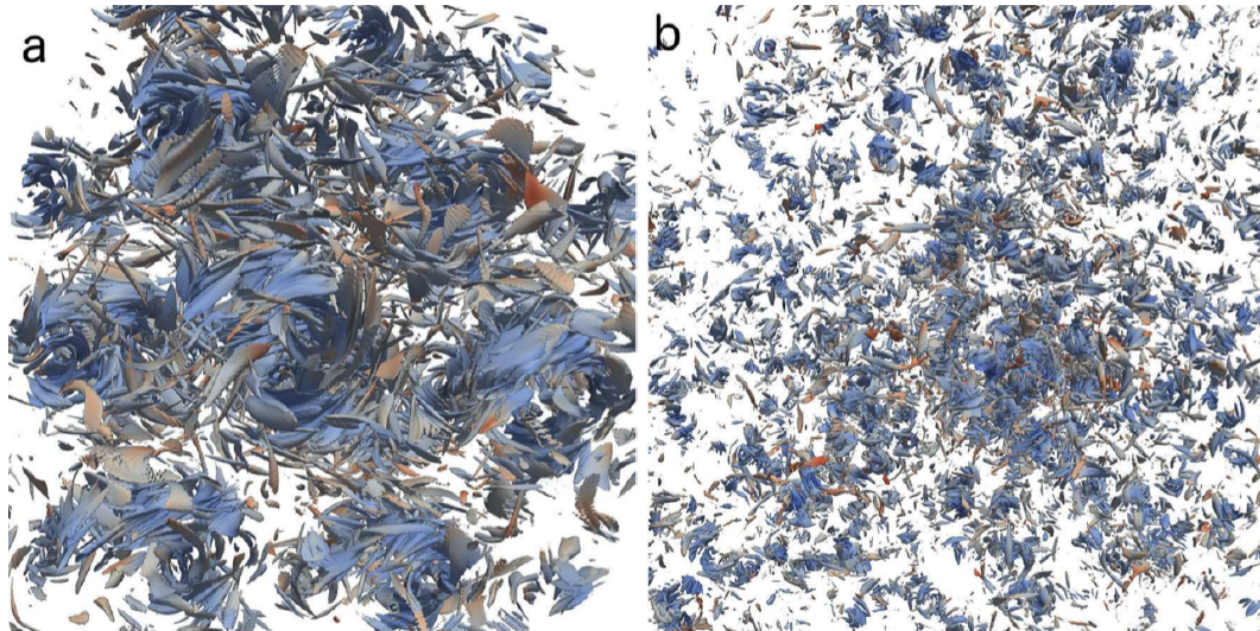


Transition from direct to inverse cascade in 3D Turbulence

Luca Biferale, Dept. Physics, INFN & CAST
University of Roma 'Tor Vergata'

biferale@roma2.infn.it

APS DENVER 2017



with **G. Sahoo** (U. Tor Vergata, Italy & U. Helsinki, Finland), **A. Alexakis** (ENS, Paris, France)



