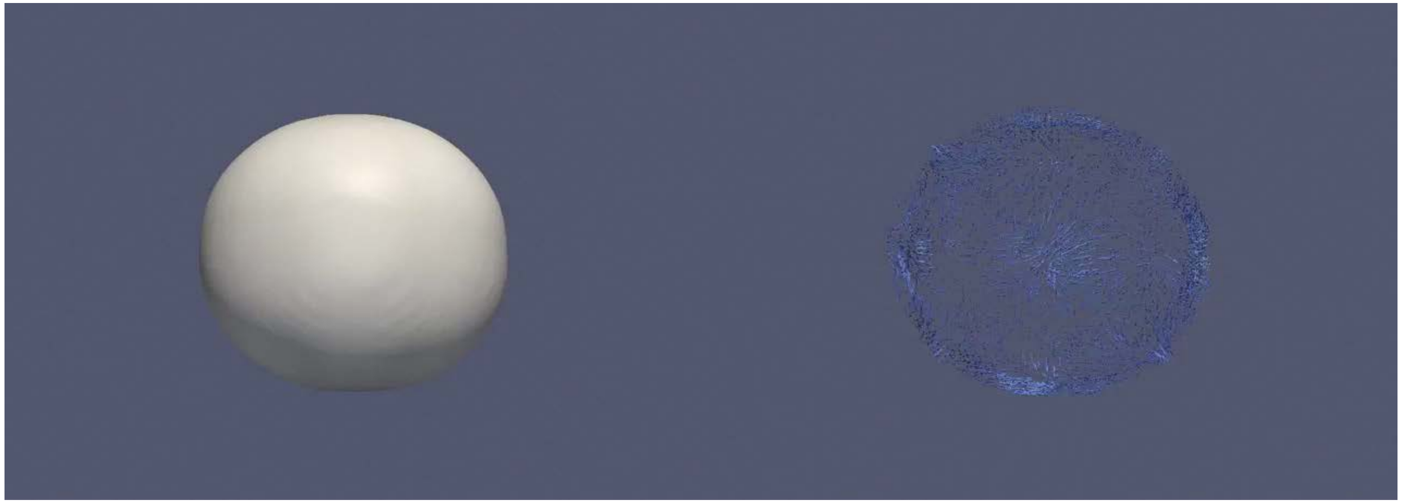
Luca Biferale, Charles Meneveau & Roberto Verzicco

Deformation statistics of small ellipsoidal drops in isotropic turbulence

VORTICAL STRUCTURES AND WALL TURBULENCE
Paolo Orlandi: A vortical and turbulent life
19th-20th September 2014



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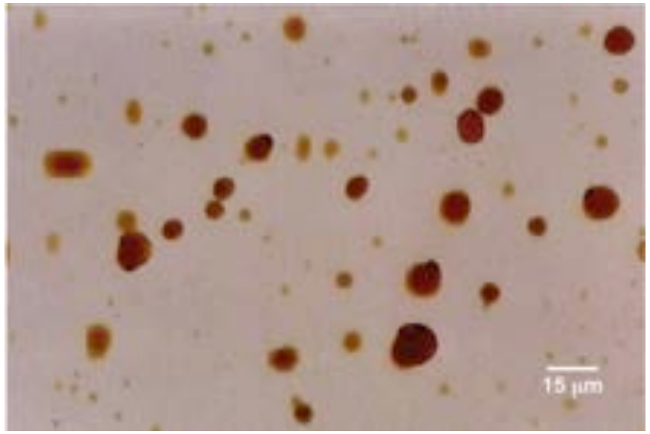
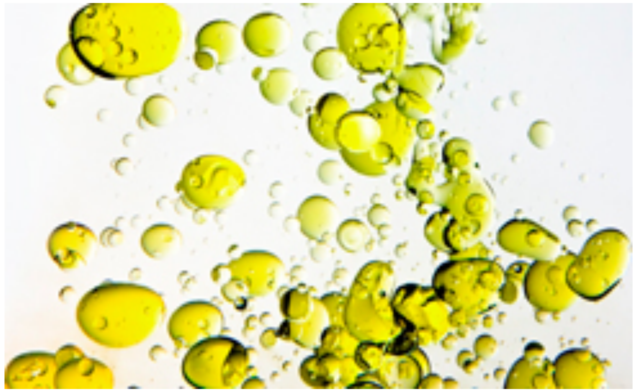
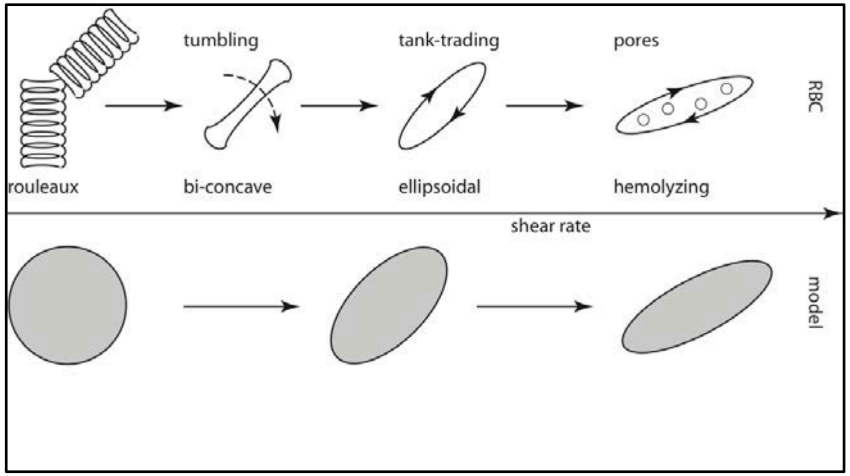




Emulsification and blending in Chemical and Food industry



Biofluids (suspensions and blood)

Environmental problem (remediation strategies for oil spill)

