APS-DFD 20^{th}-November 2017, Denver

From 2D to 3D turbulence through 2D3C configurations

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Study of the energy transfer properties

at changing the geometry of the nonlinear interactions

European Research Council

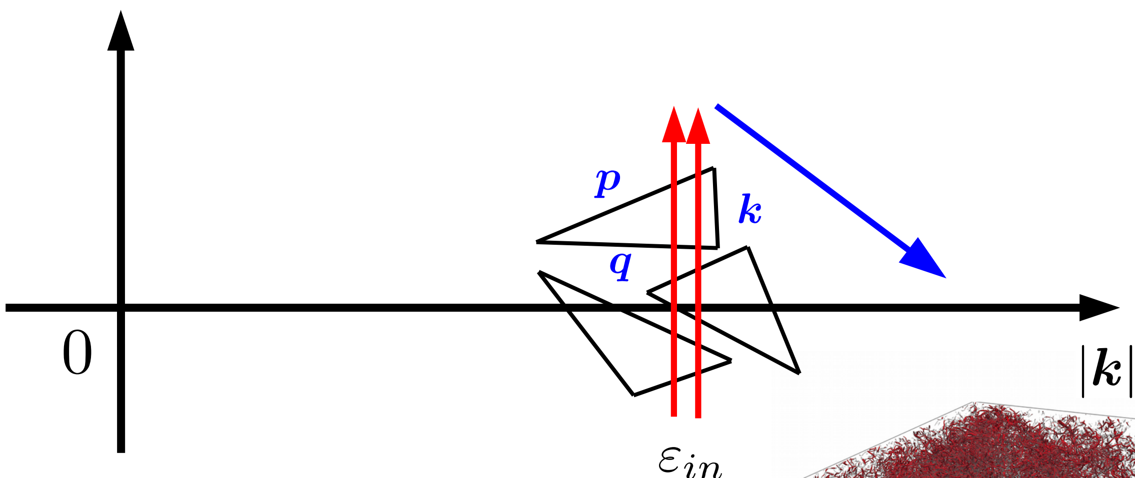
Università di Roma Tor Vergata

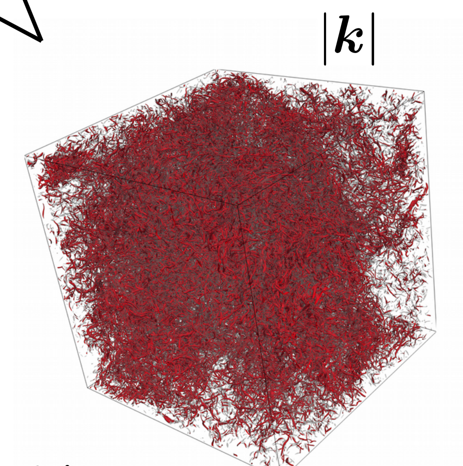
Non-linear energy transfer

$$\partial_t \int_0^k E(k',t)dk' = \Pi(k,t) - 2\nu \int_0^k k'^2 E(k',t)dk' + \varepsilon_{in}$$

$$\Pi(k) = \int_{|\boldsymbol{k}|\le k} P_{ij}(\boldsymbol{k})\ \hat{u}_i^*(\boldsymbol{k},t) \sum_{\boldsymbol{p}+\boldsymbol{q}=\boldsymbol{k}} -ik_l\ \hat{u}_j(\boldsymbol{p},t)\hat{u}_l(\boldsymbol{q},t)d\boldsymbol{k}$$

Triadic interactions





In 3D

homogeneous isotropic turbulence

a forward energy cascade

is observed

In 3D turbulence we cannot
predict the direction of the energy cascade!

Non-linear energy transfer

$$\partial_t \int_0^k E(k',t)dk' = \Pi(k,t) - 2\nu \int_0^k k'^2 E(k',t)dk' + \varepsilon_{in}$$

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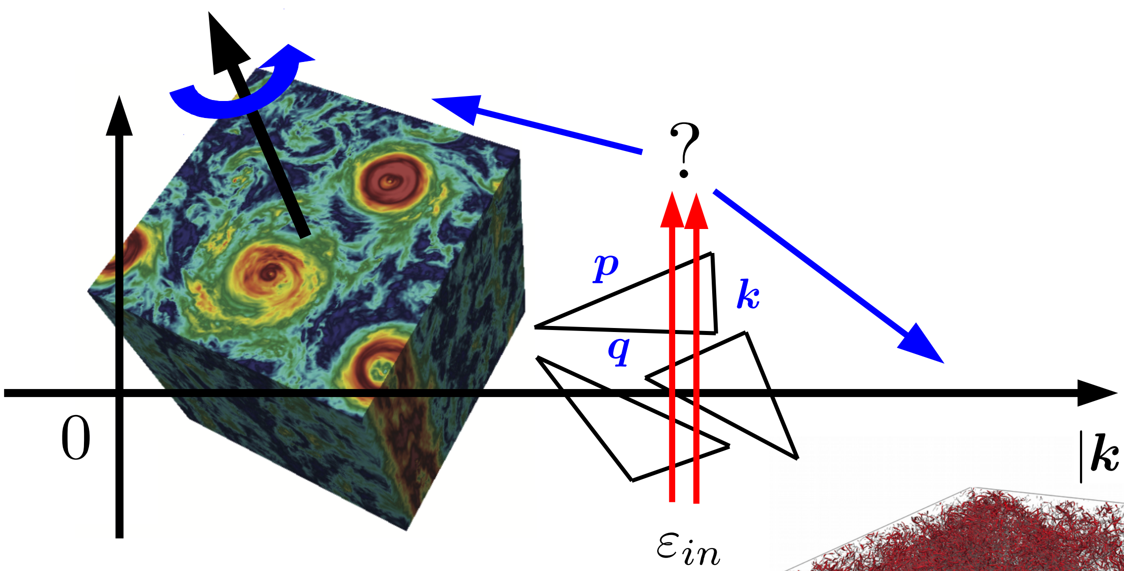
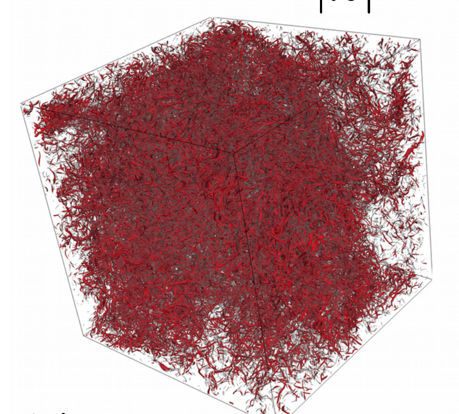
Triadic interactions





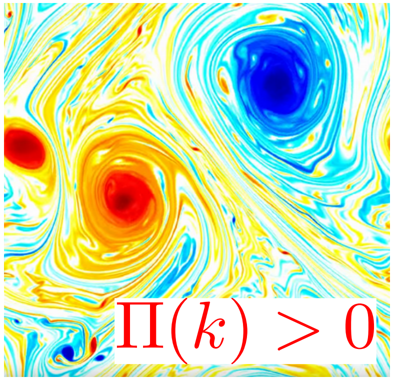
Rotation (if high enough) induces a decoupling between the 3D bulk and a 2D submanifold in Fourier space:

$$\partial_t \hat{\boldsymbol{u}} = \partial_t \hat{\boldsymbol{u}}_{2D3C} + \partial_t \hat{\boldsymbol{u}}_{bulk}$$

In 3D turbulence we cannot predict the direction of the energy cascade!

