Inverse and Direct cascades in turbulence, revisited.

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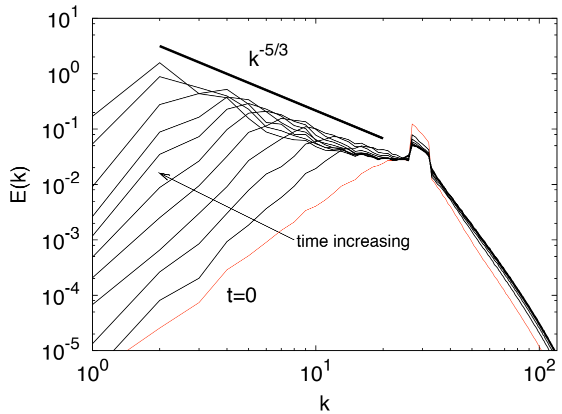

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Credits: **S. Musacchio** (CNRS-France); **F. Toschi** (University of Eindhoven, The Netherlands); **E. Titi** (Weizmann Institute of Science, Israel), **F. Bonaccorso** and **G. Sahoo** (U. Tor Vergata, Italy)

HOW TO USE UNCONVENTIONAL NUMERICS TO UNDERSTAND AND MODEL EULERIAN AND LAGRANGIAN TURBULENCE IN 2D, 3D (AND IN BETWEEN)

$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v} \cdot \partial)\mathbf{v} = -\partial P + \nu \Delta \mathbf{v} + \mathbf{F} \\ \partial \cdot \mathbf{v} = 0 \\ + \; Boundary \; Conditions \end{cases}$$

Q: CAN WE DISSECT (AND RECONSTRUCT) NS EQUATIONS TO EXTRACT INTERESTING INFORMATION FROM ITS ELEMENTARY CONSTITUENTS?

ON THE ROLE OF HELICITY IN 3D FORWARD/BACKWARD ENERGY CASCADES

A: Yes, we can! Let us look at the role played by helicity, H:

$$H = \int d^3x \, \omega \cdot \mathbf{v}$$

1) **We show that ALL flows in nature posses a class of nonlinear interactions characterized by a backward energy transfer (inverse energy cascade), triggered by the dynamics of Helicity, and that this happens even in fully isotropic, homogeneous 3D turbulence**

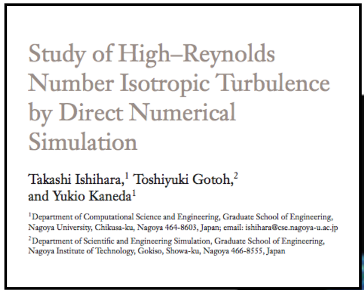
2) **Connections to small-scales intermittency**

3) **Connections to regularity of NS equations in 3D**

4) **Extensions to Magnetohydrodynamics**

5) **Other 'unique' numerical tools (Fractal Fourier Decimation)**

SMALL-SCALES INTERMITTENCY


Study of High–Reynolds Number Isotropic Turbulence by Direct Numerical Simulation

Takashi Ishihara,[1] Toshiyuki Gotoh,[2] and Yukio Kaneda[1]

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[2]Department of Scientific and Engineering Simulation, Graduate School of Engineering, Nagoya Institute of Technology, Gokiso, Showa-ku, Nagoya 466-8555, Japan

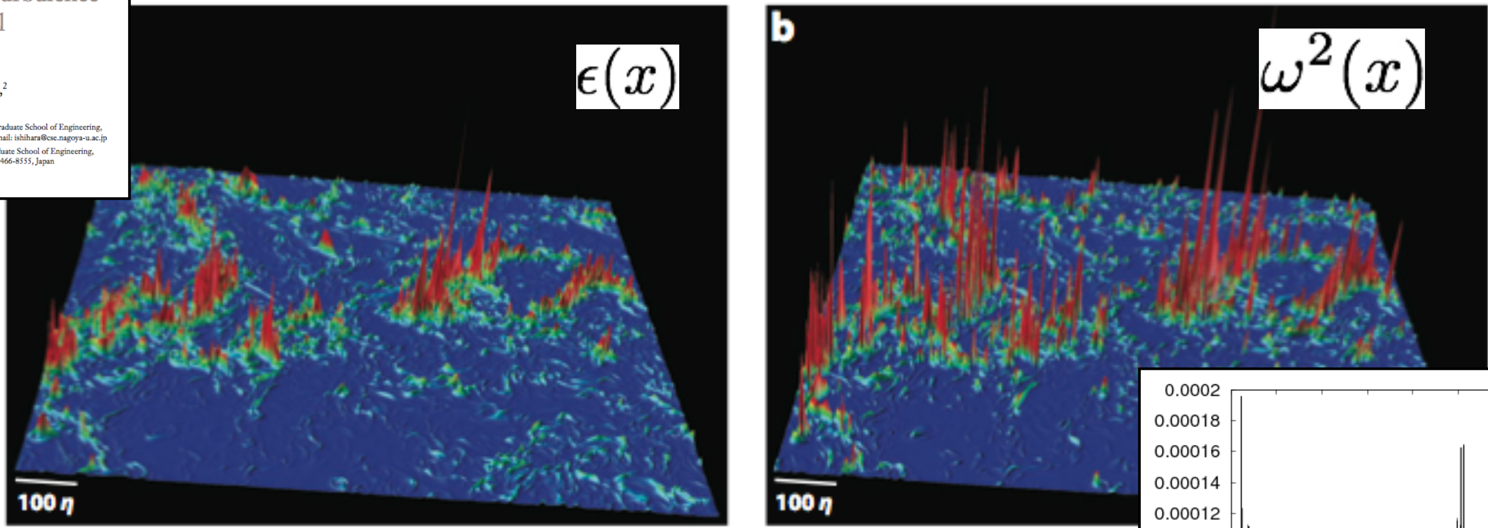
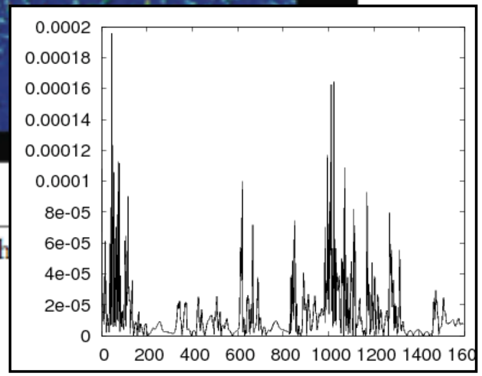
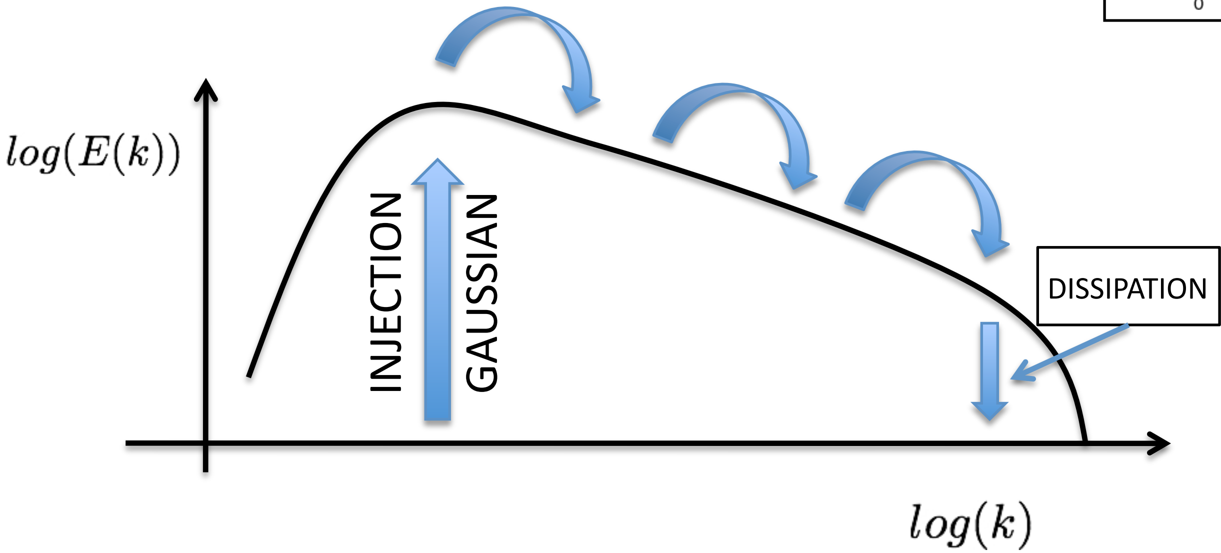


Figure 4

Snapshot of the intensity distributions of (*a*) the energy-dissipation rate $\tilde{\varepsilon} = \varepsilon/(2\nu)$ and (*b*) the enstroph
DNS-ES at $R_\lambda = 675$ in arbitrary units.

