Taming turbulence via spectral nudging

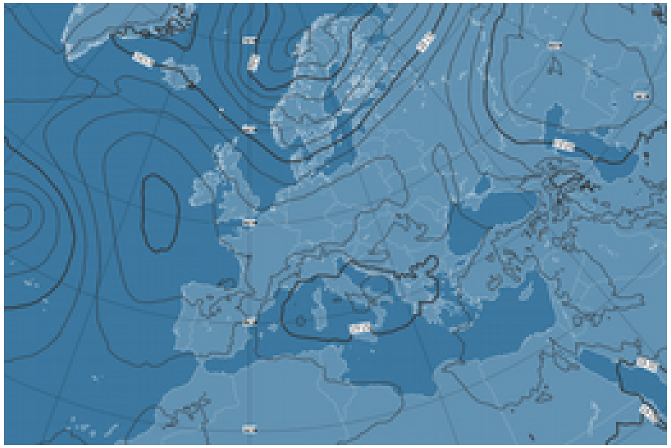
Patricio Clark Di Leoni
University of Rome “Tor Vergata”

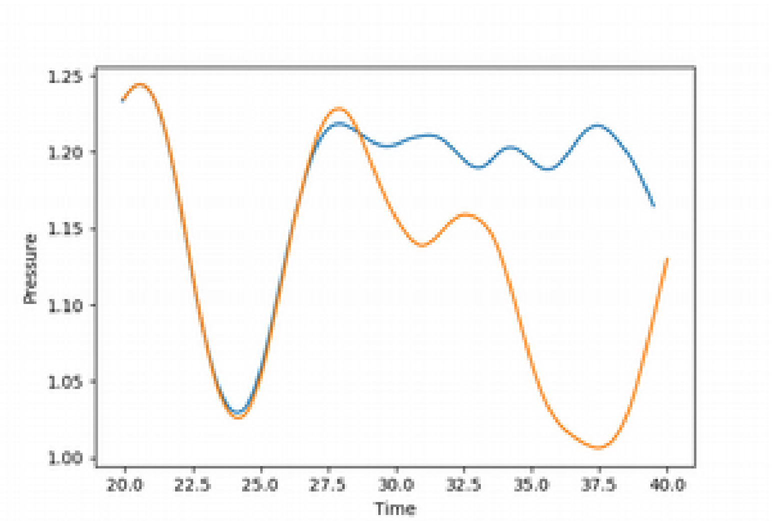
Turbulent Cascades II
Lyon, 2017

In collaboration with:
Andrea Mazzino & Luca Biferale

Data assimilation

Arises out of the need to control the evolution of a chaotic system that starts with imperfect initial conditions

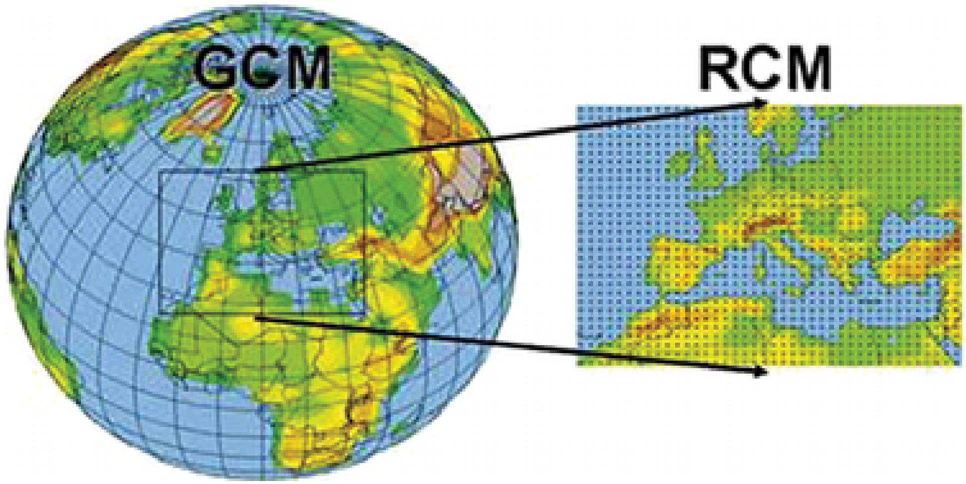




The idea is to incorporate data (measurements, observations…) into the dynamical evolution in order to "correct" it

Spectral nudging

Technique used to go from Global to Regional model [Waldron et al, (1996)] or to incorporate data from measurements [Azouani et al (2013)].



Global Circulation Model

Regional Climate Model

Consists of controlling the evolution of certain scales in the system by introducing spatially and temporally filtered information.

