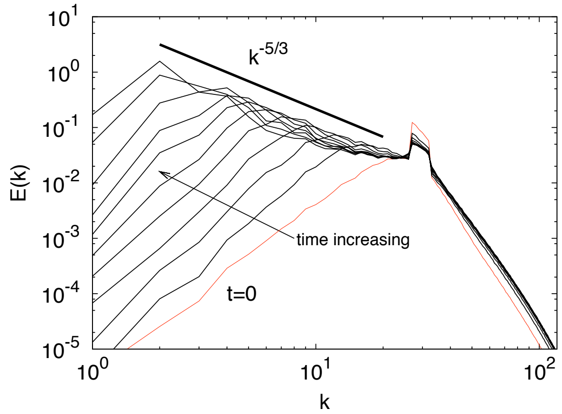
on the role of helicity in 3d turbulent flows

Luca Biferale
Dept. Physics, INFN & CAST
University of Roma 'Tor Vergata'
biferale@roma2.infn.it

EFMC10



Credits: **S. Musacchio** (CNRS-France); **F. Toschi** (University of Eindhoven, The Netherlands); **E. Titi** (Weizmann Institute of Science, Israel)

HOW TO USE UNCONVENTIONAL NUMERICS TO UNDERSTAND AND MODEL EULERIAN AND LAGRANGIAN TURBULENCE

$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v} \cdot \partial)\mathbf{v} = -\partial P + \nu \Delta \mathbf{v} + \mathbf{F} \\ \partial \cdot \mathbf{v} = 0 \\ + \; Boundary\ Conditions \end{cases}$$

DIRECT -> INVERSE ENERGY TRANSFER:

2D (Kraichnan 1966)
3D + ROTATION + HELICITY INJECTION (Mininni & Pouquet 2013)
THICK LAYER + ROTATION (Smith et al 1996)
SQUEZED DOMAINS (Celani et al 2010, Xia et al 2012)
STRONG SHEAR (Herbert et 2012)
SMALL SCALES HELICITY INJECTION (Sulem et al 1986)

