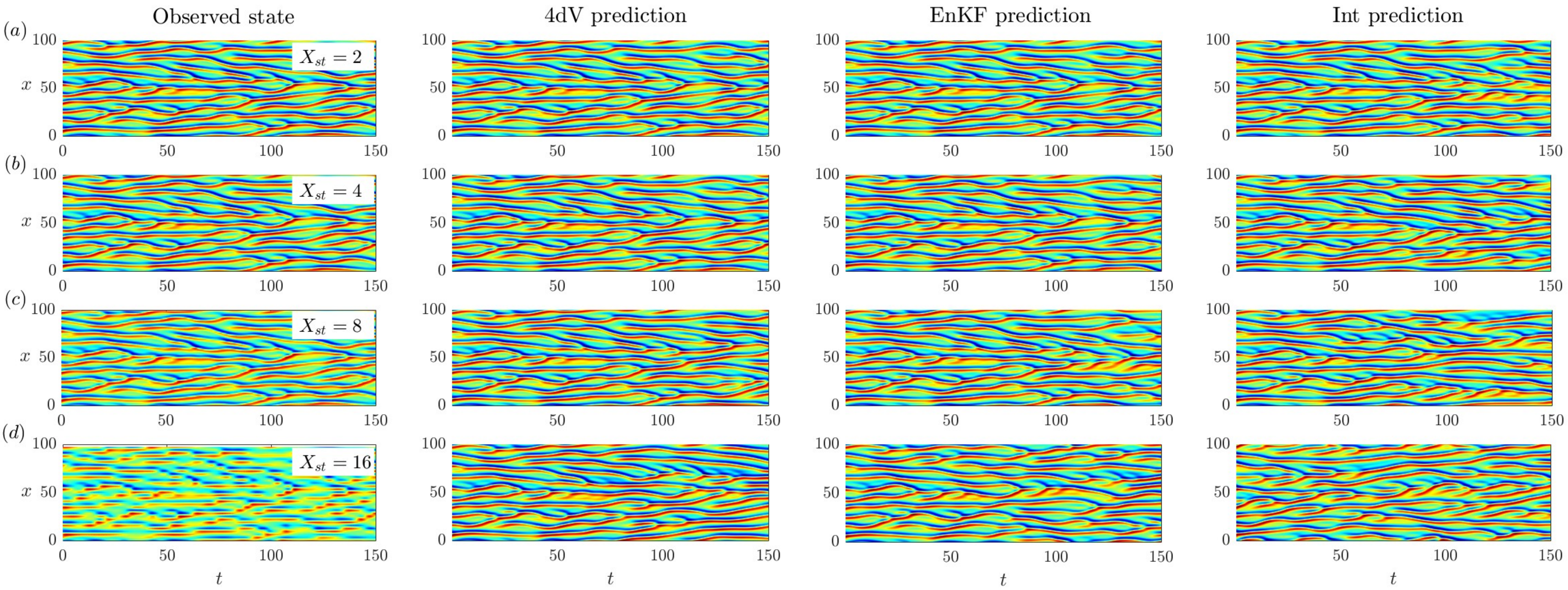


- Use data before $t = 0$ (assimilation window) to predict the future $t > 0$

Prediction accuracy variation with sparsity of observations

$\nu = 1.0, \;\; n = 512$



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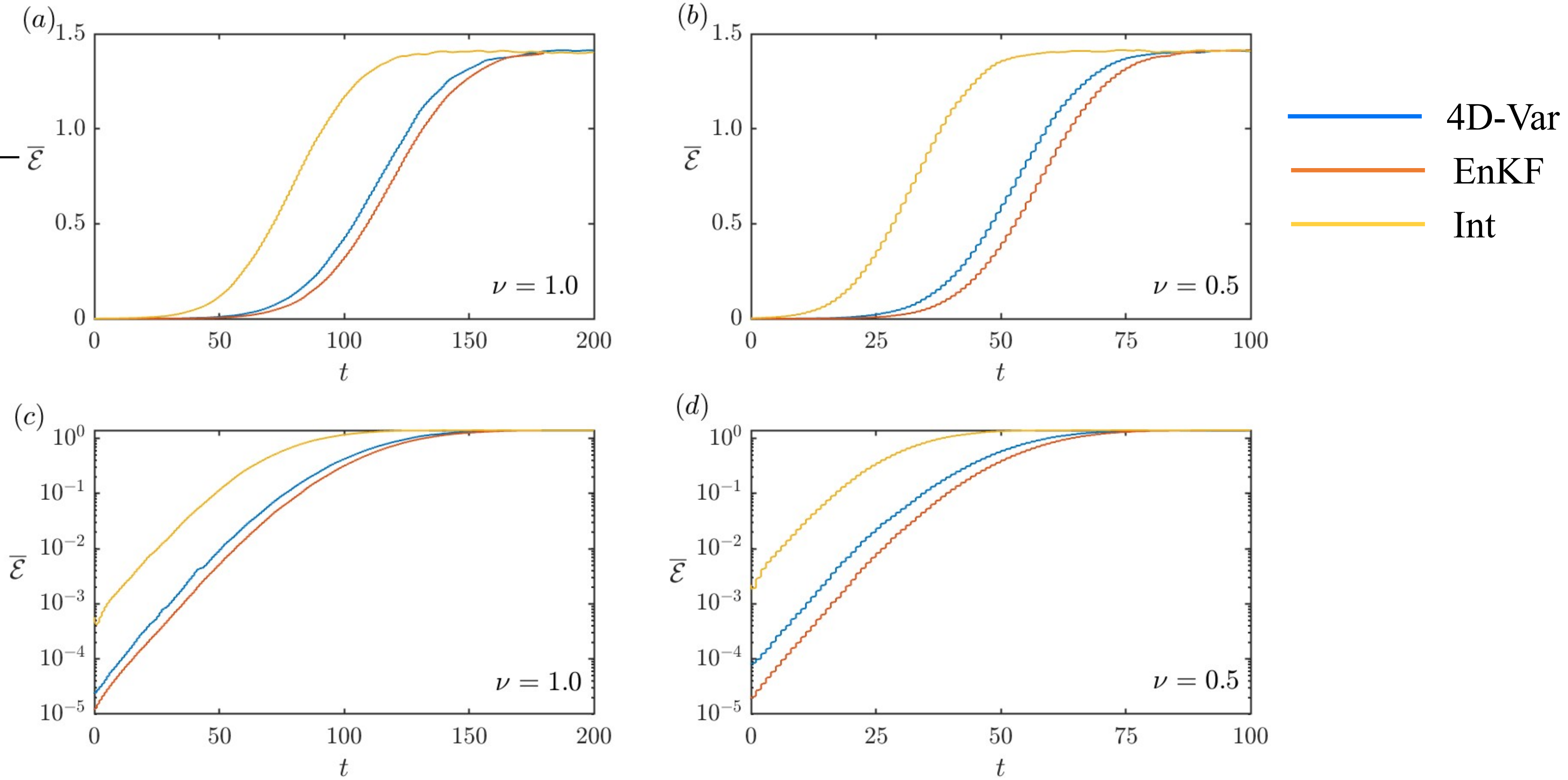
Quantification of prediction accuracy

$$u_t = -uu_x - u_{xx} - \nu u_{xxxx}$$

ν is the eddy viscosity and $L = 32\pi$ is the system size

$\nu = 1.0$ $\nu = 0.5$

This is normalised root-mean-square error averaged in space and over several calculations



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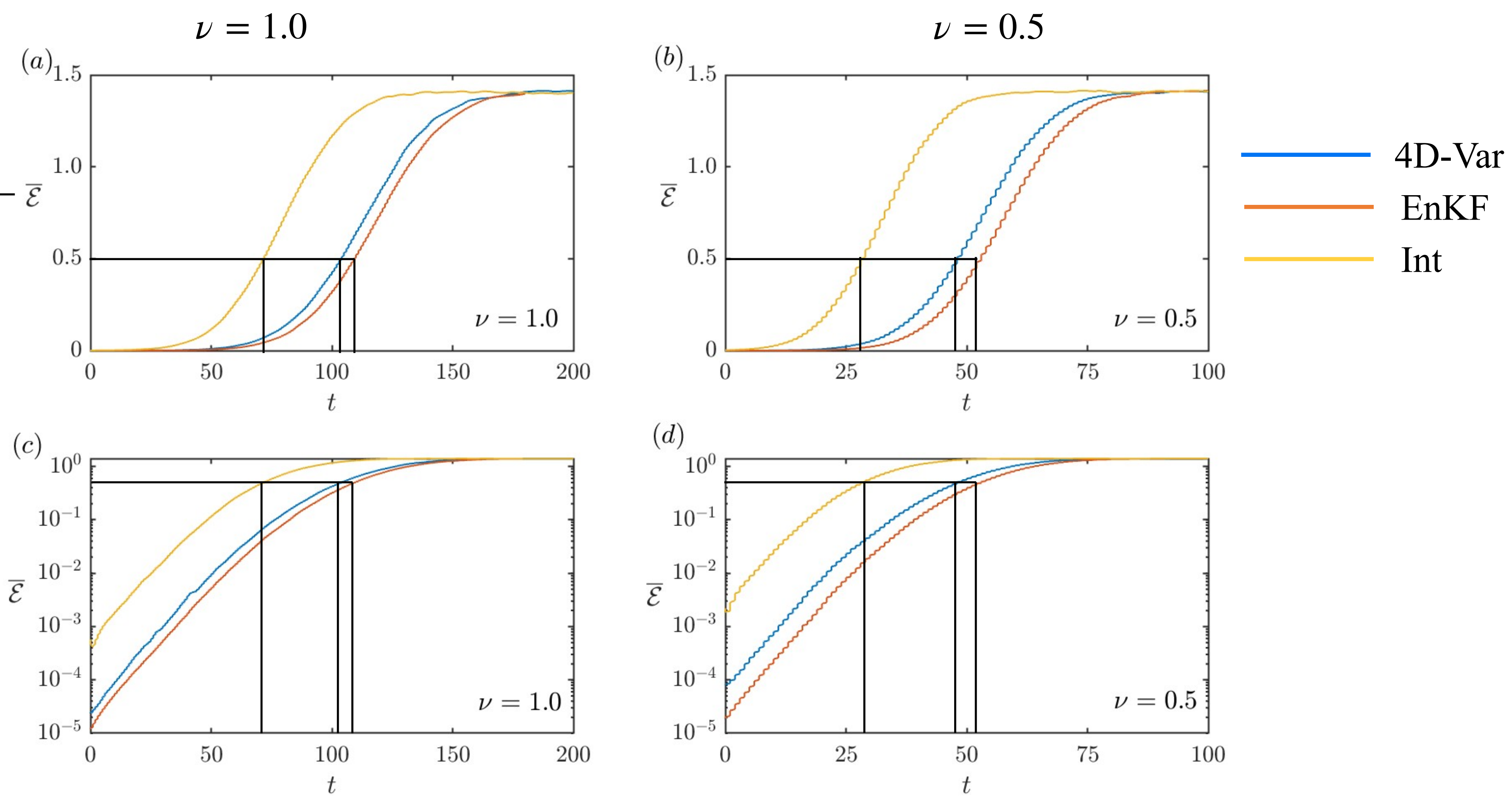
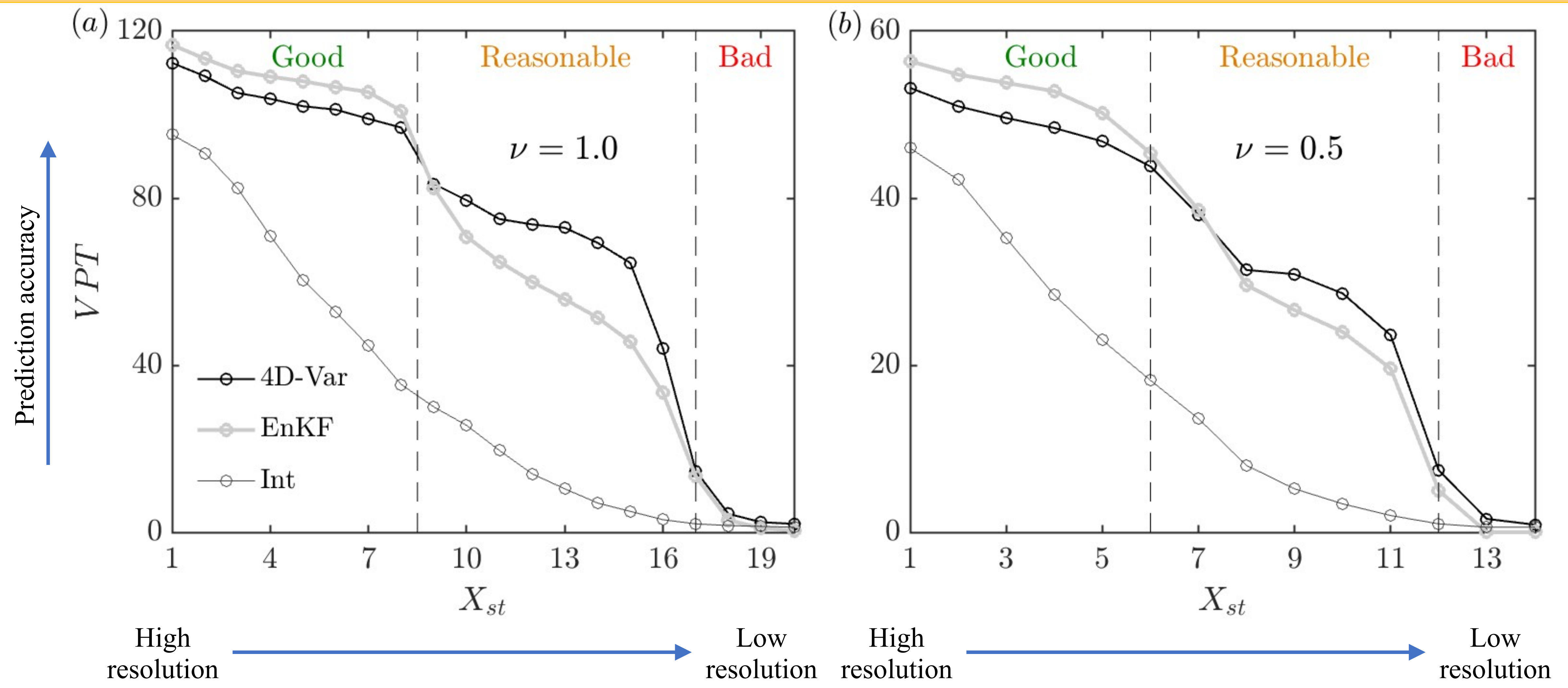
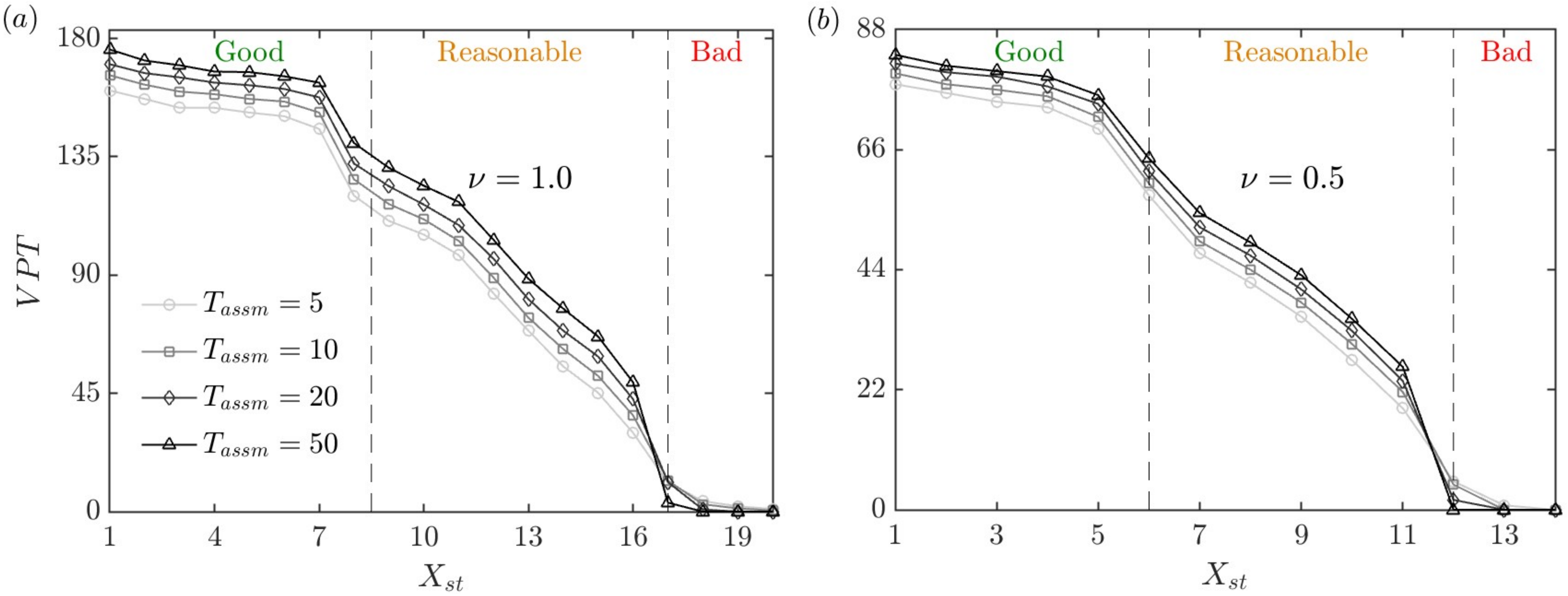


VPT - The accuracy is measured in terms of the time at which the error crosses 0.5 threshold

Variation of prediction accuracy with sparsity of observations

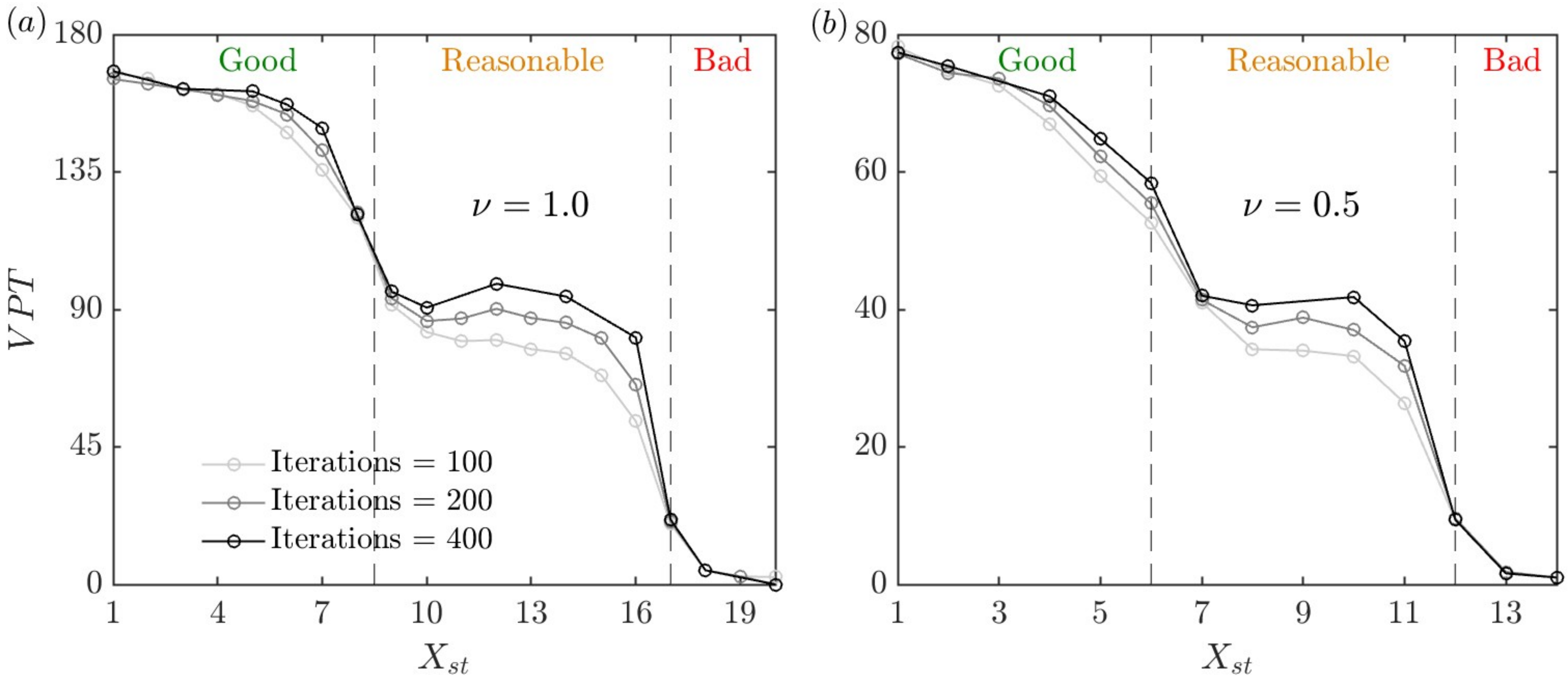


EnKF: Variation with the length of assimilation window



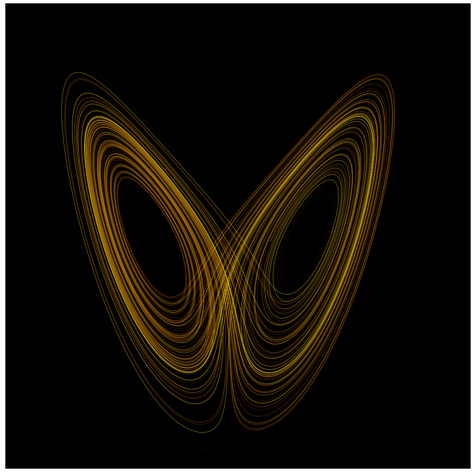
Takens' Embedding theorem: Can longer time-series compensate for spatial sparsity?