Spectral nudging

Reference simulation

$$\frac{\partial u_{\mathrm{ref}}}{\partial t} + (u_{\mathrm{ref}} \cdot \nabla) u_{\mathrm{ref}} = -\nabla p + \nu \nabla^2 u_{\mathrm{ref}} + f$$

Constant in time, isotropic, and acting on $k = [1, 2]$

Nudged simulation

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\nabla p + \nu \nabla^2 u - \alpha (u^< - u_{\mathrm{ref}}^{<,\tau})$$

Easy to implement in a pseudo-spectral code with periodic boundary conditions and $k_{\max} = N/3$

$$u(x) = \sum_{-k_{\max}}^{k_{\max}} u_k e^{-ik\cdot x} \qquad u^<(x) = \sum_{-k_c}^{k_c} u_k e^{-ik\cdot x}, \; k_c < k_{\max}$$

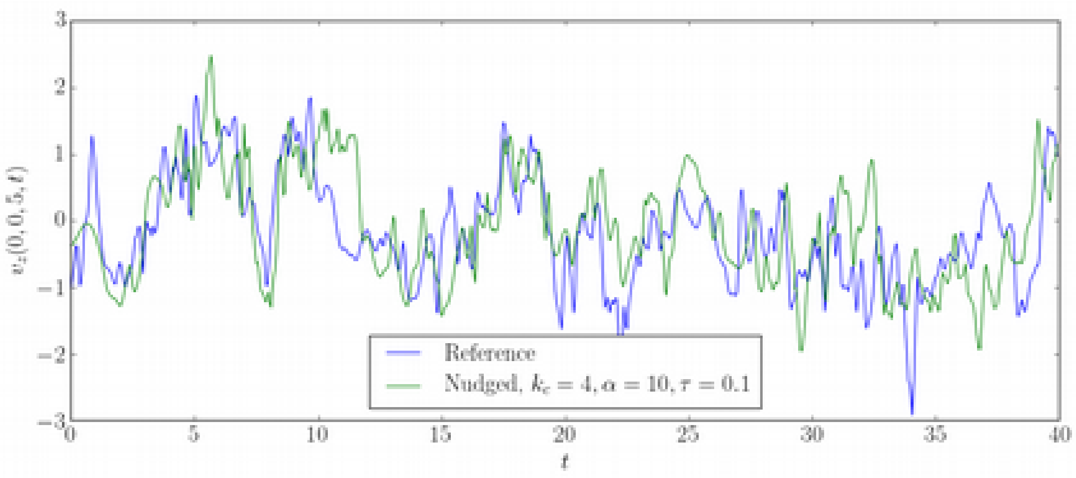
Spatial filtering

$$u_{\mathrm{ref}}^{\tau} = \frac{\tau - t}{\tau} u_1 + \frac{t}{\tau} u_2$$

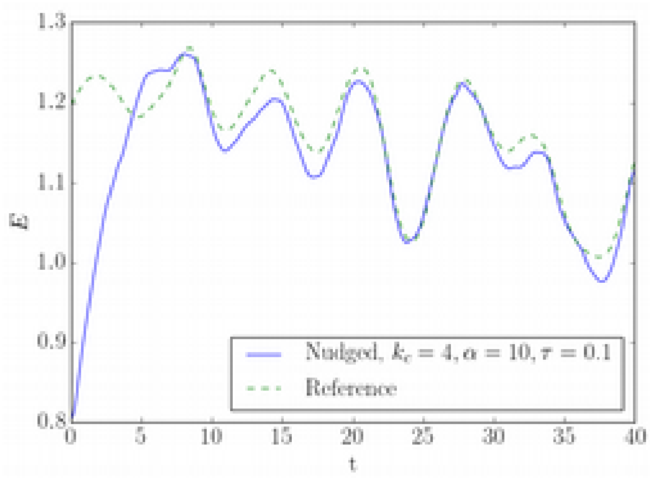
Temporal interpolation

How nudging looks like

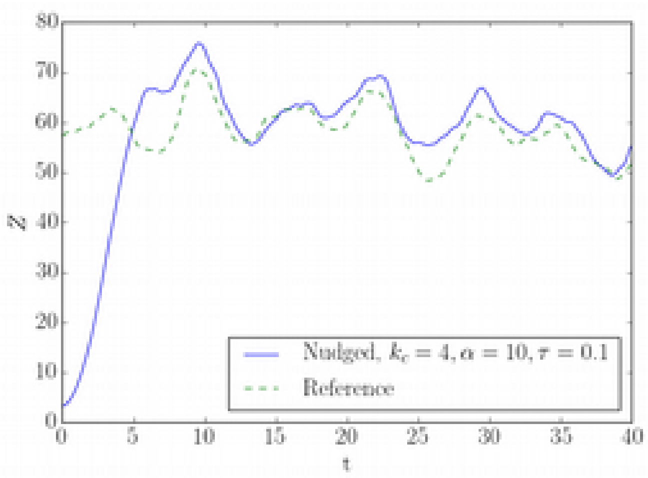
Velocity evolution



Energy

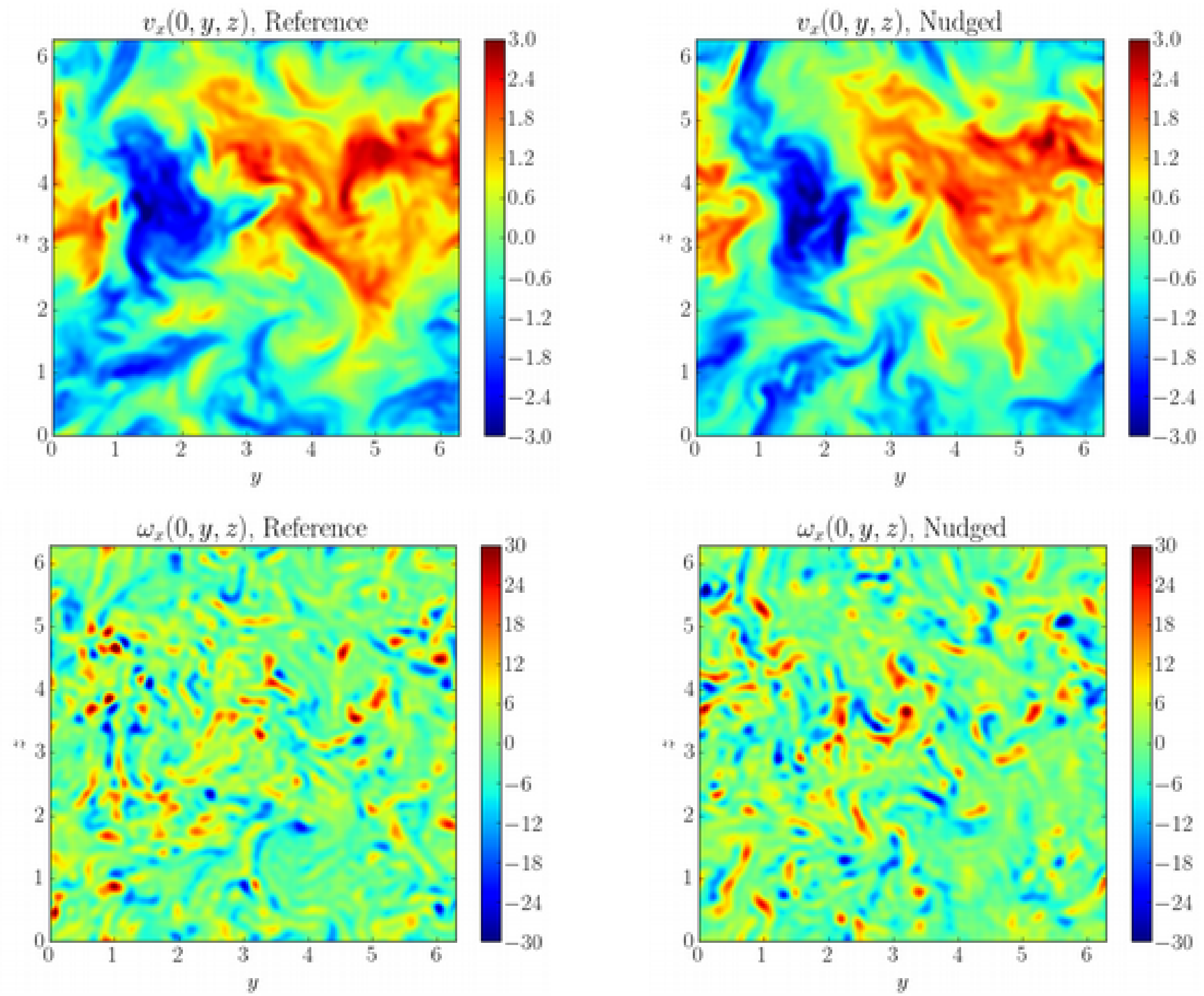


Enstrophy



Easier to control the larger scales (energy) than the smaller ones (enstrophy)

How nudging looks like

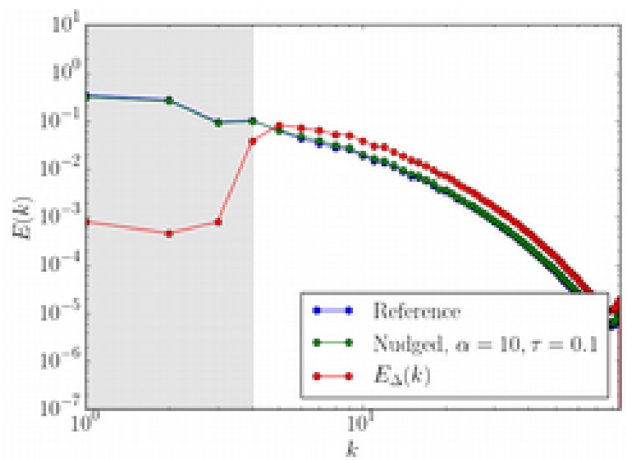


Easier to control the larger scales (velocity) than the smaller ones (vorticity)