BACKGROUND

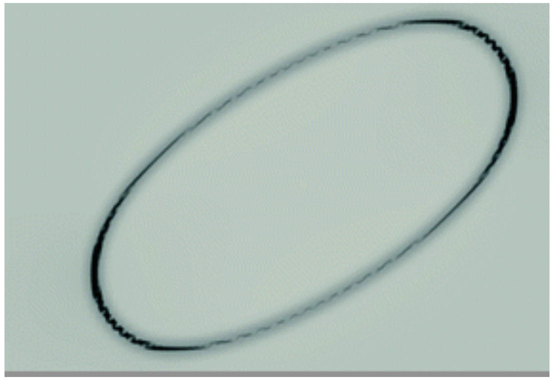
The sub-Kolmogorov size of the droplets implies that:

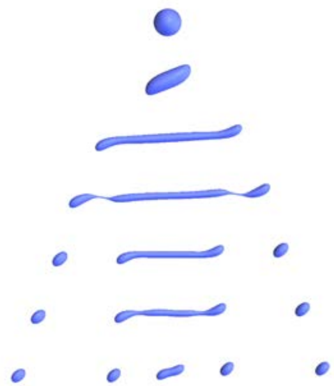
only viscous drag induced by the shear can distort the droplet shape (no inertial forces)

the distortion is resisted by the surface tension that tends to restore the spherical shape

Initial study by Taylor (1932) with drops in a laminar flow

Frijters et al. (2012)





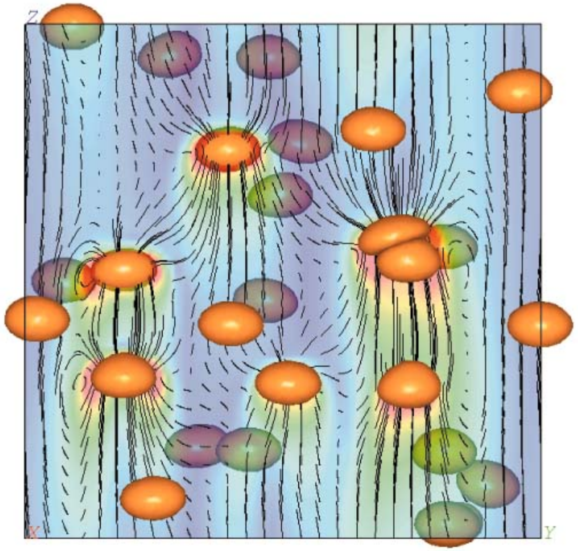
Komrakova et al. (2012)

Equilibrium configuration or droplet break-up

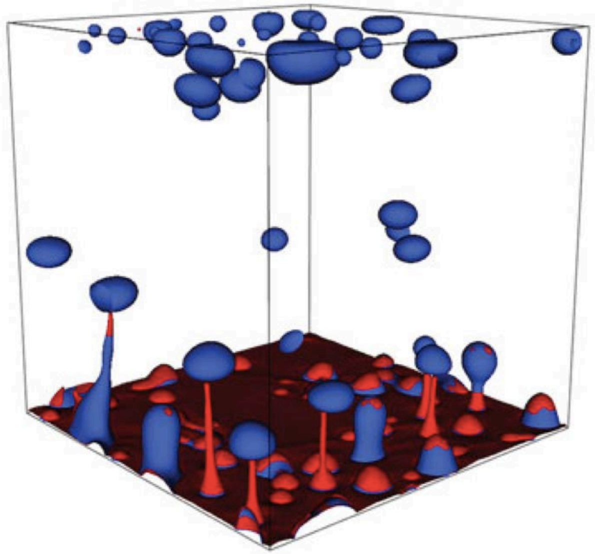
See Kolmogorov (1949) and Hinze (1955) or Lasheras et al. (2002) for larger drops with inertia forces

BACKGROUND

Detailed simulations resort to DNS of turbulence coupled with boundary integral methods (or similar) [Cristini et al. (2003), Terashima & Tryggvason (2009) and Can & Prosperetti (2012)]