4D-Var: variation with the number of iterations



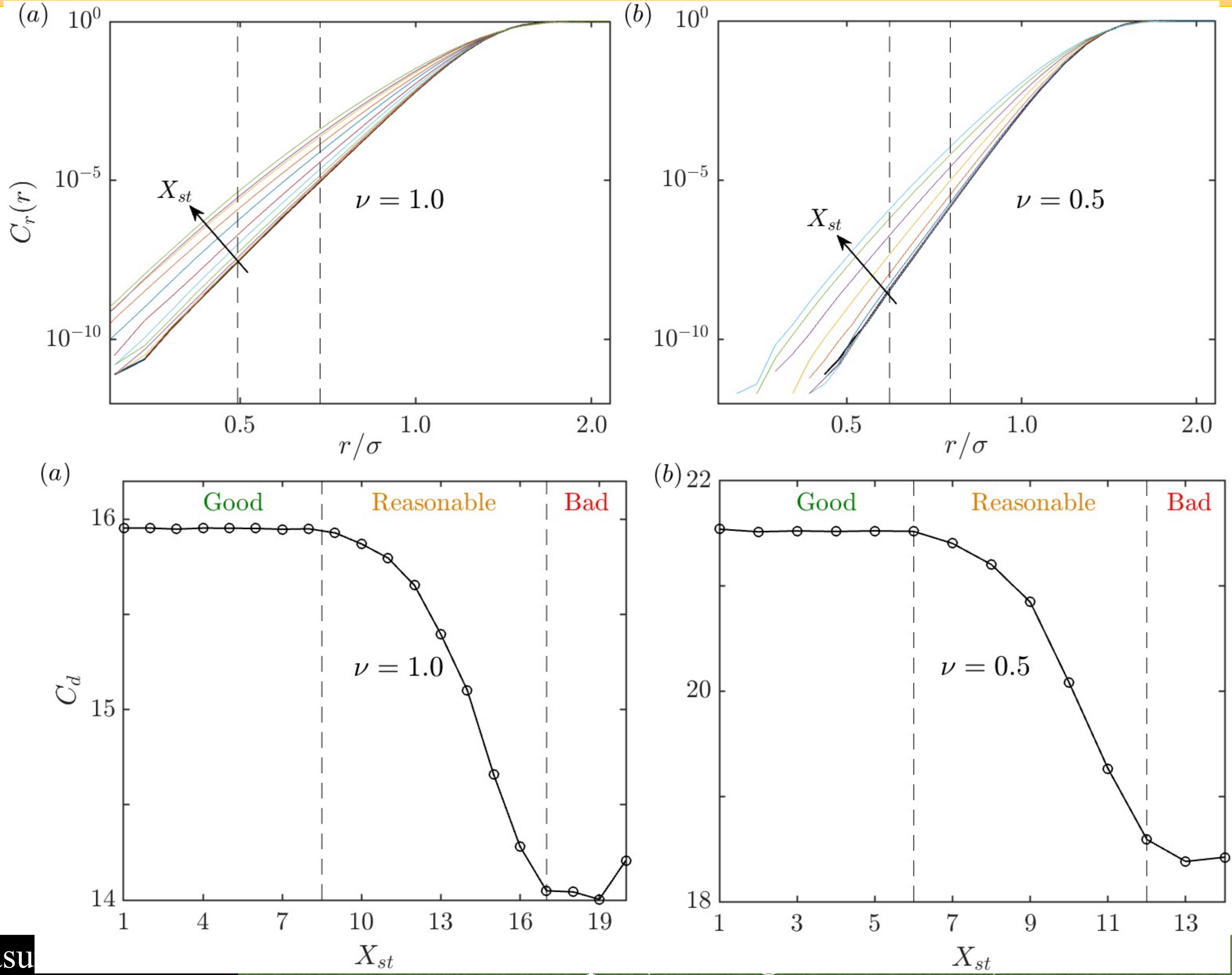
Can we iterate longer to compensate for spatial sparsity?

Correlation dimension: measure of system dynamics



- Measure the spatial complexity of the system in phase space using correlation dimension (C_d)

- Beyond some level, the information on system complexity is lost?

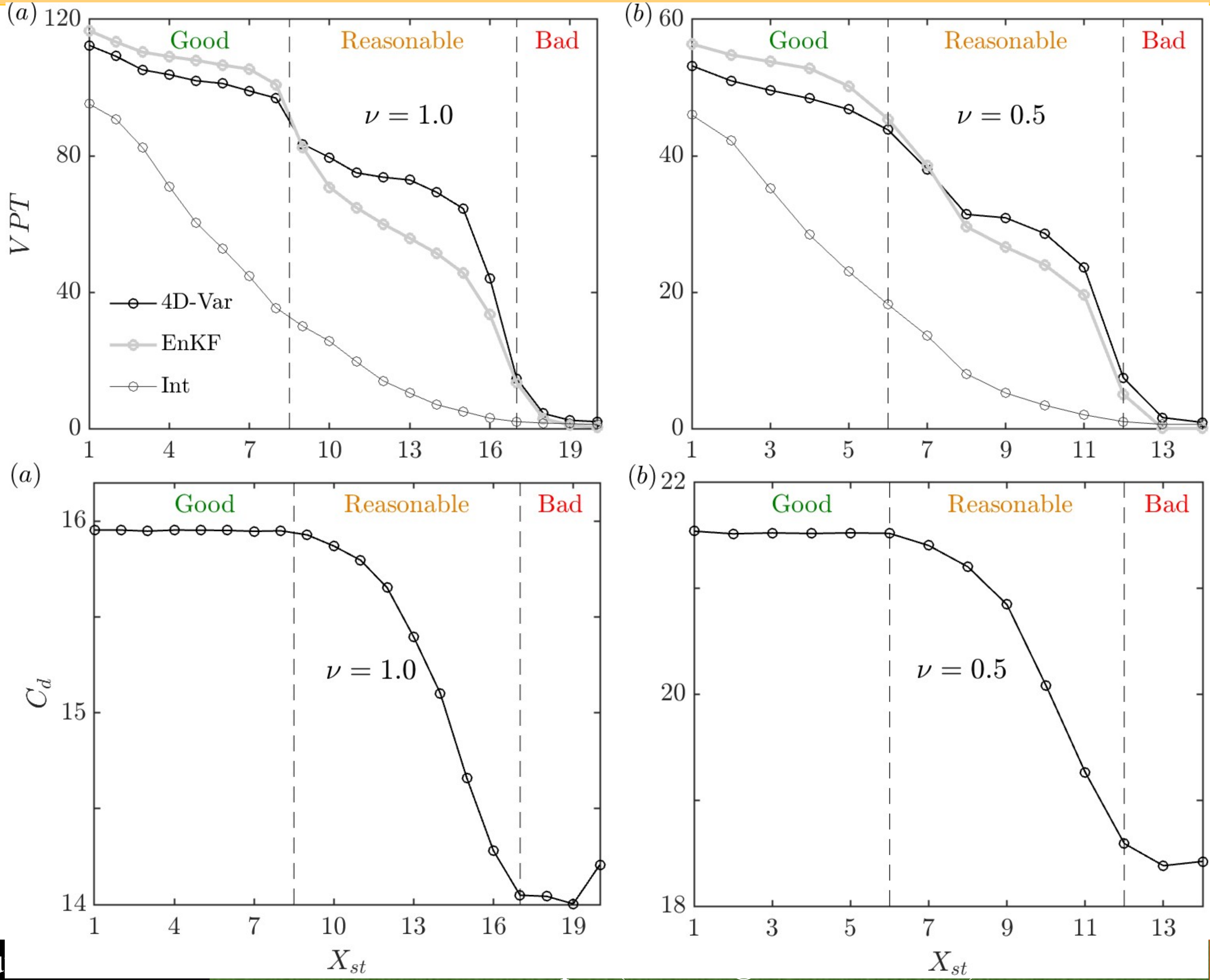


Correlation dimension: measure of system dynamics

- The level of sparsity up to which complexity can be captured matches well with the conditions for predictability

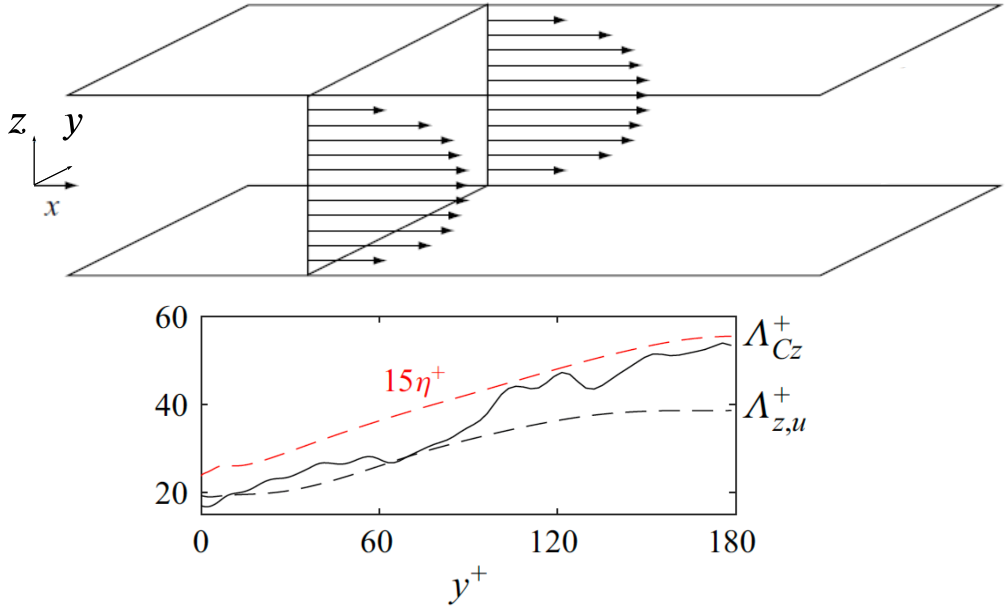
- Measure the spatial complexity of the system in phase space using correlation dimension (C_d)

- Beyond some level, the information on system complexity is lost?



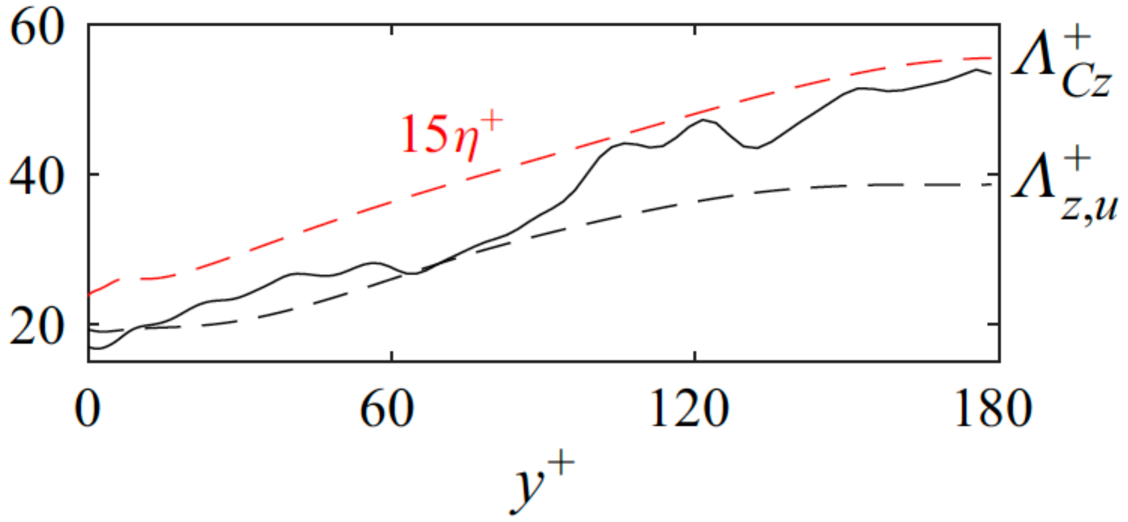
Wild extrapolation: How about fully developed turbulence?

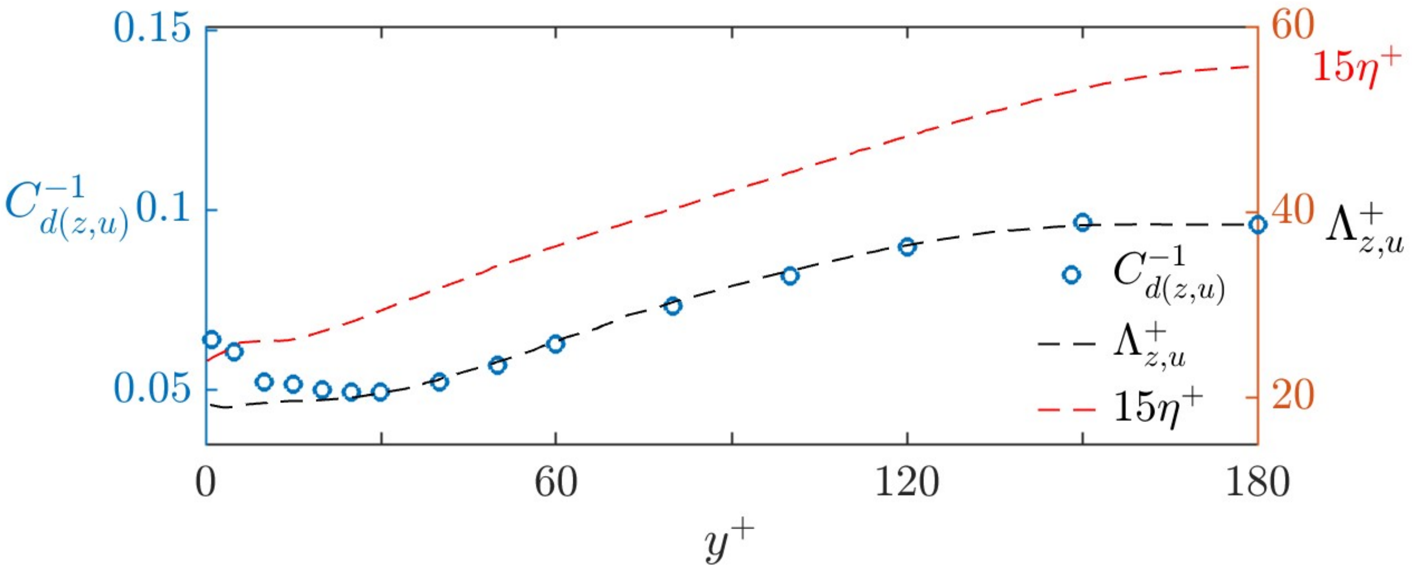
Recovering fine-scales from coarse grained data (Yoshida *et al*. 2005, PRL; Lalescu *et al*. 2013, PRL).



Recent study on turbulent channel shows that the Kolmogorov length-scale-based requirement is closely followed in super-resolution of turbulent channel flow via 4D-Var, but Taylor micro-scale gives a better criterion (Wang & Zaki 2021, JFM)

Wild extrapolation: Not perfect but satisfying





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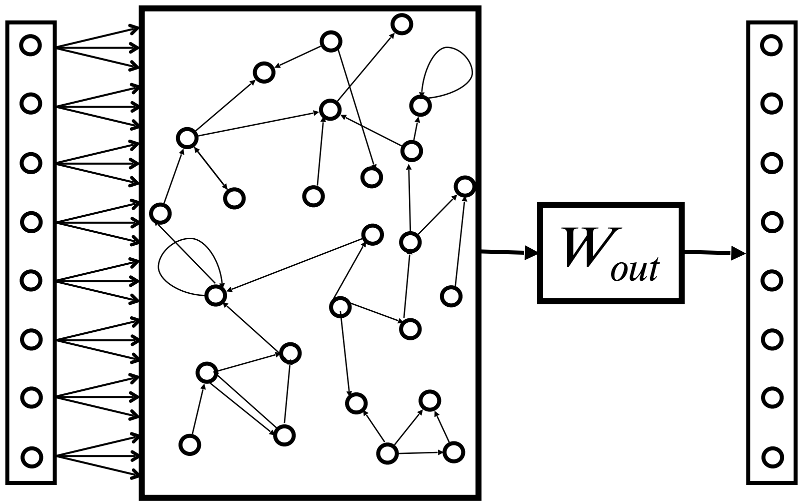
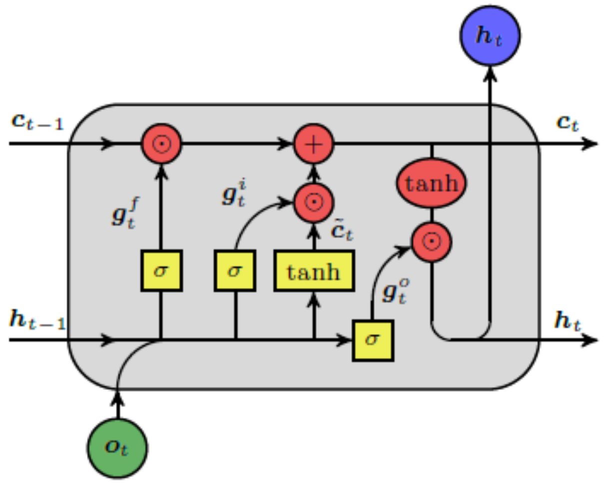
Neural networks: model-free methods for learning turbulent systems

Reservoir-computing-based RNN (shallow network)