Comparing the simulations

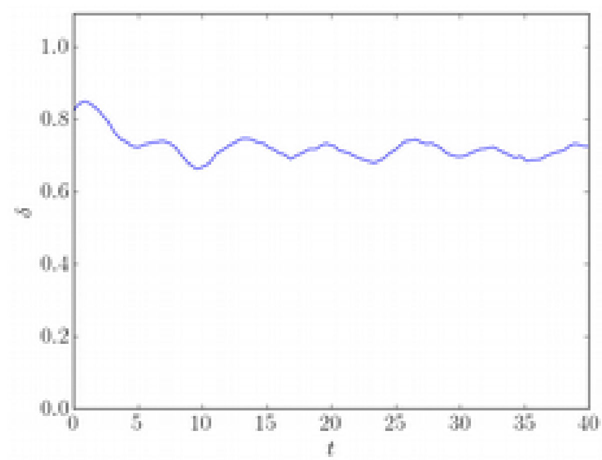
What should we analyze to determine if the nudging is working?
And if so, how well it is doing it?

Spectrum of differences



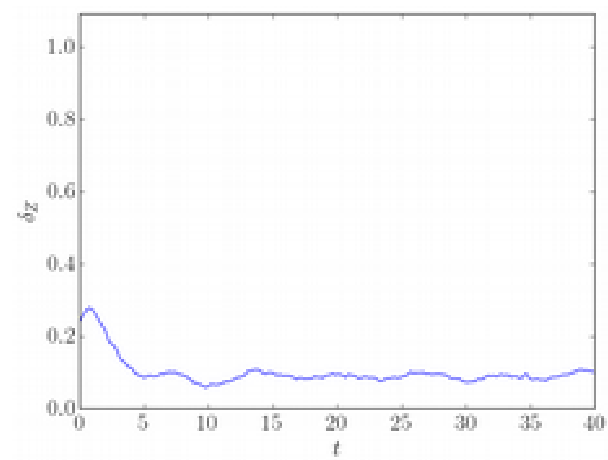
$$E_\Delta(k) = \sum_{|k|=k} (\hat{u} - \hat{u}_{\mathrm{ref}})^2(k)$$

Velocity correlations



$$\delta = \frac{\langle u \cdot u_{\mathrm{ref}} \rangle}{\sqrt{\langle u^2 \rangle \langle u_{\mathrm{ref}}^2 \rangle}}$$

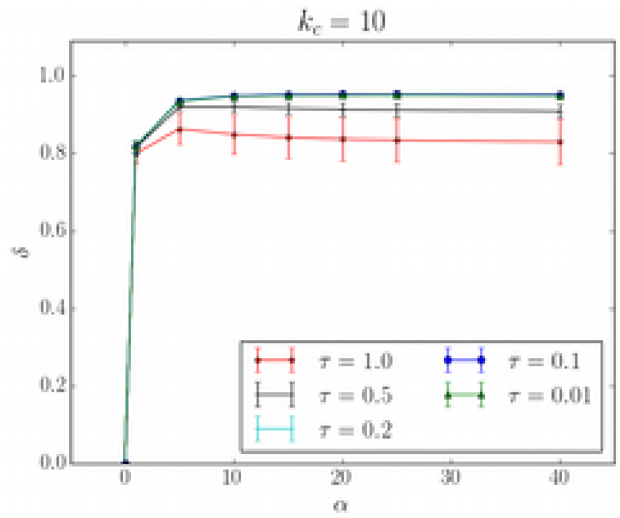
Vorticity correlations

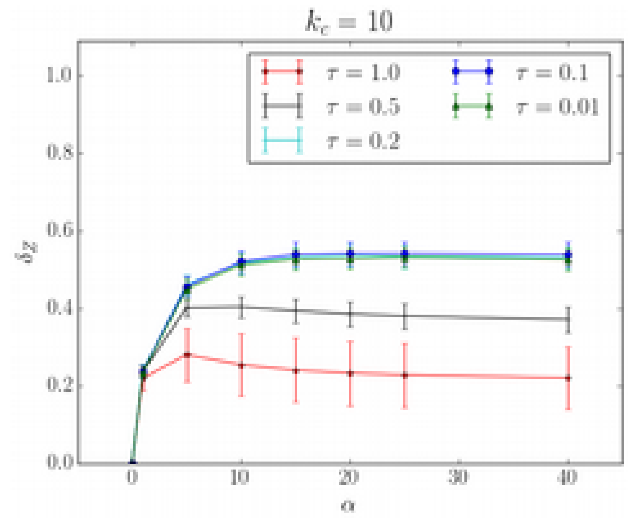


$$\delta_Z = \frac{\langle \omega \cdot \omega_{\mathrm{ref}} \rangle}{\sqrt{\langle \omega_u^2 \rangle \langle \omega_{\mathrm{ref}}^2 \rangle}}$$