ON THE ROLE OF INVISCID INVARIANTS (HELICITY & ENERGY) IN 3D FORWARD/ BACKWARD ENERGY CASCADES

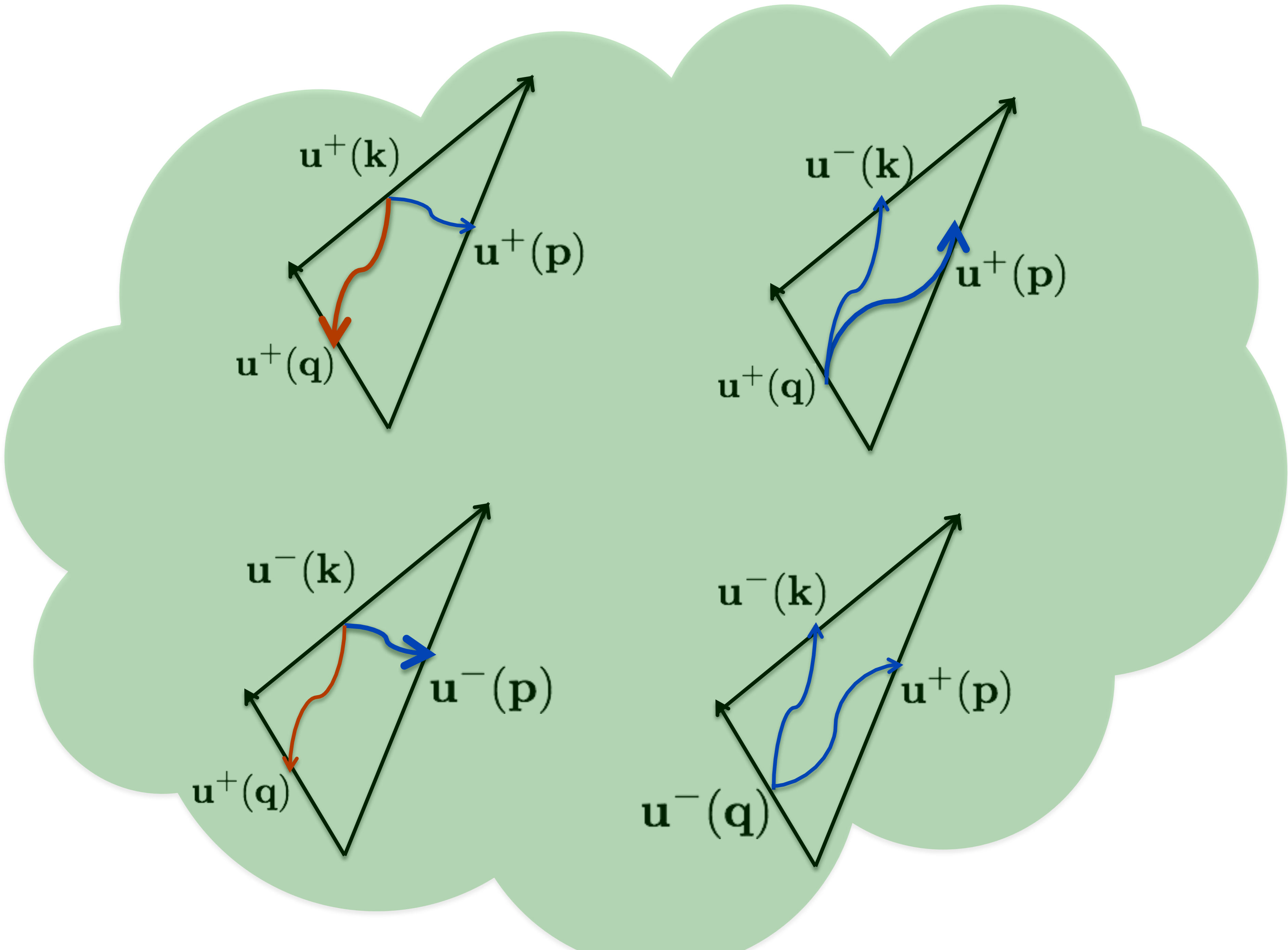
$$H = \int d^3x \, \omega \cdot \mathbf{v}$$

$$\boldsymbol{u}(\boldsymbol{k}) = u^+(\boldsymbol{k})\boldsymbol{h}^+(\boldsymbol{k}) + u^-(\boldsymbol{k})\boldsymbol{h}^-(\boldsymbol{k})$$