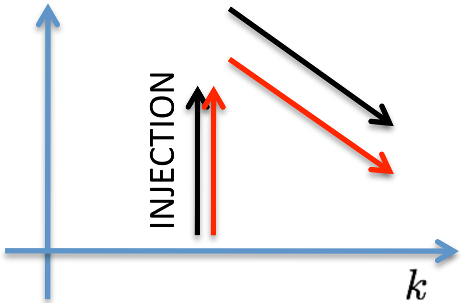




both Energy & Helicity forward cascades

$$E(k) \propto \epsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$$

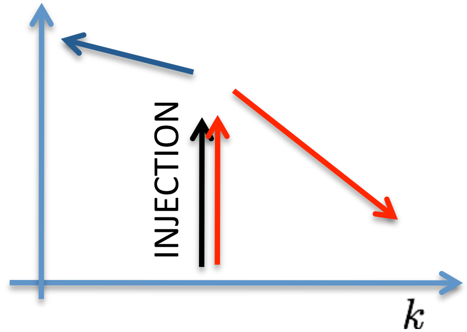
$$H(k) \propto \eta \epsilon^{-\frac{1}{3}} k^{-\frac{5}{3}}$$



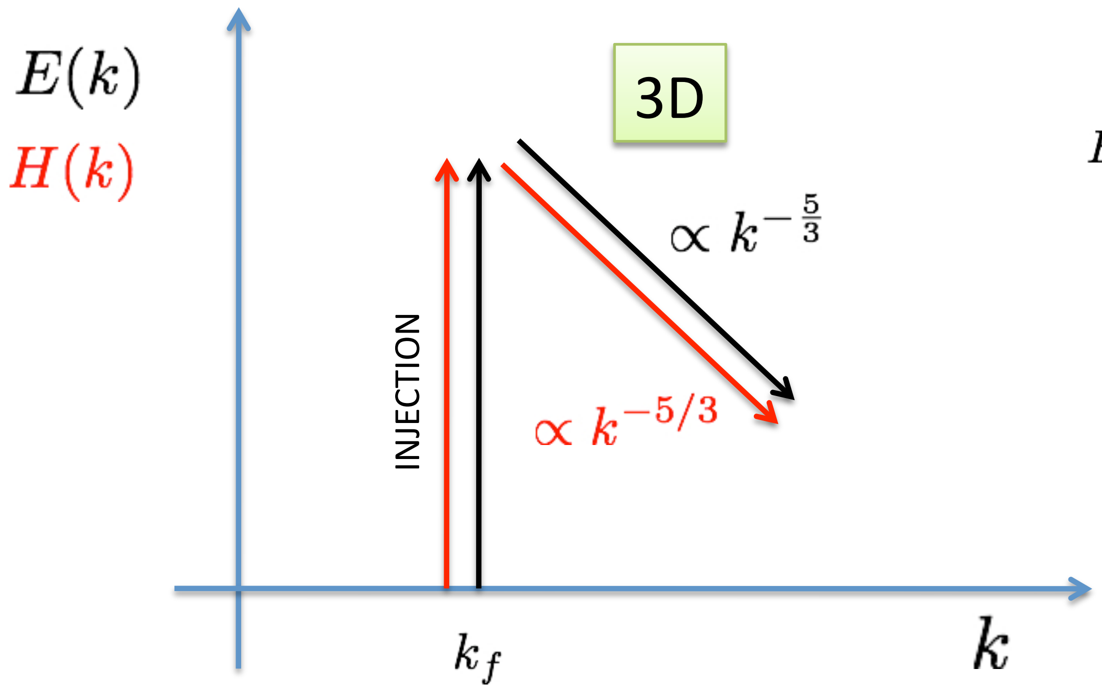
Helicity forward cascade
Energy inverse cascade

$$E(k) \propto \epsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$$

$$H(k) \propto \eta k^{-\frac{4}{3}}$$



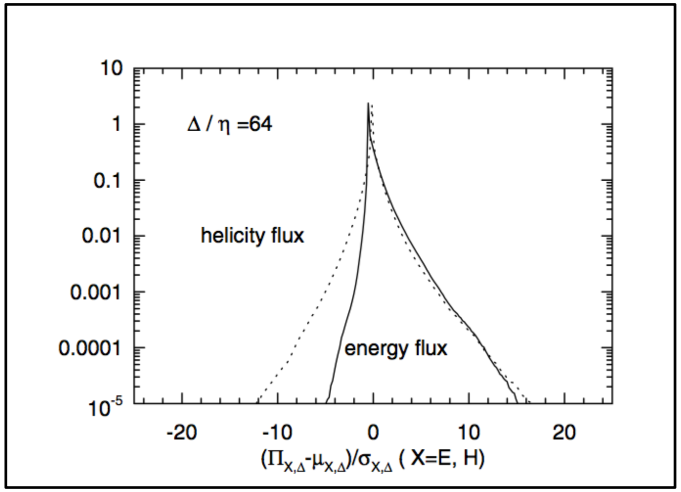
However, a problem arises with a pure helicity cascade: it appears difficult to inject helicity into the fluid without at the same time injecting some energy. Possibly this difficulty can be overcome, as for two-dimensional turbulence, by assuming that energy and helicity are fed into the fluid at a certain wavenumber k_i; helicity then cascades toward large wavenumbers according to (8) while energy cascades toward small wavenumbers (inverse cascade) according to the usual Kolmogoroff law. In the energy inverse cascade range,



2 invariants

$$E = \int d^3x\, \mathbf{v}\cdot\mathbf{v} \qquad H = \int d^3x\, \omega\cdot\mathbf{v}$$

$$H(k) \propto \eta\epsilon^{-\frac{1}{3}} k^{-\frac{5}{3}}$$
$$E(k) \propto \epsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$$



The joint cascade of energy and helicity in three-dimensional turbulence

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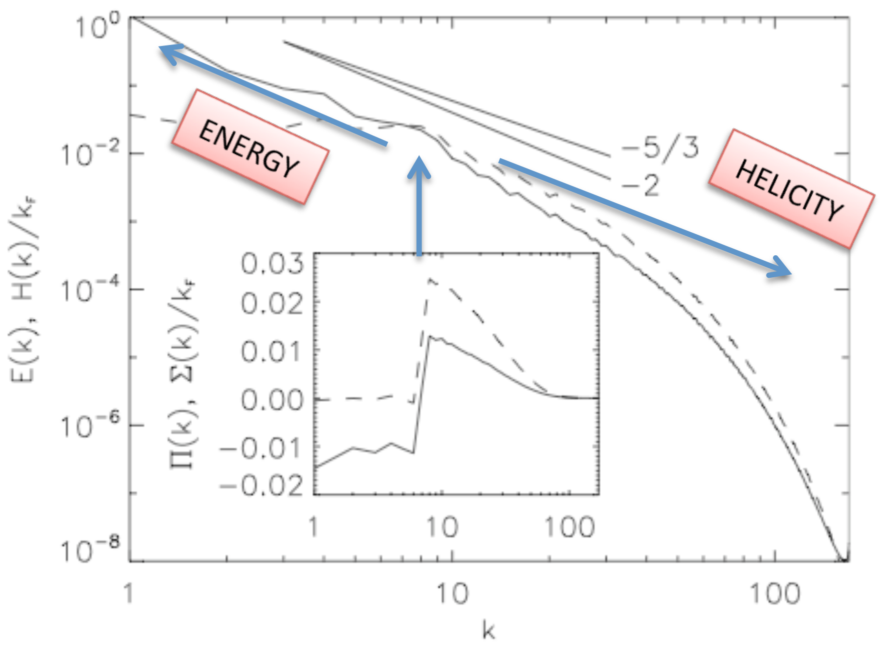
The role of helicity in three-dimensional turbulence is, in our opinion, still somewhat mysterious. In particular, it is still unclear how energy and helicity dynamics interact in detail. The role of helicity in geophysical flows has been considered[3]—without being fully resolved—while its appearance and influence in engineering applications is still largely unexplored. We hope that this work will be a helpful step in the direction of better understanding the subtle manifestations of helicity in three-dimensional turbulence.

see also:
J.C. Andre and M. Lesieur. Journ. Fluid Mech. **81,** 187 (1997)

SPLIT ENERGY-HELICITY CASCADES

P.D. Mininni and A. Pouquet. Phys. Rev. E **79,** 026304 (2009)



...un A3 with ...r. Different ...energy and ... an inverse

Dual local and non-local cascades in 3D turbulent Beltrami flows

E. Herbert, F. Daviaud, B. Dubrulle,[1] S. Nazarenko,[2] and A. Naso[3]

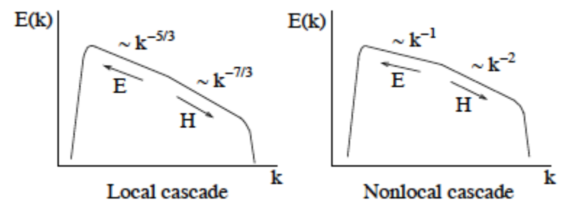


FIG. 1: Summary of the two different possibilities for dual helicity and energy cascades as a function of the wavenumber k in a Beltrami flow. Left : local case; right : non-local case.