http://www3.nd.edu/~gtryggva/MCFD/



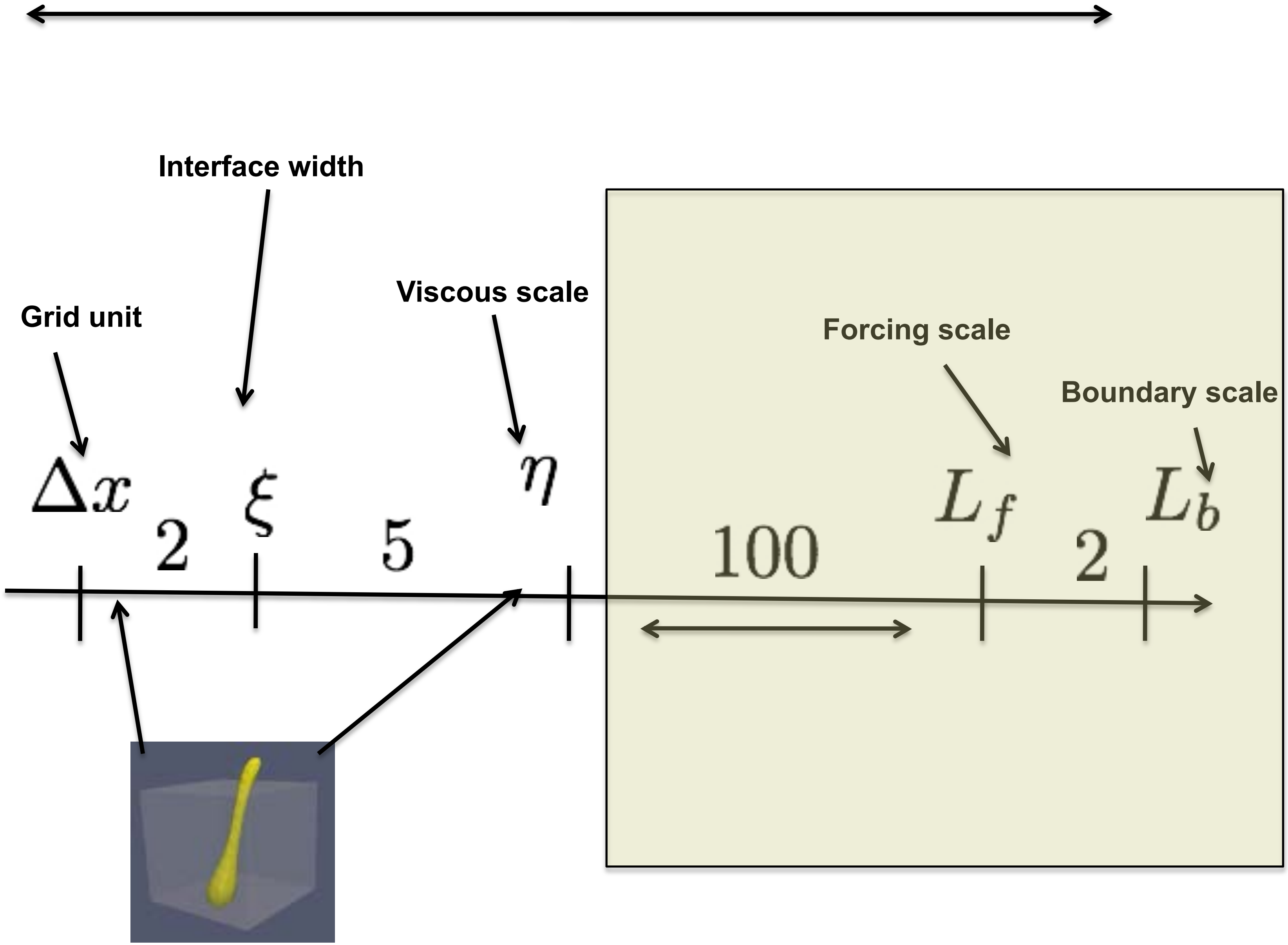
L.B. M. Sbragaglia, P. Perlekar and F. Toschi PRL (2012)

☺ Highly complex droplets shapes, instabilities, necks and satellite droplets are captured

☹ Turbulence Re_λ only moderately high and the number of droplets is few tens or hundreds

THE HIERARCHY BOTTLENECK

$$L_b/\Delta x \sim 1000 - 2000$$



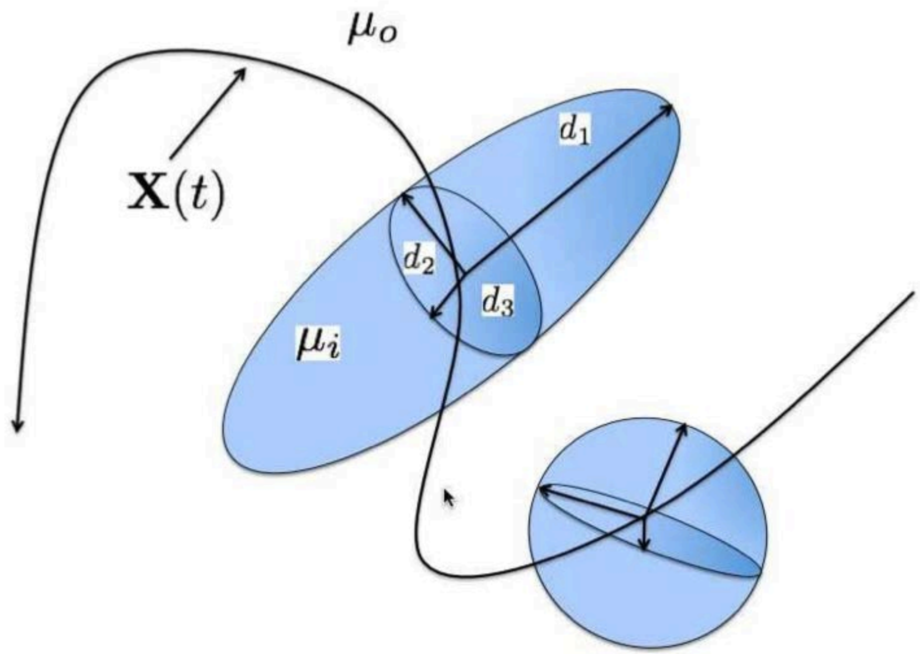
The Model

Pointwise Lagrangians with some physics built around

A fluid of viscosity μ_0 in turbulent motion

Droplets of an immiscible fluid of viscosity μ_1

Surface tension Λ at the interface



Initially spherical droplets can deform only to (triaxial) ellipsoids

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{F}, \qquad \nabla \cdot \boldsymbol{u} = 0.$$

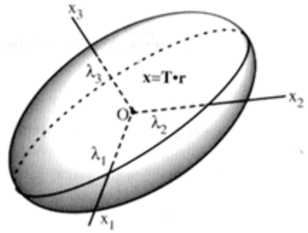
$$\dot{X} = \boldsymbol{u}(X(t), t),$$

Maffettone, Minale, J. Non-Newtonian Fluid Mech., 78 (1998) 227–241

ELLIPSOIDAL DROP DETERMINED BY A SYMMETRIC POSITIVE DEFINED SECOND RANK TENSOR

M’s eigenvalues: squared semiaxis of the ellipsoid

$$G_t = \left\langle \left(\frac{\partial u_1}{\partial x_1} \right)^2 \right\rangle^{1/2}.$$