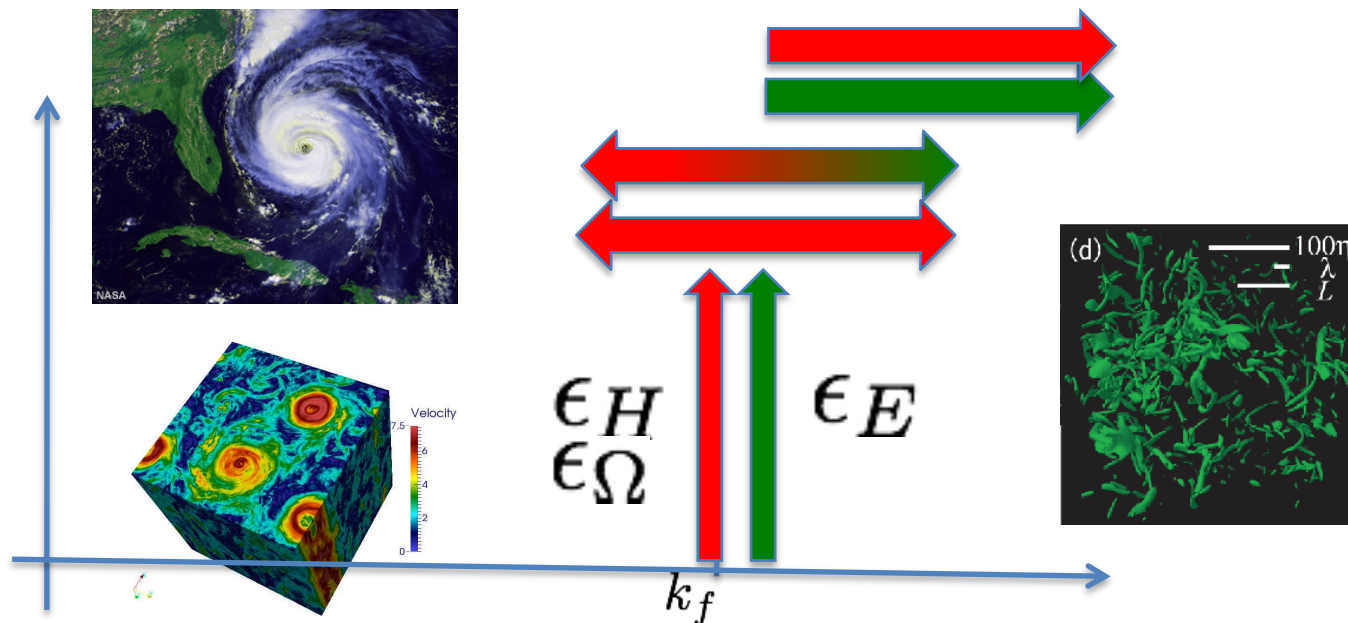
MOTIVATIONS:

A TALE ABOUT TRANSFER PROPERTIES OF INVISCID CONSERVED QUANTITIES, KINETIC ENERGY, HELICITY ENSTROPY, MAGNETIC HELICITY ETC...

Q1: HOW TO PREDICT THE DIRECTION OF THE TRANSFER (FORWARD/BACKWARD) AND ITS ROBUSTNESS UNDER EXTERNAL PERTURBATION (FORCING/BOUNDARY CONDITIONS)?

Q2: HOW MUCH THE FLUCTUATIONS AROUND THE MEAN TRANSFER ARE INTENSE AND SELF-SIMILAR (INTERMITTENCY AND ANOMALOUS SCALING) ?

AS A MATTER OF FACT, FOR 3D NAVIER STOKES EQUATIONS, WE DO NOT KNOW HOW TO PREDICT NEITHER THE SIGN OF THE MEAN ENERGY TRANSFER NOR THE INTENSITY OF THE FLUCTUATIONS AROUND IT.



Q: CAN WE DISSECT 3D NS EQUATIONS TO EXTRACT INTERESTING INFORMATION FROM ITS ELEMENTARY CONSTITUENTS?

R: PLAY WITH MIRROR SYMMETRY

$$\left\{ \begin{array}{l} \partial_t \mathbf{v} + (\mathbf{v} \cdot \partial) \mathbf{v} = -\partial P + \nu \Delta \mathbf{v} + \mathbf{F} \\ \partial \cdot \mathbf{v} = 0 \\ + \textit{Boundary Conditions} \end{array} \right.$$



Piero della Francesca ~ 1450 C.E. Monterchi IT

Commun. Math. Phys. 115, 435–456 (1988)

The Beltrami Spectrum for Incompressible Fluid Flows

Peter Constantin^{1,*} and Andrew Majda^{2,**}

The nature of triad interactions in homogeneous turbulence

Fabian Waleffe

Center for Turbulence Research, Stanford University–NASA Ames, Building 500,
Stanford, California 94305-3030

(Received 24 July 1991; accepted 22 October 1991)

$$u(\mathbf{k}) = u^+(\mathbf{k})h^+(\mathbf{k}) + u^-(\mathbf{k})h^-(\mathbf{k})$$