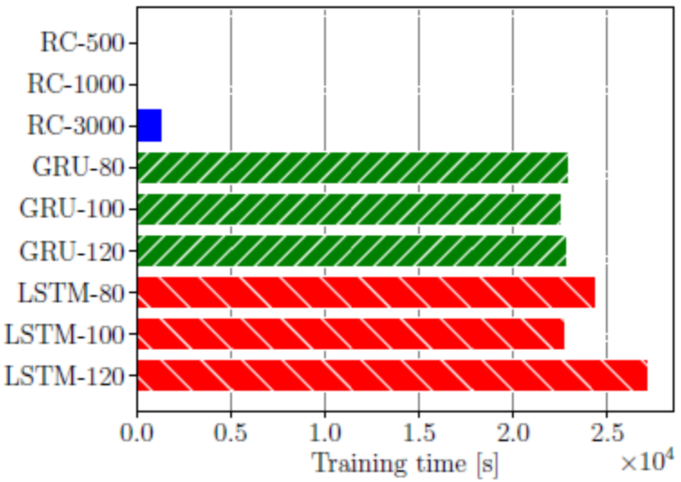


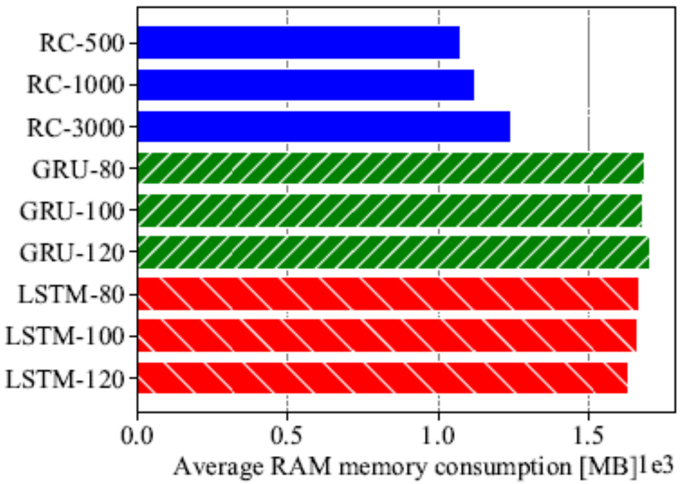
Long-short-term memory RNN (deep learning)



Reservoir-computing-based RNN

Deep learning RNN

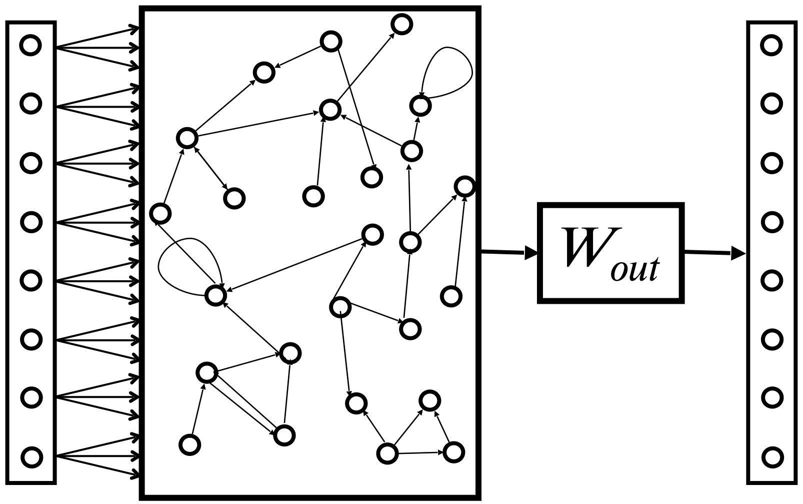




Vlachas, Pathak, Hunt, Sapsis, Girvan, Ott and Koumoutsakos 2020

Neural networks: model-free methods for learning turbulent systems

Reservoir-computing-based RNN (shallow network)



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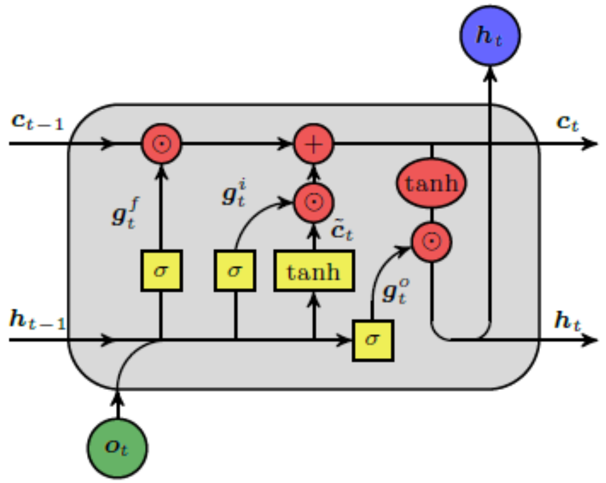
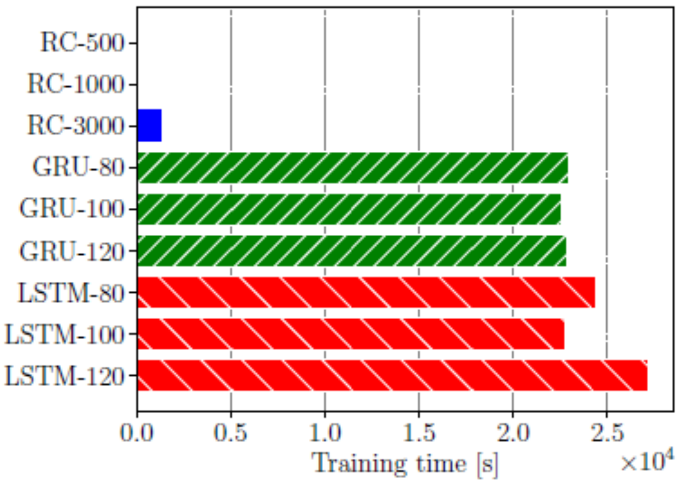
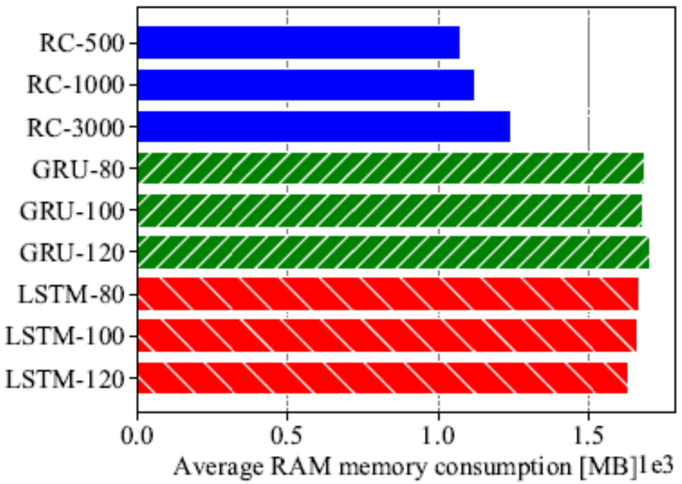


Reservoir-computing-based RNN

Deep learning RNN

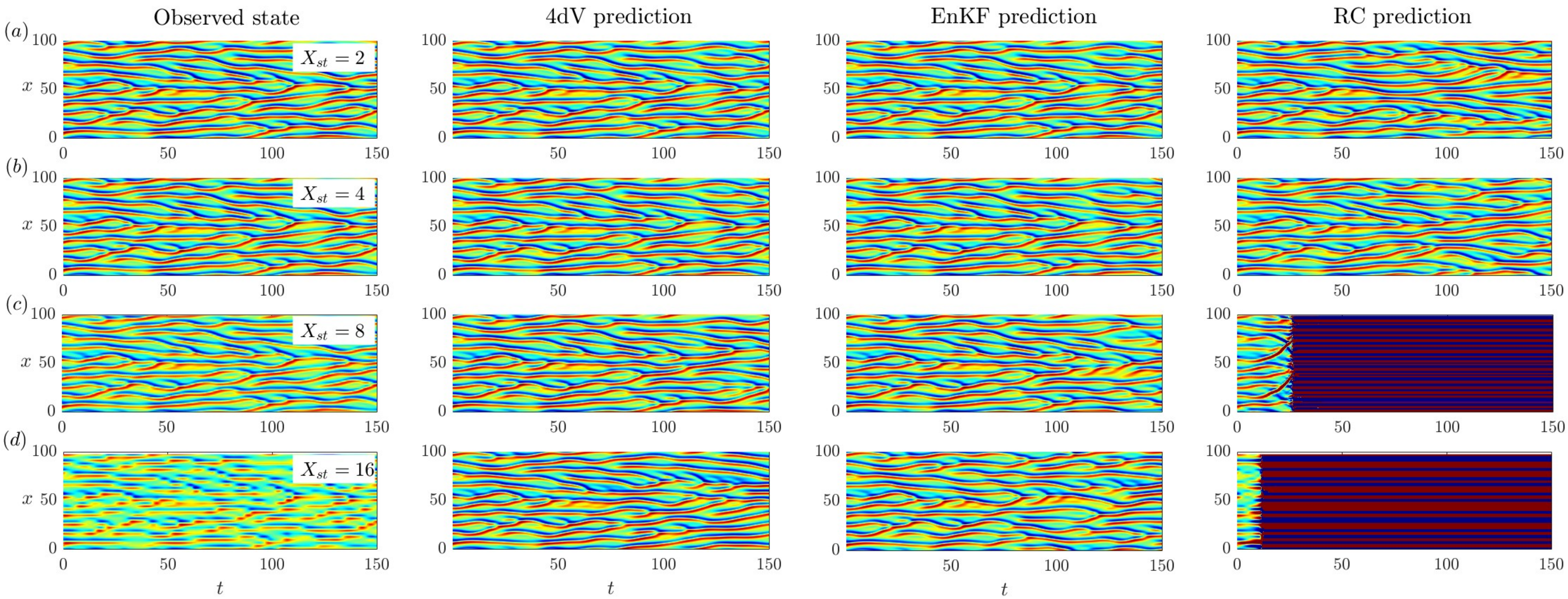




Vlachas, Pathak, Hunt, Sapsis, Girvan, Ott and Koumoutsakos 2020

Spatial resolution required for learning the system and making the predictions

$\nu = 1.0, \; n = 512$



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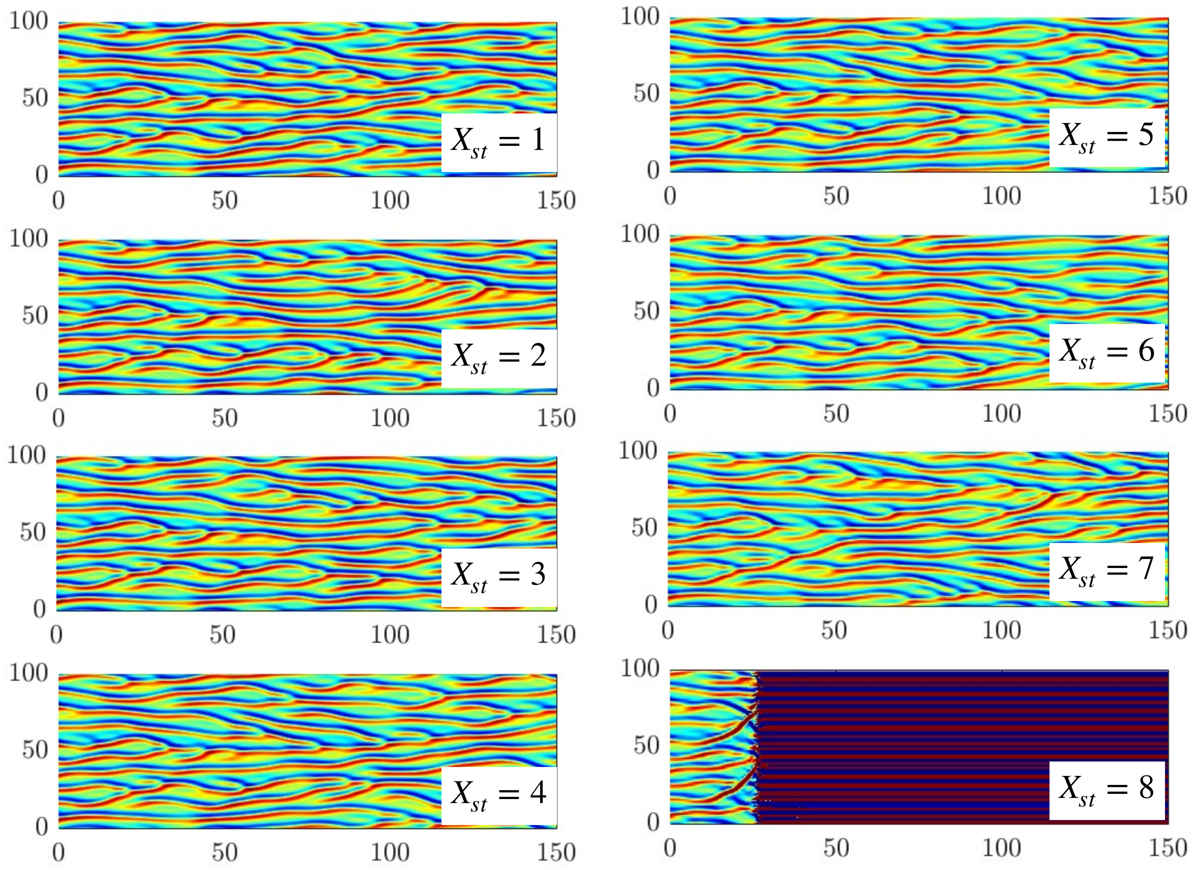
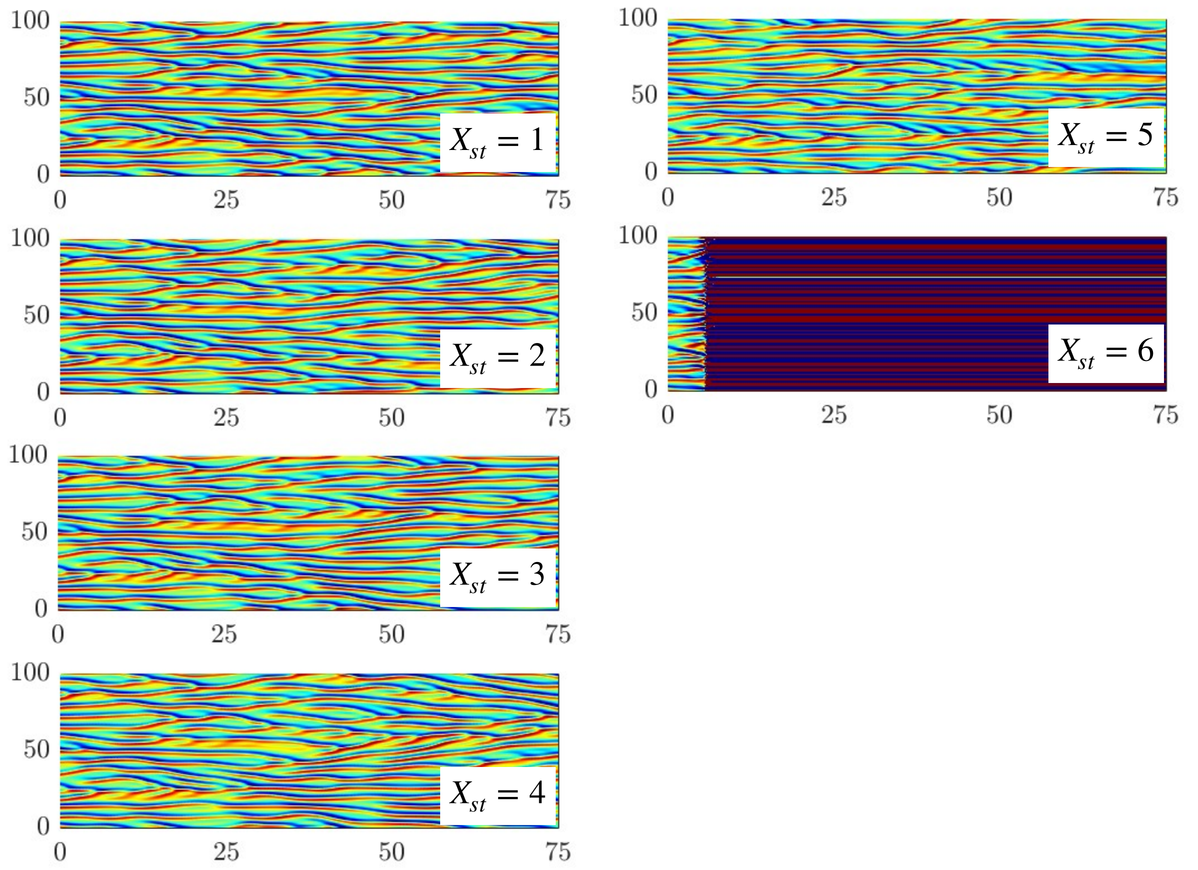


$\nu = 0.5, \;\; n = 512$



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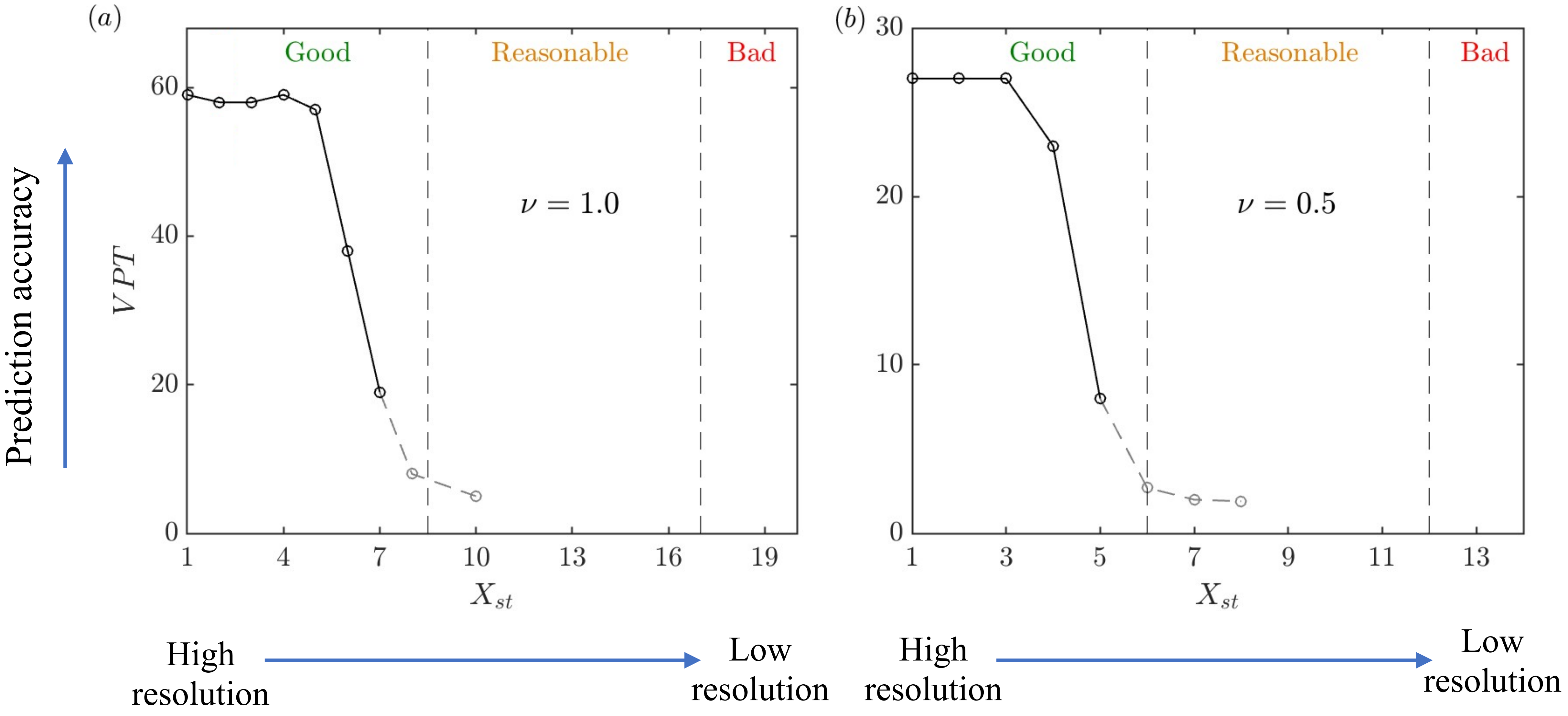