$$Ca = \frac{\mu_o R G_t}{\Lambda} = \tau G_t.$$

$$\frac{dM_{ij}}{dt} = \Omega_{ik} M_{kj} - M_{ik} \Omega_{kj} + f_2 (S_{ik} M_{kj} + M_{ik} S_{kj}) - \frac{f_1}{\tau_c} (M_{ij} - g(II_M, III_M) \delta_{ij}),$$

$$\frac{\mathrm{d}M_{ij}}{\mathrm{d}t'} = \left[f_2 (S'_{ik} M_{kj} + M_{ik} S'_{kj}) + \Omega'_{ik} M_{kj} - M_{ik} \Omega'_{kj} \right] - \frac{f_1}{Ca} \left(M_{ij} - 3 \frac{\mathrm{III}_M}{\mathrm{II}_M} \delta_{ij} \right).$$

$$\Omega_{ij} = \partial_i v_j - \partial_j v_i \qquad S_{ij} = \partial_i v_j + \partial_j v_i$$

$$M_{ij} = I_{ij} + Ca M'_{ij}$$

C.E. Chaffey, H. Brenner,
J. Colloid Interface Sci. 24(1967) 258–269.

$$f_1 = \frac{40(\lambda + 1)}{(2\lambda + 3)(19\lambda + 16)}, \qquad \lambda = \mu_1 / \mu_2$$

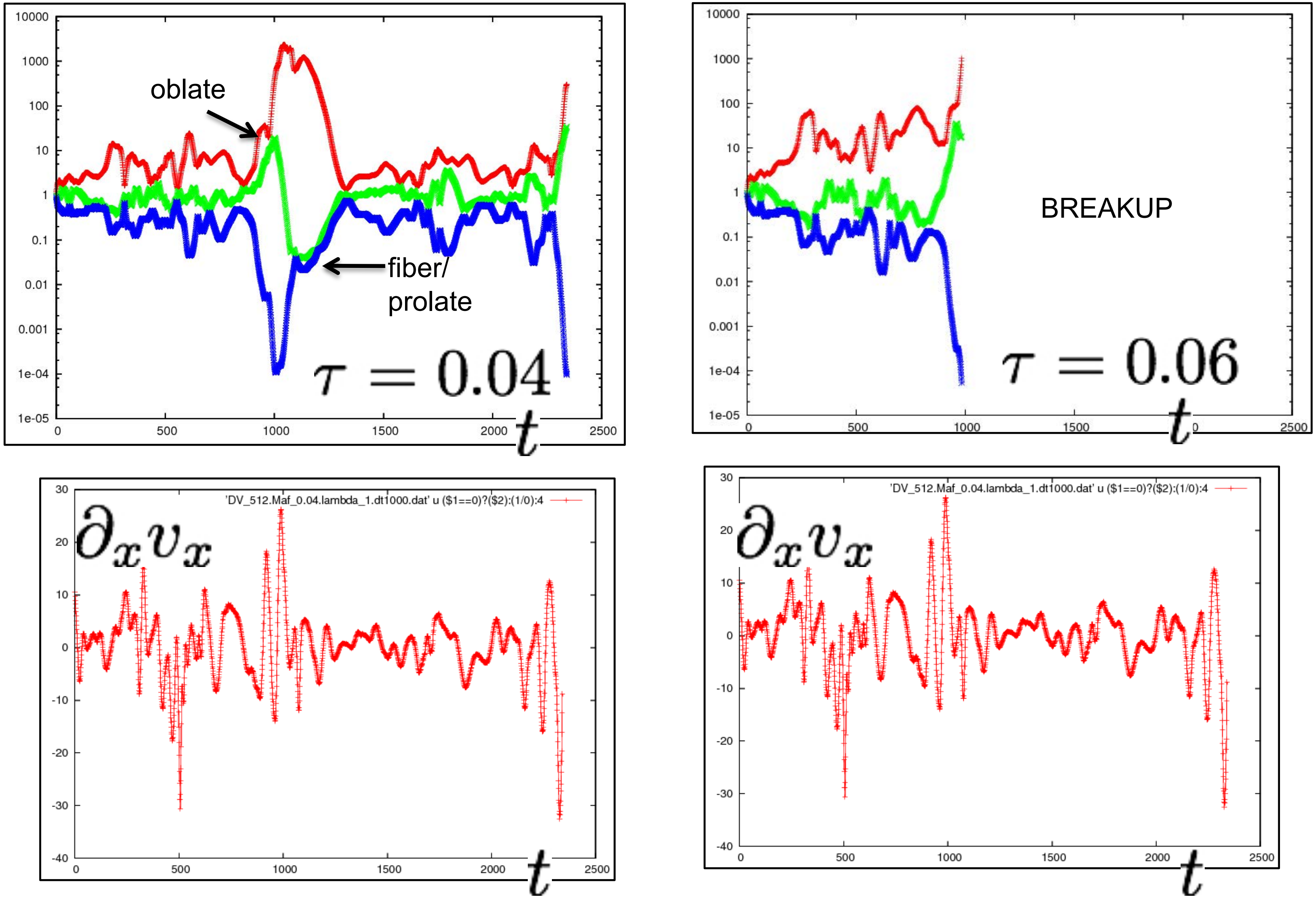
$$f_2 = \frac{5}{2\lambda + 3}.$$

G.I. Taylor,
Proc. R. Soc. A 138 (1932) 41–48.