Parameter scan

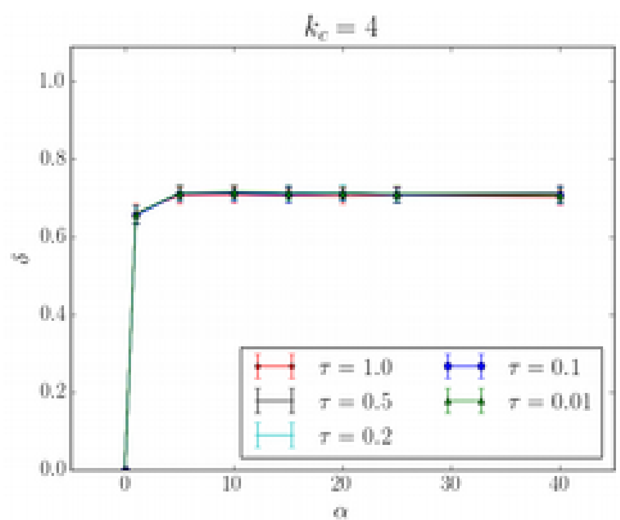
Effects of varying parameters in simulations with N=256, $k_{\max} = 85$

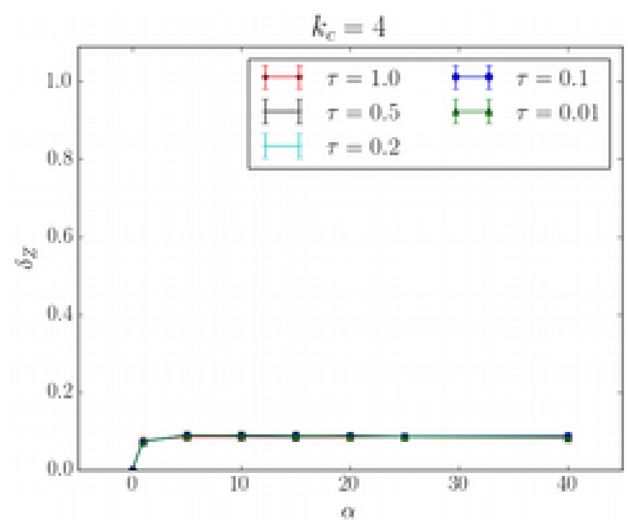




There's a critical τ that coincides with decorrelation time of the mode $\hat{u}(k = 10)$

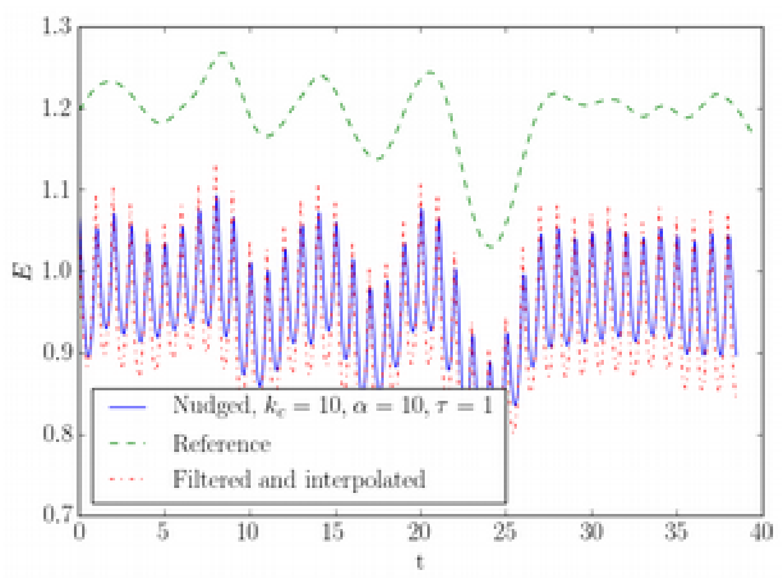
Increasing α to values larger than $1/\tau$ doesn't affect much