ROTATION 3D -> 2D

HELICITY ?
-> forward cascade

•ANISOTROPY
•HELICITY ≠ 0

The nature of triad interactions in homogeneous turbulence

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(Received 24 July 1991; accepted 22 October 1991)

$$\boldsymbol{u}(\boldsymbol{k}) = u^{+}(\boldsymbol{k})\boldsymbol{h}^{+}(\boldsymbol{k}) + u^{-}(\boldsymbol{k})\boldsymbol{h}^{-}(\boldsymbol{k})$$

$$\boldsymbol{h}^{\pm} = \hat{\boldsymbol{\nu}} \times \hat{\boldsymbol{k}} \pm i\hat{\boldsymbol{\nu}}$$

$$\hat{\boldsymbol{\nu}} = \boldsymbol{z} \times \boldsymbol{k}/||\boldsymbol{z} \times \boldsymbol{k}||$$

$$i\boldsymbol{k} \times \boldsymbol{h}^{\pm} = \pm k\boldsymbol{h}^{\pm}$$

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^{+}(\boldsymbol{k})|^2 + |u^{-}(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^{+}(\boldsymbol{k})|^2 - |u^{-}(\boldsymbol{k})|^2). \end{cases}$$

$$u^{s_k}(\mathbf{k},t) \quad (s_k = \pm 1)$$

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_p p - s_q q) \times [u^{s_p}(\mathbf{p}) u^{s_q}(\mathbf{q})]^*. \quad (15)$$

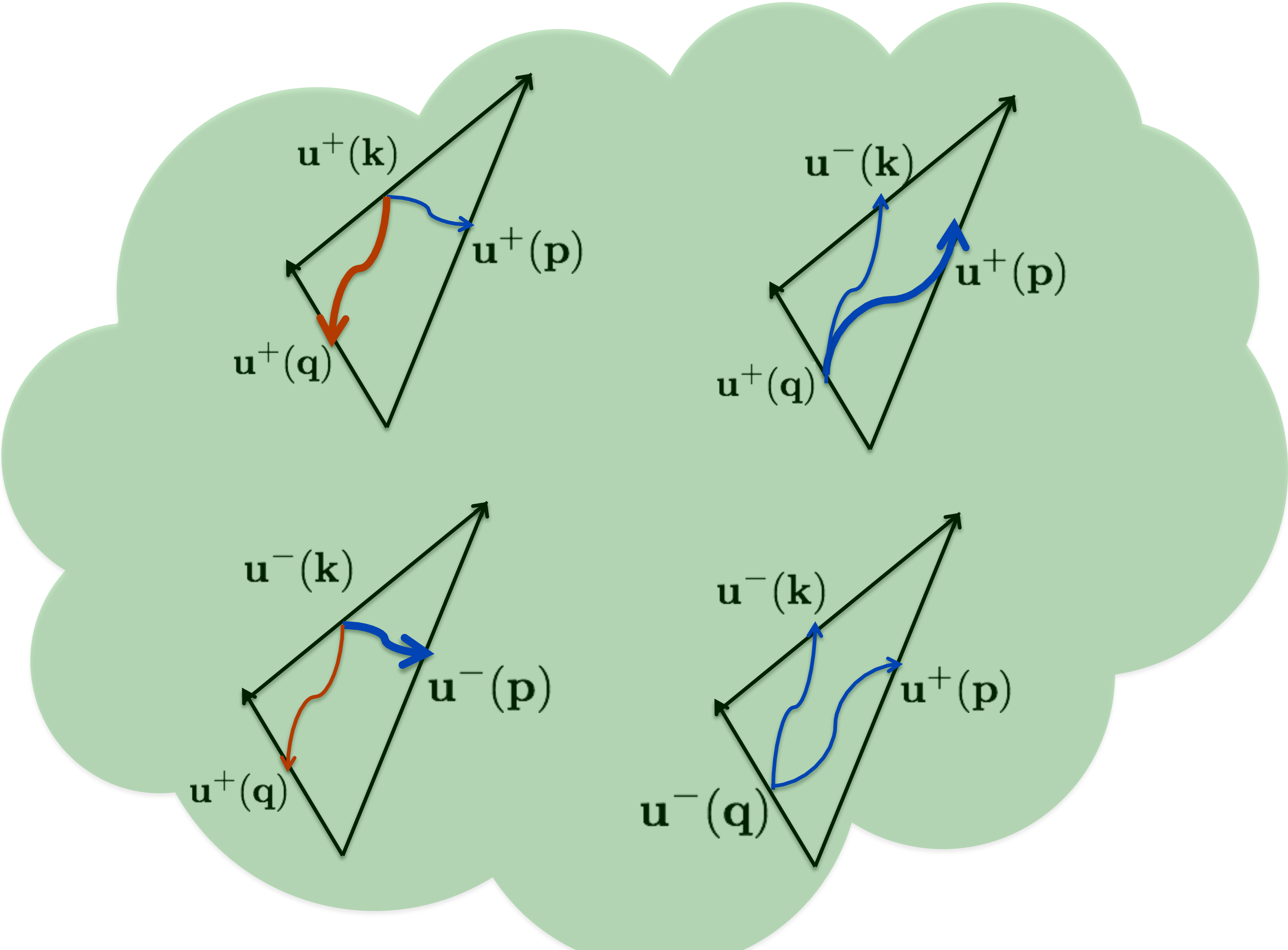
Eight different types of interaction between three modes $u^{s_k}(\mathbf{k})$, $u^{s_p}(\mathbf{p})$, and $u^{s_q}(\mathbf{q})$ with $|\mathbf{k}|<|\mathbf{p}|<|\mathbf{q}|$ are allowed according to the value of the triplet (s_k, s_p, s_q)

$$\dot{u}^{s_k} = r(s_p p - s_q q)\frac{s_k k + s_p p + s_q q}{p}(u^{s_p} u^{s_q})^*,$$

$$\dot{u}^{s_p} = r(s_q q - s_k k)\frac{s_k k + s_p p + s_q q}{p}(u^{s_q} u^{s_k})^*,$$