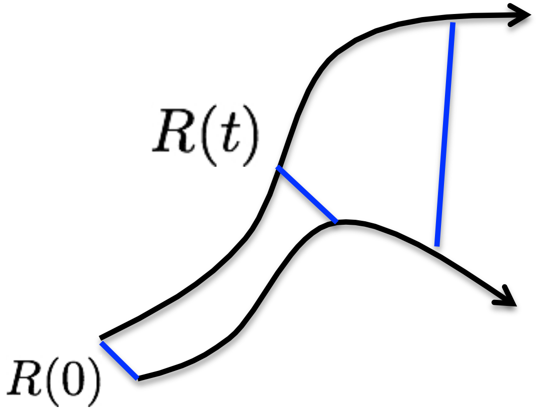
	N	Re_λ	η	δx	ε	ν	τ_η	t_{dump}	δt	T_L	G_t
Run I	512	185	0.01	0.012	0.9	0.002	0.047	0.004	0.0004	2.2	5.48
Run II	2048	400	0.0026	0.003	0.88	0.00035	0.02	0.00115	0.000115	2.2	11.4

TABLE 1. Eulerian parameters for the two sets of data from the DNS of homogeneous and isotropic turbulence. Here N is the number of grid points in each spatial direction; Re_λ is the Taylor-scale Reynolds number; η is the Kolmogorov dissipative scale; $\delta x = \mathcal{L}/N$ is the grid spacing, with $\mathcal{L} = 2\pi$ denoting the physical size of the numerical domain; $\tau_\eta = (\nu/\varepsilon)^{1/2}$ is the Kolmogorov dissipative timescale; ε is the average rate of energy injection; $\nu = \mu_0/\rho$ is the kinematic viscosity; t_{dump} is the time interval between two successive data recordings along particle trajectories; δt is the time step of the model integration; $T_L = L/U_0$ is the eddy turnover time at the integral scale $L = \pi$, U_0 is the typical large-scale root-mean-square velocity and G_t is the reference inverse turbulent timescale. Averages are performed over two large eddy turnover times.

$$d_1^2, d_2^2, d_3^2 \qquad Re_\lambda \sim 300$$



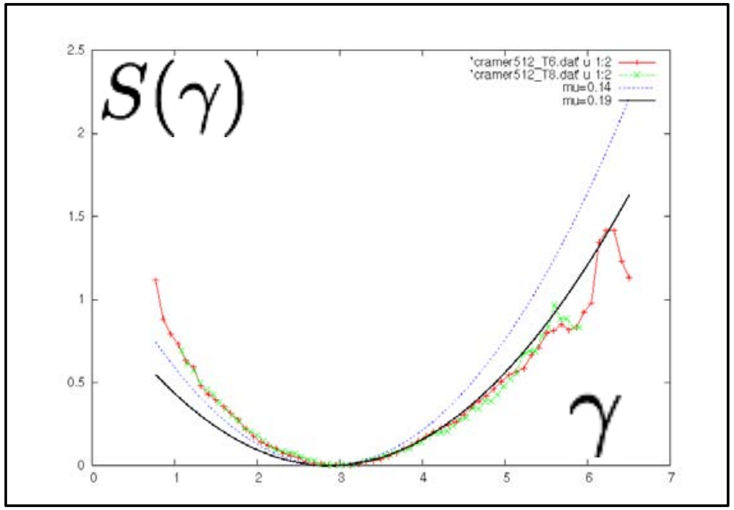
$$\frac{\mathrm{d}R_i}{\mathrm{d}t} = A_{ik}R_k$$



$$\frac{R(t)}{R(0)} \sim \exp^{\int_0^t d\tau \partial_x u(\tau)} = \exp^{t(\frac{1}{t}\int_0^t d\tau \partial_x u(\tau))} = \exp^{t\gamma(t)}$$

$$\gamma(t) = \frac{1}{t}\log\left(\frac{|\boldsymbol{R}(t)|}{|\boldsymbol{R}(0)|}\right),$$

CRAMER FUNCTION



$$P(\gamma, t) \sim exp(-tS(\gamma))$$

$$S(\gamma) = (\gamma - \lambda_L)^2/(2\sigma),$$

DROPLET DEFORMATION/STRETCHING STATISTICS

$$\frac{dM_{ij}}{dt} = \boxed{\partial_i v_k M_{ik} + \partial_j v_k M_{ki}} - \frac{f_1}{\tau}(M_{ij} - g(M)I) \qquad \lambda = 1$$

Finite Time Lyapunov Exponent: local stretching rate

$$\begin{cases} Tr(M(t)) \sim exp(2\gamma t) \\ P(\gamma, t) \sim exp(-tS(\gamma)) \end{cases}$$

$$\langle (Tr(M(t)))^q \rangle \sim \int d\gamma \, exp(t(2q\gamma - S(\gamma))) \sim exp(tL(2q))$$

$$L(2q) = \max_{\gamma}(2q\gamma - S(\gamma))$$

$$\langle (Tr(M(t)))^q \rangle \sim exp(-t\frac{f_1}{\tau})$$

RELAXATION

$$\frac{dM_{ij}}{dt} = \partial_i v_k M_{ik} + \partial_j v_k M_{ki} - \frac{f_1}{\tau}(M_{ij} - g(M)I)$$

Finite Time Lyapunov Exponent: local stretching rate

$$\begin{cases} Tr(M(t)) \sim exp(2\gamma t) \\ P(\gamma, t) \sim exp(-tS(\gamma)) \end{cases}$$

$$\langle (Tr(M(t)))^q \rangle \sim \int d\gamma\, exp(t(2q\gamma - S(\gamma))) \sim exp(tL(2q))$$

$$L(2q) = \max_{\gamma}(2q\gamma - S(\gamma))$$

$$\langle (Tr(M(t)))^q \rangle \sim exp(-t\frac{f_1}{\tau})$$

BALANCE

$$\langle (Tr(M(t)))^q \rangle \sim exp(-t(L(2q) - \frac{qf_1}{\tau}))$$

STATIONARITY: POWER LAW TAIL

$$\begin{cases} P(Tr(M)) \sim Tr(M)^{-(\tilde{q}+1)} \\ L(2\tilde{q}) = \dfrac{\tilde{q}f_1}{\tau} \end{cases}$$