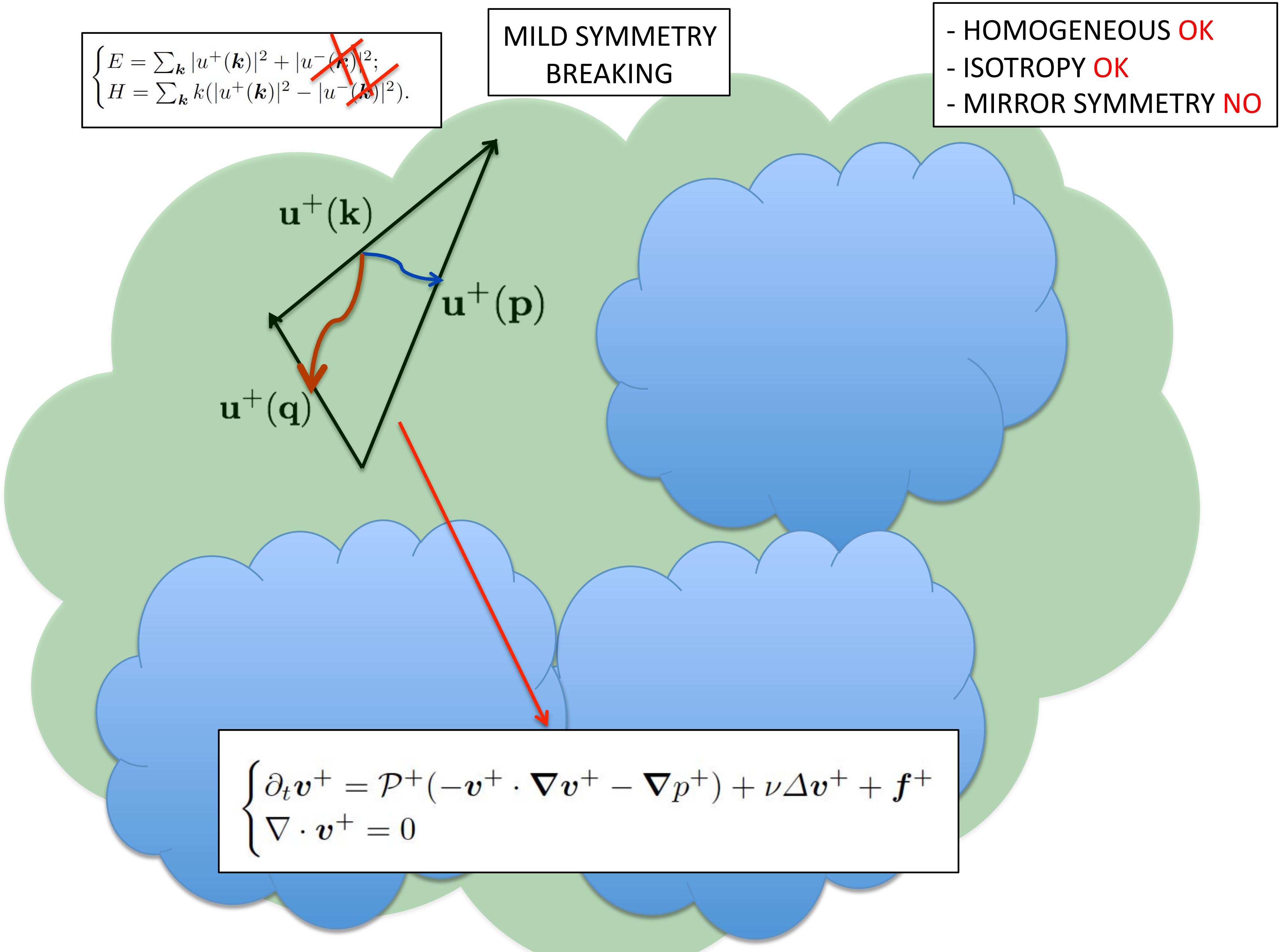
$$i\boldsymbol{k} \times \boldsymbol{h}^\pm = \pm k \boldsymbol{h}^\pm$$

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) = \sum_{s_p=\pm, s_q=\pm} g_{s_k,s_p.s_q} \sum_{p+q=k} u^{s_p}(\mathbf{p})u^{s_q}(\mathbf{q}) - \nu k^2 u^{s_k}(\mathbf{k})$$