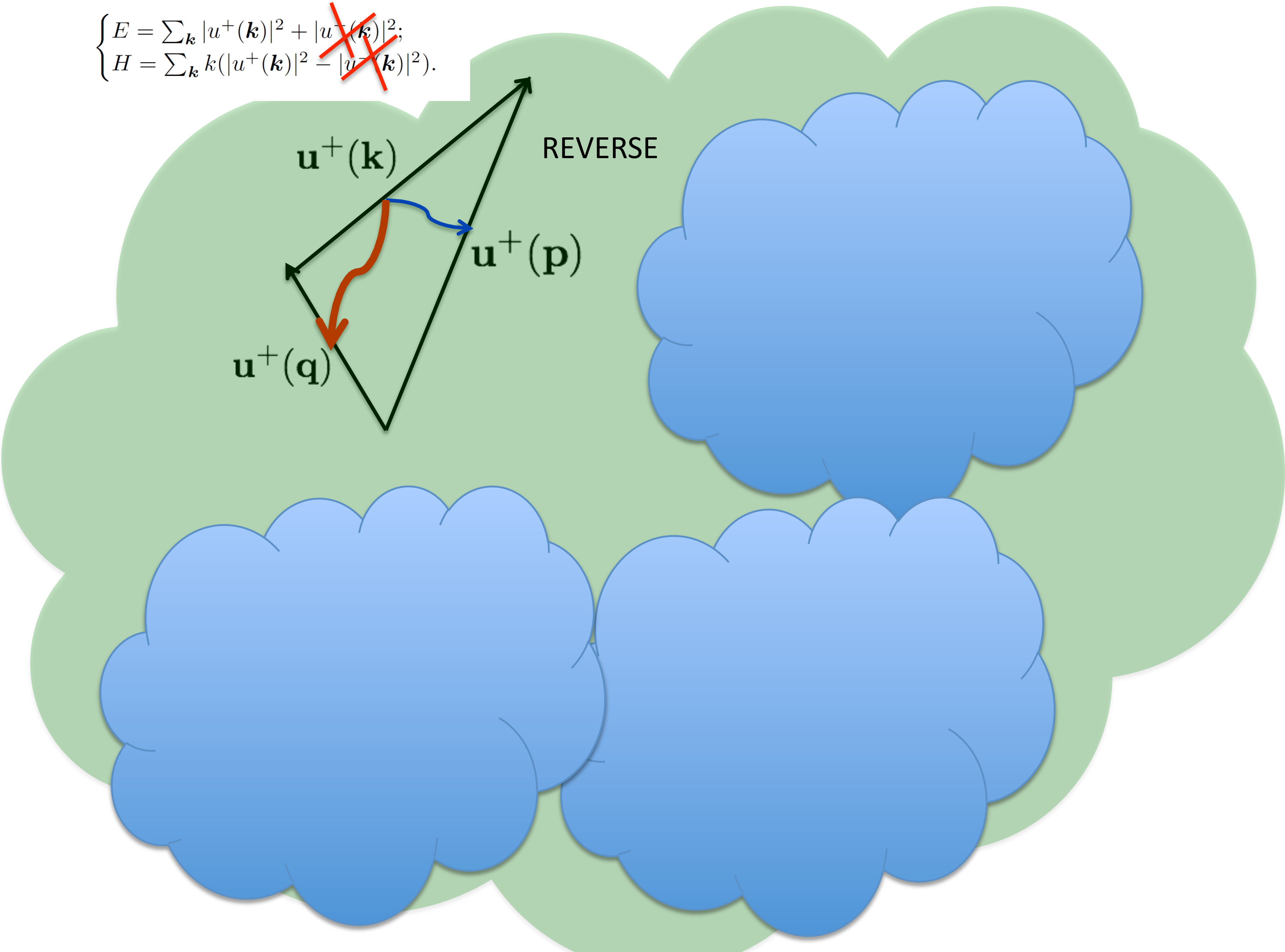
$$\dot{u}^{s_q} = r(s_k k - s_p p)\frac{s_k k + s_p p + s_q q}{p}(u^{s_k} u^{s_p})^*.$$