COIL-STRETCH TRANSITION (DROPLET BREAK-UP)

$$L(2q) \sim 2\bar{\gamma}q \qquad 2\bar{\gamma} = f_1/\tau_c$$

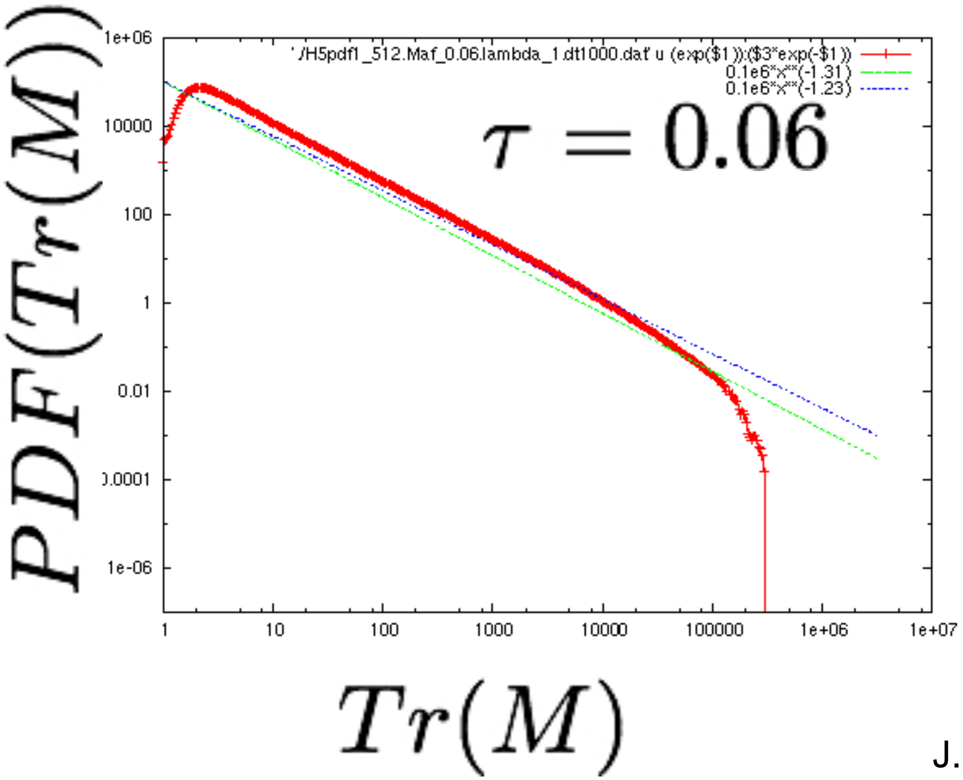
VOLUME 91, NUMBER 3 PHYSICAL REVIEW LETTERS week ending 18 JULY 2003

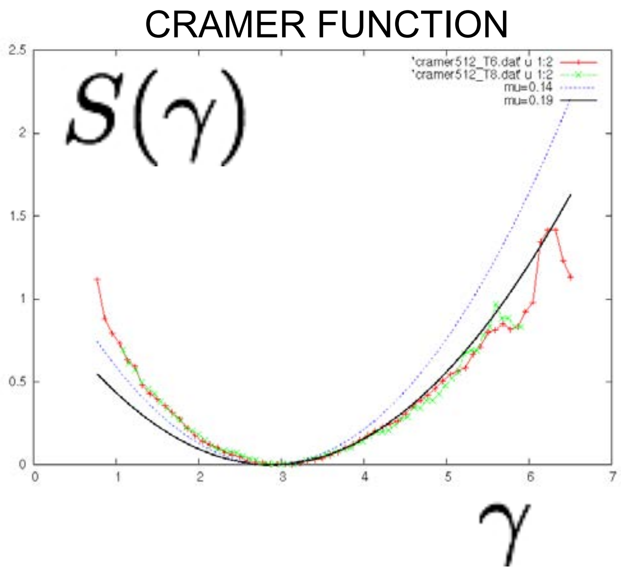
Two-Dimensional Turbulence of Dilute Polymer Solutions

Guido Boffetta,[1] Antonio Celani,[2] and Stefano Musacchio[1]