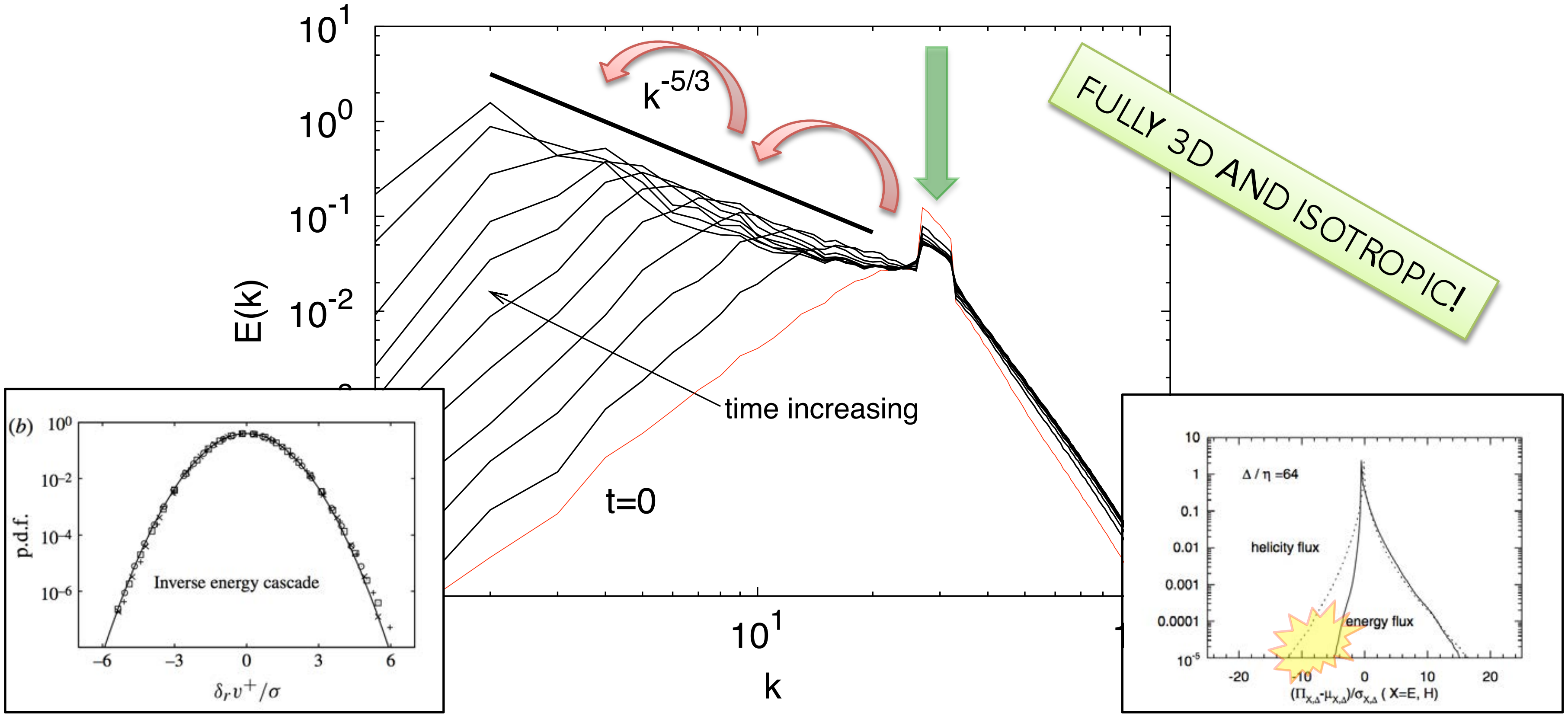
$\mathbf{u}^+(\mathbf{k})$
$\mathbf{u}^+(\mathbf{p})$
$\mathbf{u}^+(\mathbf{q})$
$\mathbf{u}^-(\mathbf{k})$
$\mathbf{u}^+(\mathbf{p})$
$\mathbf{u}^+(\mathbf{q})$
$\mathbf{u}^-(\mathbf{k})$
$\mathbf{u}^-(\mathbf{p})$
$\mathbf{u}^+(\mathbf{q})$
$\mathbf{u}^-(\mathbf{k})$
$\mathbf{u}^+(\mathbf{p})$
$\mathbf{u}^-(\mathbf{q})$



$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$
MILD SYMMETRY BREAKING
- HOMOGENEOUS OK
- ISOTROPY OK
- MIRROR SYMMETRY NO
$\mathbf{u}^+(\mathbf{k})$
$\mathbf{u}^+(\mathbf{p})$
$\mathbf{u}^+(\mathbf{q})$
$\begin{cases} \partial_t \boldsymbol{v}^+ = \mathcal{P}^+(-\boldsymbol{v}^+ \cdot \boldsymbol{\nabla} \boldsymbol{v}^+ - \boldsymbol{\nabla} p^+) + \nu \Delta \boldsymbol{v}^+ + \boldsymbol{f}^+ \\ \nabla \cdot \boldsymbol{v}^+ = 0 \end{cases}$

INVERSE ENERGY FLUX: FROM SMALL TO LARGE SCALES in 3D!