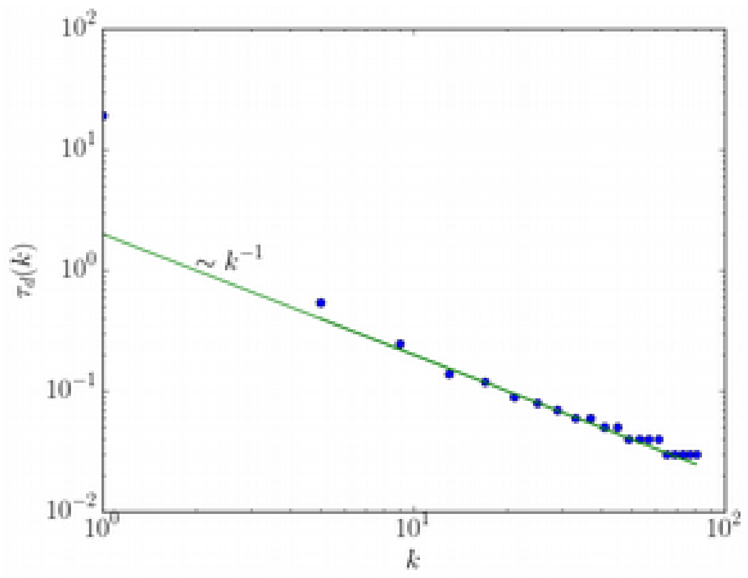




The general behaviour is replicated at different k_c

Choosing a bad interpolation time





Comes from doing $u^2 = (au_1 + bu_2)^2$
with $u_1 \cdot u_2 = 0$

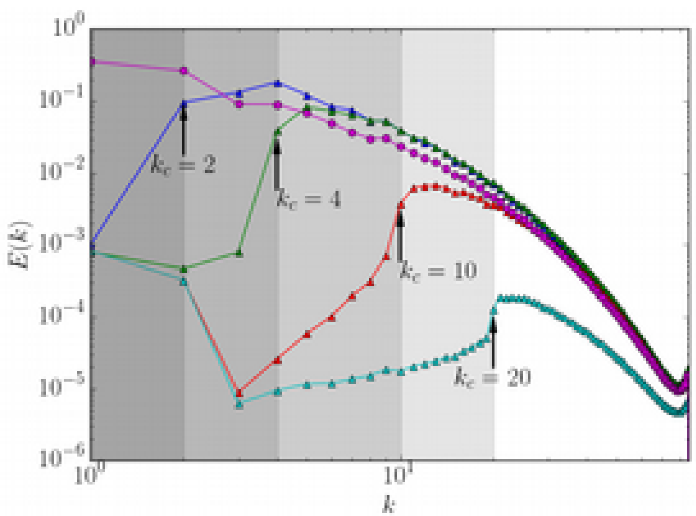
$$a + b = 1$$

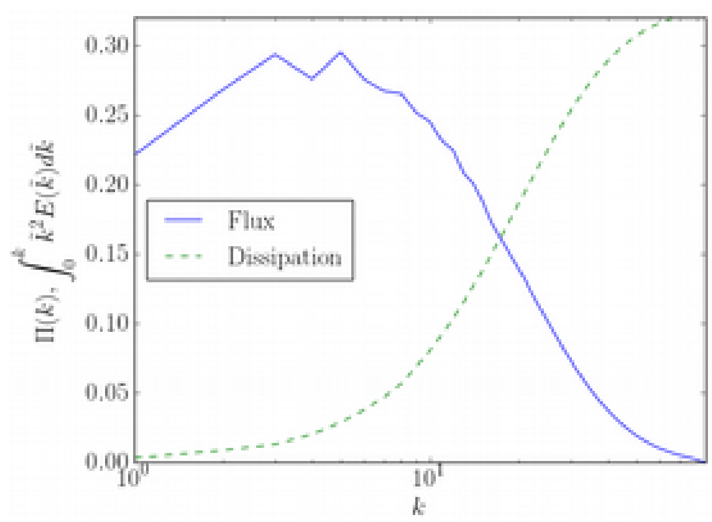
Decorrelation times for each fourier mode obtained from the autocorrelation functions $\langle \hat{u}^*(k, t + t') \cdot \hat{u}(k, t) \rangle_t$

It is important to choose an interpolation time smaller than the decorrelation time of the smallest scale you are nudging

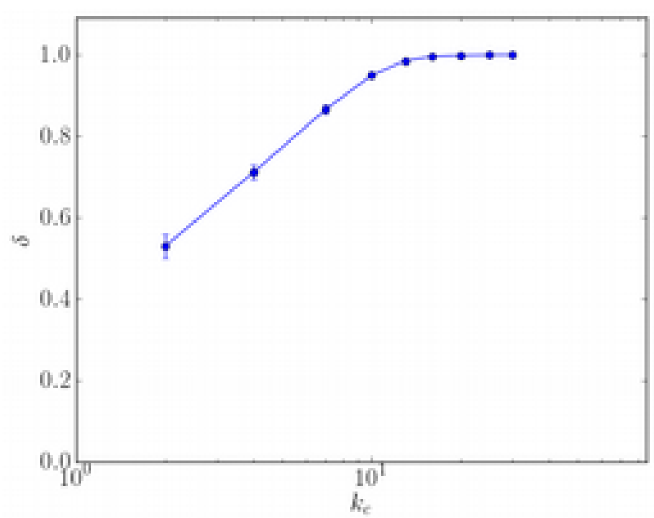
The decorrelation time in homogeneous isotropic turbulence is determined by the sweeping time