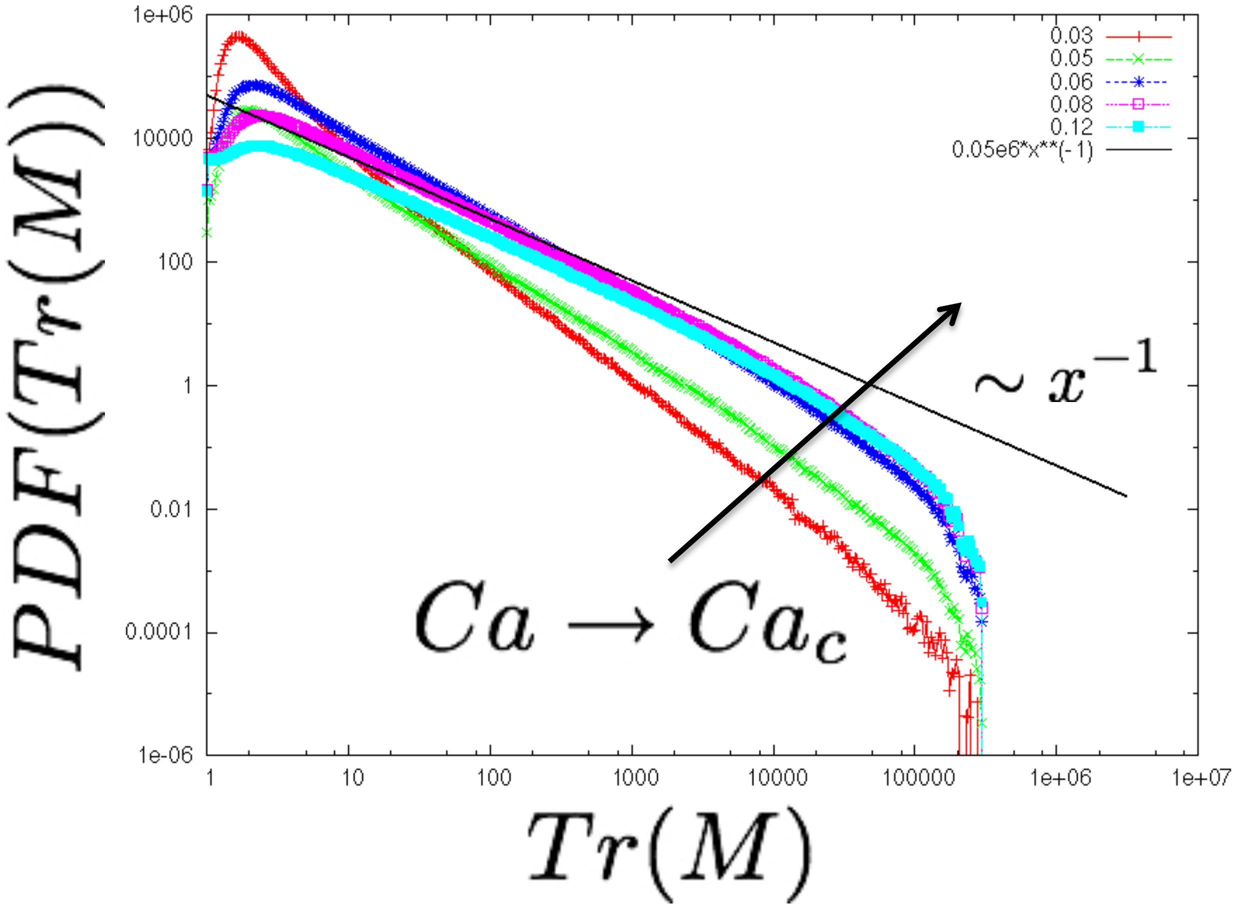
E. Balkovsky, A. Fouxon, and V. Lebedev, Phys. Rev. Lett. **84,** 4765 (2000).

	$\tau = 0.03$	$\tau = 0.05$	$\tau = 0.06$	$\tau = 0.08$
$\tilde{q}\|\mu = 0.14/\tau_\eta$	1.6	0.57	0.31	0
$\tilde{q}\|\mu = 0.19/\tau_\eta$	1.2	0.41	0.23	0





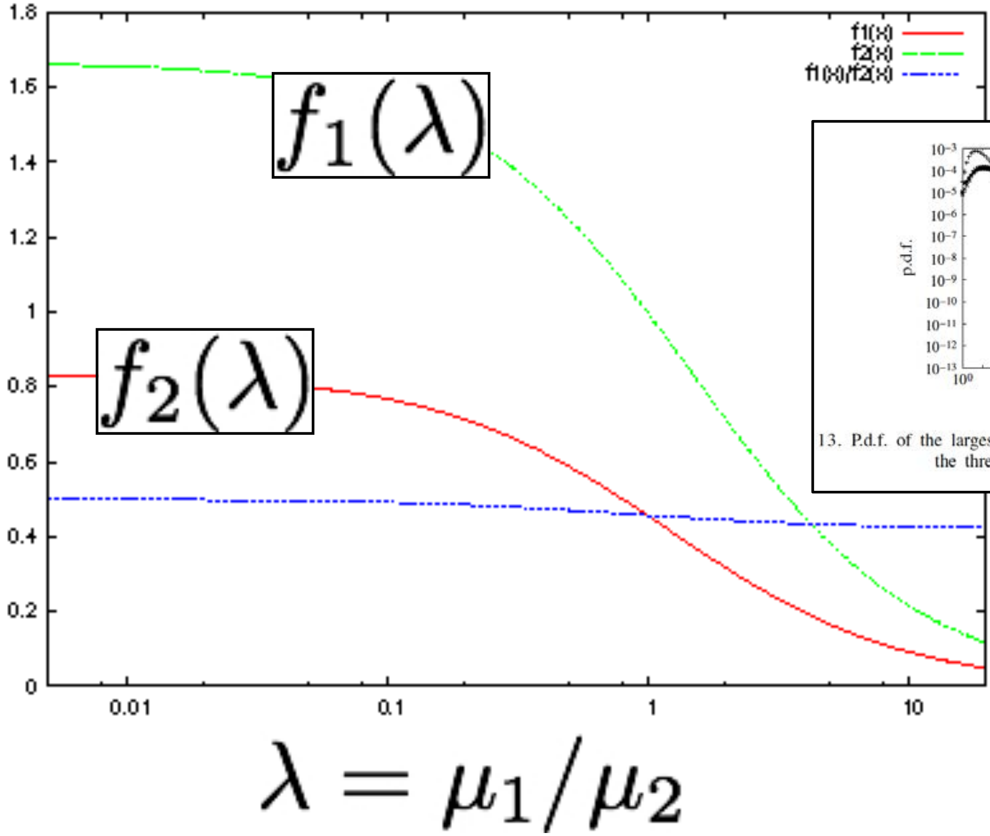
J. Bec, L. B., G. Boffetta, M. Cencini, A. Lanotte, S. Musacchio, & F. Toschi Phys. Fluids 18, 091702, 2006.