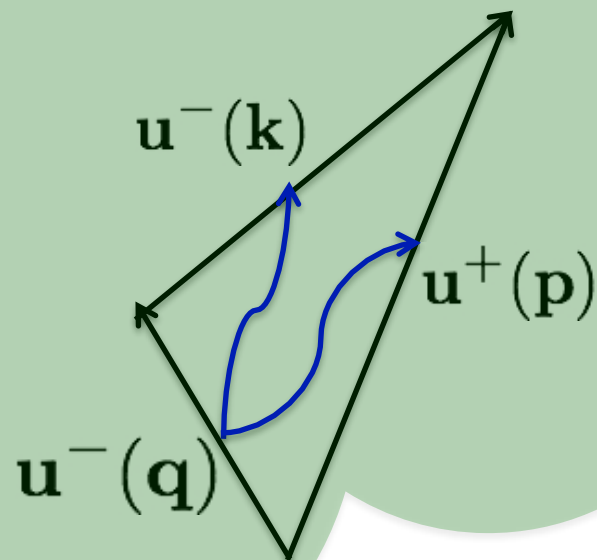
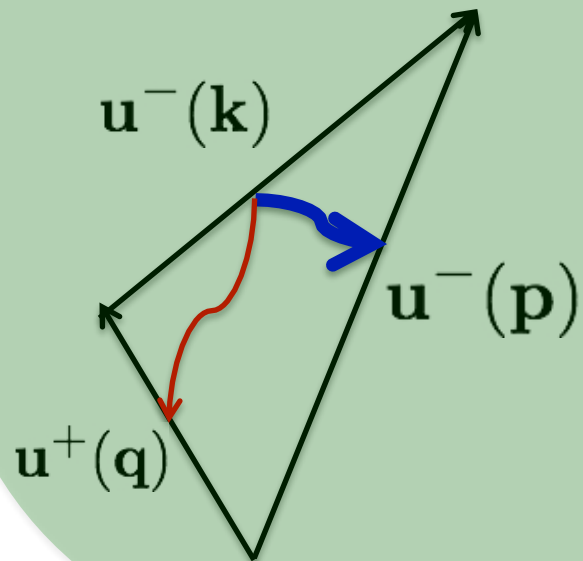
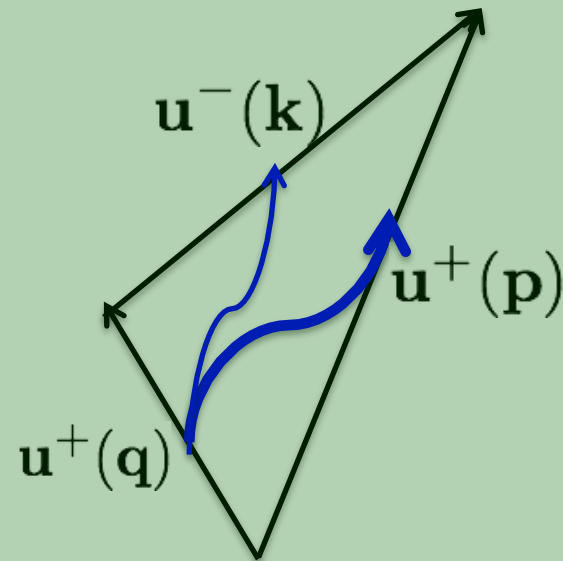
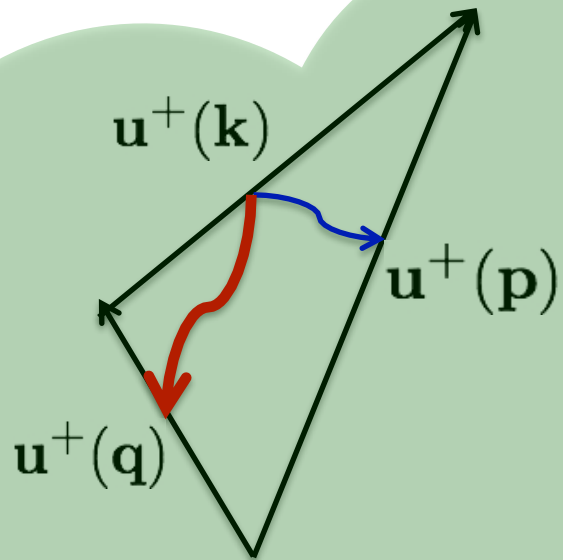
$$h^\pm = \hat{\nu} \times \hat{\mathbf{k}} \pm i\hat{\nu}$$