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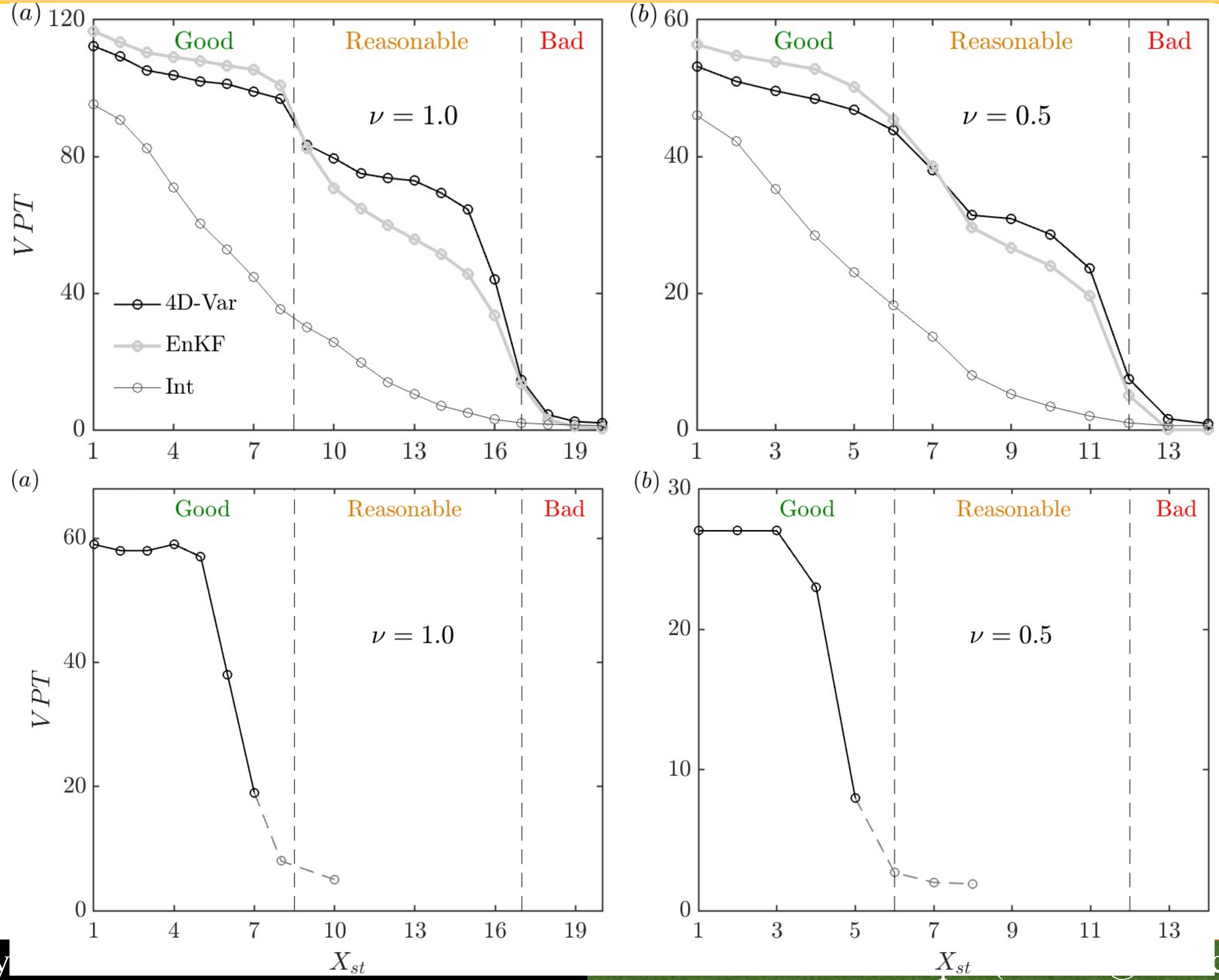
Data assimilation vs Neural networks
Model-based vs Model-free methods



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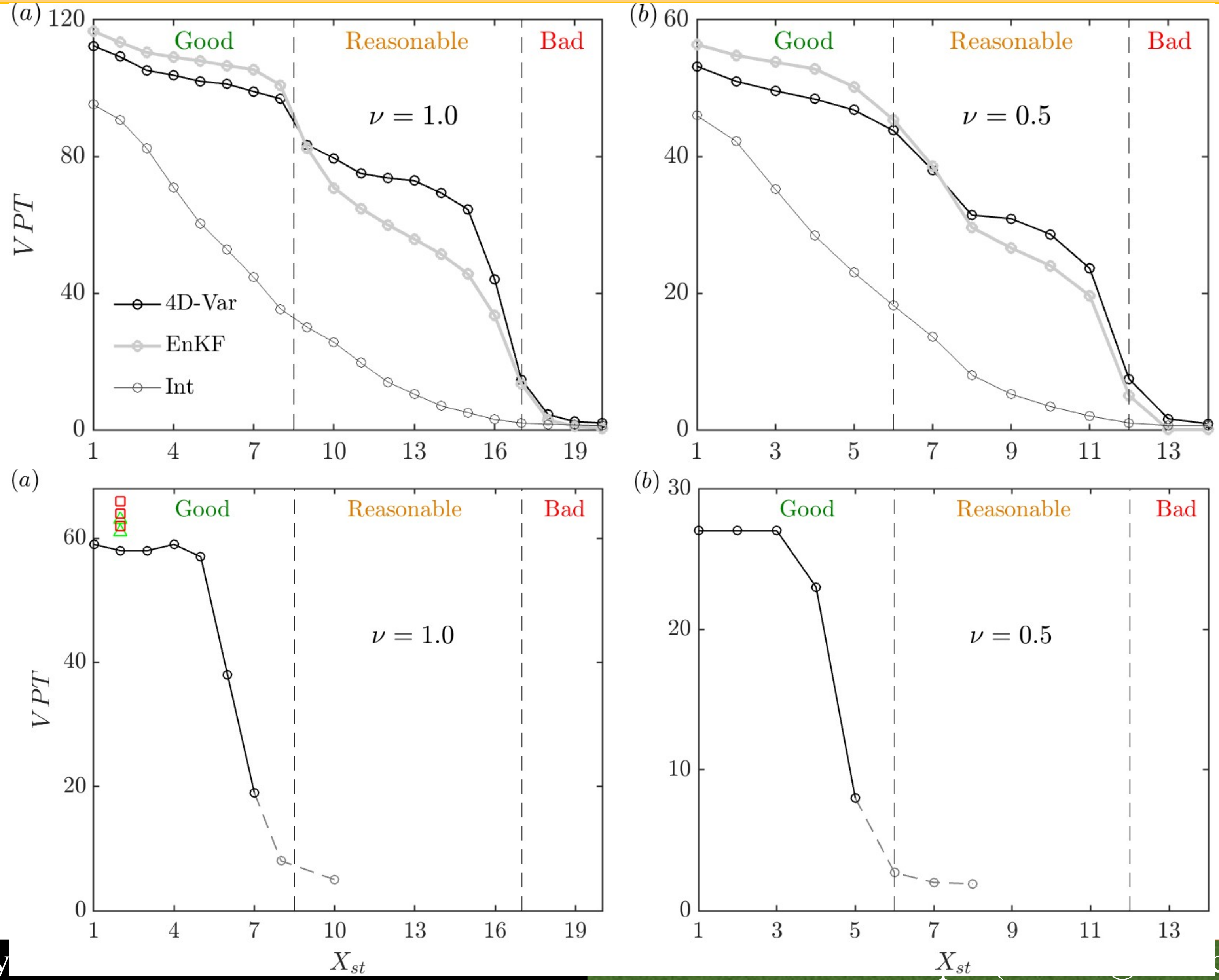
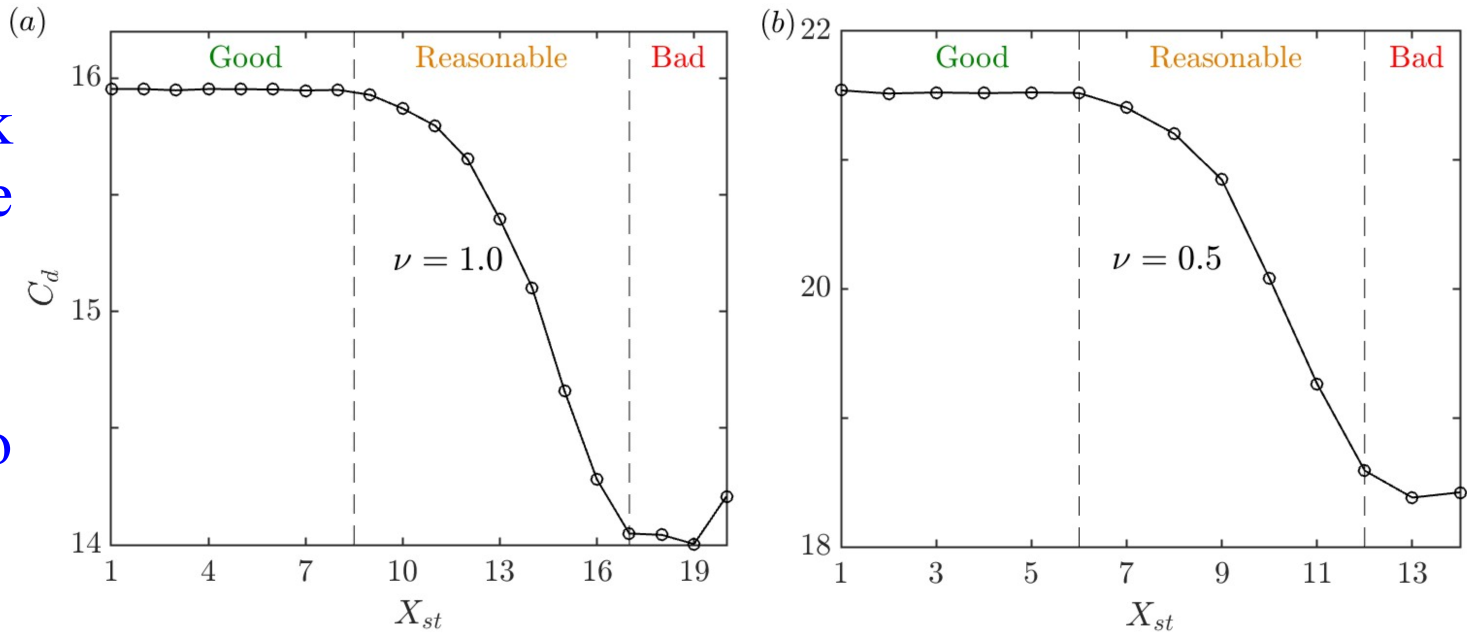


Summary of data-driven methods part

- Is there a limit of spatial resolution beyond which data assimilation methods cannot estimate/ predict?
- Do model-free machine learning methods need higher or lower resolution?
- Can classical methods still be useful when data-driven methods fail?

- Data assimilation methods can work only up to the resolution at which the system's complexity is captured.
- Machine learning methods need higher resolution because they need to learn the system dynamics from data.



Caution: data-driven methods are still a decade or two away for practical turbulent systems

- Is there a limit of spatial resolution beyond which data assimilation methods cannot estimate/ predict?
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- **Can classical methods still be useful when data-driven methods fail?**

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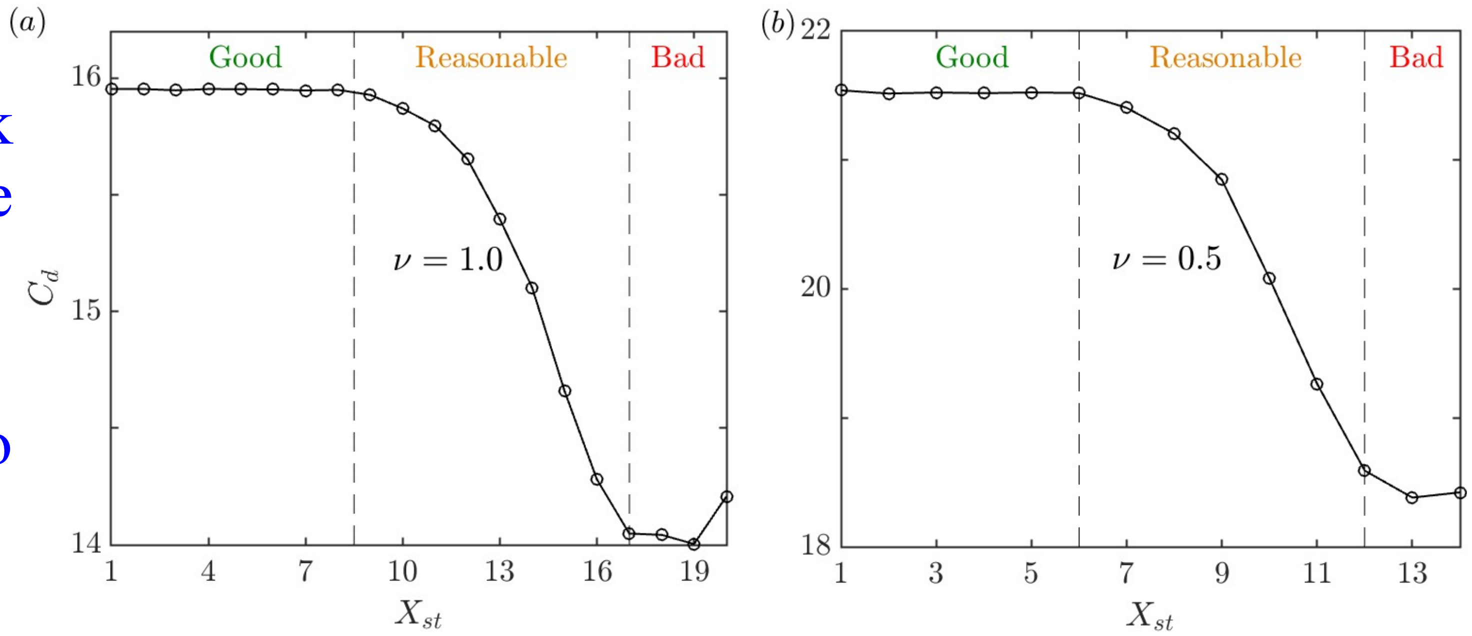

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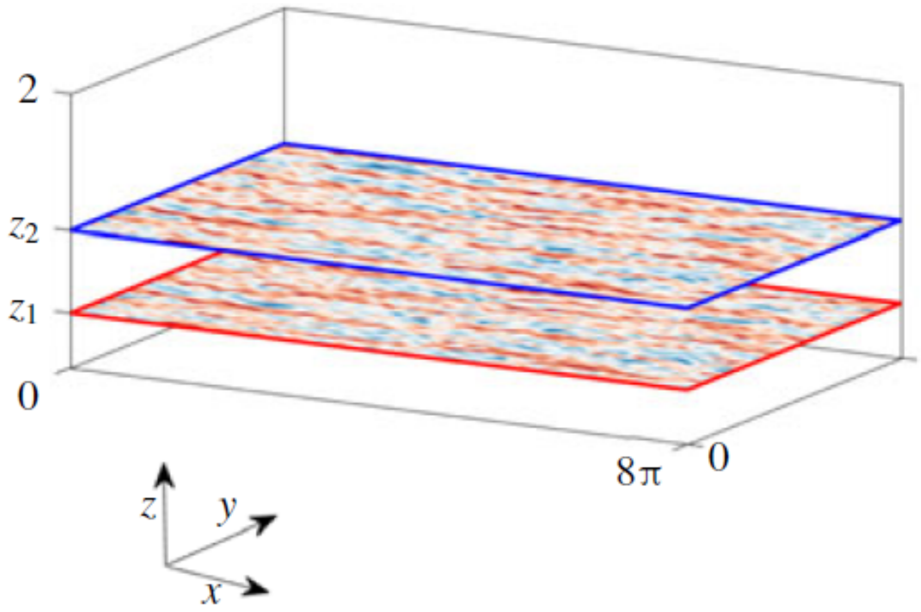
Linearised models: Low data requirement and high physical interpretability



Measurement plane

Need estimation at other planes

Data assimilation and neural networks may not work with such limited measurements.

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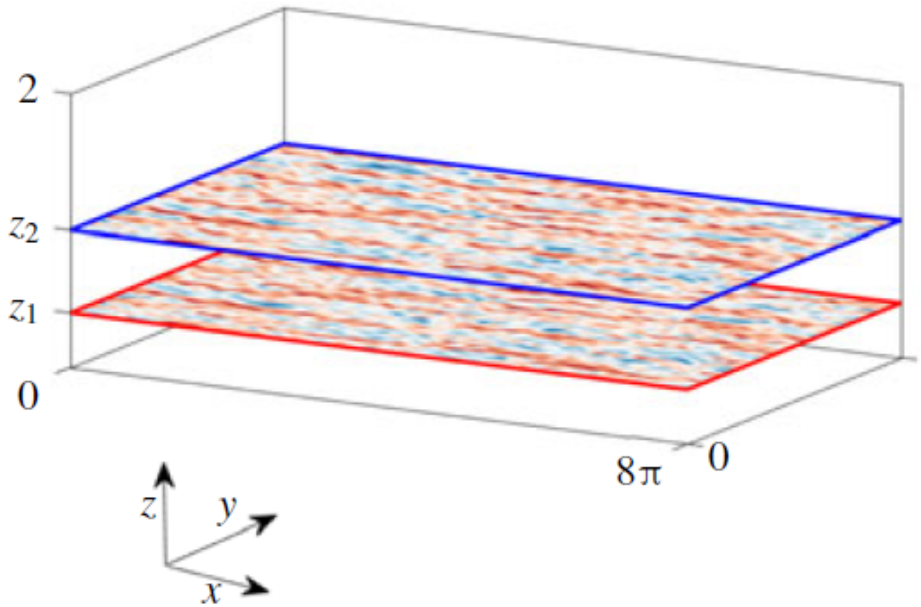
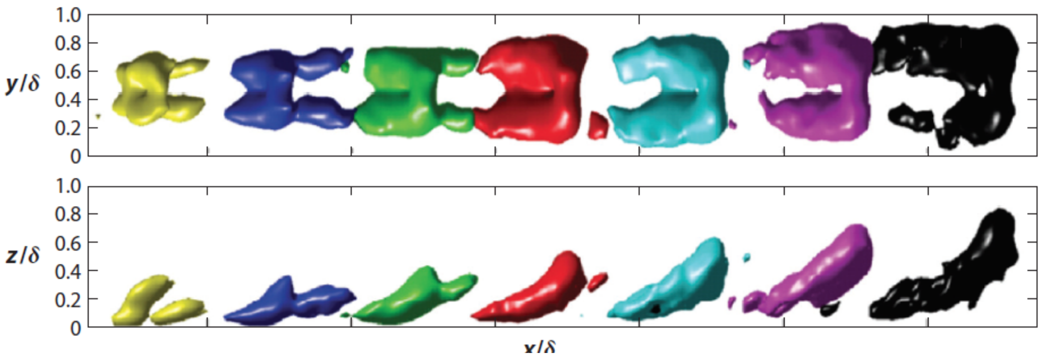
Linearised models: Low data requirement and high physical interpretability





Dominant flow structures (attached eddies), experimental eduction by Dennis & Nickels (JFM, 2011)

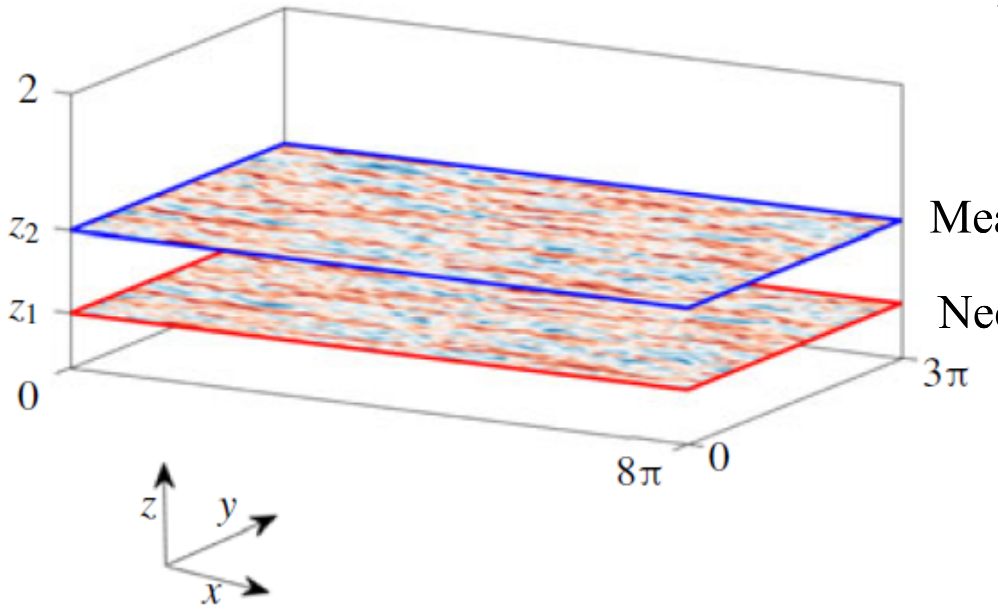
Mechanism: del Alamo & Jimenez (JFM 2006), Pujals et al. (PoF 2009), McKeon & Sharma (JFM 2010), Pickering et al. (JFM 2020), etc.