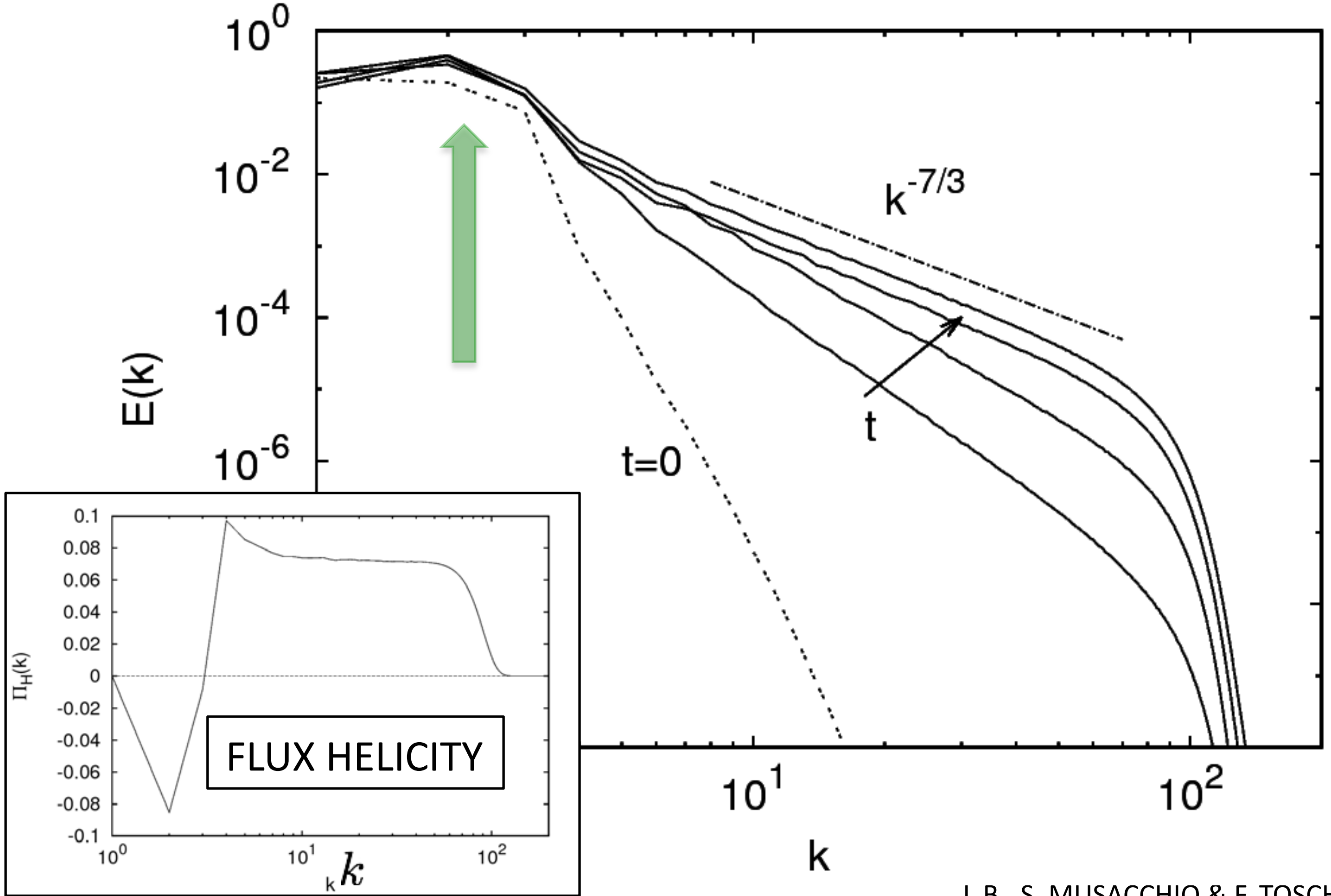
$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$



L.B., S. MUSACCHIO & F. TOSCHI Phys. Rev. Lett. **108** 164501 (2012); JFM **730**, 309 (2013)

LARGE SCALES FORCING: DIRECT HELICITY CASCADE

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$



L.B., S. MUSACCHIO & F. TOSCHI
730, 309 (2013) (2013)

ESISTENCE AND UNIQUENESS OF WEAK SOLUTIONS OF THE HELICAL-DECIMATED NSE

$$\begin{cases} \partial_t \boldsymbol{v}^+ = \mathcal{P}^+(-\boldsymbol{v}^+ \cdot \boldsymbol{\nabla} \boldsymbol{v}^+ - \boldsymbol{\nabla} p^+) + \nu \Delta \boldsymbol{v}^+ + \boldsymbol{f}^+ \\ \nabla \cdot \boldsymbol{v}^+ = 0 \end{cases}$$