$$\hat{\nu} = \mathbf{z} \times \mathbf{k} / \|\mathbf{z} \times \mathbf{k}\|$$

$$i\mathbf{k} \times \mathbf{h}^\pm = \pm k\mathbf{h}^\pm$$

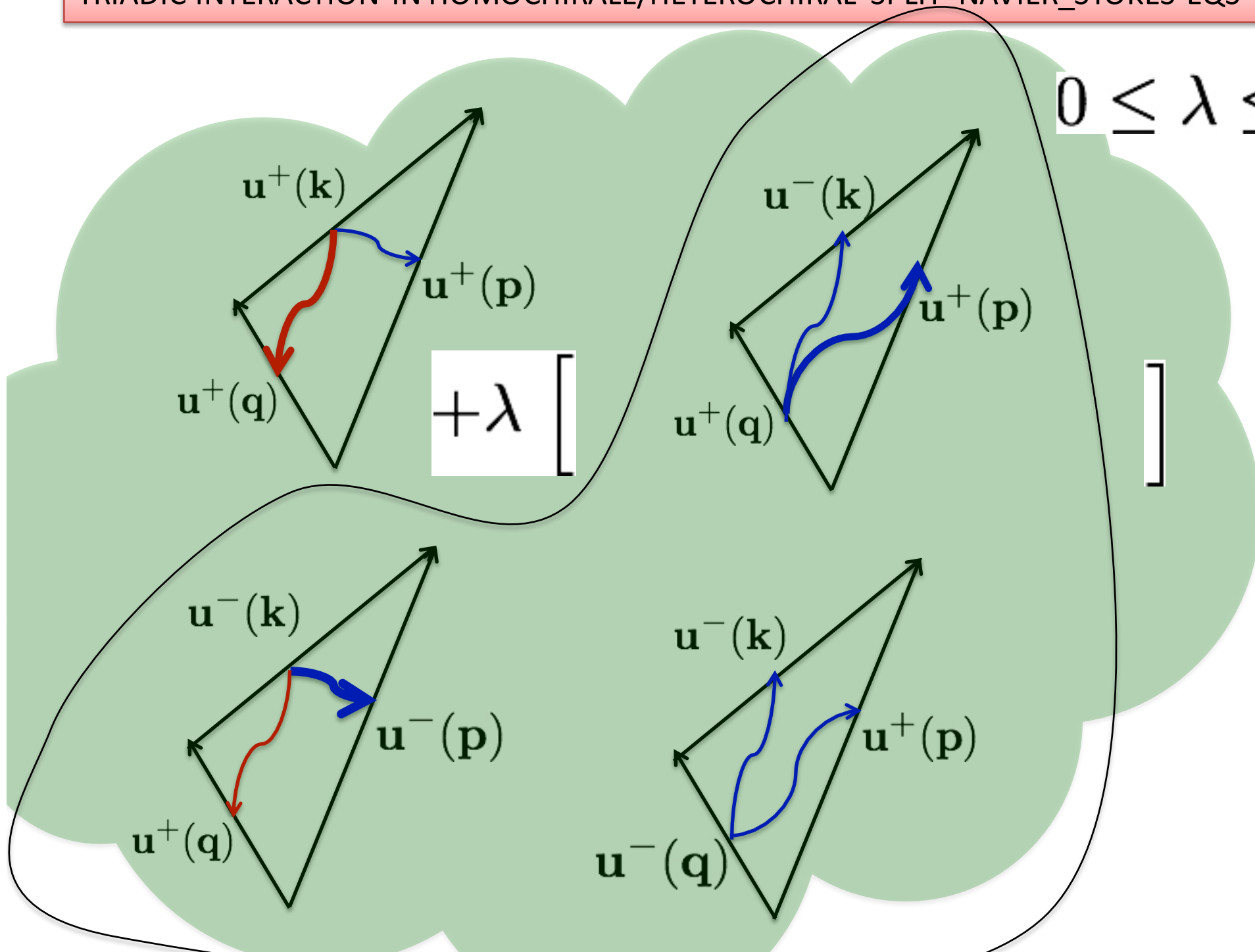
$$\begin{cases} E = \sum_{\mathbf{k}} |u^+(\mathbf{k})|^2 + |u^-(\mathbf{k})|^2; \\ H = \sum_{\mathbf{k}} k(|u^+(\mathbf{k})|^2 - |u^-(\mathbf{k})|^2). \end{cases}$$