Estimation: Zare et al. (JFM 2017), Illingworth et al. (JFM 2018), Towne et al. (JFM 2020), Gupta et al. (JFM 2021), Wu & He (JFM 2023), etc.

Control: Semeraro et al. (JFM 2013), Jin et al. (2020), Jafari et al. (JFM 2023), etc.

Linearised models: Low data requirement and high physical interpretability

How do we obtain such linear models?



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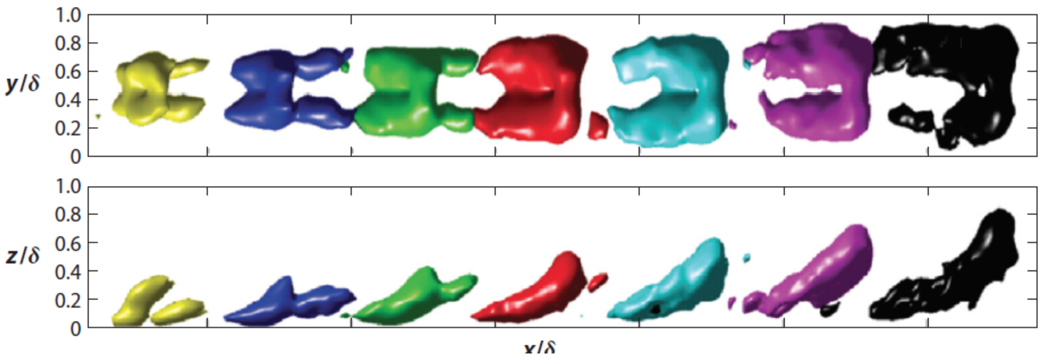

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Decompose the governing equations in terms of the mean and fluctuating parts

$$\frac{\partial u_i}{\partial t} = -u_k\frac{\partial u_i}{\partial x_k} - \frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

$$\frac{\partial u_k}{\partial x_k} = 0$$

We will try to understand the formation of energetic structures via the mean flow instabilities of the governing Navier—Stokes equations.

$$u_i = U_i + u_i, \quad p = P + p$$

(U_i, P) is the base flow state

$$\frac{\partial u_i}{\partial t} = -U_k\frac{\partial u_i}{\partial x_k} - u_k\frac{\partial U_i}{\partial x_k} - \frac{\partial u_k u_i}{\partial x_k} - \frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

Evolution of disturbance (u_i, p) over the base flow state

$$E(t) = \frac{1}{2}\int u_i u_i dV$$

Volume integral of the disturbance kinetic energy

Role of the linear terms

$$u_i \frac{\partial u_i}{\partial t} = -U_k u_i \frac{\partial u_i}{\partial x_k} - u_k u_i \frac{\partial U_i}{\partial x_k} - u_i \frac{\partial u_k u_i}{\partial x_k} - u_i \frac{\partial p}{\partial x_i} + \frac{u_i}{Re} \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

Use the divergence free condition and apply the volume integral

We will try to understand the formation of energetic structures via the mean flow instabilities of the governing Navier—Stokes equations.

$$u_i = U_i + u_i, \quad p = P + p$$

(U_i, P) is the base flow state

$$\frac{\partial u_i}{\partial t} = -U_k \frac{\partial u_i}{\partial x_k} - u_k \frac{\partial U_i}{\partial x_k} - \frac{\partial u_k u_i}{\partial x_k} - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

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$$\frac{\partial}{\partial x_k}\left[-\frac{1}{2}u_i u_k U_k - \frac{1}{2}u_i u_i u_k - u_i p \delta_{ik}\right]$$

Use the divergence free condition and apply the volume integral

$$\frac{dE}{dt} = -\int u_i u_k \frac{\partial U_i}{\partial x_k} dV - \frac{1}{Re}\int \frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_k} dV$$

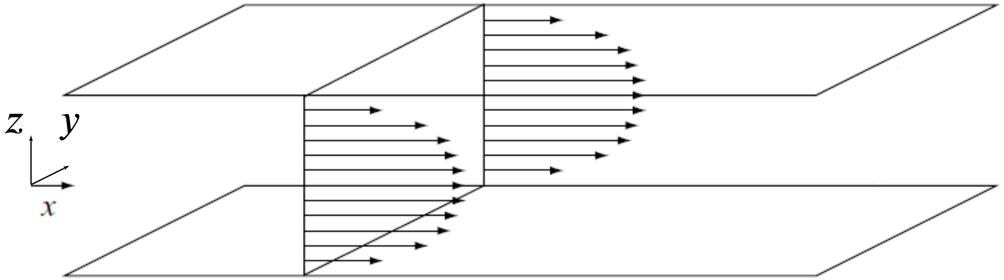
Reynolds-Orr equation

Transfer of energy from the basic flow

Viscous dissipation

The energy for developing and sustaining turbulence must come through the **linear** energy amplification mechanism.

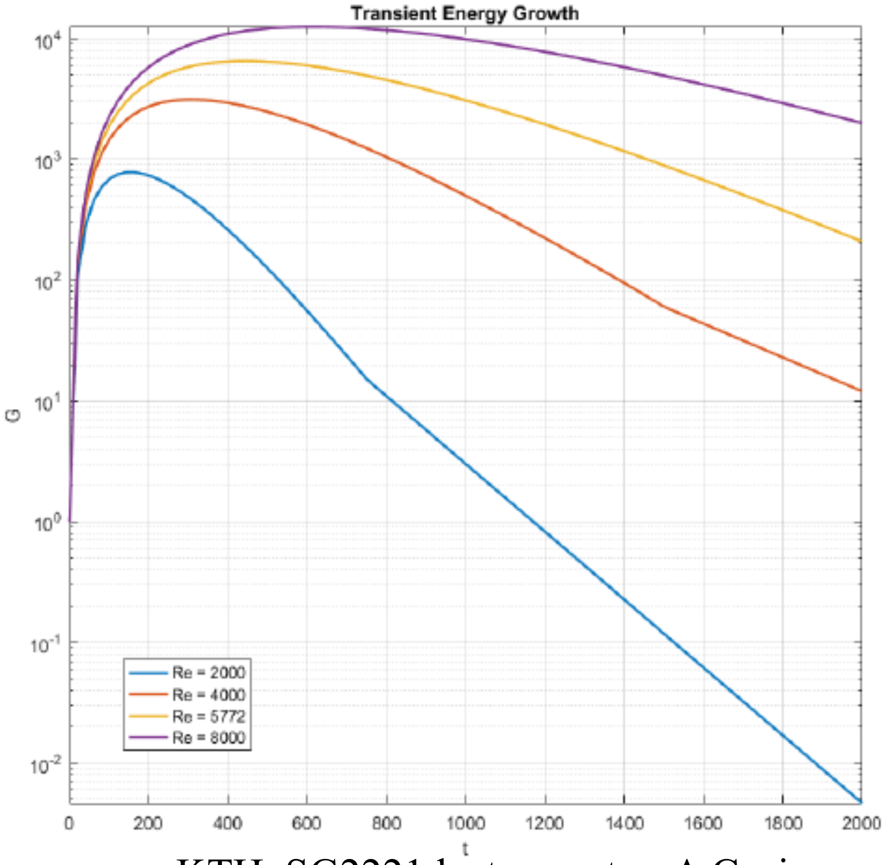
Turbulent channel flow: linear amplification mechanism



$U(z)$ is the mean flow velocity

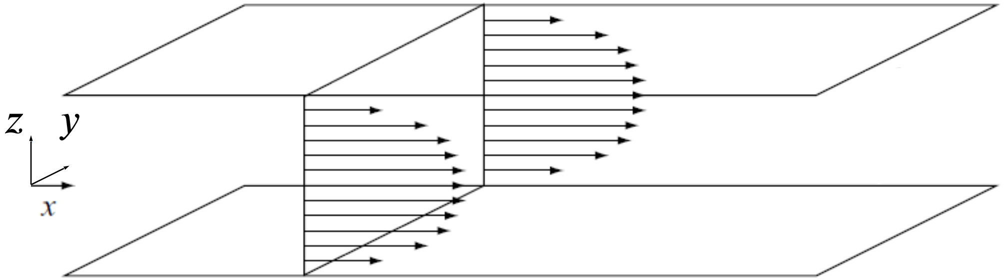
$\langle u_k u_i \rangle$ is the mean Reynolds stress

$$\frac{\partial u_i}{\partial t} = -U_k \frac{\partial u_i}{\partial x_k} - u_k \frac{\partial U_i}{\partial x_k} - \cancel{\frac{\partial (u_k u_i - \langle u_k u_i \rangle)}{\partial x_k}} - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$



KTH, SG2221 lecture notes, A Ceci

Turbulent channel flow: linear amplification mechanism



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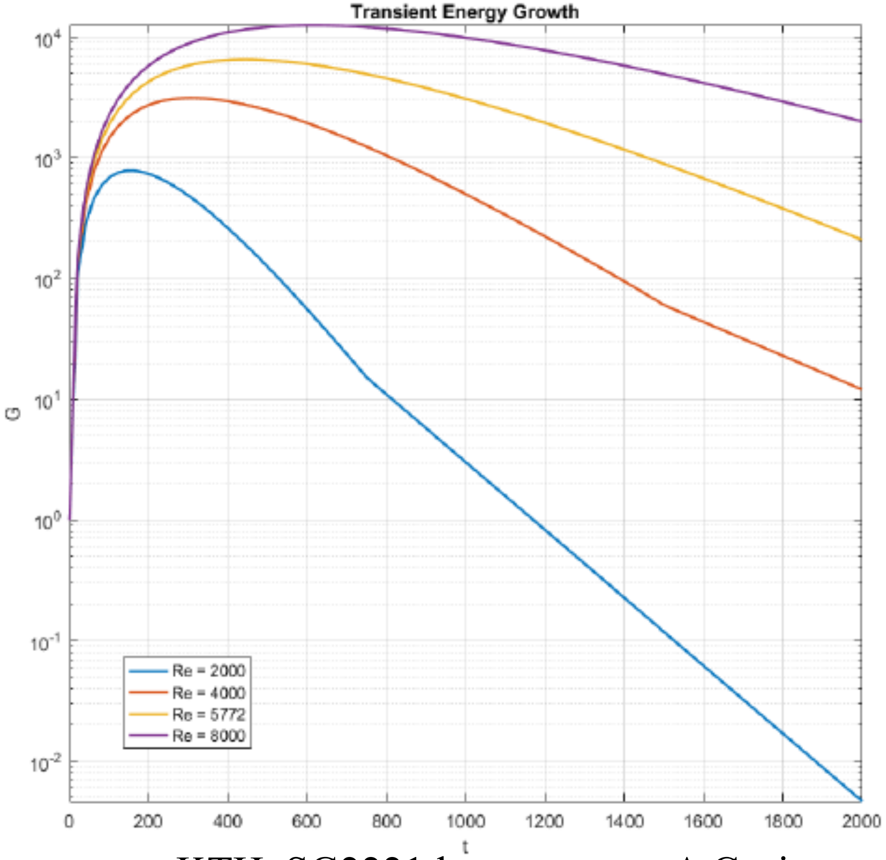
Problem: Without the nonlinear term, there is no engine to sustain the turbulence.



KTH, SG2221 lecture notes, A Ceci

Turbulent channel flow: approximation of the nonlinear term

$$\frac{\partial u_i}{\partial t} = -U_k \frac{\partial u_i}{\partial x_k} - u_k \frac{\partial U_i}{\partial x_k} - \frac{\partial (u_k u_i - \langle u_k u_i \rangle)}{\partial x_k} - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

Damping term
del Alamo & Jimenez (2006)

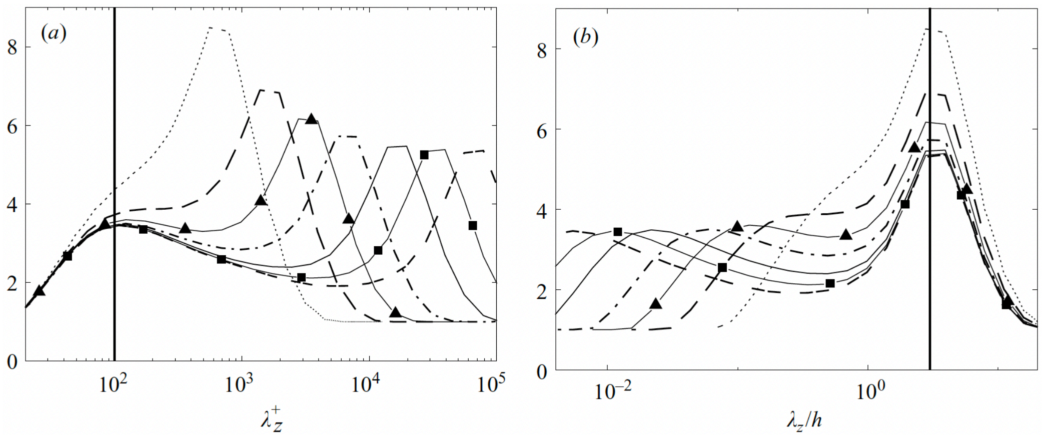
Unknown forcing term
McKeon & Sharma (2010)

Vikrant Gupta (vikrant@sustech.edu.cn)

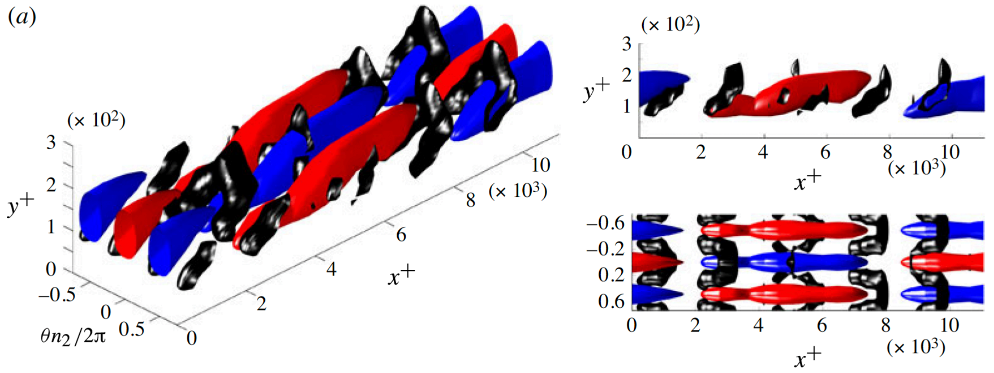
Turbulent channel flow: approximation of the nonlinear term

$$\frac{\partial u_i}{\partial t} = -U_k \frac{\partial u_i}{\partial x_k} - u_k \frac{\partial U_i}{\partial x_k} - \frac{\partial (u_k u_i - \langle u_k u_i \rangle)}{\partial x_k} - \frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

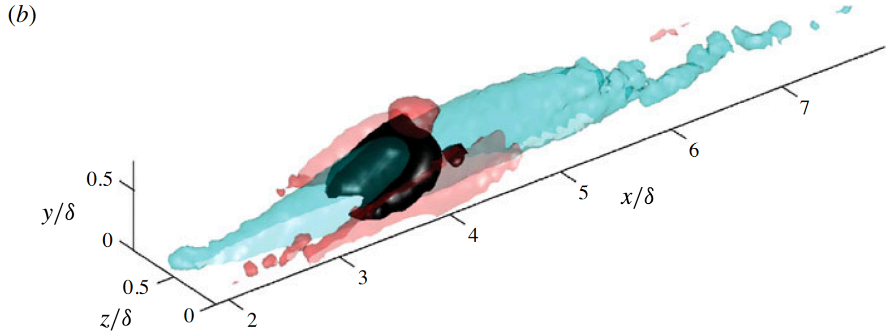
Baseline model $$\frac{\partial u_i}{\partial t} = -U_k \frac{\partial u_i}{\partial x_k} - u_k \frac{\partial U_i}{\partial x_k} + d(x,t) + \frac{\partial}{\partial x_k}\left(\nu_t\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)\right) - \frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_k \partial x_k}$$



del Alamo & Jimenez (2006) - linear model captures the inner and outer peaks



Sharma & McKeon (2013) - linear model



Dennis & Nickels (2011) - experimental study

Refining the linear model for quantitative accuracy

$$\frac{\partial u_i}{\partial t} = -U_k \frac{\partial u_i}{\partial x_k} - u_k \frac{\partial U_i}{\partial x_k} - \frac{\partial (u_k u_i - \langle u_k u_i \rangle)}{\partial x_k} - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

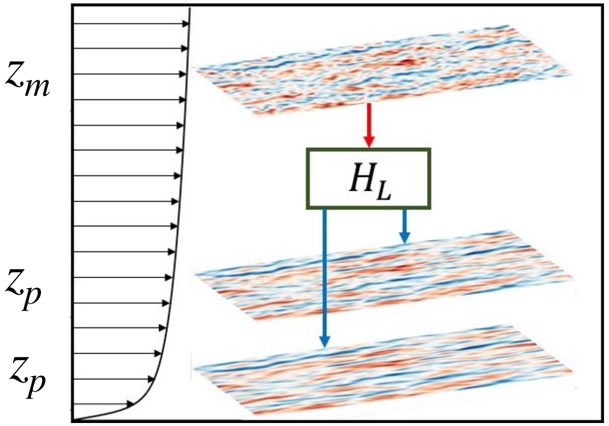
Baseline model
$$\frac{\partial u_i}{\partial t} = -U_k \frac{\partial u_i}{\partial x_k} - u_k \frac{\partial U_i}{\partial x_k} + d(x,t) + \frac{\partial}{\partial x_k}\left(\nu_t \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)\right) - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

Two main approaches to refine the models are:

Statistical: Zare et al. (2017), Majda & Qi (2018), etc.

Phenomenological: Gupta et al. (2021), Wu and He (2023), etc.