TRIADIC INTERACTION IN THE WHOLE NAVIER_STOKES EQS

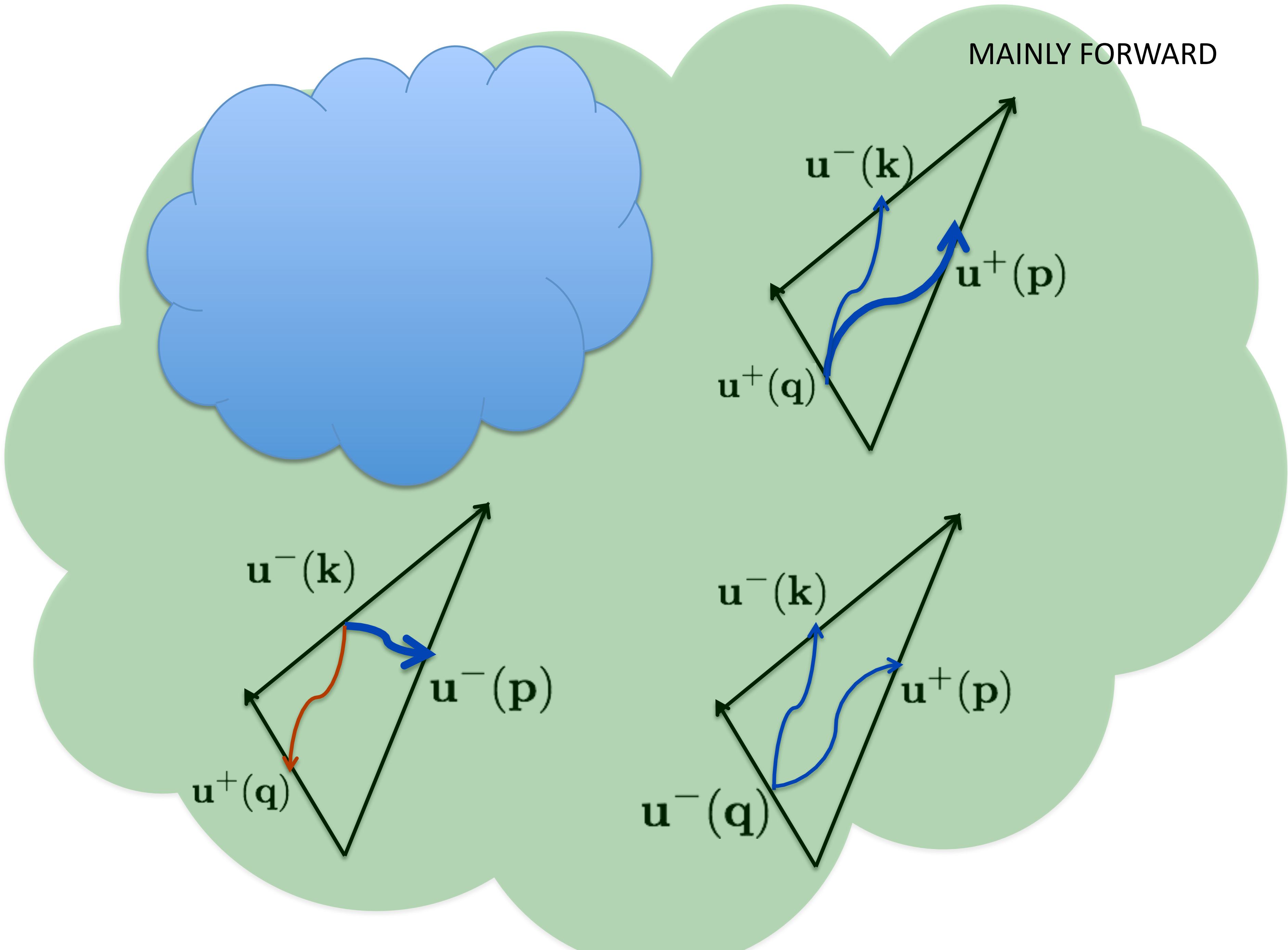


TRIADIC INTERACTION IN DECIMATED NAVIER_STOKES EQS

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$

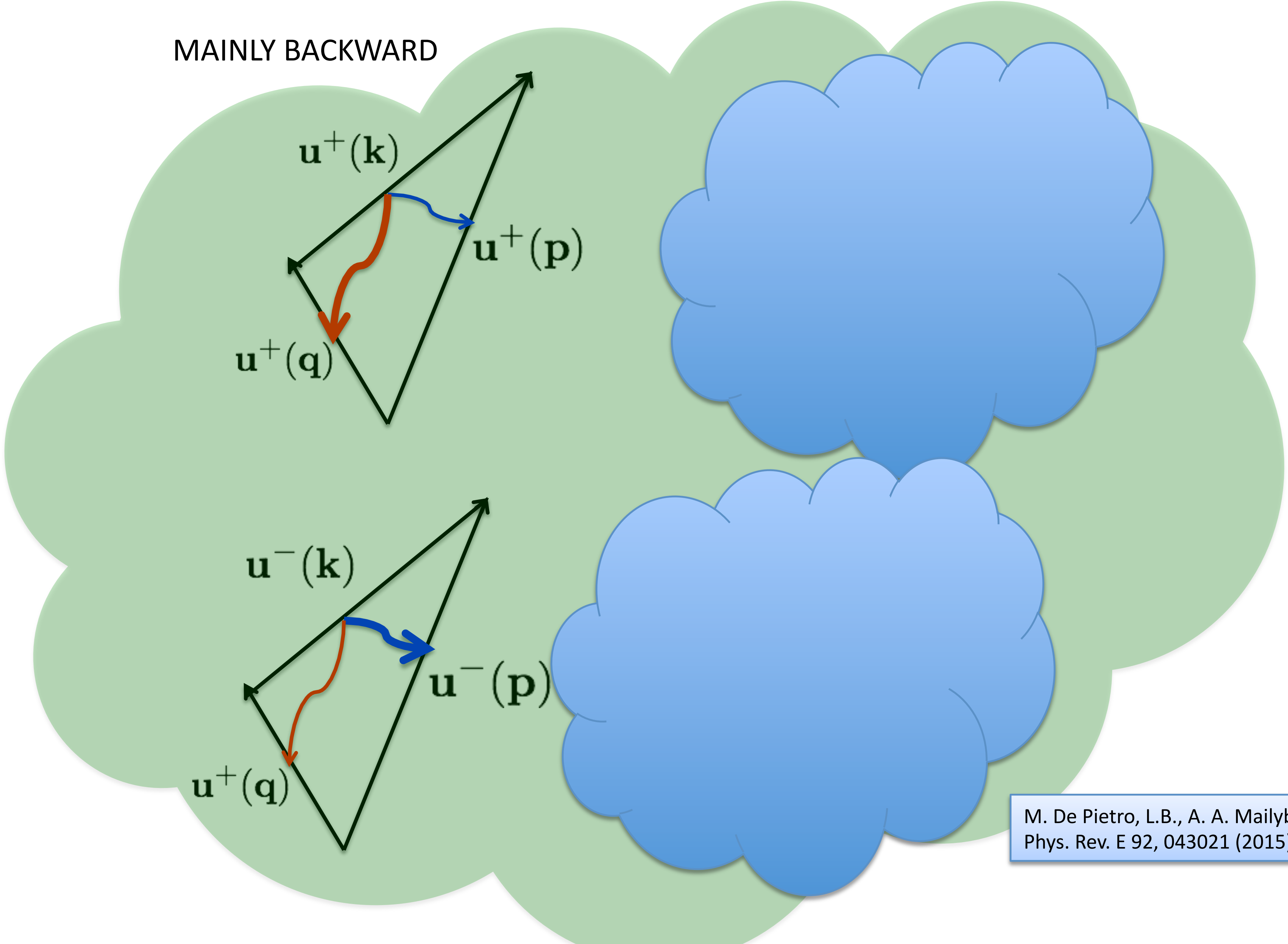


TRIADIC INTERACTION IN DECIMATED NAVIER_STOKES EQS



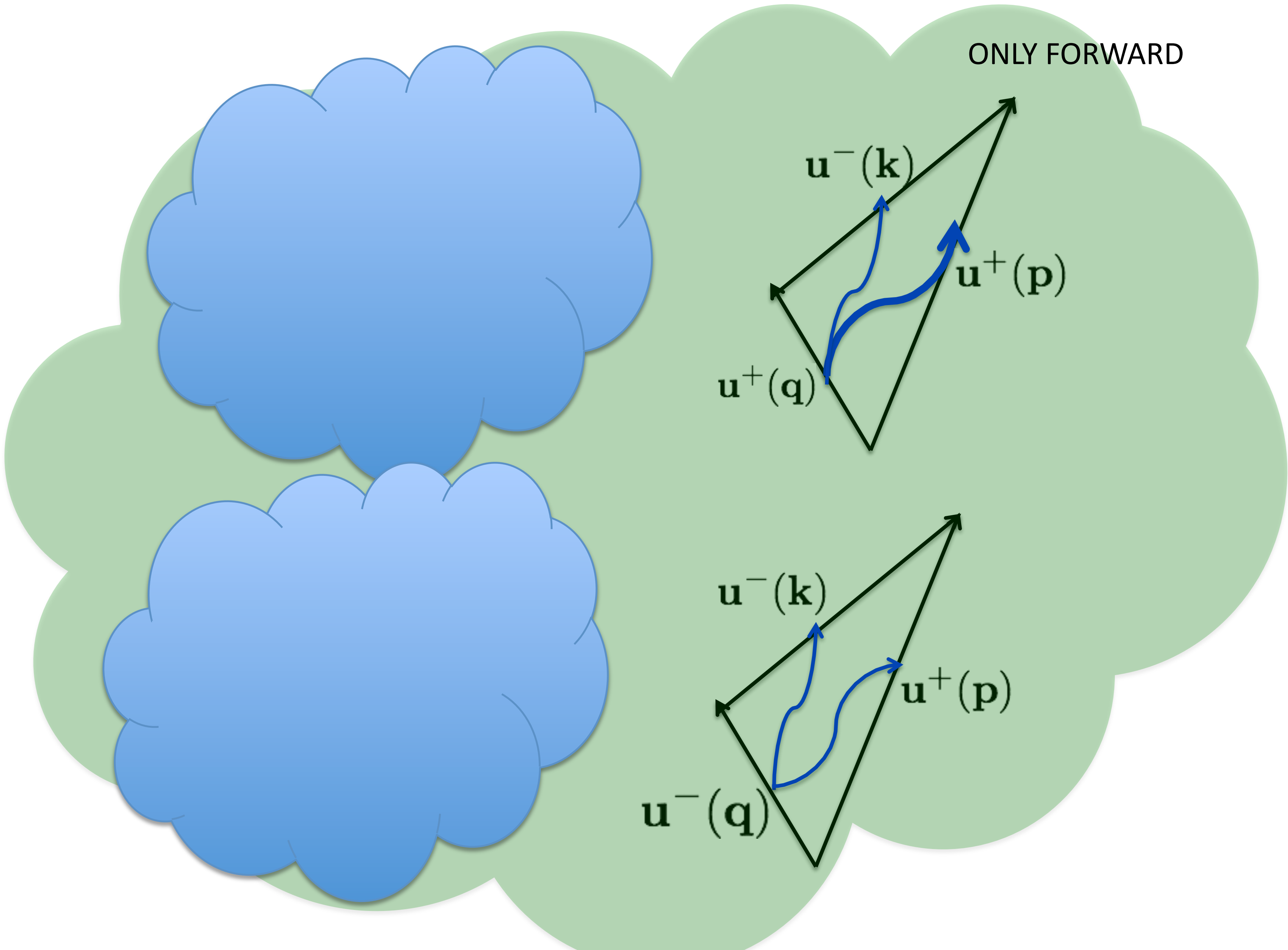
TRIADIC INTERACTION IN DECIMATED NAVIER_STOKES EQS

MAINLY BACKWARD



M. De Pietro, L.B., A. A. Mailybaev
Phys. Rev. E 92, 043021 (2015)

TRIADIC INTERACTION IN DECIMATED NAVIER_STOKES EQS



ONLY REVERSE

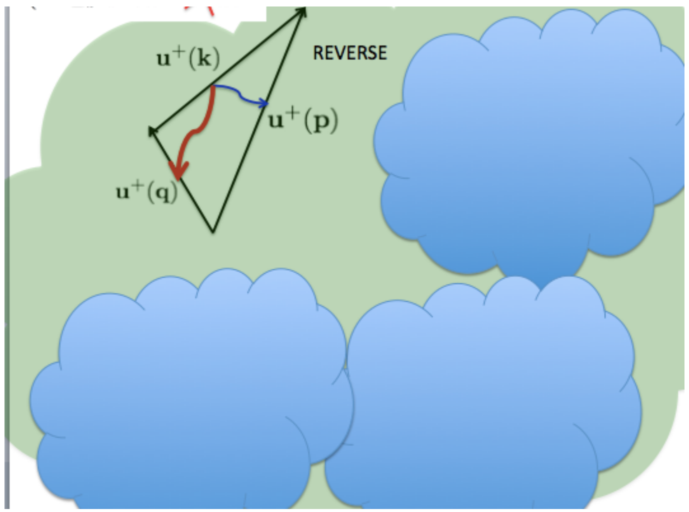
$$\mathcal{P}^{\pm} \equiv \frac{\boldsymbol{h}^{\pm} \otimes \overline{\boldsymbol{h}^{\pm}}}{\overline{\boldsymbol{h}^{\pm}} \cdot \boldsymbol{h}^{\pm}}. \qquad \boldsymbol{v}^{\pm}(\boldsymbol{x}) \equiv \sum_{\boldsymbol{k}} \mathcal{P}^{\pm} \boldsymbol{u}(\boldsymbol{k});$$

$$\boldsymbol{u}(\boldsymbol{k}) = u^{+}(\boldsymbol{k})\boldsymbol{h}^{+}(\boldsymbol{k}) + u^{-}(\boldsymbol{k})\boldsymbol{h}^{-}(\boldsymbol{k})$$

LOCAL BELTRAMIZATION (IN FOURIER)