HELICAL-FOURIER TRIADIC INTERACTION IN NAVIER-STOKES EQS



TRIADIC INTERACTION IN HOMOCHIRALE/HETEROCHIRAL SPLIT NAVIER-STOKES EQS

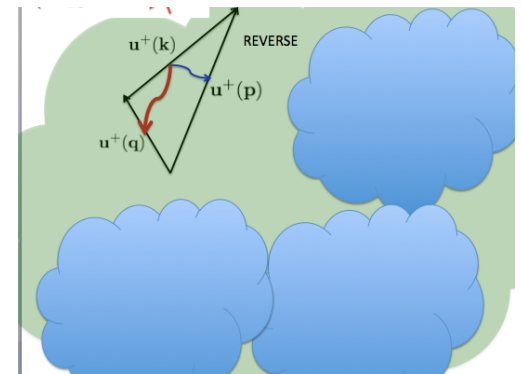
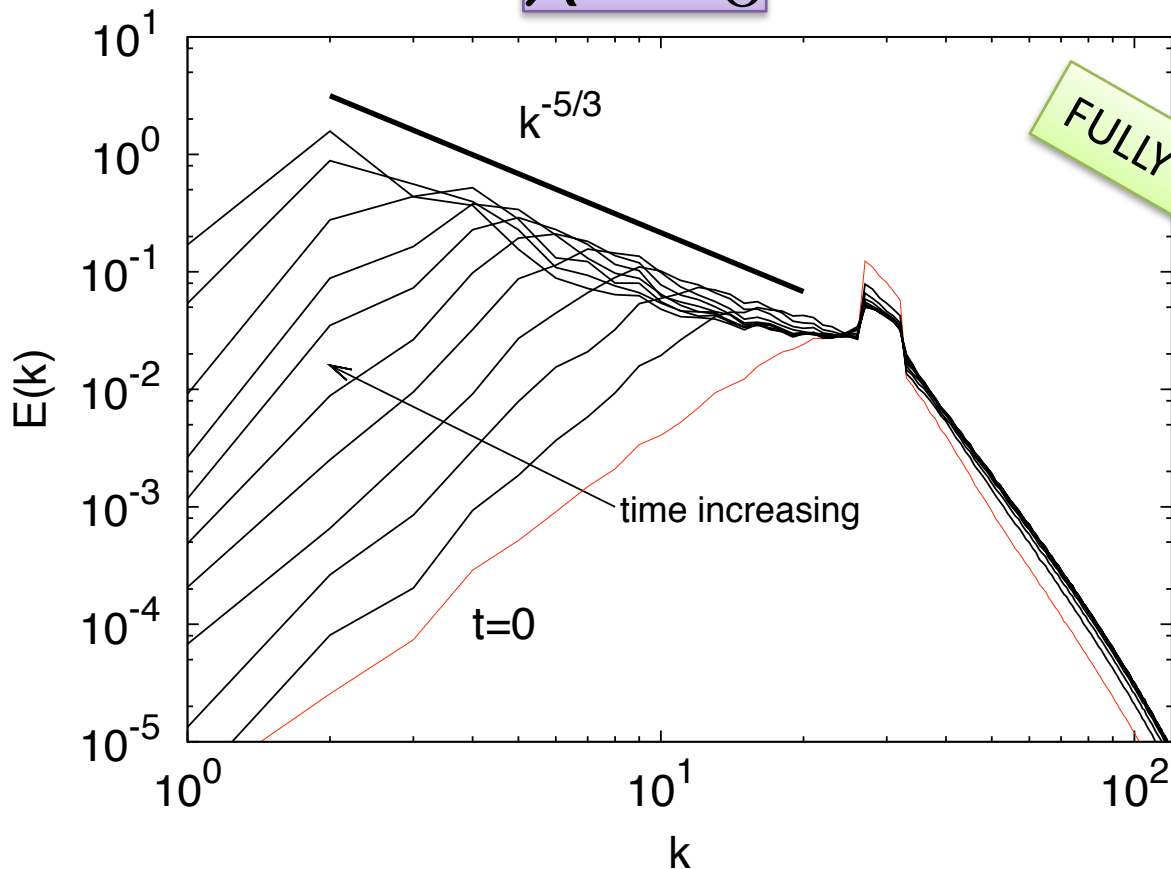
$$0 \leq \lambda \leq 1$$