Two mechanisms are known to produce **backward cascade:**

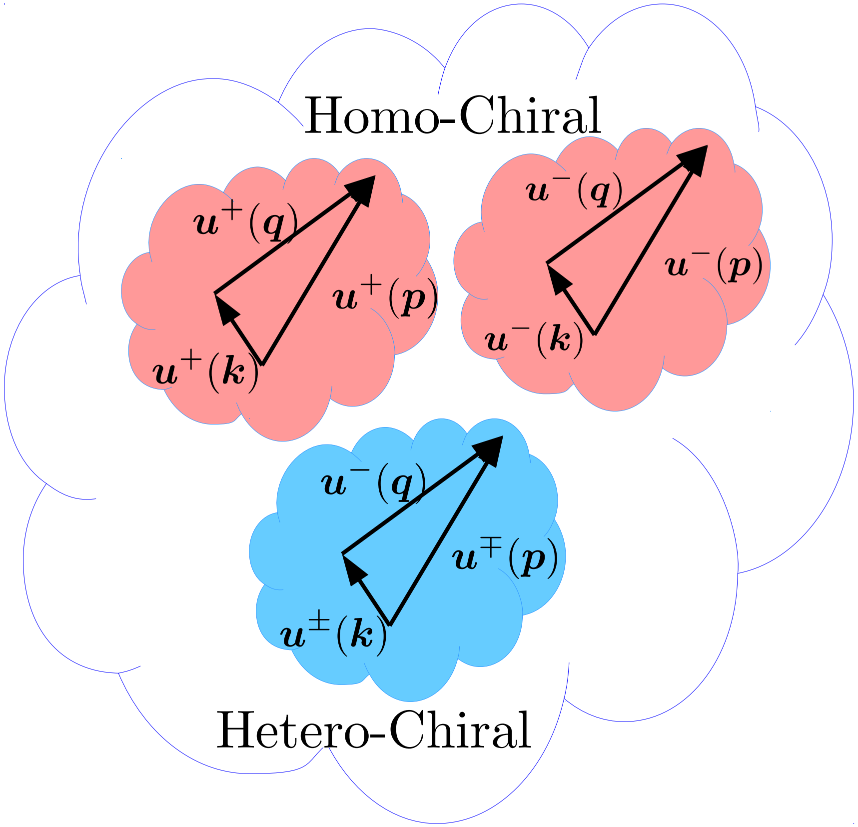
1) by 2D interactions



two positive definite quadratic invariants

$$E = \frac{1}{2}\langle u_i u_i \rangle \qquad \Omega = \frac{1}{2}\langle \omega_i \omega_i \rangle$$

2) by 3D "Homo-Chiral" interactions



second invariant is not positive definite

$$E = \frac{1}{2}\langle u_i u_i \rangle \qquad H = \frac{1}{2}\langle u_i \omega_i \rangle$$

Waleffe, F. (**1992**). PoF A: Fluid Dynamics, 4(2), 350-363.

$$\boldsymbol{u}(\boldsymbol{k}) = u^+(\boldsymbol{k})\boldsymbol{h}^+(\boldsymbol{k}) + u^-(\boldsymbol{k})\boldsymbol{h}^-(\boldsymbol{k})$$

$$i\boldsymbol{k} \times \boldsymbol{h}^\pm = \pm k \boldsymbol{h}^\pm$$

homo- and hetero-chiral triads conserve energy separately:

$$\Pi(k) = \underset{> 0}{\Pi_{HOM}(k)} + \underset{< 0}{\Pi_{HET}(k)}$$

2D3C velocity field

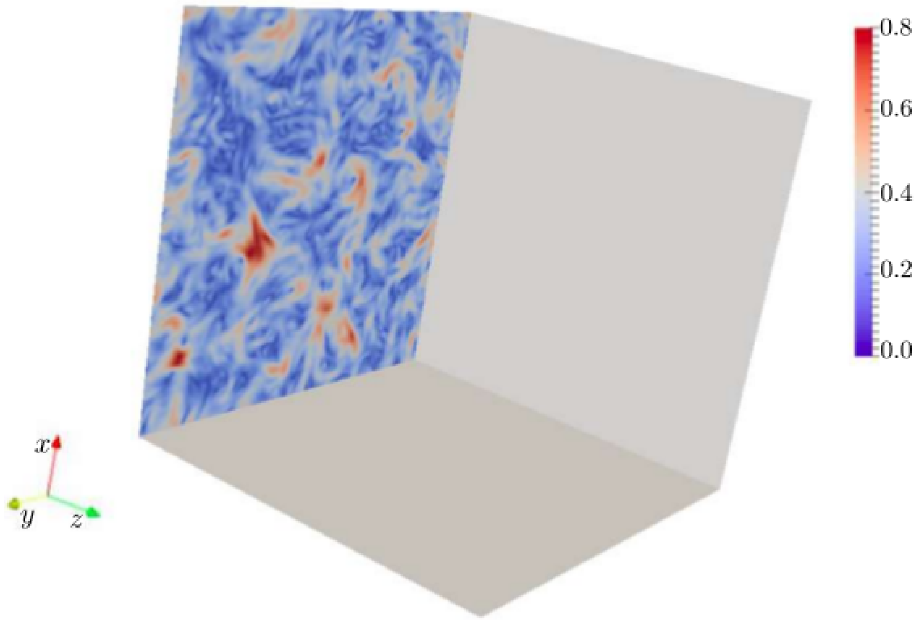
$\boldsymbol{u}(x, y, t)$ → **three** components; → **constant** in the z-direction

$$\boldsymbol{u} = \boldsymbol{u}^{2\mathrm{D}} + \theta$$

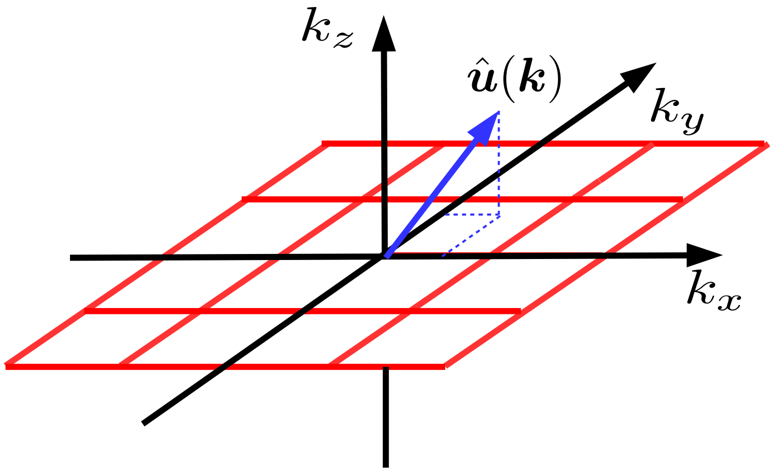
$$\boldsymbol{u}^{2\mathrm{D}} = \begin{pmatrix} u_x \\ u_y \\ 0 \end{pmatrix}; \qquad \theta = \begin{pmatrix} 0 \\ 0 \\ u_z \end{pmatrix}$$

$$\begin{cases} \partial_t \boldsymbol{u}^{2\mathrm{D}} & = -(\boldsymbol{u}^{2\mathrm{D}} \cdot \nabla)\boldsymbol{u}^{2\mathrm{D}} - \nabla P + \nu \Delta \boldsymbol{u}^{2\mathrm{D}} \\ \partial_t \theta & = -(\boldsymbol{u}^{2\mathrm{D}} \cdot \nabla)\theta + \nu \Delta \theta \\ \nabla \cdot \boldsymbol{u} & = 0 \end{cases}$$

physical space:



Fourier space:



2D3C velocity field

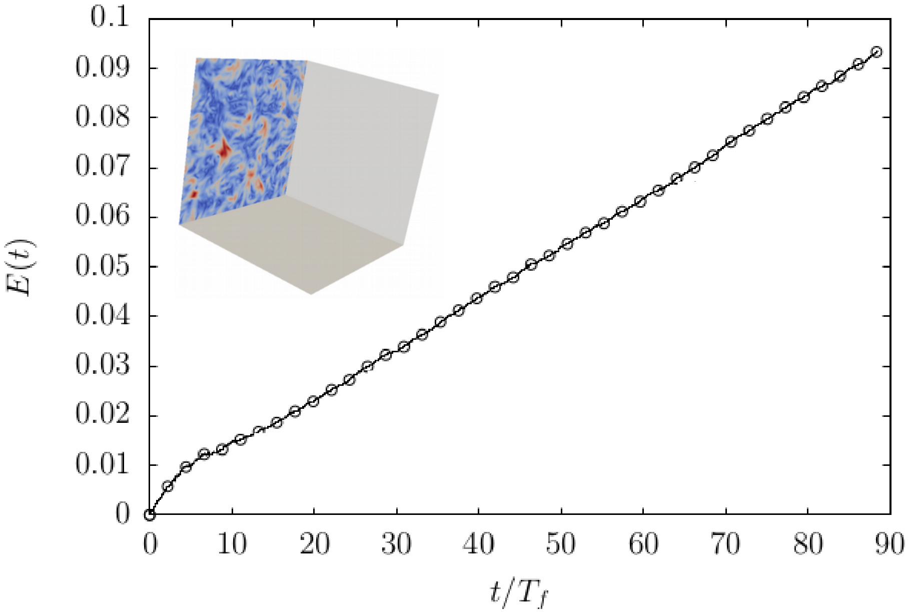
$\boldsymbol{u}(x, y, t)$ → **three** components; → **constant** in the z-direction

$$\boldsymbol{u} = \boldsymbol{u}^{2D} + \theta$$

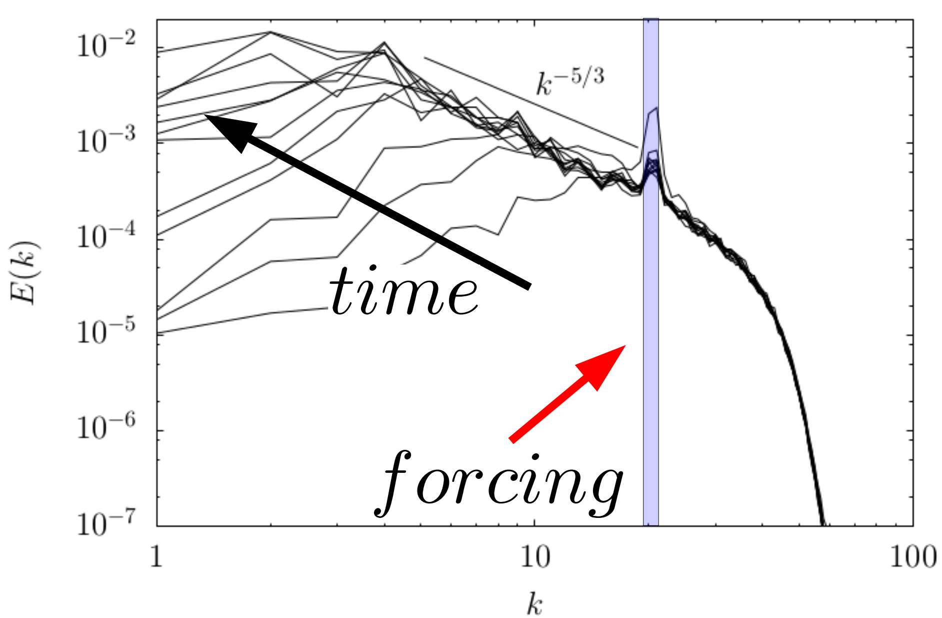
$$\boldsymbol{u}^{2D} = \begin{pmatrix} u_x \\ u_y \\ 0 \end{pmatrix}; \qquad \theta = \begin{pmatrix} 0 \\ 0 \\ u_z \end{pmatrix}$$

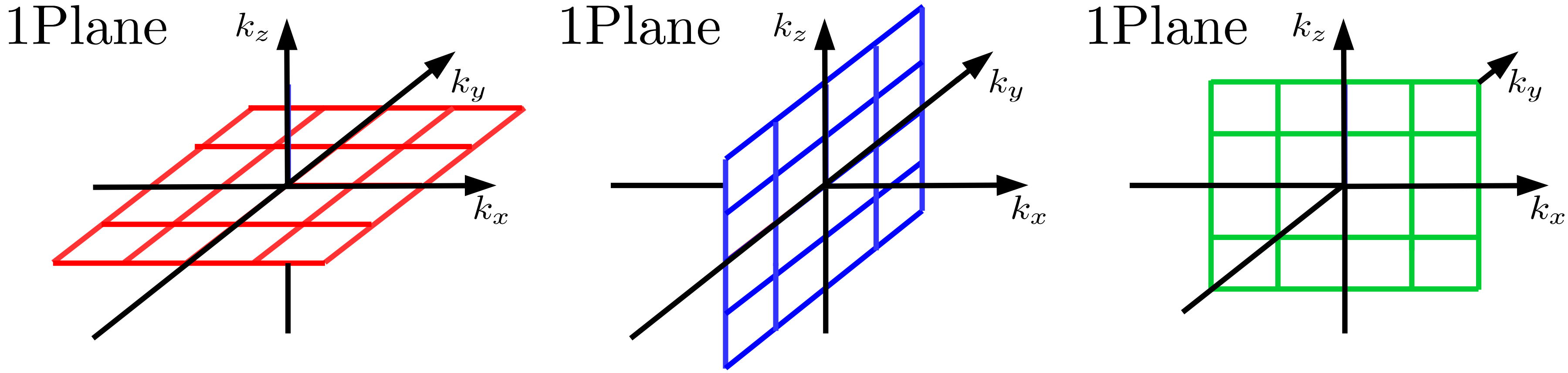
$$\begin{cases} \partial_t \boldsymbol{u}^{2D} &= -(\boldsymbol{u}^{2D} \cdot \nabla)\boldsymbol{u}^{2D} - \nabla P + \nu \Delta \boldsymbol{u}^{2D} + \boldsymbol{f} \\ \partial_t \theta &= -(\boldsymbol{u}^{2D} \cdot \nabla)\theta + \nu \Delta \theta + f_z \\ \nabla \cdot \boldsymbol{u} &= 0 \end{cases}$$

Total energy:

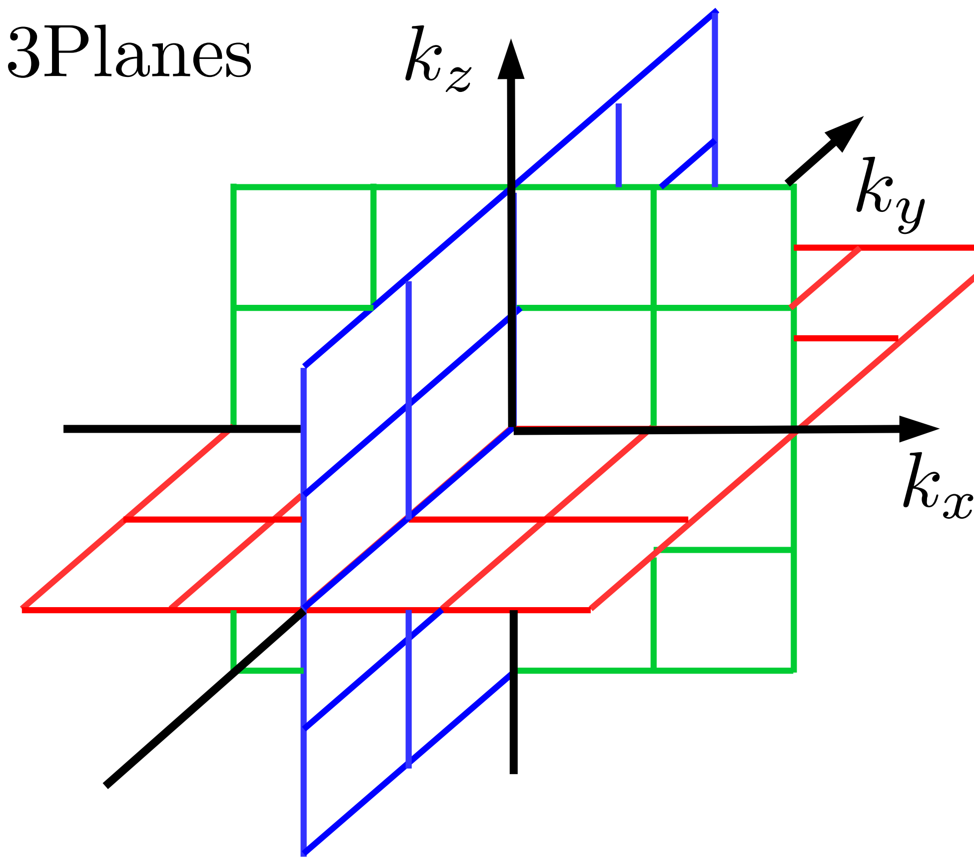


Energy **spectrum:**





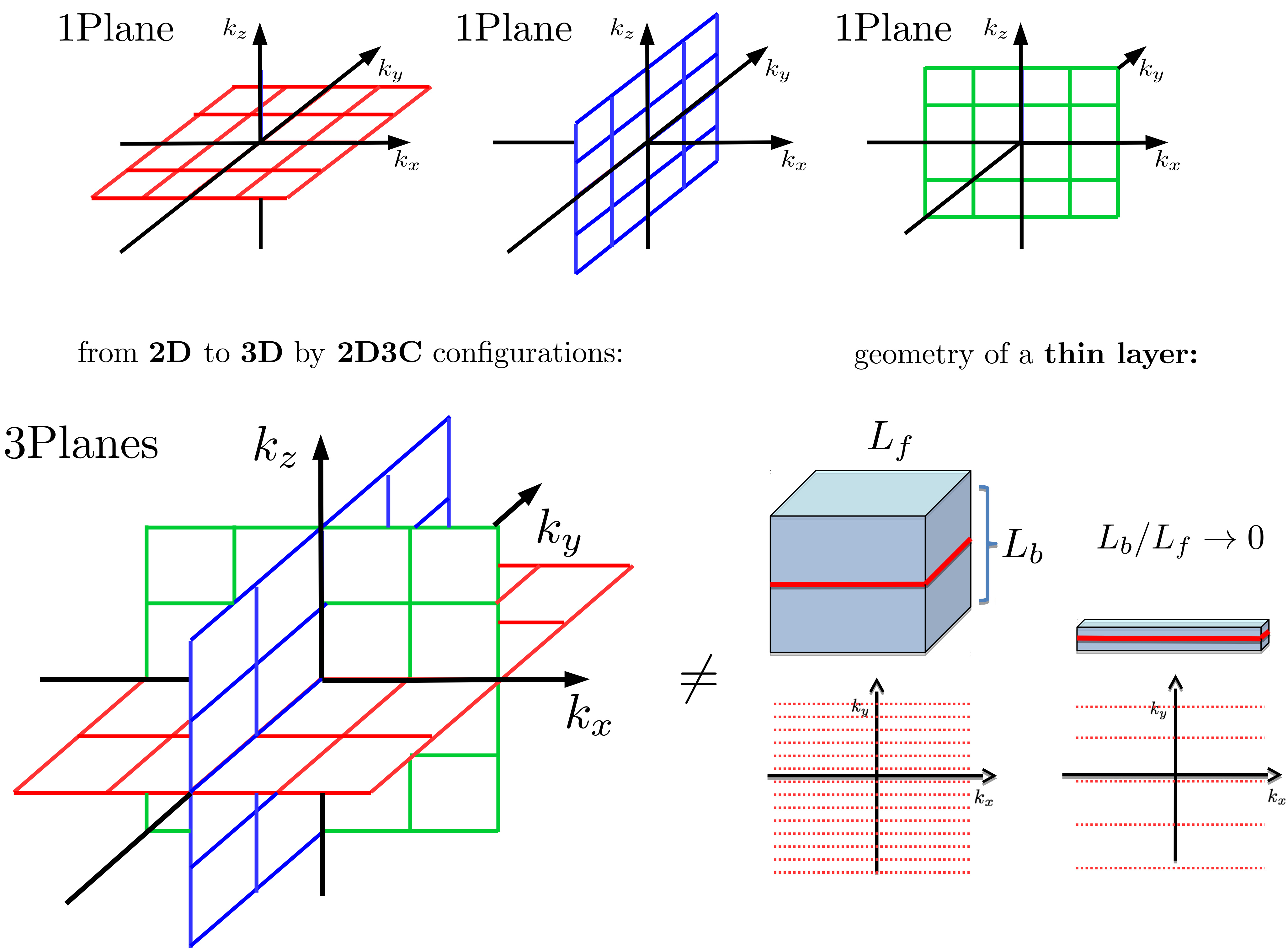
from **2D** to **3D** by **2D3C** configurations:

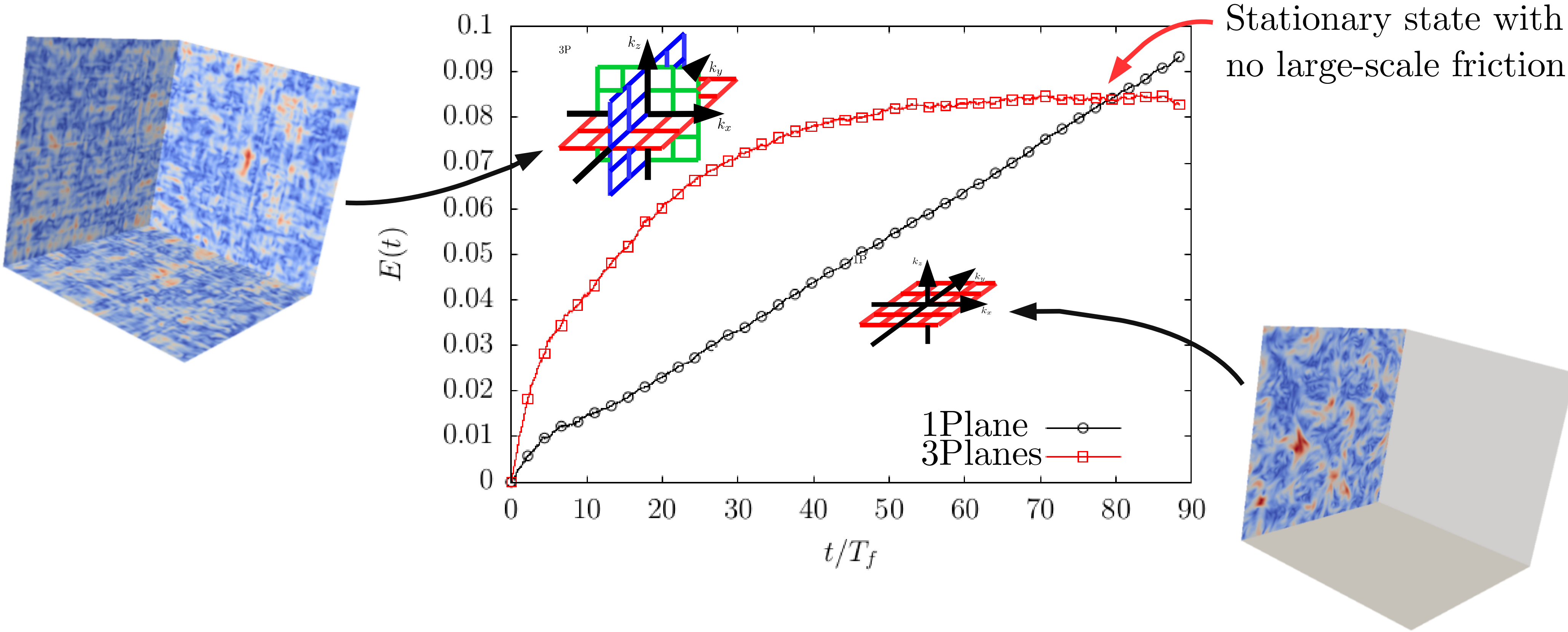


Main properties:

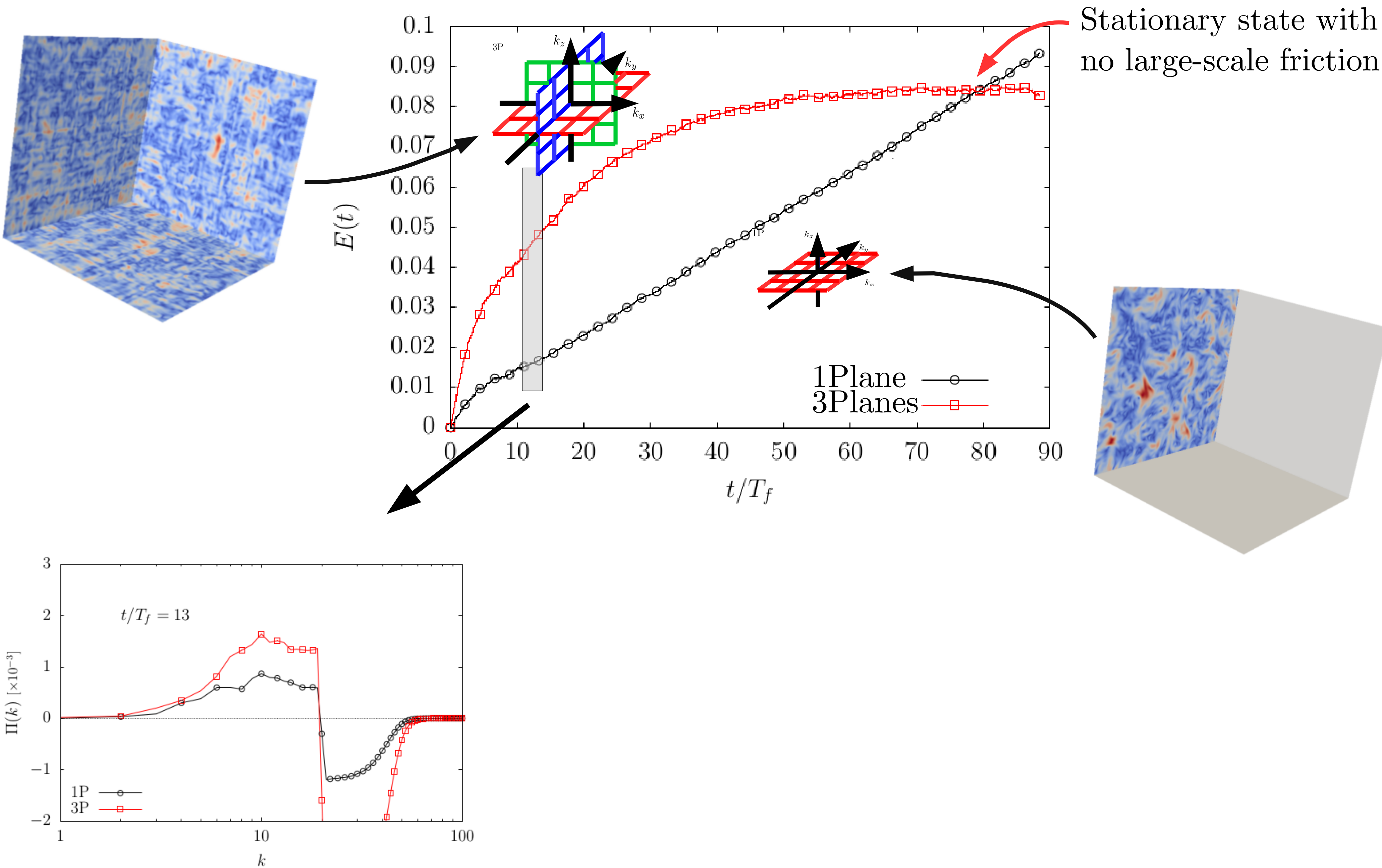
three-dimensionality

isotropy
under discrete rotations

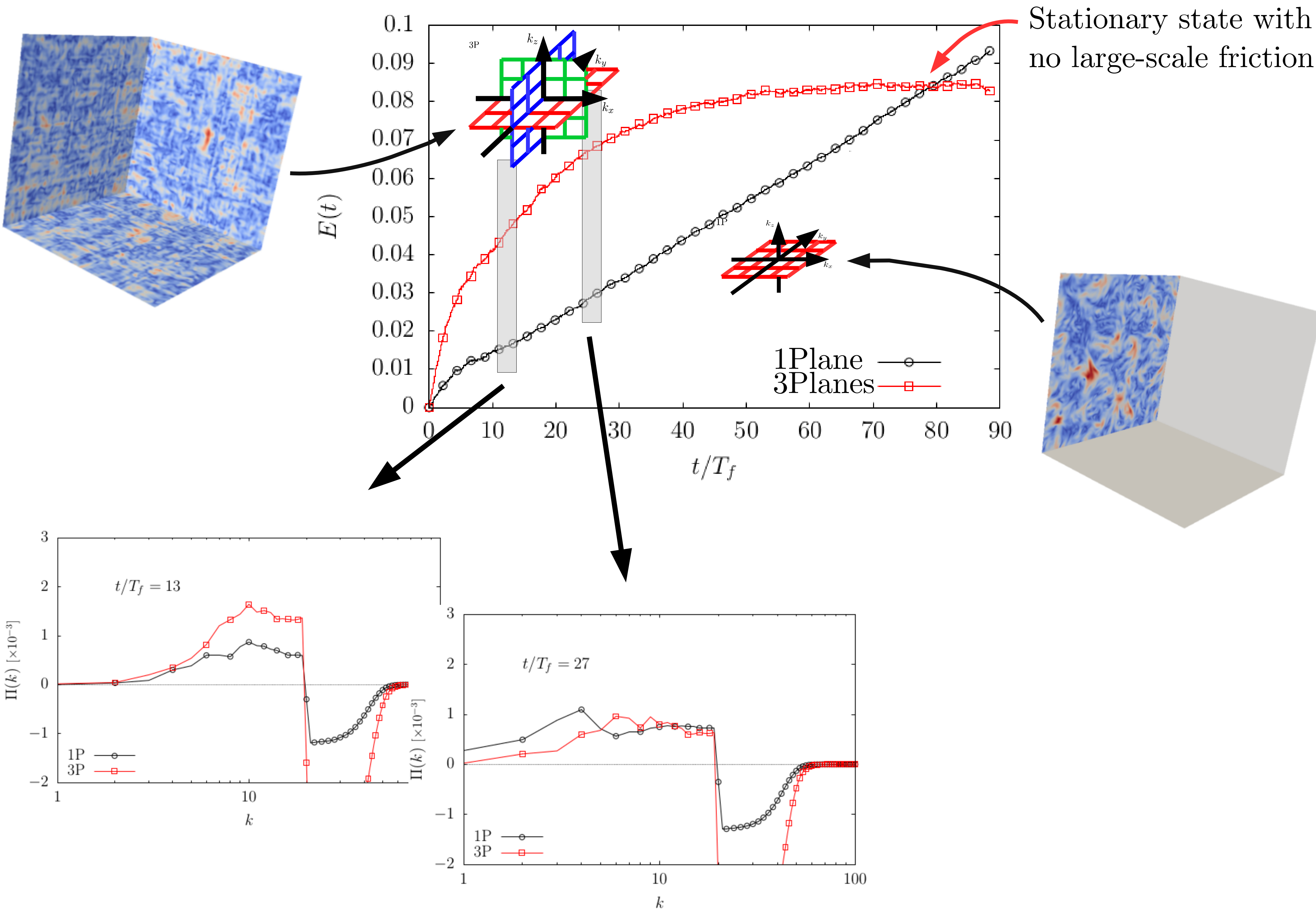

1Plane
k_z
k_y
k_x
1Plane
k_z
k_y
k_x
1Plane
k_z
k_y
k_x
from **2D** to **3D** by **2D3C** configurations:
geometry of a **thin layer:**
3Planes
k_z
k_y
k_x
$\neq$
L_f
L_b
$L_b/L_f \to 0$
k_y
k_x
k_y
k_x



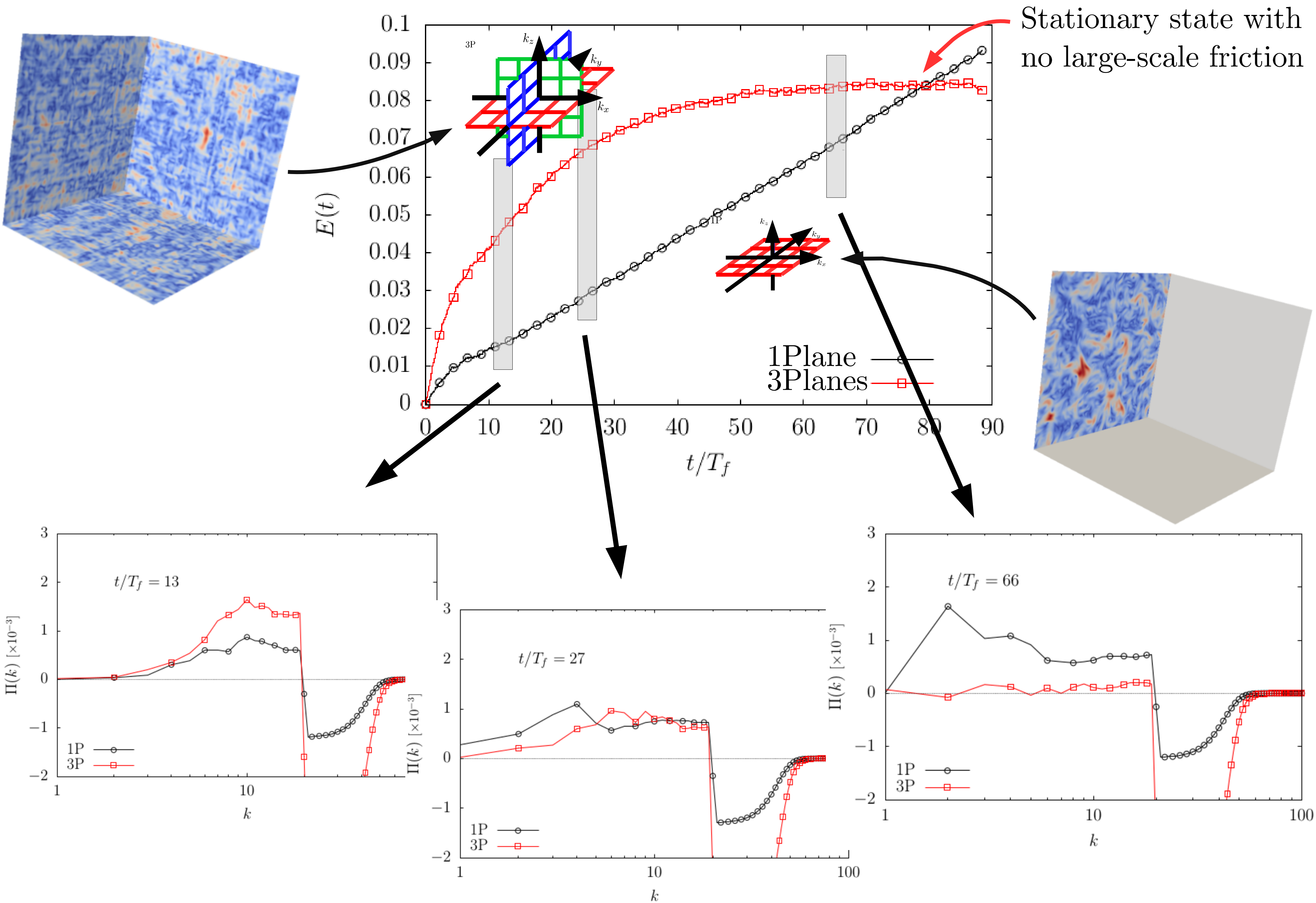
Stationary state with
no large-scale friction
3P
k_z
k_y
k_x
1P
$E(t)$
t/T_f
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
10
20
30
40
50
60
70
80
90
1Plane
3Planes



Stationary state with
no large-scale friction
E(t)
t/T_f
1Plane
3Planes
3P
1P
k_x
k_y
k_z
t/T_f = 13
Π(k) [×10^-3]
k
1P
3P