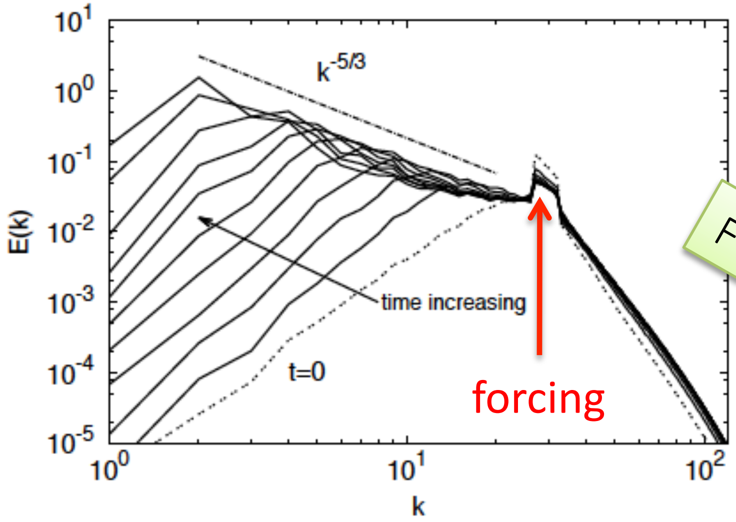
$$\partial_t v^{+} + \mathcal{P}^{+} B[v^{+}, v^{+}] = \nu \Delta v^{+} + \mathbf{f}^{+}$$