Application to estimation of large scales in turbulent channel flow



z_m - measurement plane location

z_p - estimation plane location

$$\frac{\partial u_i}{\partial t} = -U_k \frac{\partial u_i}{\partial x_k} - u_k \frac{\partial U_i}{\partial x_k} + d(x,t) + \frac{\partial}{\partial x_k}\left(\nu_t \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)\right) - \frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

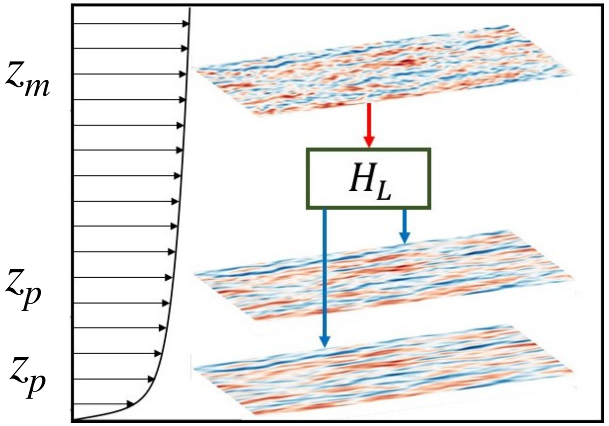
$$\frac{d\mathbf{q}}{dt} = A\mathbf{q} + B\mathbf{d}$$

$$\mathbf{y}_m = C\mathbf{q}$$

Estimate **q** based on observation $\mathbf{y}_m$

Vikrant Gupta (vikrant@sustech.edu.cn)

Application to estimation of large scales in turbulent channel flow



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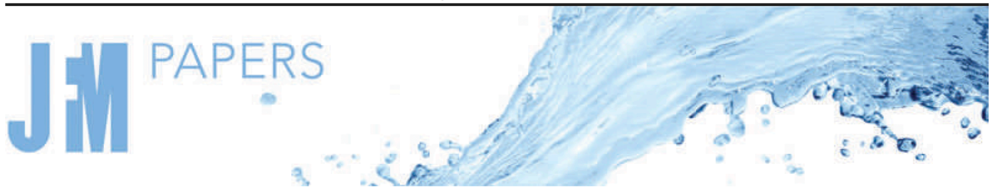
J. Fluid Mech. (2021), *vol.* 925, A18, doi:10.1017/jfm.2021.671

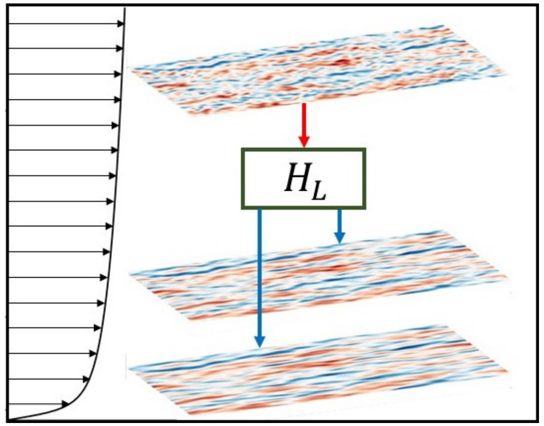


Linear-model-based estimation in wall turbulence: improved stochastic forcing and eddy viscosity terms

Vikrant Gupta[1,2,3], Anagha Madhusudanan[4], Minping Wan[1,2,3,†], Simon J. Illingworth[4] and Matthew P. Juniper[5]

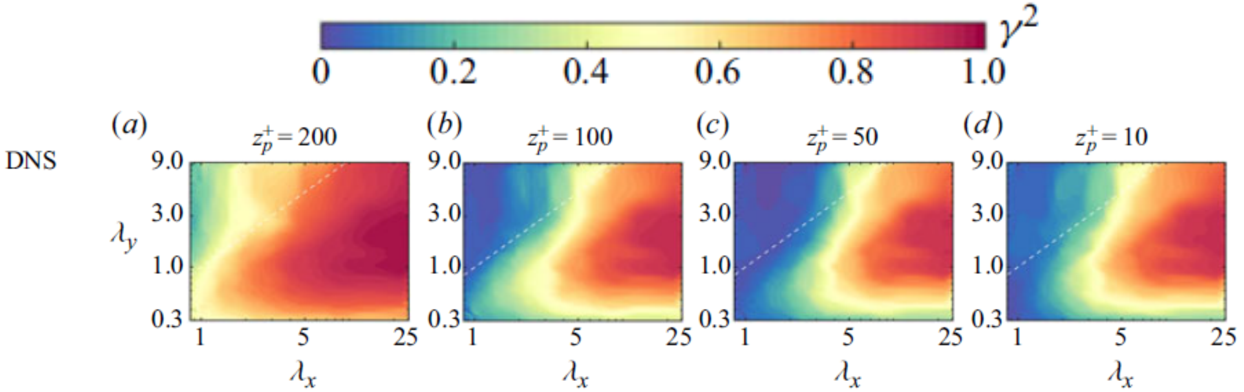
We improve the stochastic forcing term $d(x,t)$ and the eddy viscosity term ν_t in the baseline model to obtain accurate estimation of the large-scale fluctuations in wall turbulence

Comparison with coherence factor from DNS data

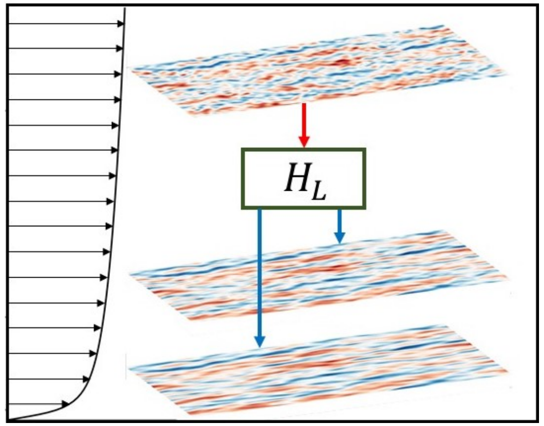


$z_m^+ = 300$

DNS results
(data from UPM)



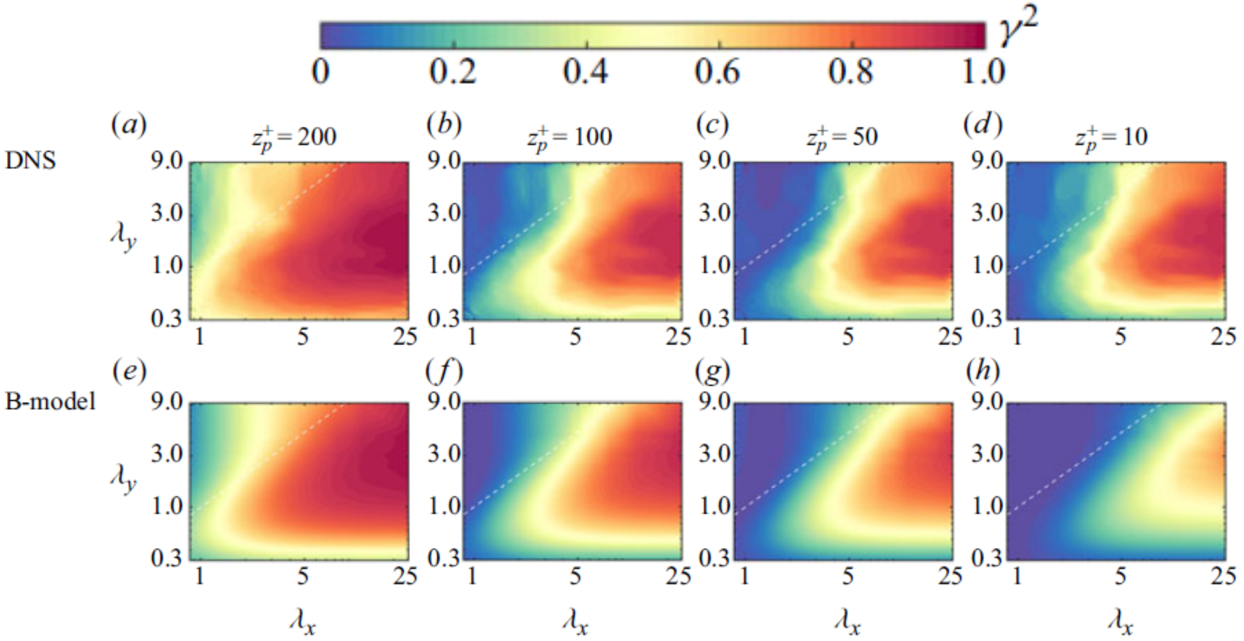
Comparison with coherence factor from DNS data



$z_m^+ = 300$

DNS results
(data from UPM)

$$d(x,t) + \frac{\partial}{\partial x_k}\left(\nu_t\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)\right)$$



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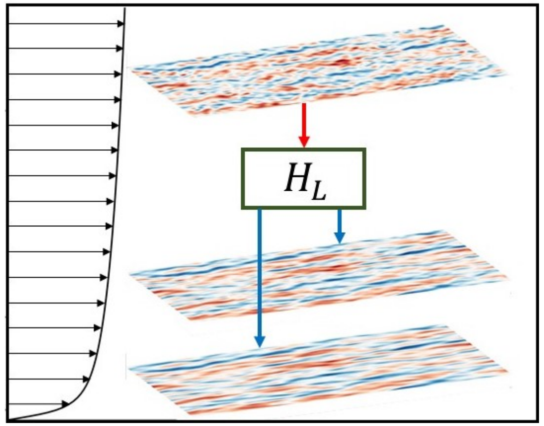

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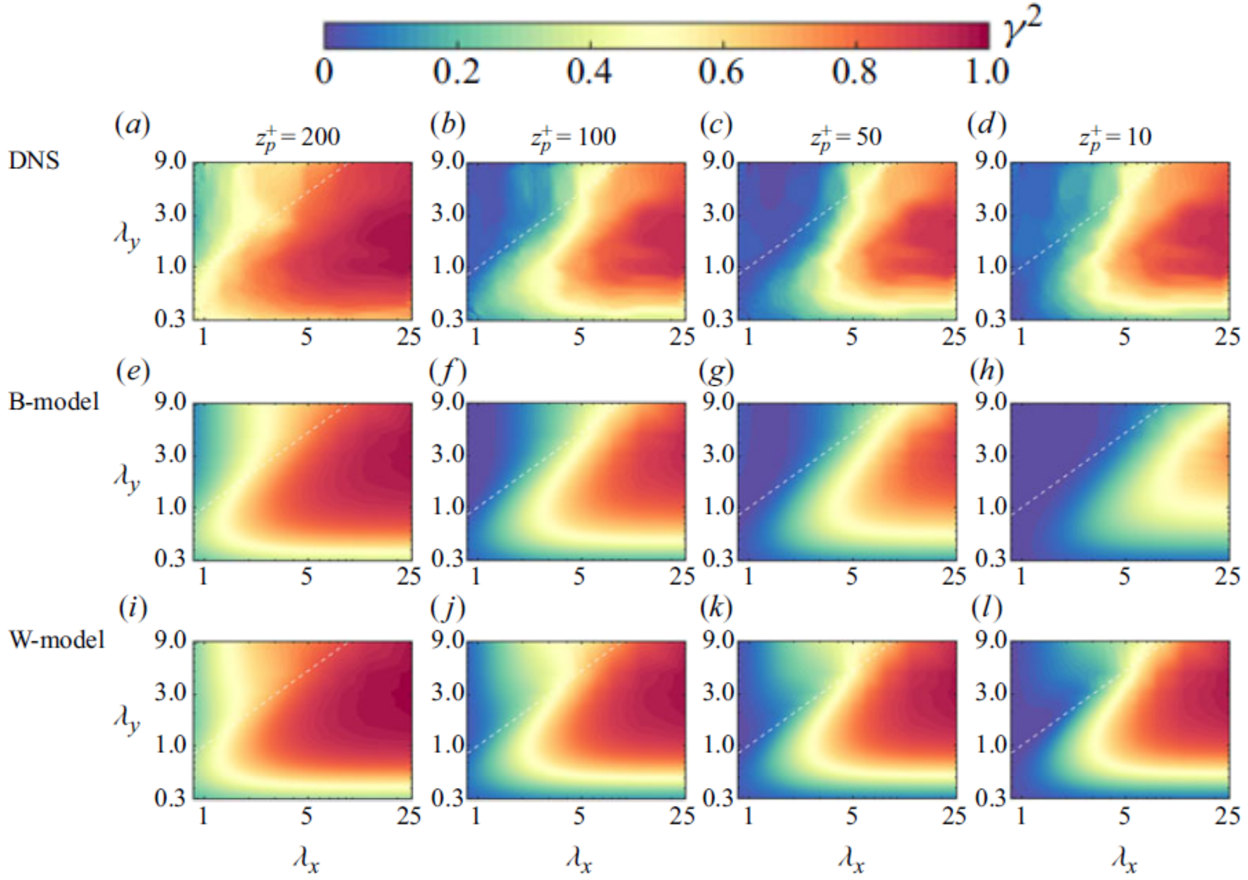
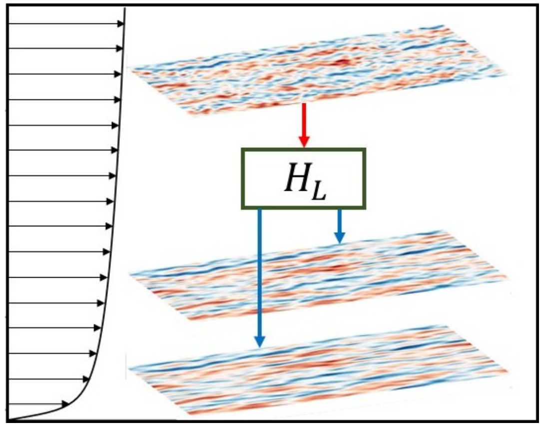
Wall-dependence implemented such that the forcing is proportional to the damping term



Comparison with coherence factor from DNS data



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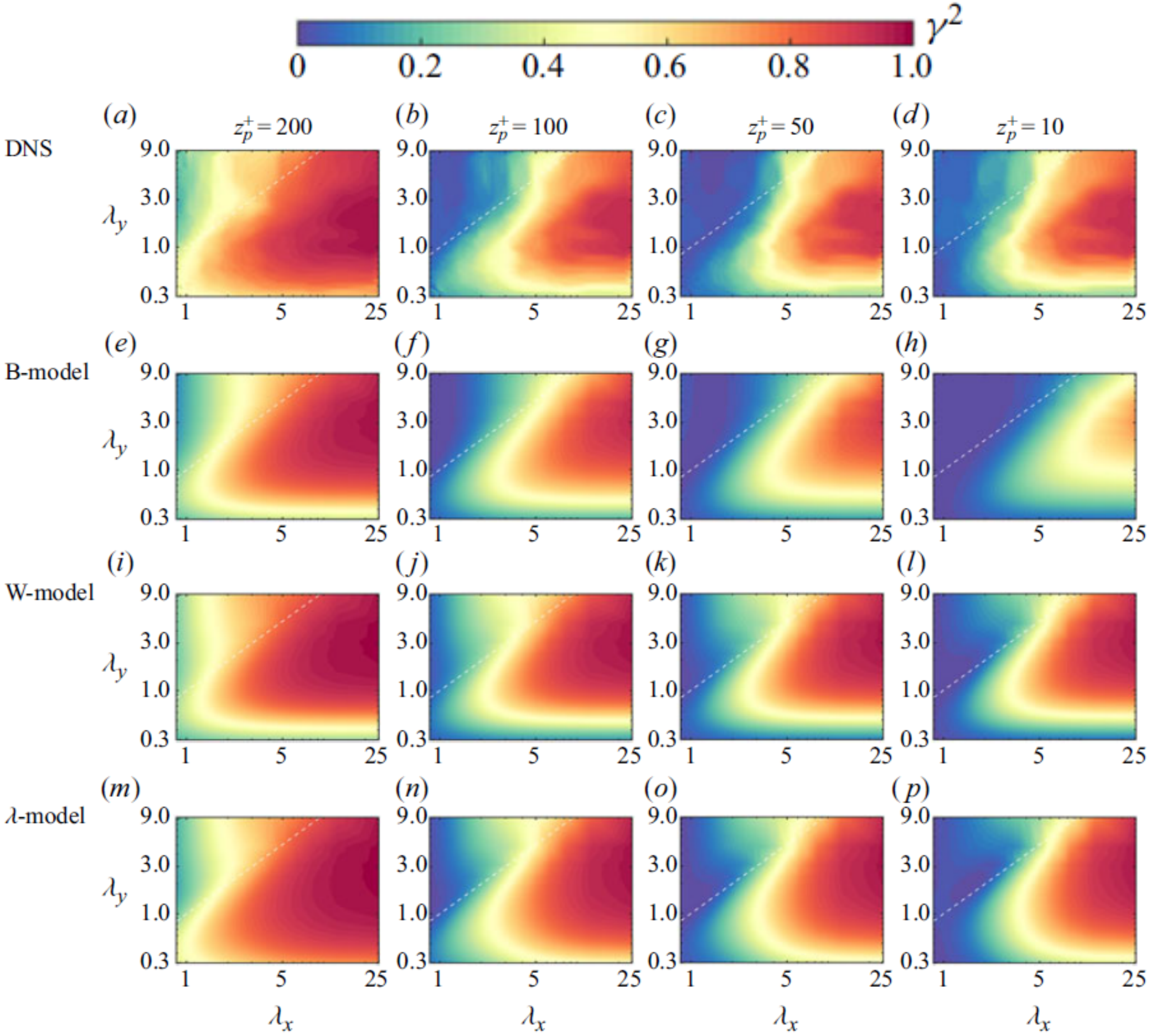
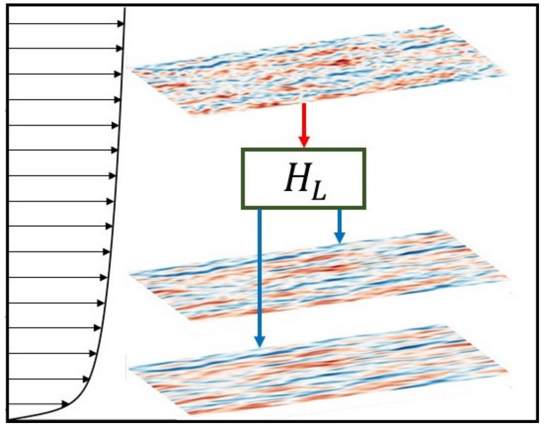


$$d(x,t) + \frac{\partial}{\partial x_k}\left(\nu_t\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)\right)$$

Wall-dependence implemented such that the forcing is proportional to the damping term

Scale-dependence implemented such that the energy transfers are proportional to the length-scales

Comparison of the ratio of fluctuations magnitude from DNS data



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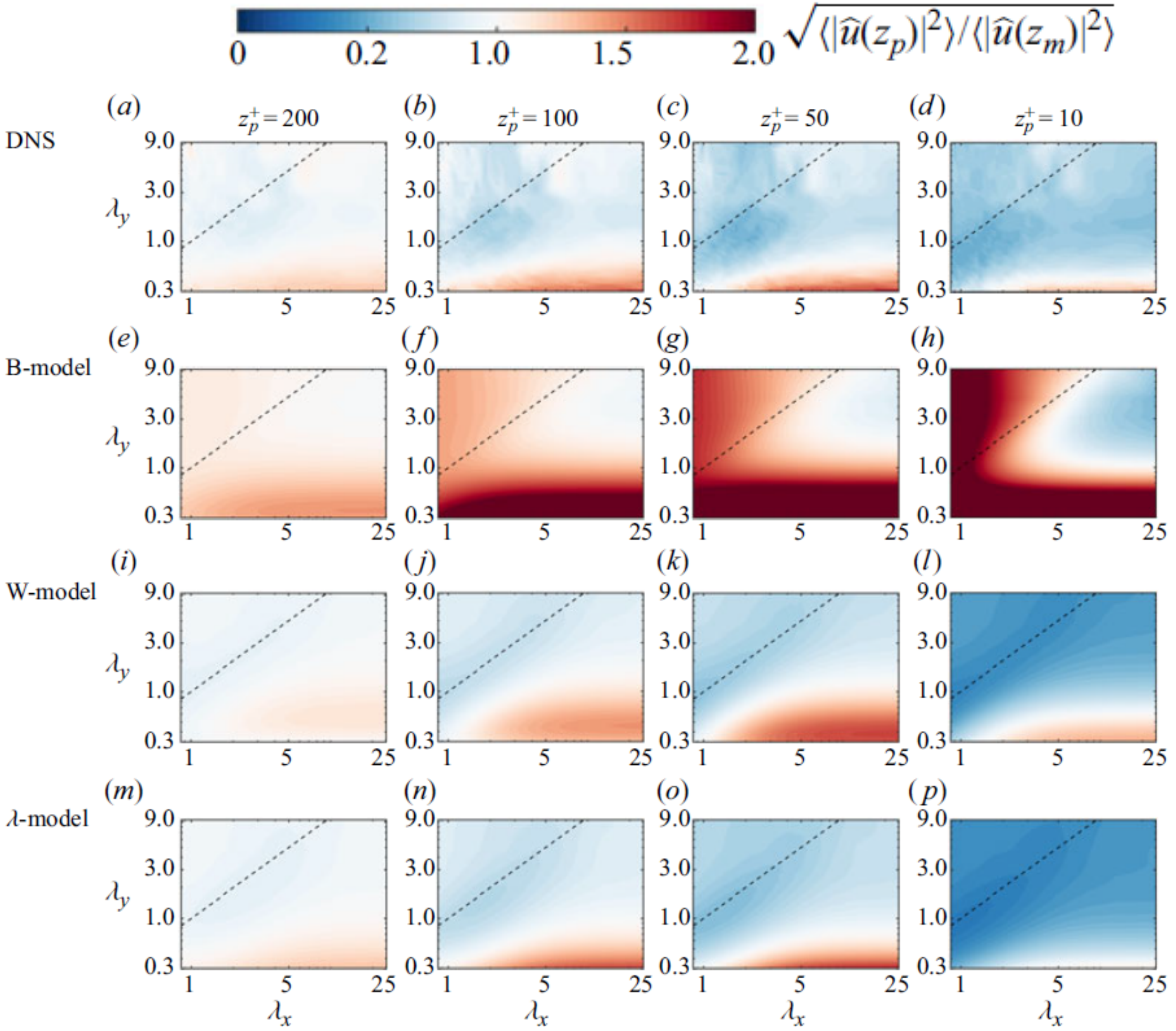
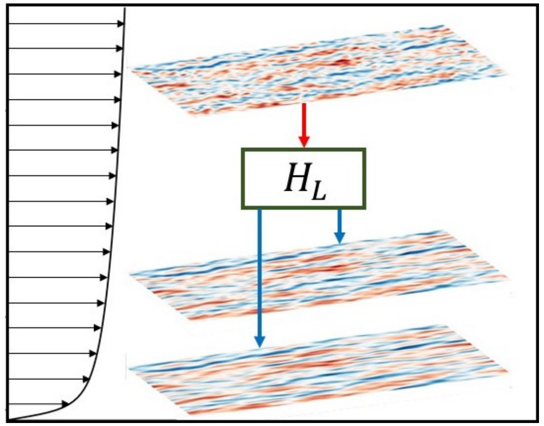