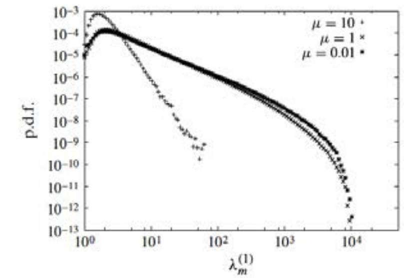


$PDF(Tr(M))$
1e+06
10000
100
1
0.01
0.0001
1e-06
1
10
100
1000
10000
100000
1e+06
1e+07
$Tr(M)$
0.03
0.05
0.06
0.08
0.12
0.05e6*x**(-1)
$\sim x^{-1}$
$Ca \to Ca_c$

For μ≠1 (different viscosities) the importance of strain and rotation is unequal and the analogy with polymers fails (**the prediction underestimates Ca_c**)

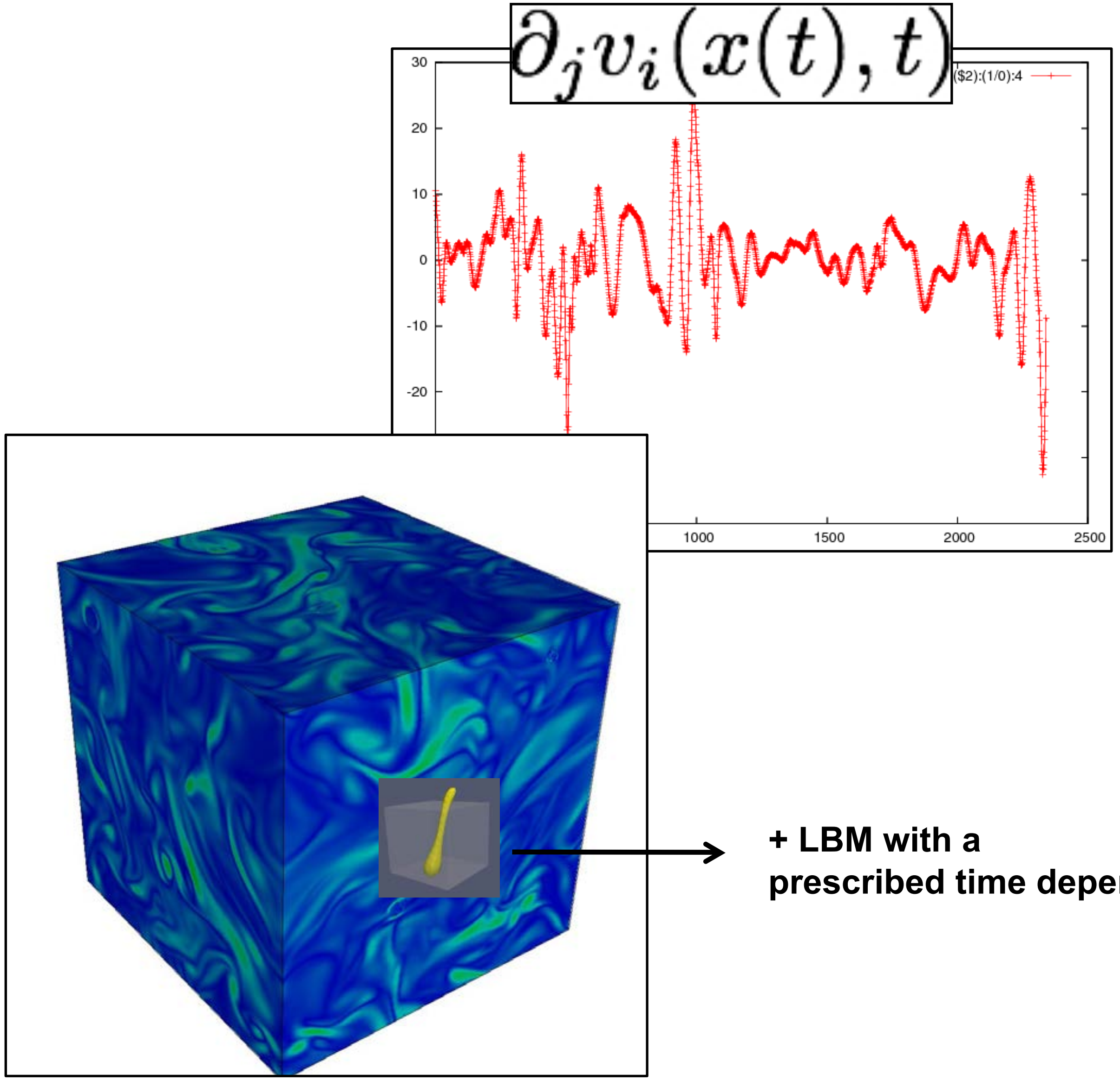
$$\frac{dM_{ij}}{dt} = \Omega_{ik}M_{kj} - M_{ik}\Omega_{kj} + f_2(S_{ik}M_{kj} + M_{ik}S_{kj}) - \frac{f_1}{\tau_c}(M_{ij} - g(II_M, III_M)\delta_{ij}),$$





13. P.d.f. of the largest eigenvalue of the morphology tensor for $Ca = 0.32$ of the three viscous ratios, $\mu = 0.01$, 1, 10.

MULTI_SCALE COUPLING: PSEUDOSPECTRAL AT HIGH REYNOLDS TO EXTRACT



**+ LBM with a
prescribed time dependent shear?**

CONCLUSIONS

SMALL DROPLETS/BUBBLES DYNAMICS: BREAK-UP TRANSITION + PDF OF DEFORMATION -> Large Deviation Theory.

PROBLEMS WHEN ROTATION RATE IS EFFICIENT?

COARSE GRAINED DYNAMICS FOR INTERIAL RANGE BUBBLES/DROPLETS ?

ESTIMATE APRIORI FEEDBACK ON THE FLOW BY DROPLET DEFORMATION -> BUBBLE-DRAG REDUCTION?

ESTIMATE EFFECTS OF INERTIA: FOLLOWING TRAJECTORIES OF HEAVY/LIGHT OBJECTS

DROPLET BREAK-UP IN NON-HOMOGENEOUS TURBULENCE?

J. Fluid Mech. (2014), *vol.* 754, *pp.* 184–207. © Cambridge University Press 2014 184
doi:10.1017/jfm.2014.366

Deformation statistics of sub-Kolmogorov-scale ellipsoidal neutrally buoyant drops in isotropic turbulence

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