decimated-NSE

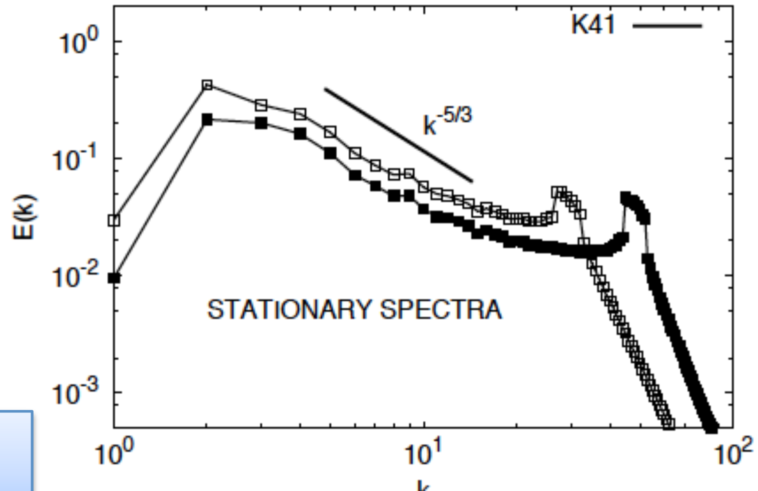


SMALL SCALES FORCING: INVERSE ENERGY CASCADE

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$



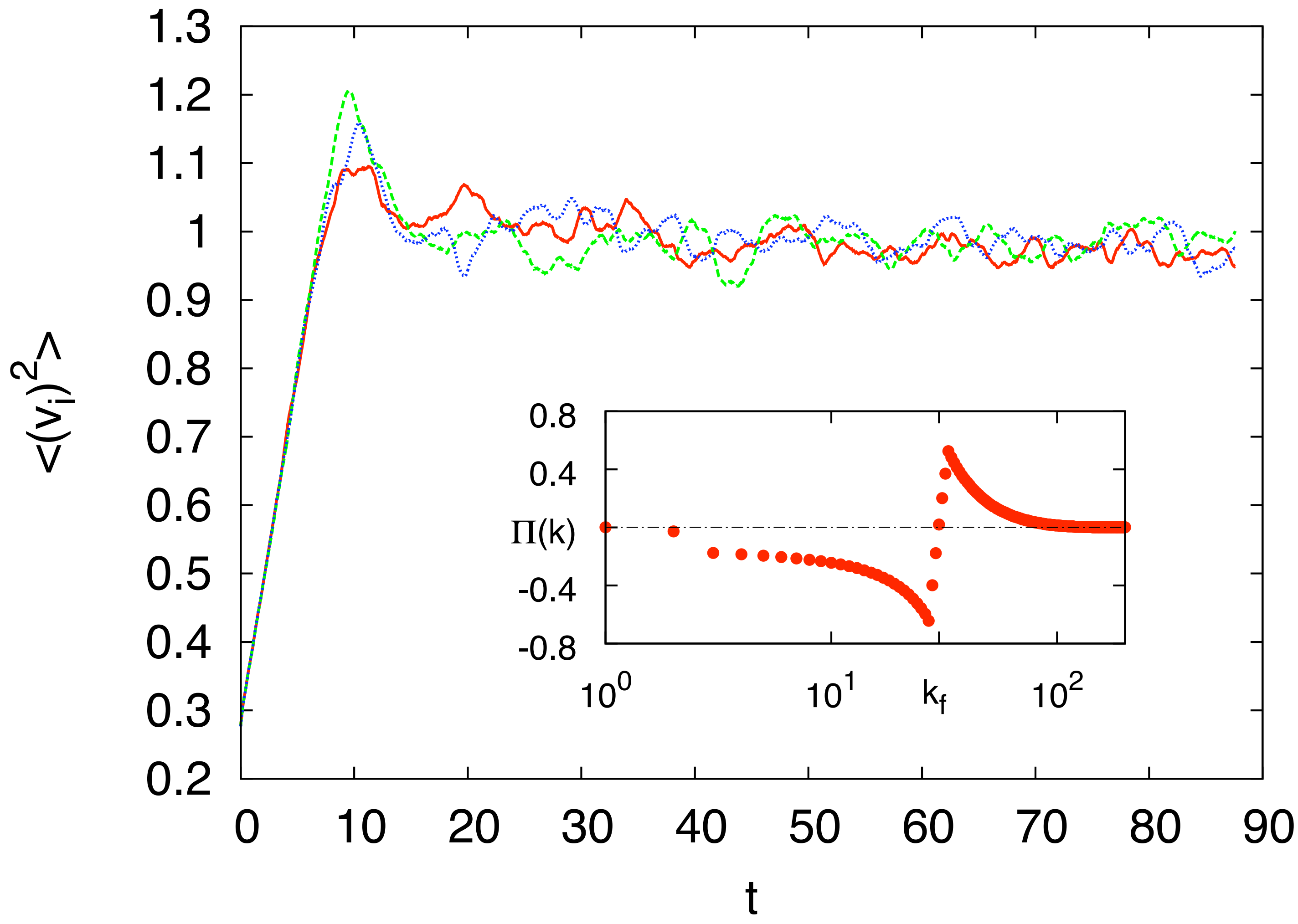
FULLY 3D AND ISOTROPIC



L. B., S. Musacchio and F. Toschi
Phys. Rev. Lett. 108 164501, 2012

SMALL SCALES FORCING: INVERSE ENERGY CASCADE

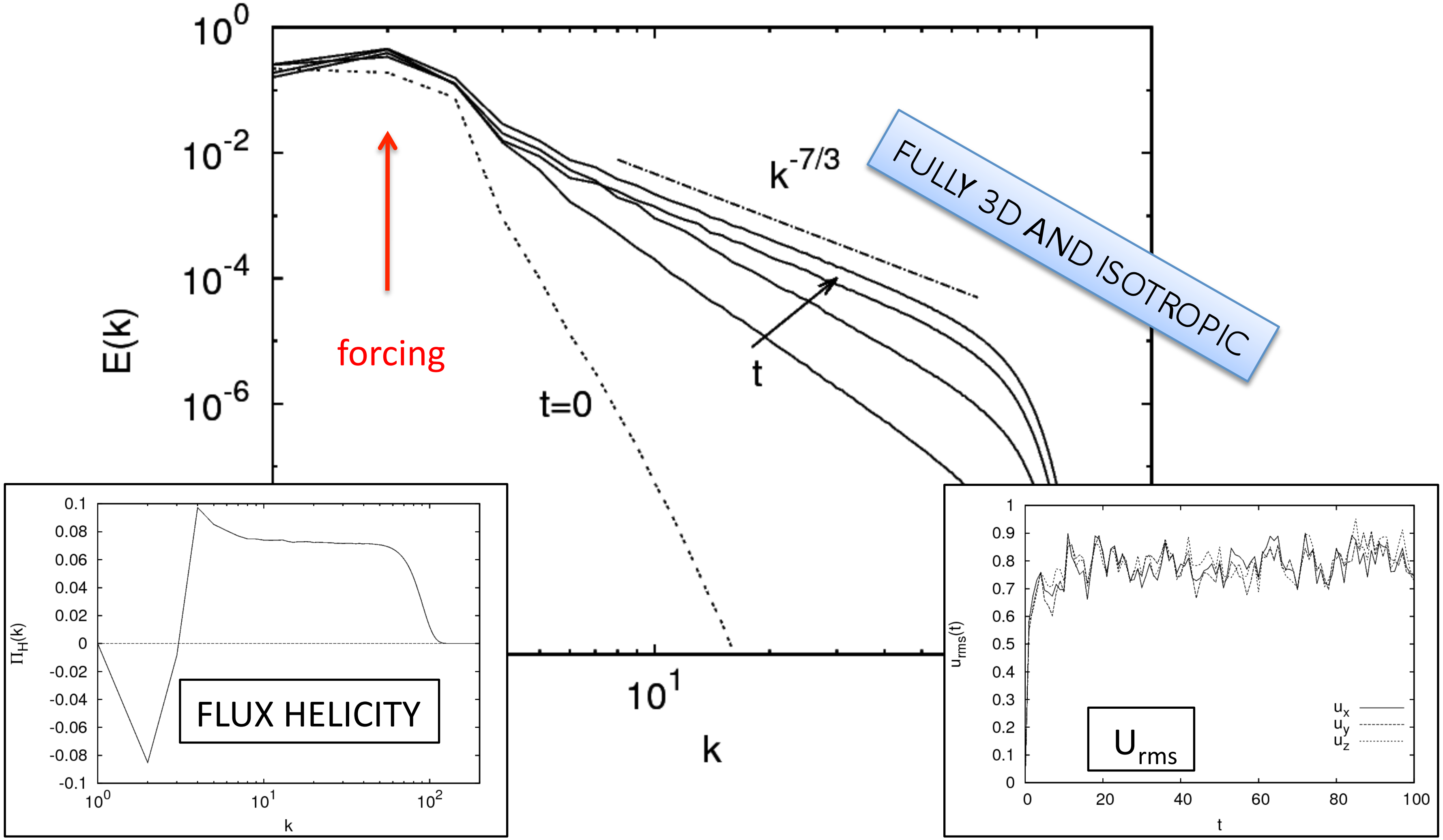
1) PERFECTLY 3D AND PERFECTLY ISOTROPIC



LARGE SCALES FORCING: DIRECT HELICITY CASCADE

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$

L. B., S. Musacchio and F. Toschi
J. Fluid Mech. 730, 309 (2013)

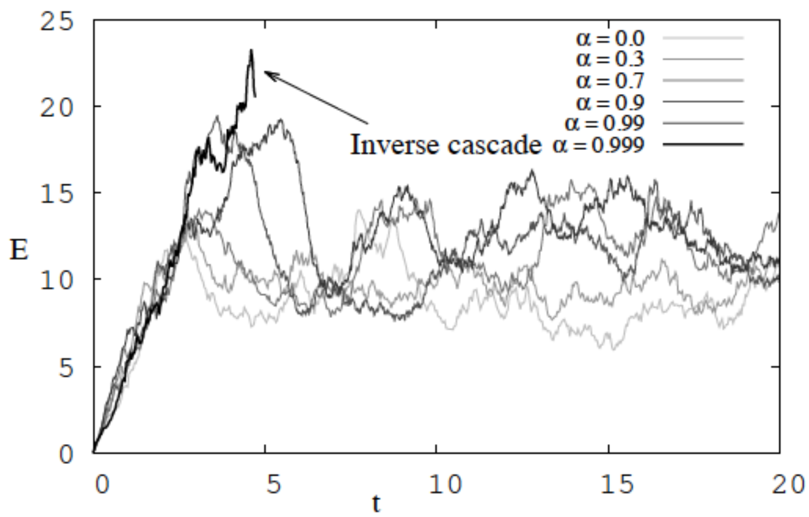


FROM NS TO FULLY-HELICAL

$$\boldsymbol{u}^{\alpha}(\boldsymbol{x}) \equiv D^{\alpha}\boldsymbol{u}(\boldsymbol{x}) \equiv \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\boldsymbol{x}}\, \mathcal{D}^{\alpha}_{\boldsymbol{k}} \boldsymbol{u}_{\boldsymbol{k}}, \qquad (4)$$

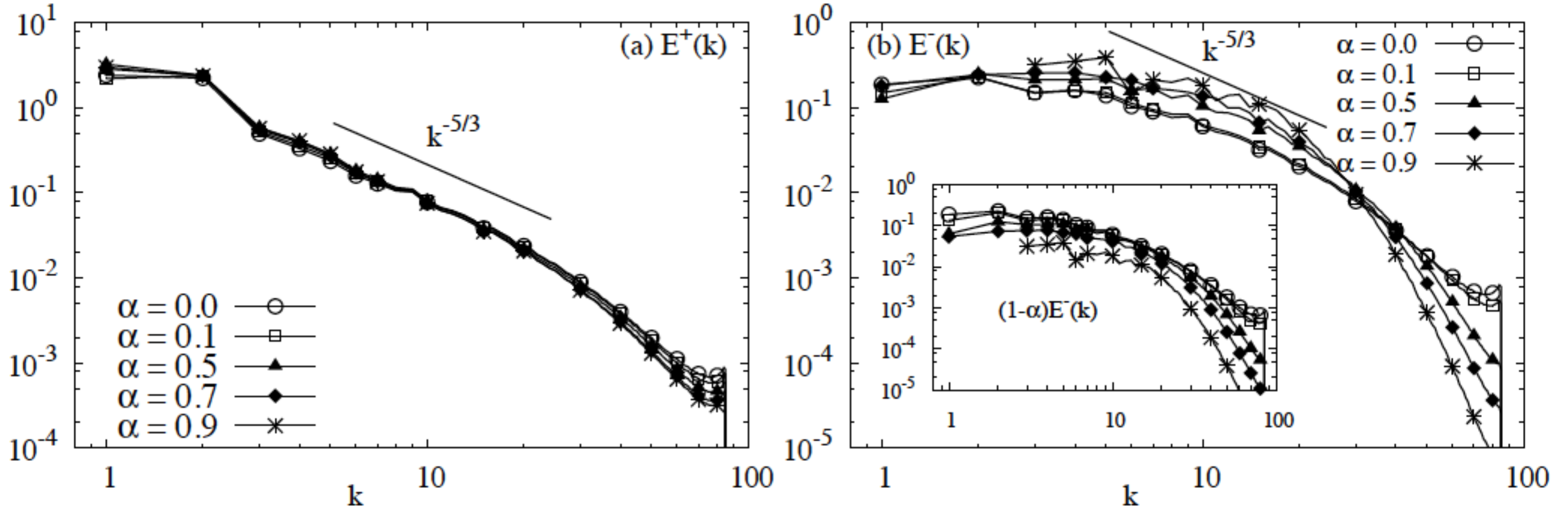
where $\mathcal{D}^{\alpha}_{\boldsymbol{k}} \equiv (1-\gamma^{\alpha}_{\boldsymbol{k}}) + \gamma^{\alpha}_{\boldsymbol{k}}\mathcal{P}^{+}_{\boldsymbol{k}}$ and $\gamma^{\alpha}_{\boldsymbol{k}} = 1$ with probability α or $\gamma^{\alpha}_{\boldsymbol{k}} = 0$ with probability $1-\alpha$. The α-decimated Navier-Stokes equations (α-NSE) are

$$\partial_t \boldsymbol{u}^{\alpha} = D^{\alpha}[-\boldsymbol{u}^{\alpha}\cdot\boldsymbol{\nabla}\boldsymbol{u}^{\alpha} - \boldsymbol{\nabla}p^{\alpha}] + \nu\Delta\boldsymbol{u}^{\alpha}, \qquad (5)$$

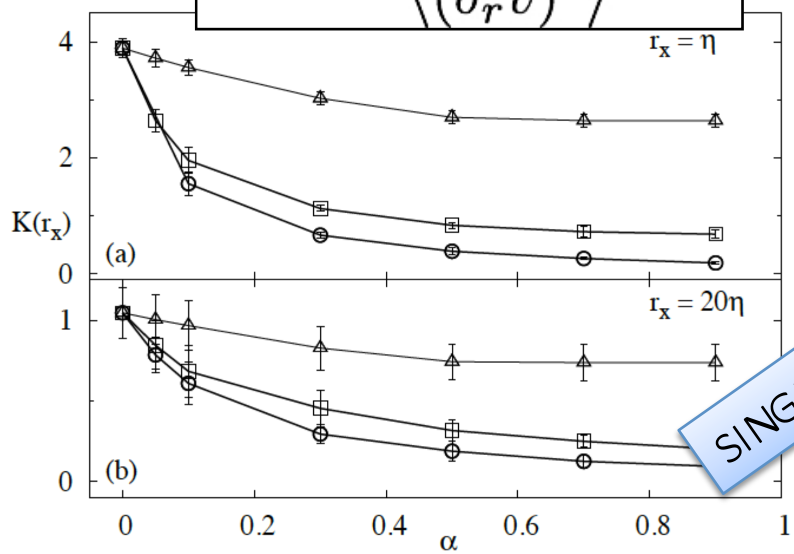


G. Sahoo, F. Bonaccorso and L. B. Phys. Rev. E (2015)

NEGATIVE (OPPOSITE SIGN) HELICAL MODES -> CATALYZER



$$K(r) = \frac{\langle (\delta_r v)^4 \rangle}{\langle (\delta_r v)^2 \rangle^2} - 3$$