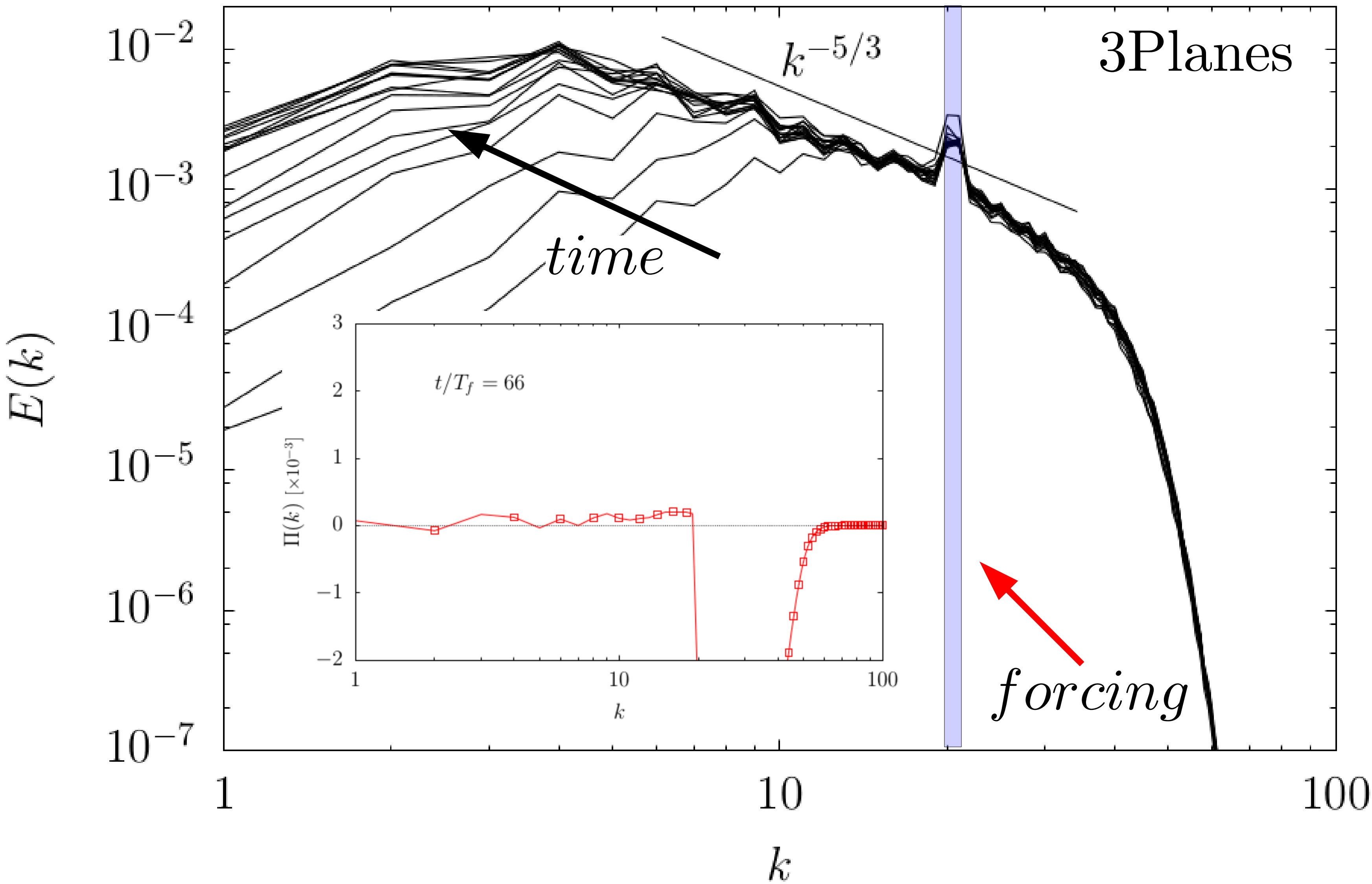


Stationary state with
no large-scale friction
$E(t)$
t/T_f
3P
1P
k_x
k_y
k_z
1Plane
3Planes
$t/T_f = 13$
$t/T_f = 27$
$\Pi(k)$ [$\times 10^{-3}$]
k
1P
3P



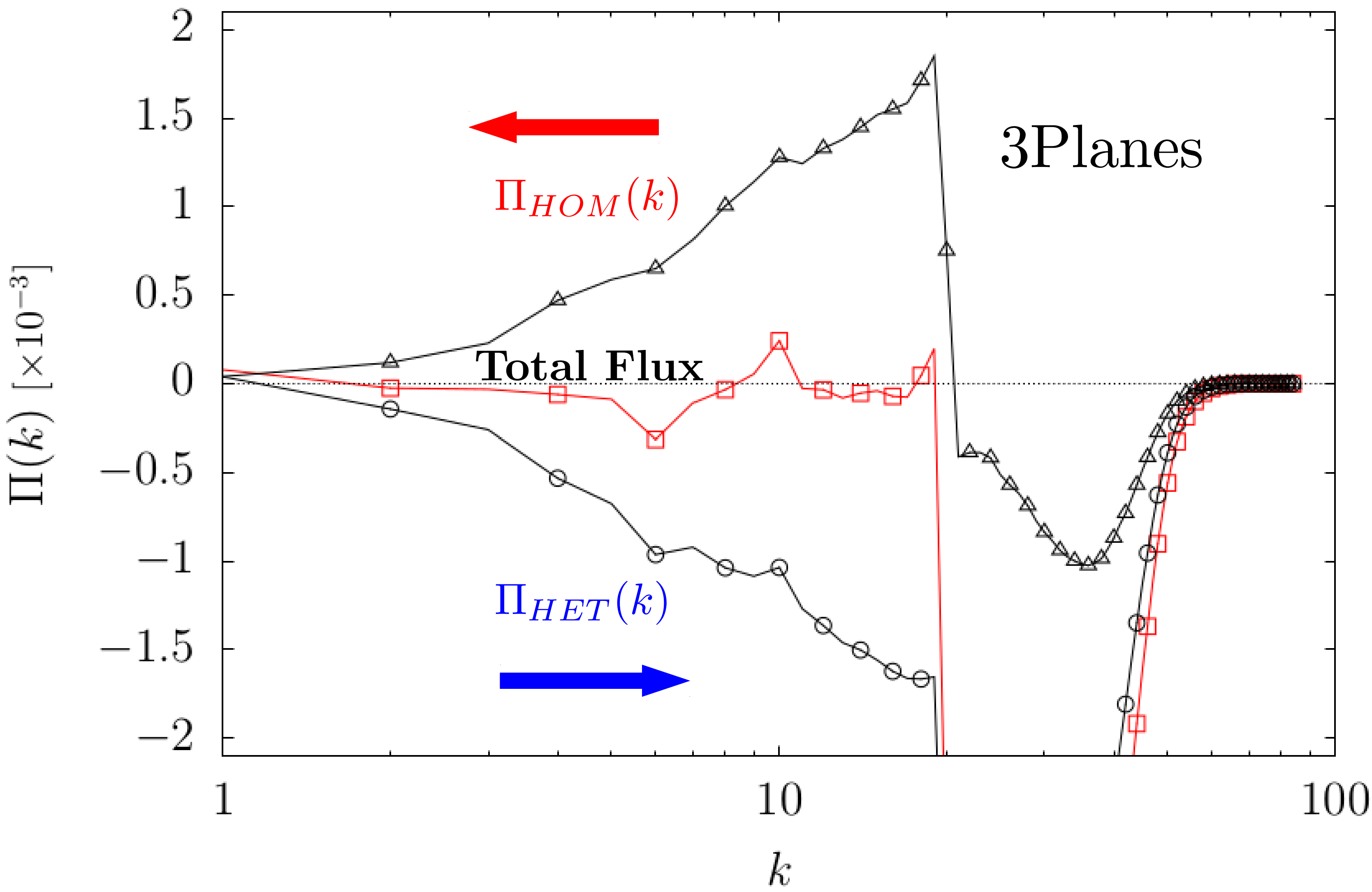
Stationary state with
no large-scale friction
E(t)
t/T_f
1Plane
3Planes
3P
1P
t/T_f = 13
t/T_f = 27
t/T_f = 66
Π(k) [×10^{-3}]
k
1P
3P

zero Flux, no Equilibrium!