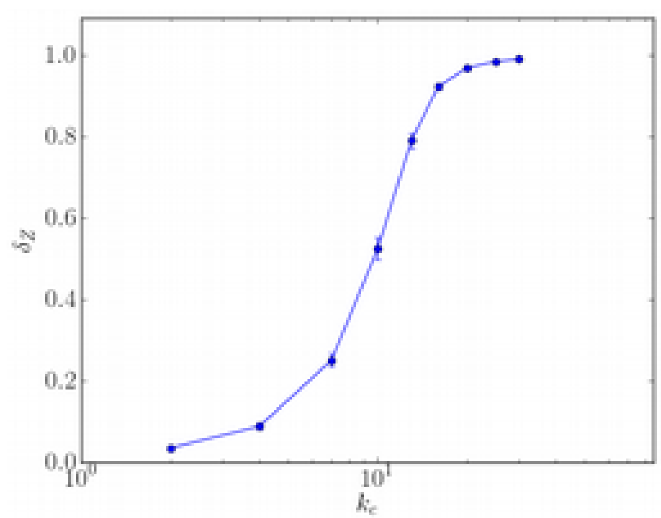
Nudging at different scales





In order to achieve good correlation in the large and the small scales one needs to nudge up to the scales when dissipation starts becoming important



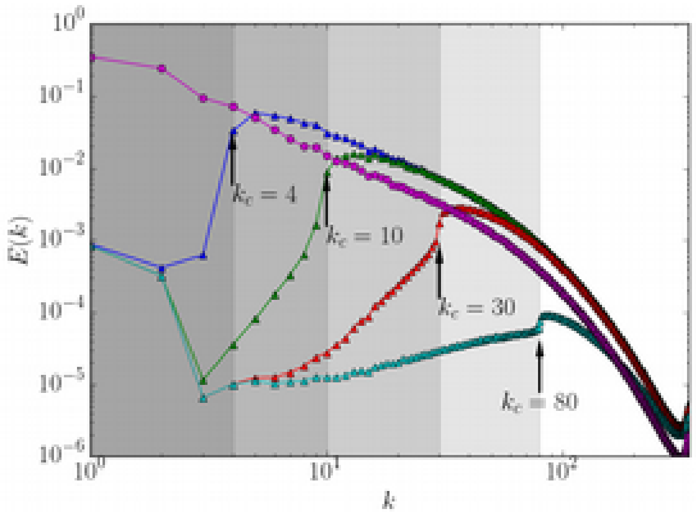


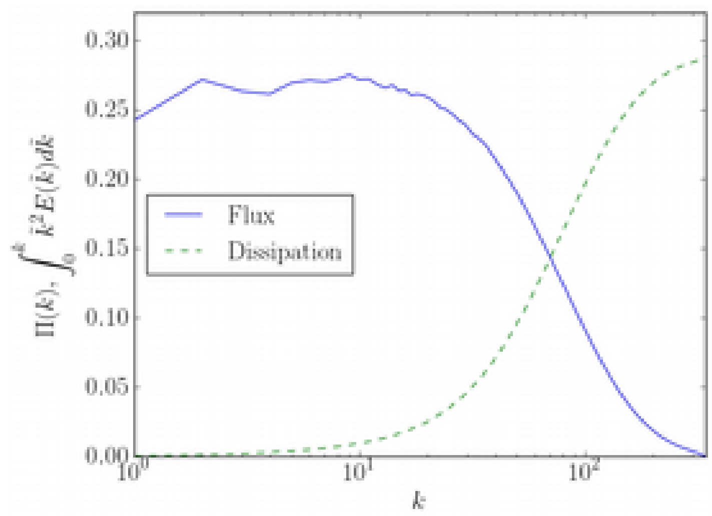
Note that the number of nudged modes is still smal compared to the total