HILBERT-NORM COINCIDES WITH THE SIGN-DEFINITE HELICTY

$$||g||_{H^{1/2}} = \sum_{\boldsymbol{k}} k|g(k)|^2$$

CONSERVATION HELICITY: NEW APRIORI BOUND ON THE VELOCITY

$$\frac{1}{2}\partial_t \sum_{\boldsymbol{k}} k|u^+(\boldsymbol{k},t)|^2 + \frac{\nu}{2}\sum_{\boldsymbol{k}} k^3|u^+(\boldsymbol{k},t)|^2 \leq \frac{1}{2\nu}\sum_{\boldsymbol{k}} |f^+(\boldsymbol{k})|^2 k^{-1}.$$

$$\frac{1}{2}\partial_t ||v^+||^2_{H^{\frac{1}{2}}} + \frac{\nu}{2}||v^+||^2_{H^{\frac{3}{2}}} \leq \frac{1}{2\nu}\sum_{\boldsymbol{k}} |f^+(\boldsymbol{k})|^2 k^{-1}.$$

$$v^+ \in L^\infty_t H^{\frac{1}{2}}_x; \qquad \sqrt{\nu} v^+ \in L^2_t H^{\frac{3}{2}}_x$$

L.B. & E. TITI J. Stat. Phys. **151**, 1089 (2013)

ON THE ROLE OF HELICITY IN 3D FORWARD/BACKWARD TURBULENT ENERGY TRANSFER

Q: Can we dissect (and reconstruct) NS equations to extract interesting information from its elementary constituents?

A: Yes, we can!

1) **We showed that ALL flows in nature posses a class of nonlinear interactions characterized by a backward energy transfer (inverse energy cascade), triggered by the dynamics of Helicity, and that this happens even in fully isotropic, homogeneous 3D turbulence**
2) **Connections to small-scales intermittency ?**
3) **Connections to regularity of NS equations in 3D ?**
4) **Extensions to Magnetohydrodynamics ?**
5) **Connections to energy reversal in rotating turbulence?**

$$\mathbf{u}(\mathbf{k}) = u^+(\mathbf{k})\mathbf{h}^+(\mathbf{k}) + \epsilon(\mathbf{k})u^-(\mathbf{k})\mathbf{h}^-(\mathbf{k})$$

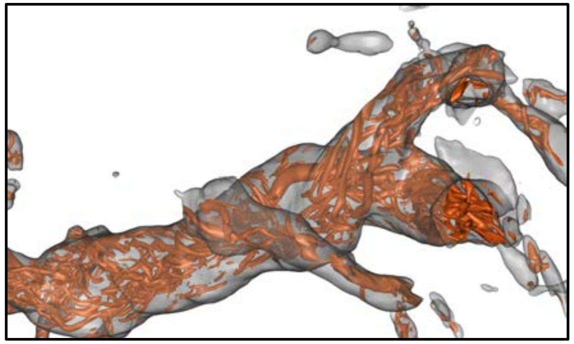
HOW TO USE UNCONVENTIONAL NUMERICS TO UNDERSTAND AND MODEL EULERIAN AND LAGRANGIAN TURBULENCE

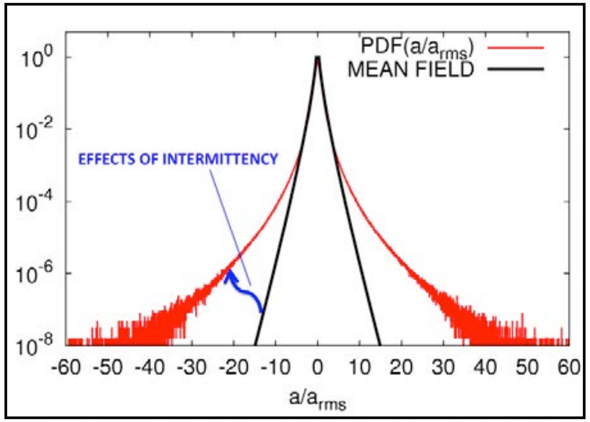
$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v}\cdot\partial)\mathbf{v} = -\partial P + \nu\Delta\mathbf{v} + \mathbf{F} \\ \partial\cdot\mathbf{v} = 0 \\ + \textit{Boundary Conditions} \end{cases}$$

Prob. 1: STRONGLY OUT-OF-EQUILIBRIUM

Prob. 2: STRONGLY NON-GAUSSIAN STATISTICS

Prob. 3: ~ 'INIFINITE' NUMBER OF DEGREES-OF-FREEDOM





Q1: CAN WE DISSECT (AND RECONSTRUCT) NS EQUATIONS TO EXTRACT INTERESTING INFORMATION FROM ITS ELEMENTARY CONSTITUENTS?