zero Flux, no Equilibrium!

$$\Pi(k) = \Pi_{HOM}(k) + \Pi_{HET}(k)$$



Conclusions:

-)We have studied the effect of changing the **geometry of the interactions** on the direction of the mean energy transfer in turbulence

-)We have **coupled three 2D3C flows** in order to study the transition from 2D to 3D turbulence

-)We end up with a system which reaches a stationary state without the addition of any hypo-viscous term at large scales

-)The **zero energy flux** is obtained as a results of a **non-equilibrium** dynamics thanks to the cancellation of different fluxes following different channels

-)The complete transition from 2D3C to 3D dynamics can be reached adding a small percentage ($\sim 10\%$) of modes in the Fourier space domain

From two-dimensional to three-dimensional turbulence through two-dimensional three-component flows

L. Biferale,[1, a)] M. Buzzicotti,[1] and M. Linkmann[1]

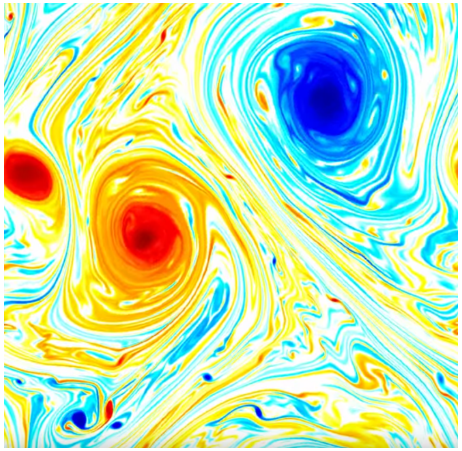
Physics of Fluids **29**, 111101 (2017)

.. in 2 dimensions

two positive definite quadratic invariants

$$E = \frac{1}{2}\langle u_i u_i \rangle \qquad \Omega = \frac{1}{2}\langle \omega_i \omega_i \rangle$$

Inverse energy transfer



$$\Pi(k') > 0$$

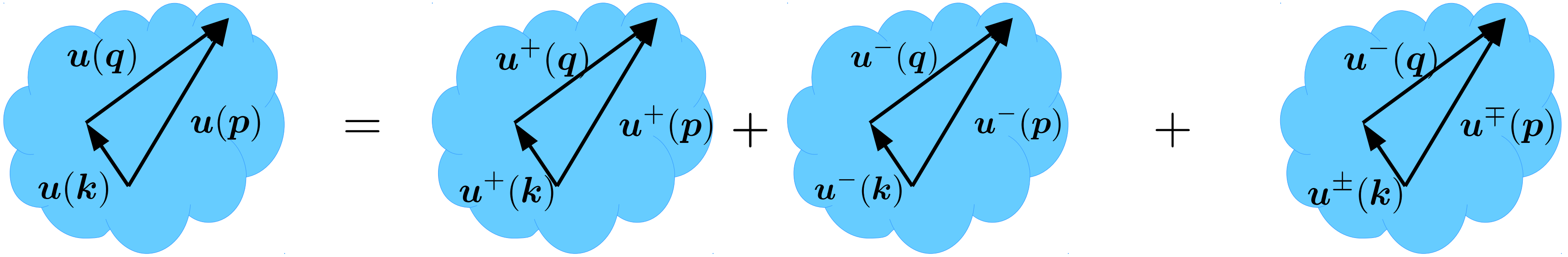
.. in 3 dimensions

second invariant is not positive definite

$$E = \frac{1}{2}\langle u_i u_i \rangle \qquad H = \frac{1}{2}\langle u_i \omega_i \rangle$$

$$\boldsymbol{u}(\boldsymbol{k}) = u^+(\boldsymbol{k})\boldsymbol{h}^+(\boldsymbol{k}) + u^-(\boldsymbol{k})\boldsymbol{h}^-(\boldsymbol{k})$$
$$i\boldsymbol{k} \times \boldsymbol{h}^\pm = \pm k \boldsymbol{h}^\pm$$

Waleffe, F. (**1992**). Physics of Fluids A: Fluid Dynamics, 4(2), 350-363.



Homo-chiral

Hetero-chiral

.. in 2 dimensions

two positive definite quadratic invariants

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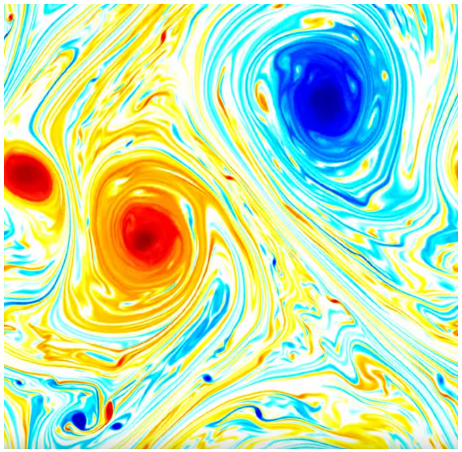

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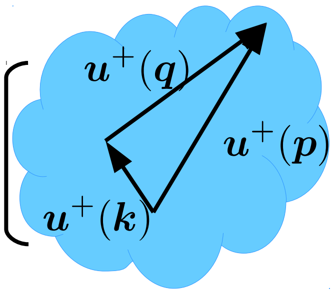
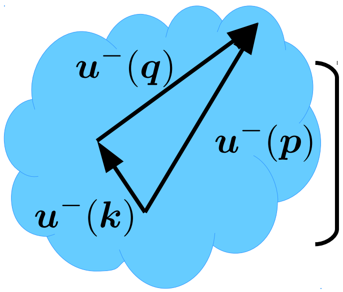
$$i\boldsymbol{k} \times \boldsymbol{h}^\pm = \pm k \boldsymbol{h}^\pm$$

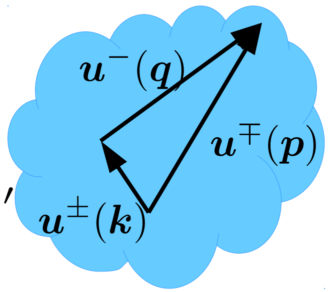
$\Pi(k') = \int_{|\boldsymbol{k}| \le k'} \Big[$



$+$



$\Big] d\boldsymbol{k} + \int_{|\boldsymbol{k}| \le k'}$



$d\boldsymbol{k}$

Homo-chiral: $\Pi_{HOM}(k') > 0$

Hetero-chiral: $\Pi_{HET}(k') < 0$