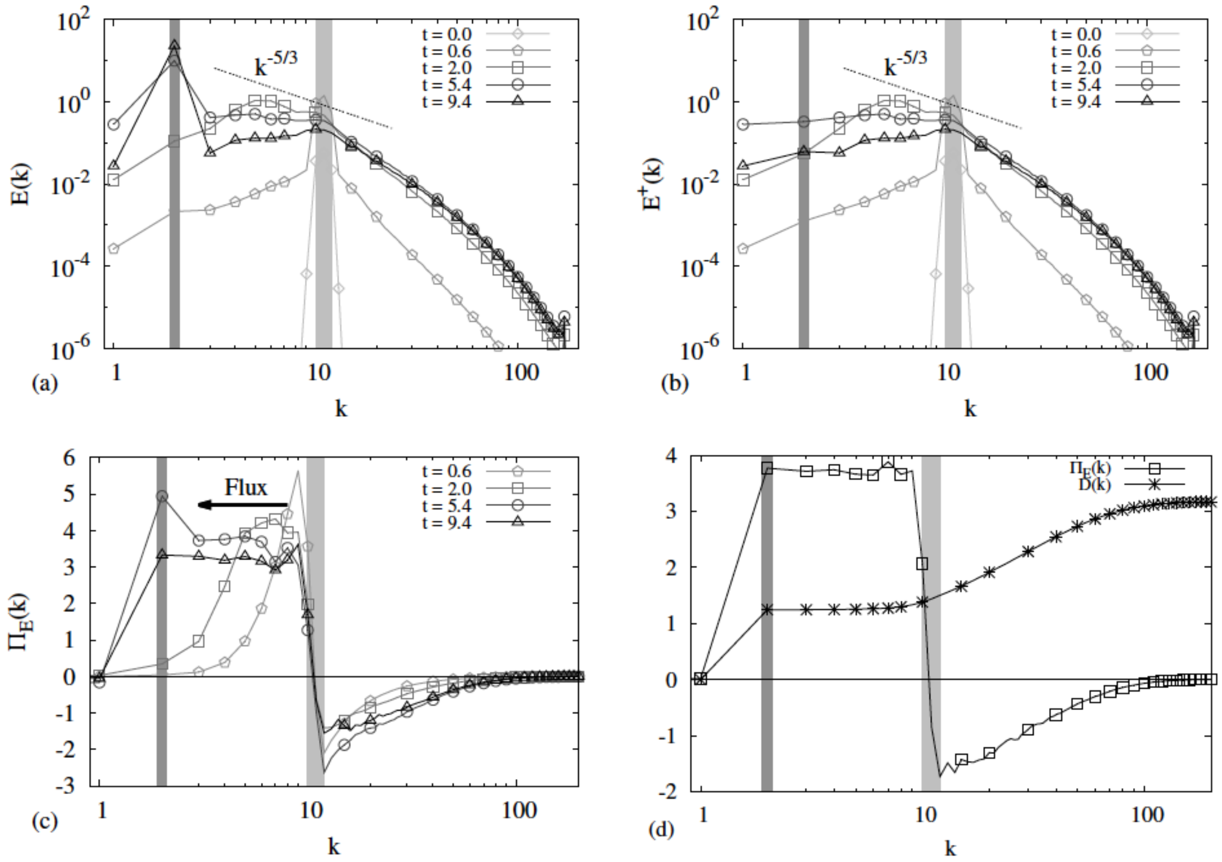
SINGULAR EFFECT ON INTERMITTENCY

G. Sahoo, F. Bonaccorso and L. B.
Phys. Rev. E (2015)

TRIAD-BY-TRAID BACKWARD -> HELICAL CONDENSATE ON THE MINORITY MODES

$$\boldsymbol{u}'(\boldsymbol{x}) \equiv \mathcal{D}_m \boldsymbol{u}(\boldsymbol{x}) \equiv \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\boldsymbol{x}} \left[(1-\gamma_{\boldsymbol{k}}) + \gamma_{\boldsymbol{k}} \mathcal{P}^{+}_{\boldsymbol{k}}\right] \hat{\boldsymbol{u}}_{\boldsymbol{k}},$$

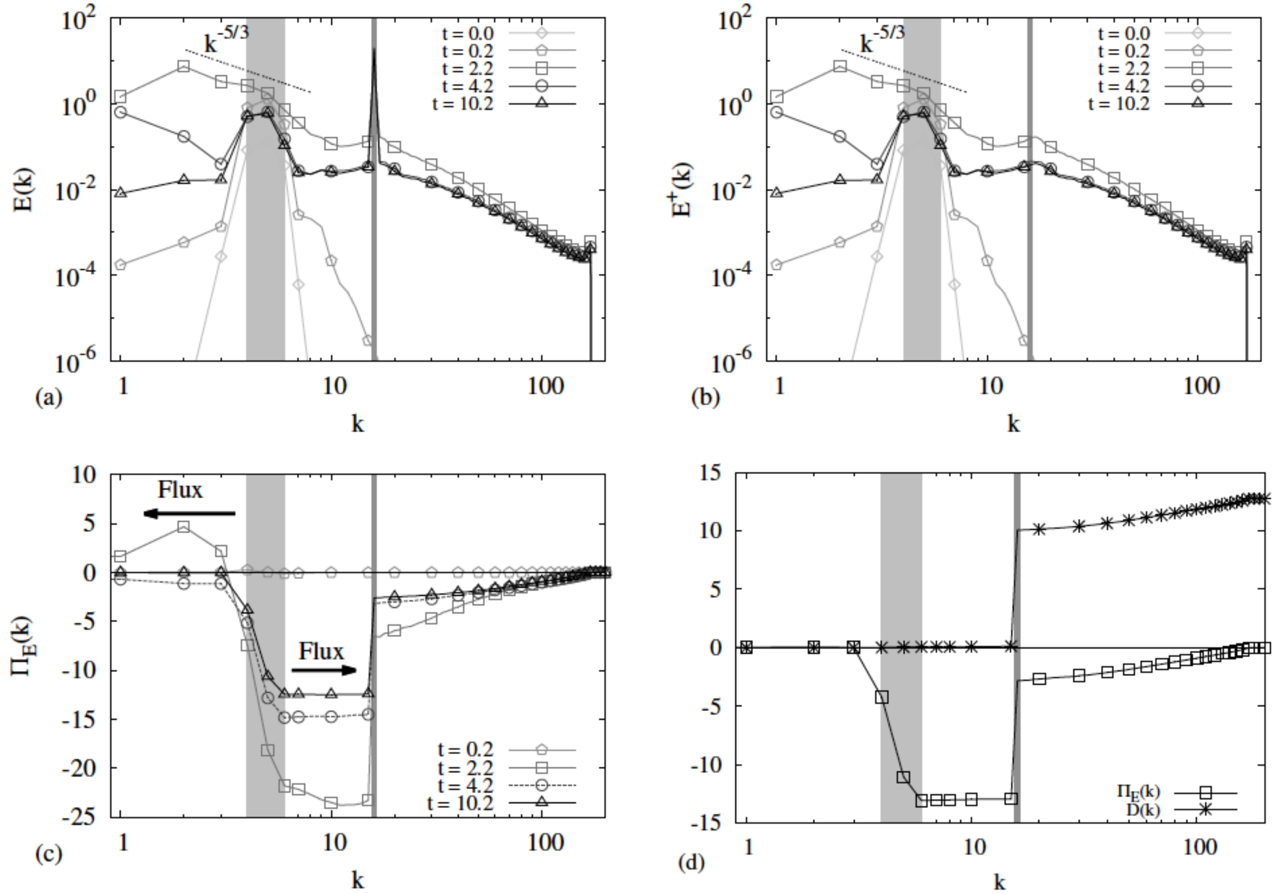
$$E = \sum_{\boldsymbol{k}} (|u^{+}_{\boldsymbol{k}}|^2 + (1-\gamma_{\boldsymbol{k}})|u^{-}_{\boldsymbol{k}}|^2),$$



G. Sahoo, L.B.

Eur. Phys. J. E (2015) 38: 114.

TRIAD-BY-TRAID FORWARD



G. Sahoo, L.B.

Eur. Phys. J. E (2015) 38: 114.

•ALL 3D FLOWS IN NATURE POSSES A SUBSET OF INTERACTIONS RESPONSIBLE OF INVERSE (QUASI-GAUSSIAN) ENERGY CASCADE AND FORWARD (NON-GAUSSIAN?) HELICITY CASCADE

•SUCH DECIMATED NS EQUATIONS ARE 'MORE' REGULAR THAN THE WHOLE SYSTEM: EXISTENCE AND UNIQUENESS OF SOLUTIONS CAN BE PROVED

•EXACT EQUATIONS (à la Karman-Howart) FOR THRID ORDER CORRELATION FUNCTIONS CAN BE DERIVED

• MINORITY MODES WITH (OPPOSITE) HELICITY SIGNS -> CATALYZER FOR THE FORWARD ENERGY TRASFER

• INTERMITTECY STRONLGY SENSITIVE TO HELICAL MODE REDUCTION

HELICAL CONDENSATES