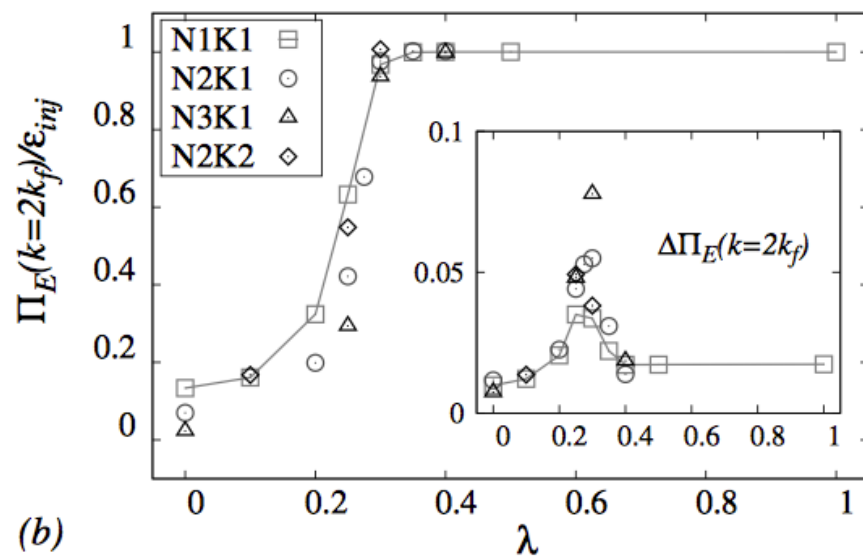
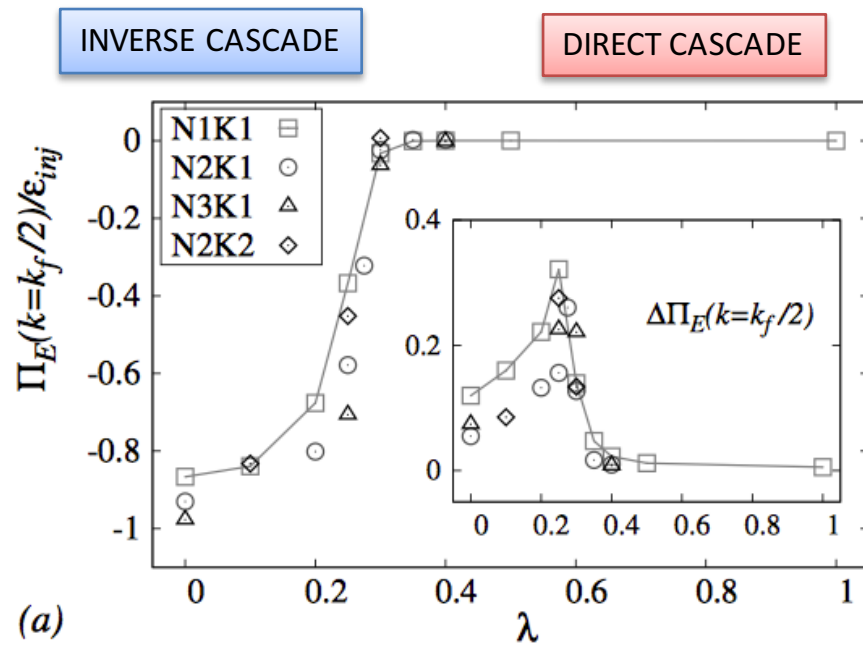
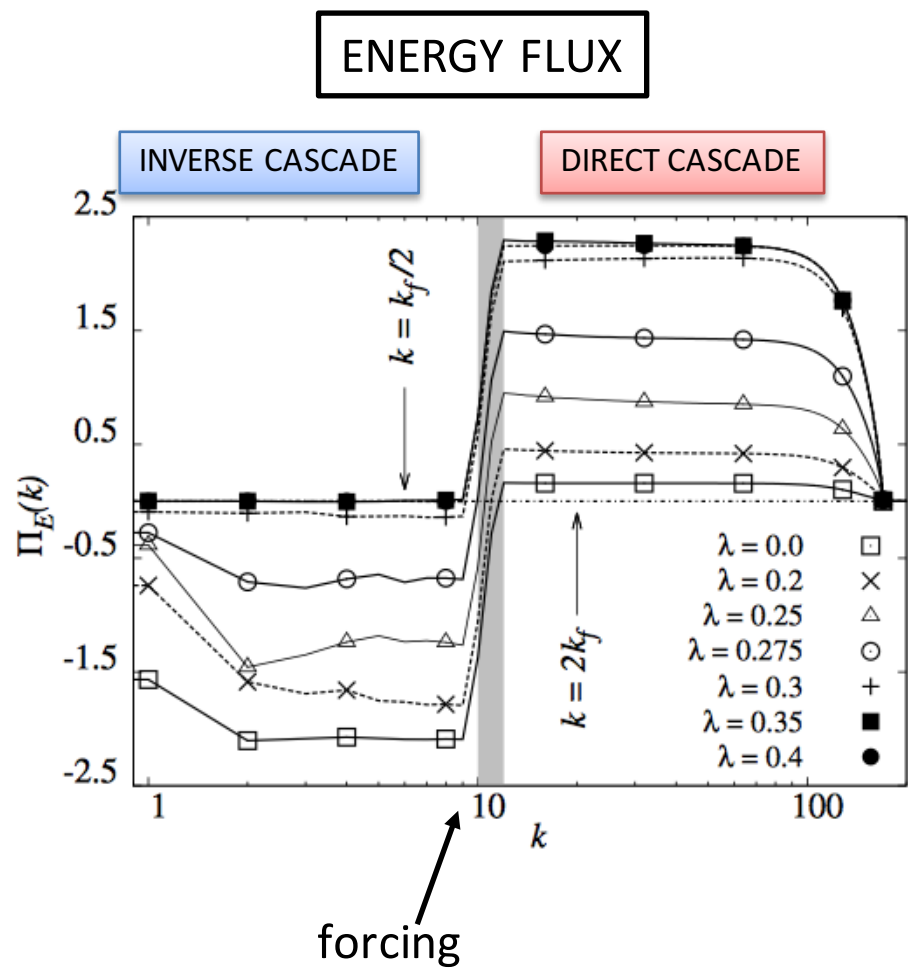