Vikrant Gupta (vikrant@sustech.edu.cn)

Comparison of the ratio of fluctuations magnitude from DNS data



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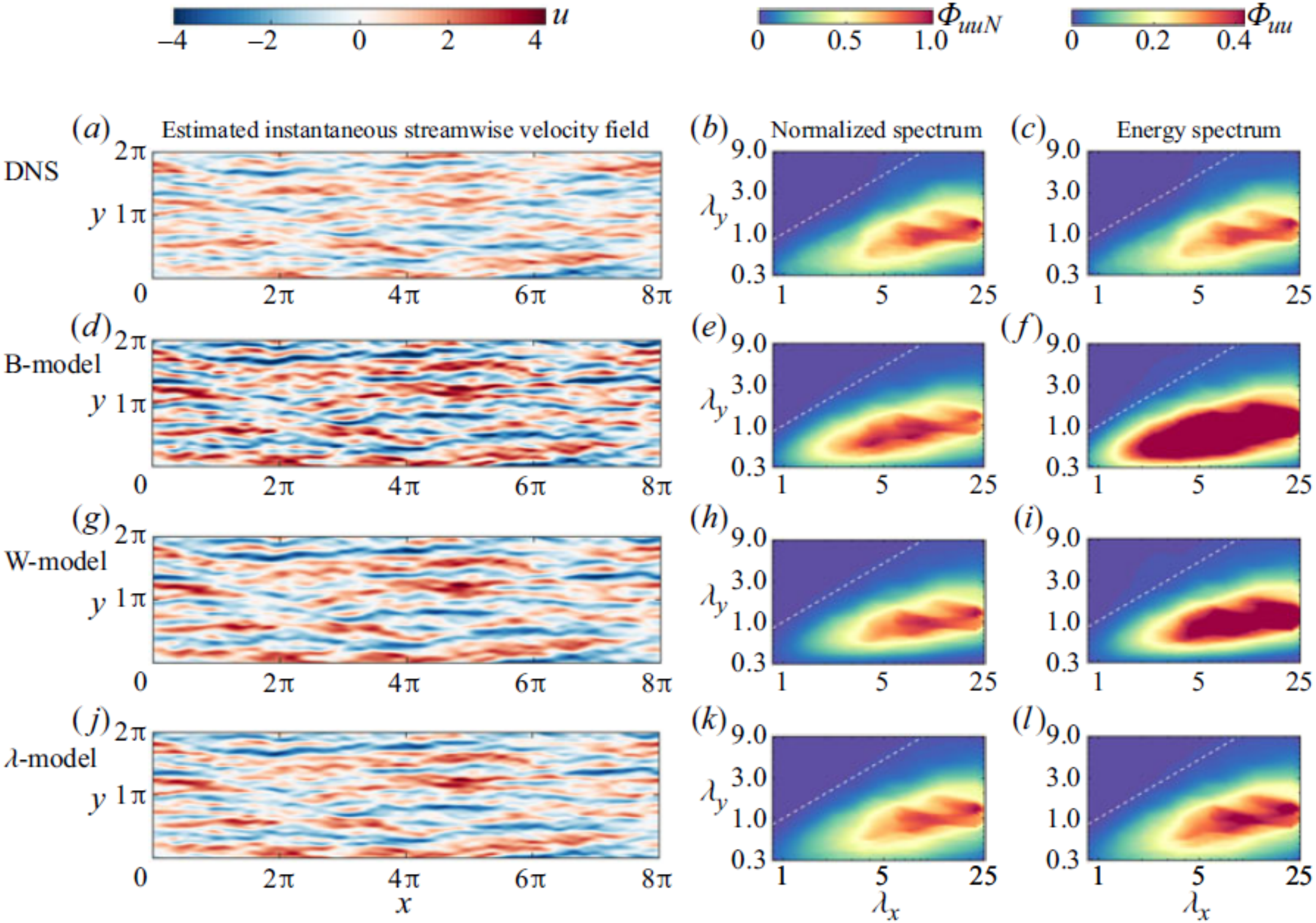
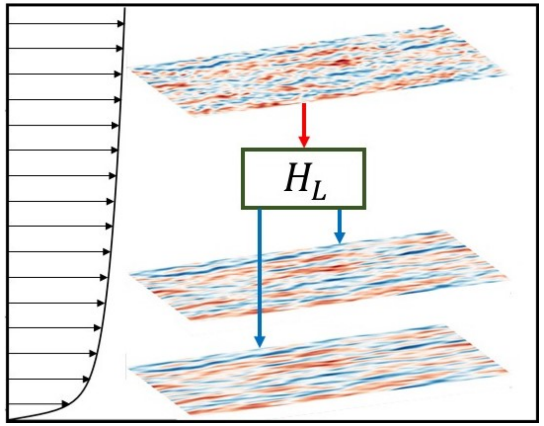


Vikrant Gupta (vikrant@sustech.edu.cn)

Physical interpretation in terms of the production term



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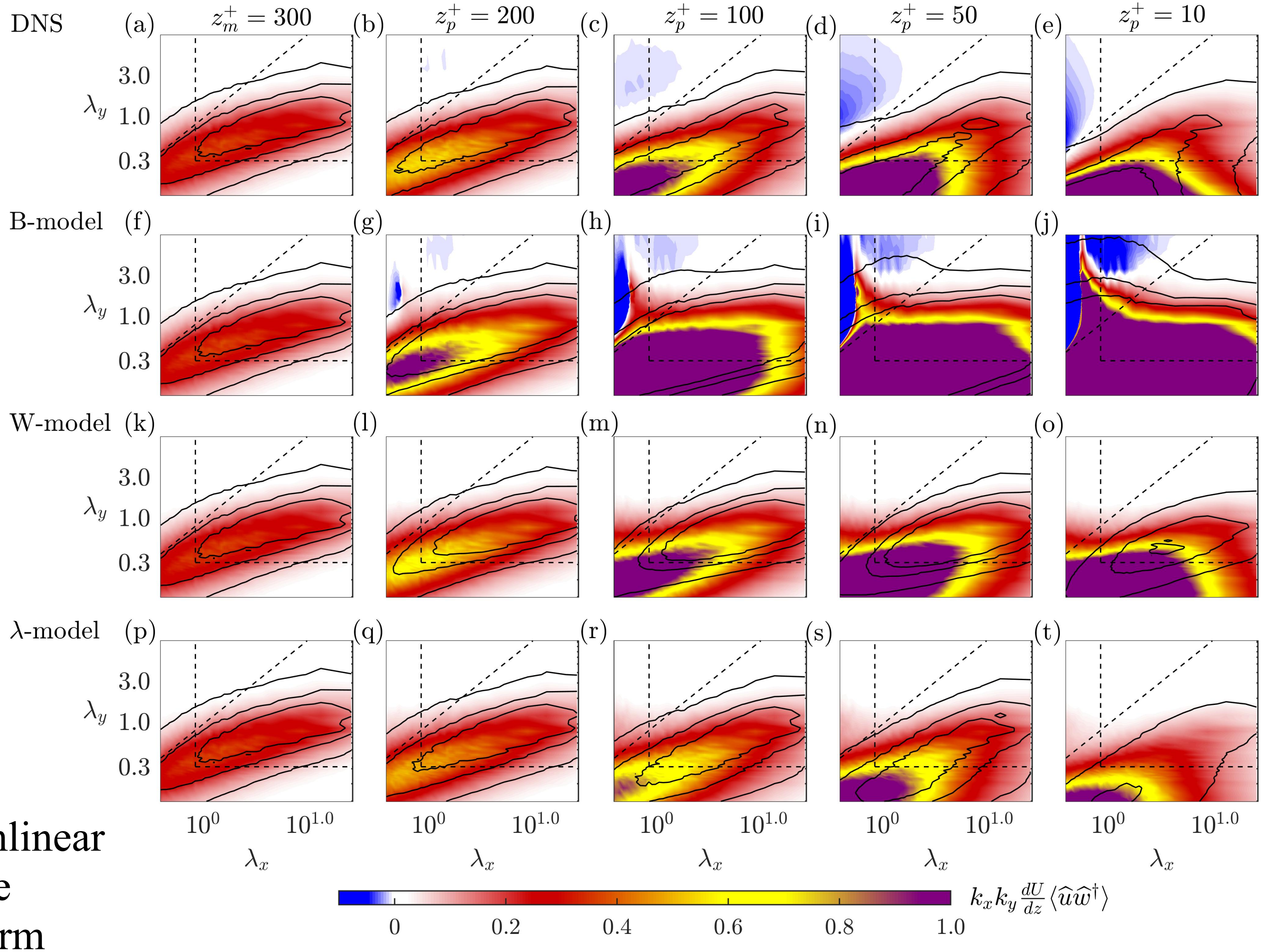
$$u_i \frac{\partial u_i}{\partial t} = -U_k u_i \frac{\partial u_i}{\partial x_k} - u_k u_i \frac{\partial U_i}{\partial x_k} - u_i \frac{\partial u_k u_i}{\partial x_k} - u_i \frac{\partial p}{\partial x_i} + \frac{u_i}{Re} \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

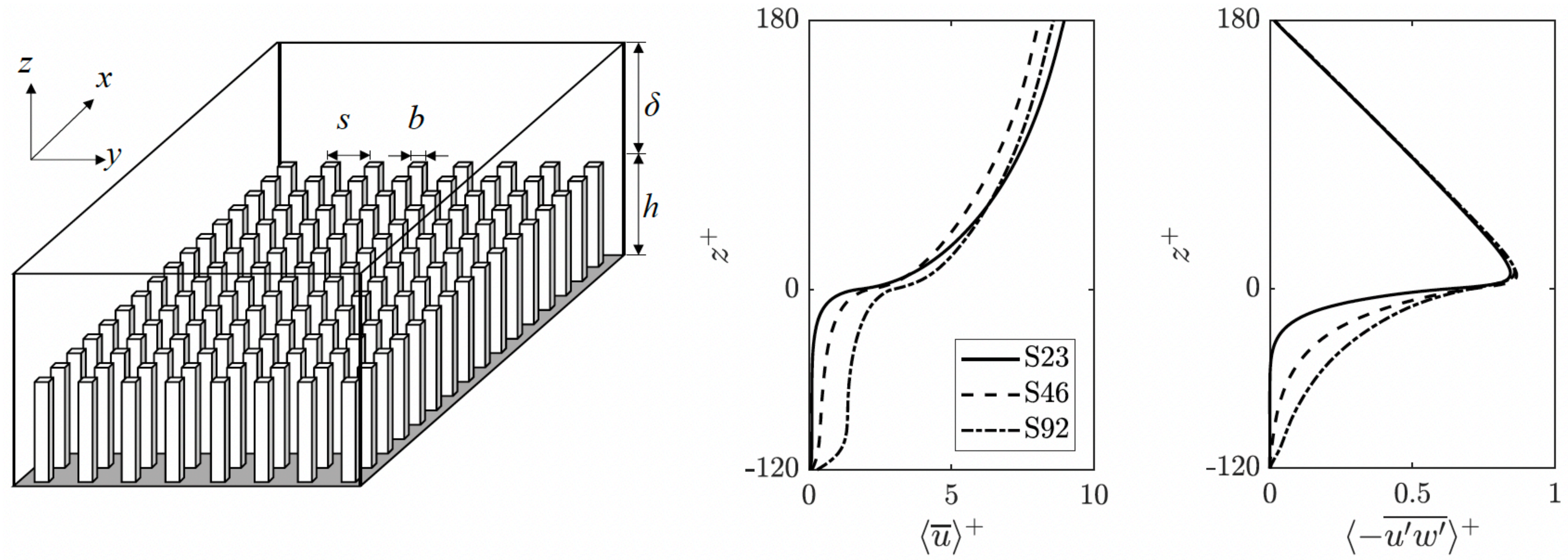
$$\frac{dE}{dt} = -\int u_i u_k \frac{\partial U_i}{\partial x_k} dV - \frac{1}{Re} \int \frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k} dV$$

Production term

Production term, although linear, will be zero if nonlinear term is ignored. The nonlinear term thus needs to be modelled appropriately to capture the production term



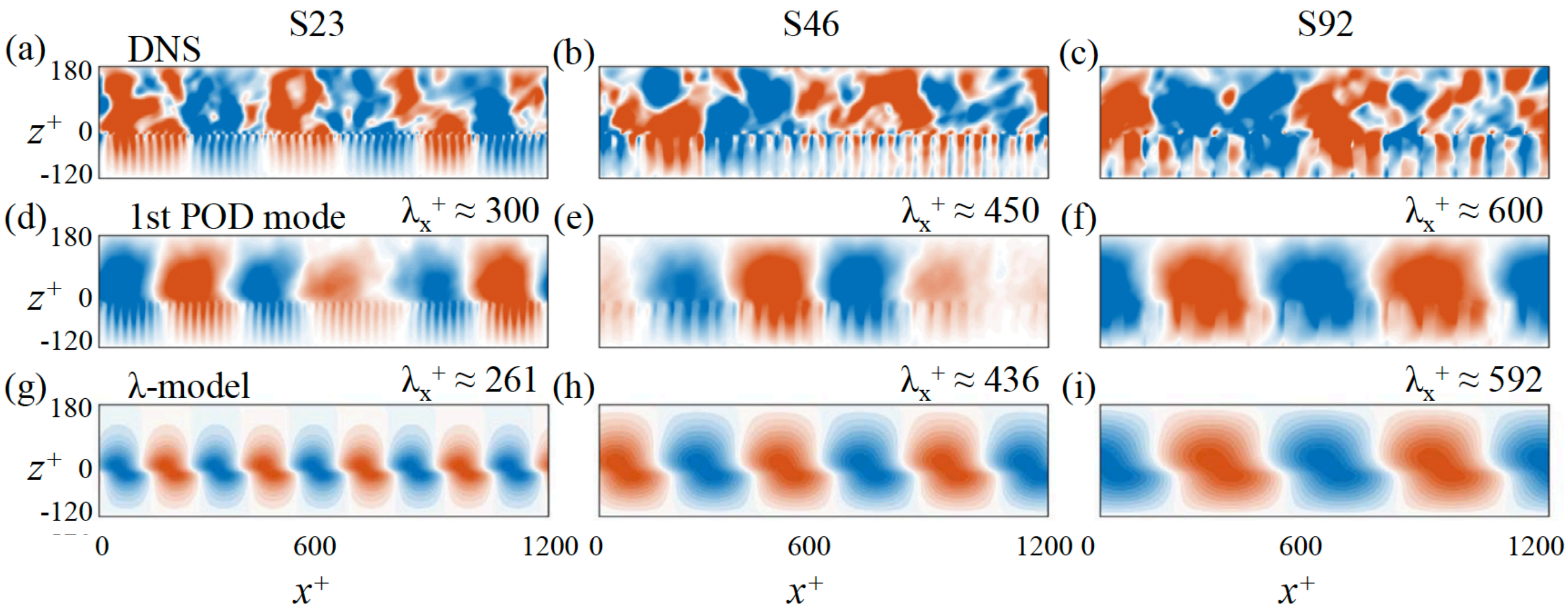
Application to flows over canopies



Collaborators:

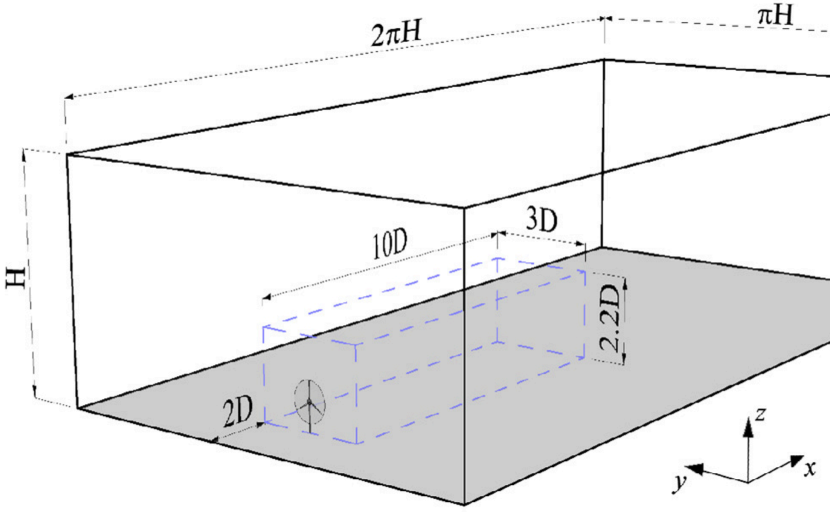
Wen Zhang, Southern University of Science & Technology, Shenzhen

Application to flows over canopies

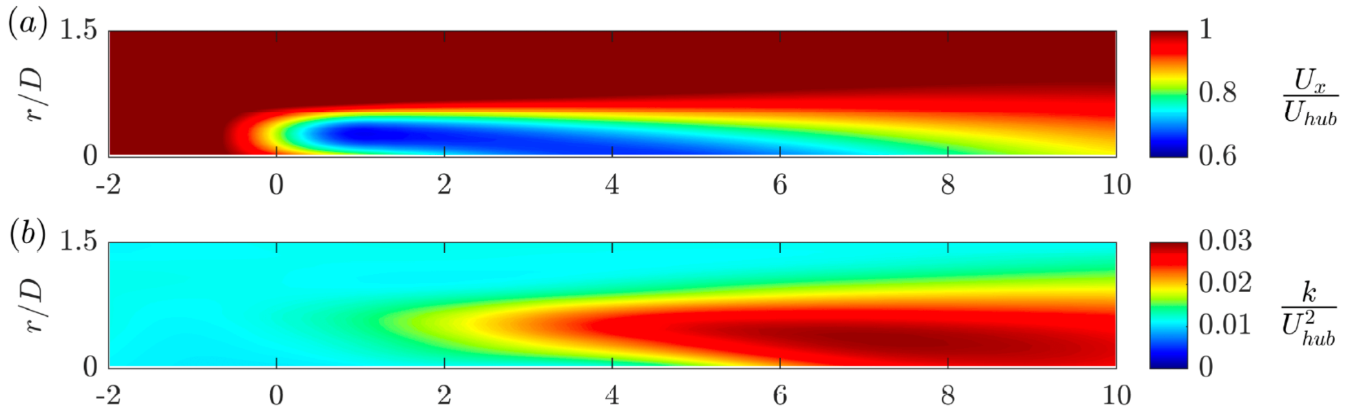


These results show the wide applicability of linear models and their ability for turbulence estimation even when very little measurement data is available

Application to wind turbine wakes: calculation of TKE generation in the far wake region



Mean streamwise velocity and TKE in the spanwise plane behind turbine



Existing engineering wake models can predict the mean streamwise velocity satisfactorily but the estimation of TKE is a challenge.