Q2: CAN WE UNDERSTAND THE ORIGIN OF THE STRONG FLUCTUATIONS EMPIRICALLY OBSERVED IN THE ENERGY TRANSFER RATE?

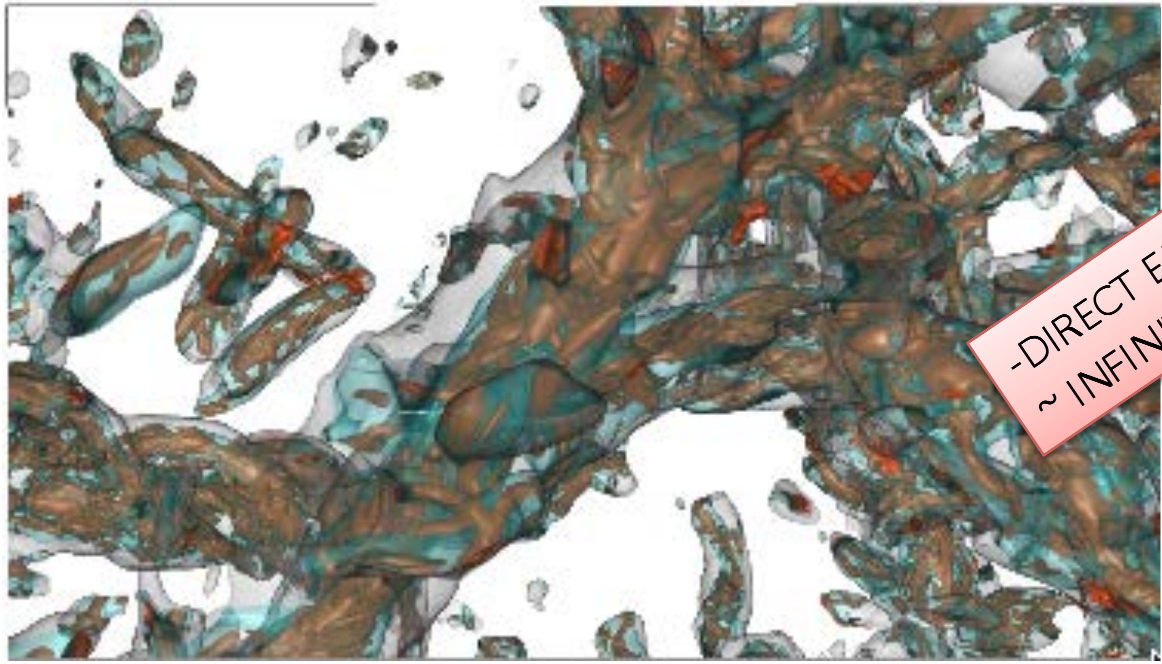
HOW TO USE UNCONVENTIONAL NUMERICS TO UNDERSTAND AND MODEL EULERIAN AND LAGRANGIAN TURBULENCE

$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v}\cdot\partial)\mathbf{v} = -\partial P + \nu\Delta\mathbf{v} + \mathbf{F} \\ \partial\cdot\mathbf{v} = 0 \\ + \; Boundary\ Conditions \end{cases}$$

Q: CAN WE DISSECT (AND RECONSTRUCT) NS EQUATIONS TO EXTRACT INTERESTING INFORMATION FROM ITS ELEMENTARY CONSTITUENTS?

1. NAVIER-STOKES 3D

$$\begin{cases} Re \to \infty \\ \nu \to 0 \end{cases}$$



-DIRECT ENERGY CASCADE
~ INFINITE $\#_{dof}$

$$\#_{dof} \propto Re^{9/4}$$

NO FLUCTUATION-DISSIPATION