HOMOCHIRAL 3D NAVIER STOKES EQS.

$$\begin{cases} E = \sum_{\mathbf{k}} |u^+(\mathbf{k})|^2 + |u^-(\mathbf{k})|^2; \\ H = \sum_{\mathbf{k}} k(|u^+(\mathbf{k})|^2 - |u^-(\mathbf{k})|^2). \end{cases}$$

$\lambda = 0$





ALL THREE DIMENSIONAL FLOWS IN NATURE POSSES FOURIER-INTERACTIONS
ABLE TO TRANSFER ENERGY EITHER BACKWARD (HOMOCHIRAL TRIADS) OR
FORWARD (HETEROCHIRAL)

IN NATURAL CONDITIONS THE HETEROCHIRAL TRIADS ARE MORE EFFICIENT AND
ENERGY CASCADES FORWARD IN HOMOGENEOUS AND ISOTROPIC TURBULENCE

IF HETEROCHIRAL TRIADS ARE LESS EFFICIENT, A SHARP REVERSE OF THE ENERGY
CASCADE IS OBSERVED **WITHOUT BREAKING OF ANY SYMMETRY**

Discontinuous Transition from Direct to Inverse Cascade in Three-Dimensional Turbulence

G. Sahoo, A Alexakis, L Biferale.

Phys. Rev. Lett. 118, 164501 (2017).

Role of helicity for large-and small-scales turbulent fluctuations

G Sahoo, F Bonaccorso, and L Biferale.

Phys. Rev. E 92, *Rapid Comm*, 051002 (2015).

Inverse energy cascade in three-dimensional isotropic turbulence

L Biferale, S Musacchio, and F Toschi.

Phys. Rev. Lett. 108, 164501 (2012).