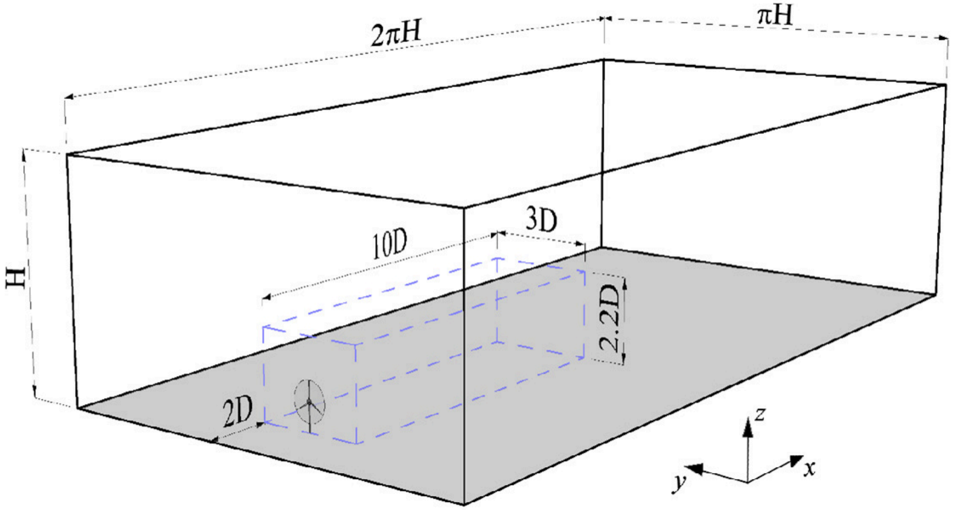
Can we use linear models for the TKE estimation?

Collaborators:

Dachuan Feng, joint PhD student at SUSTech and HKUST (now at TU Delft)

Larry K. B. Li, The Hong Kong University of Science & Technology

Application to wind turbine wakes: calculation of TKE generation in the far wake region



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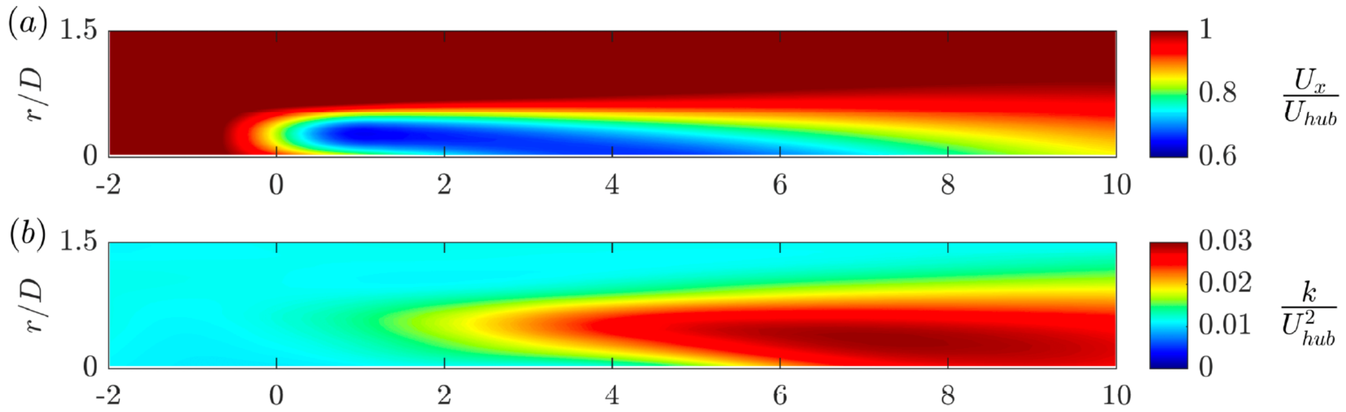
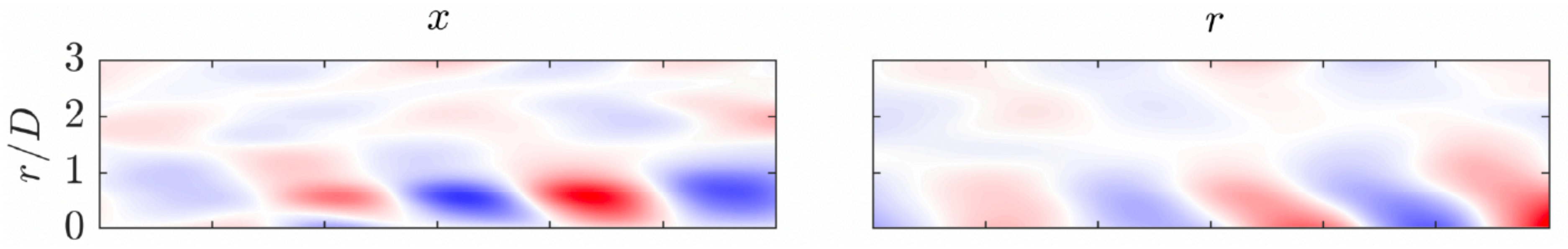
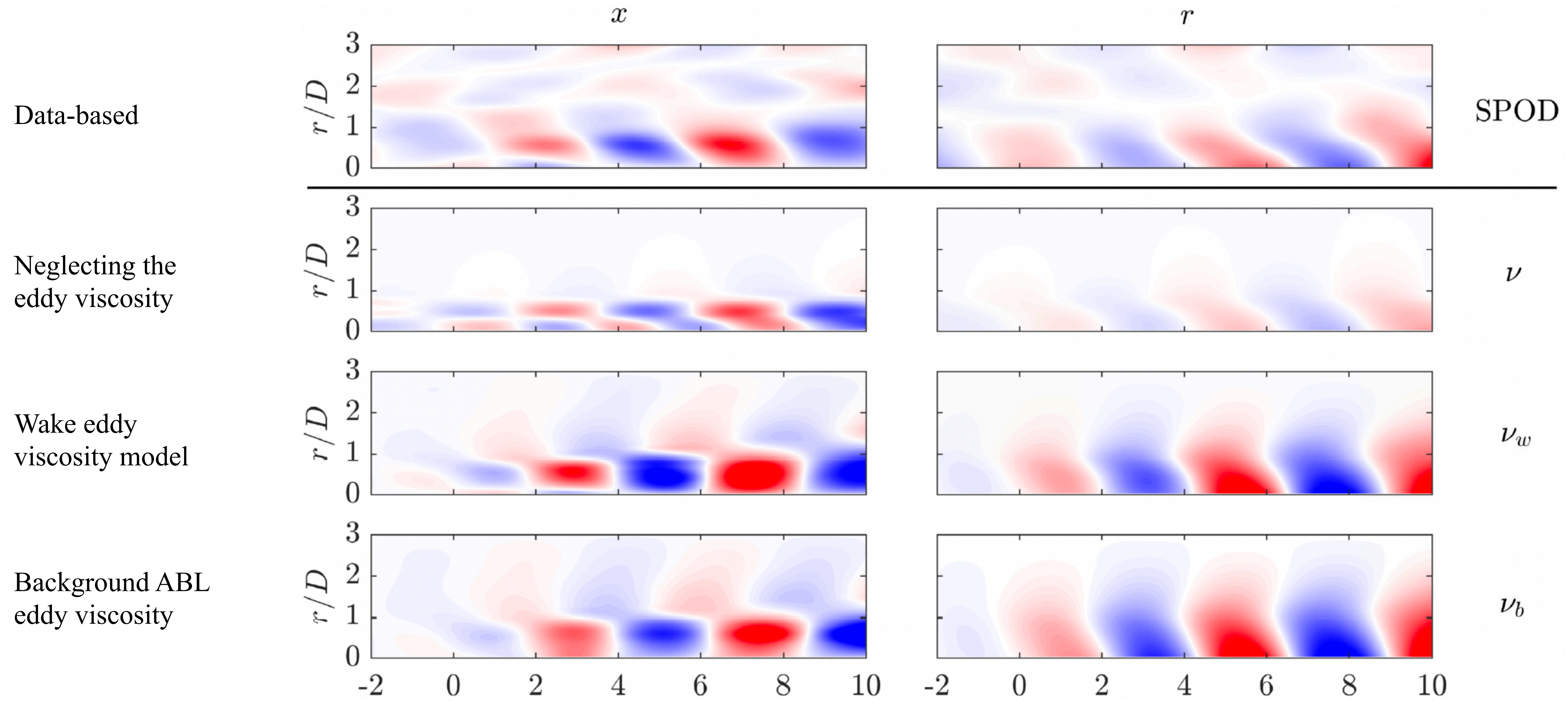


Validation results: Data-based modes via spectral proper orthogonal decomposition (SPOD) are first obtained



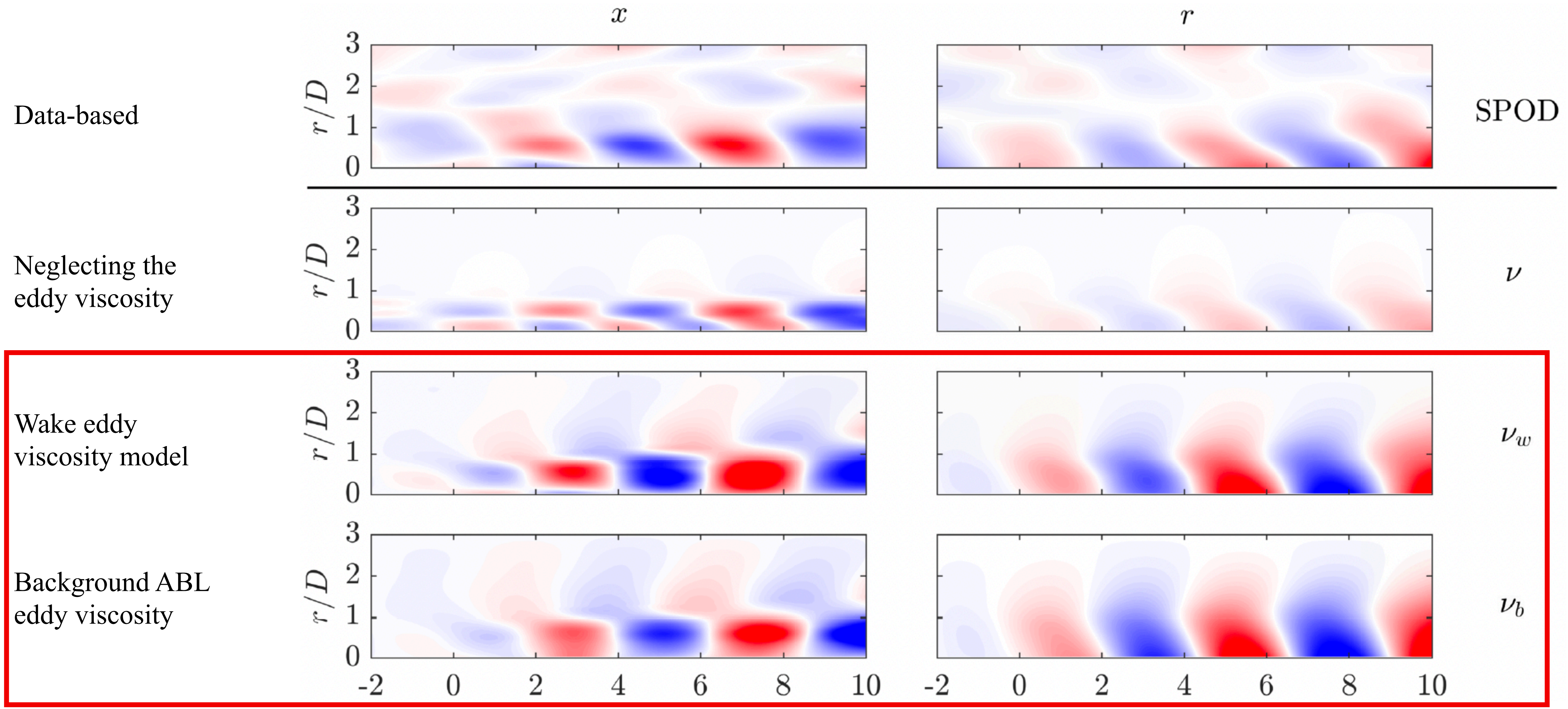
First SPOD mode at Strouhal number (St) = 0.2

Application to wind turbine wakes: calculation of TKE generation in the far wake region



Vikrant Gupta (vikrant@sustech.edu.cn)

Application to wind turbine wakes: calculation of TKE generation in the far wake region



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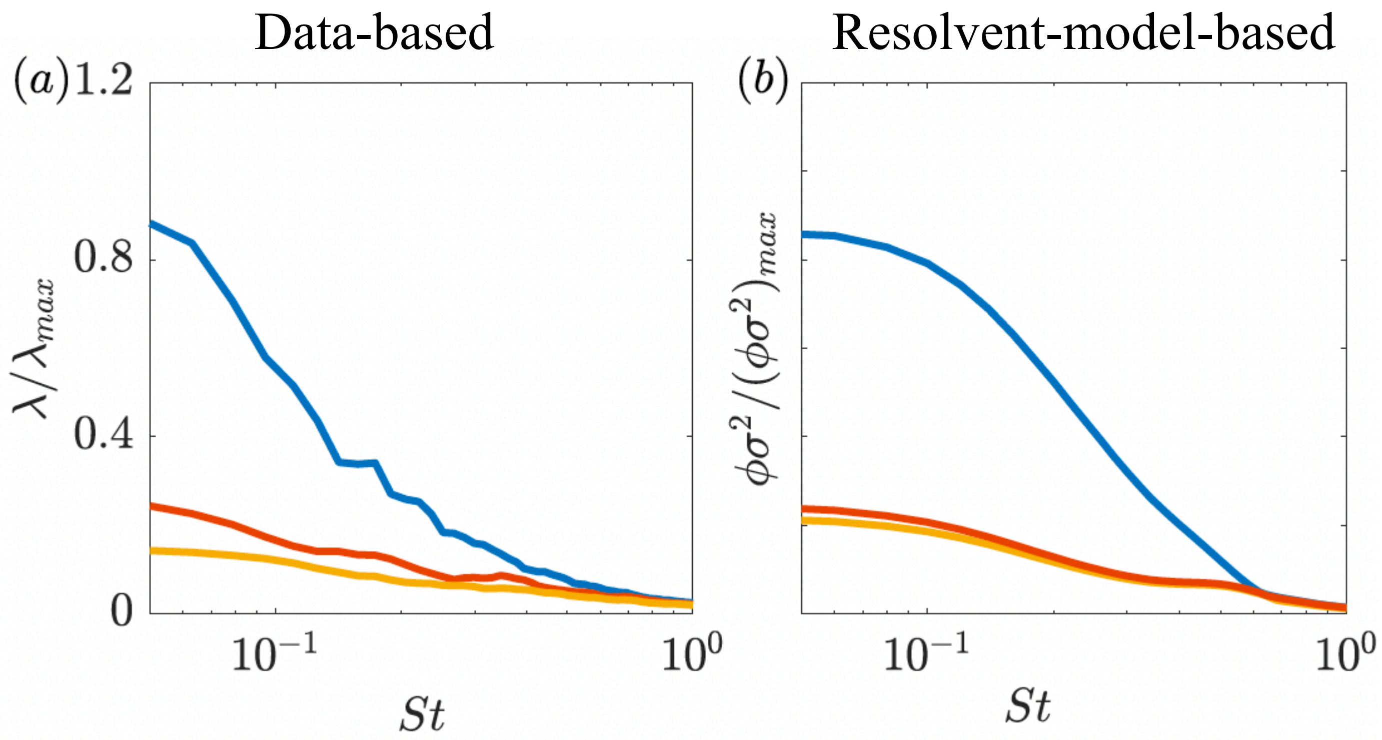
Application to wind turbine wakes: calculation of TKE generation in the far wake region



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Componentwise influence of upstream turbulence on the far-wake dynamics of wind turbines

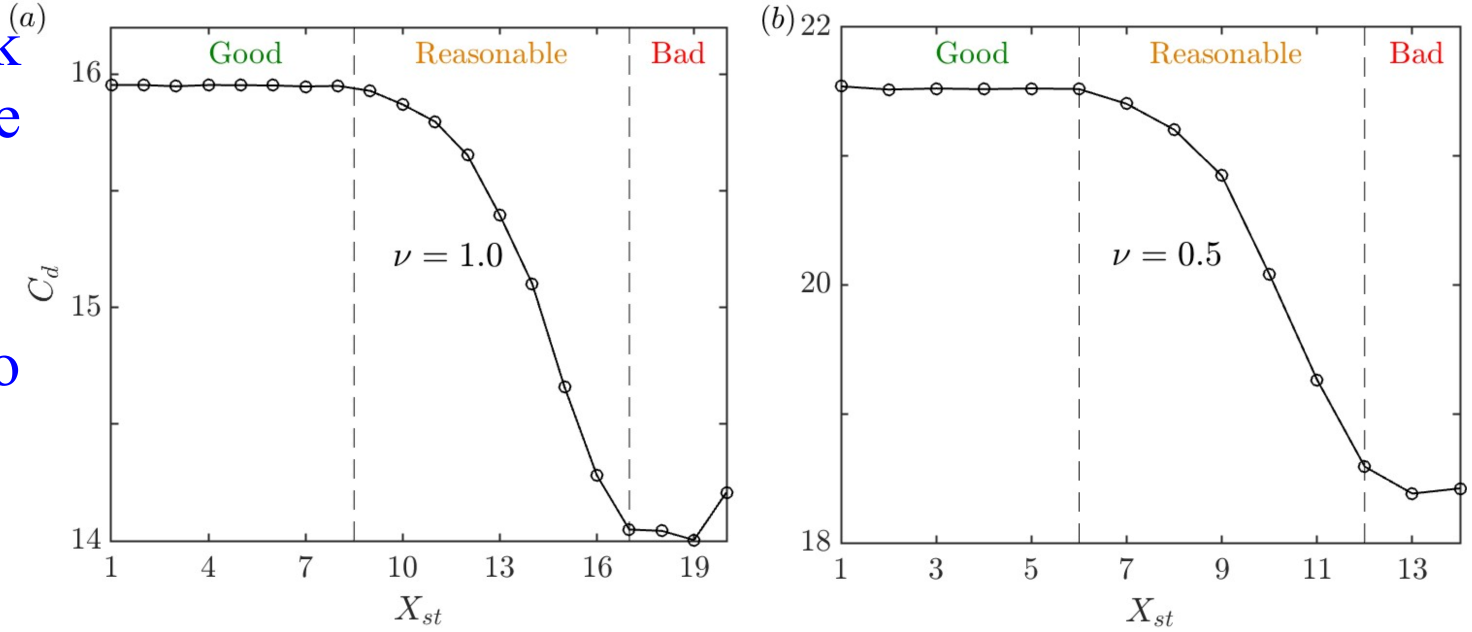
Dachuan Feng [a,b,c], Larry K.B. Li [a,d], Vikrant Gupta [b,c,e,*], Minping Wan [b,c,e,f,**]

Resolvent analysis for predicting energetic structures in the far-wake of a wind turbine, Feng, Gupta, Li & Wan, *Under review, PRF*

Caution: data-driven methods are still a decade or two away for practical turbulent systems

- Is there a limit of spatial resolution beyond which data assimilation methods cannot estimate/ predict?
- Do model-free machine learning methods need higher or lower resolution?
- **Can classical methods still be useful when data-driven methods fail?**

- Data assimilation methods can work only up to the resolution at which the system's complexity is captured.
- Machine learning methods need higher resolution because they need to learn the system dynamics from data.
- Classical methods still have a role to play.



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