$$\left(\frac{17}{81}\right)^3 \approx 0.01$$

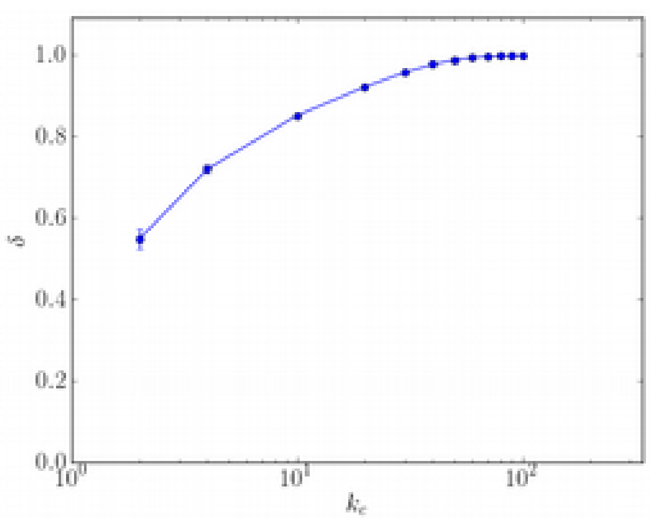
Increasing Reynolds number

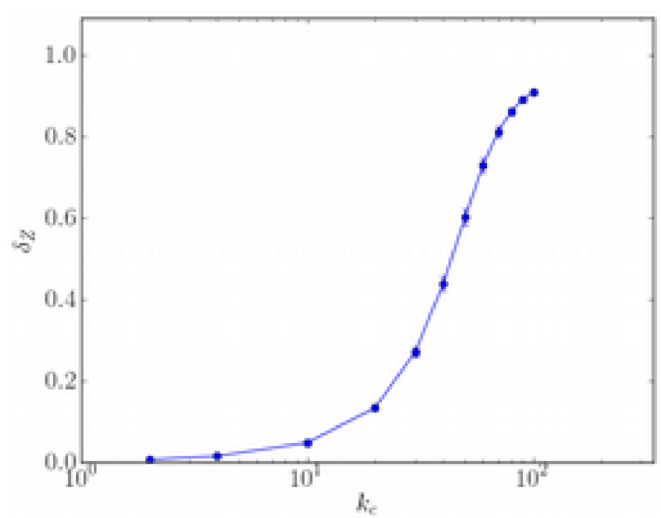
Simulations with N=1024, $k_{\max} = 341$





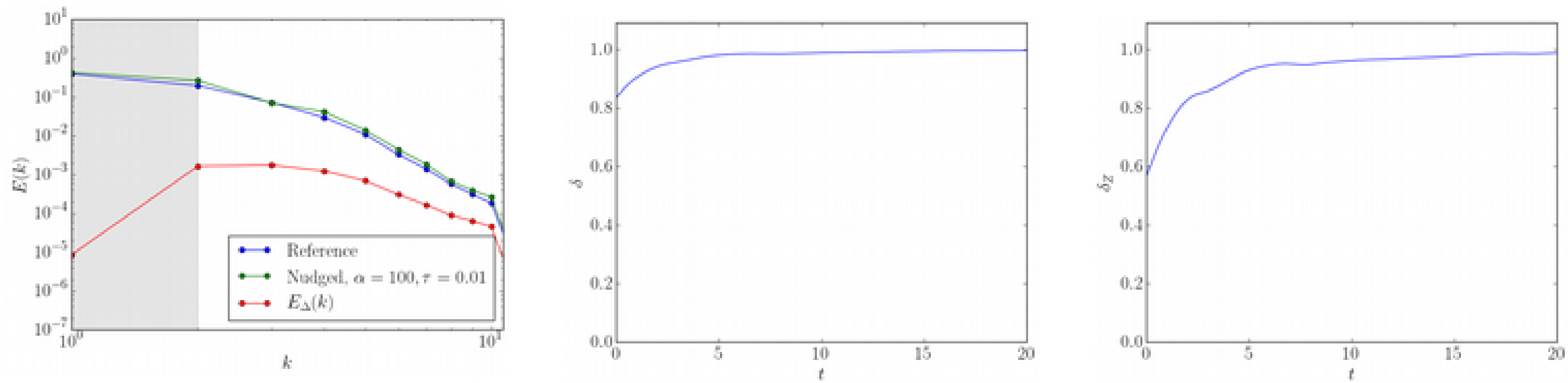
Increasing Reynolds number does not make things any easier





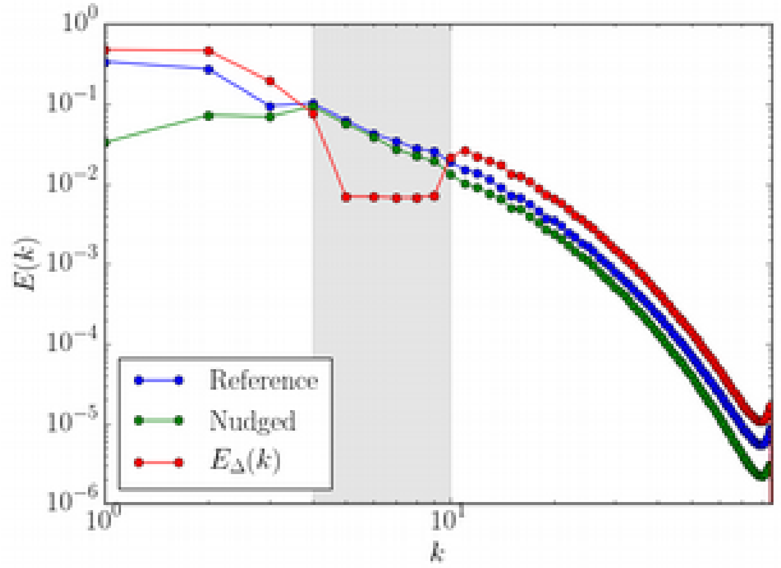
Decreasing Reynolds number

Simulation performed at N=32, $k_{\max} = 10$



At low Reynolds number, where there is spatiotemporal chaos, but not a real turbulent scaling it is easier to control the flow

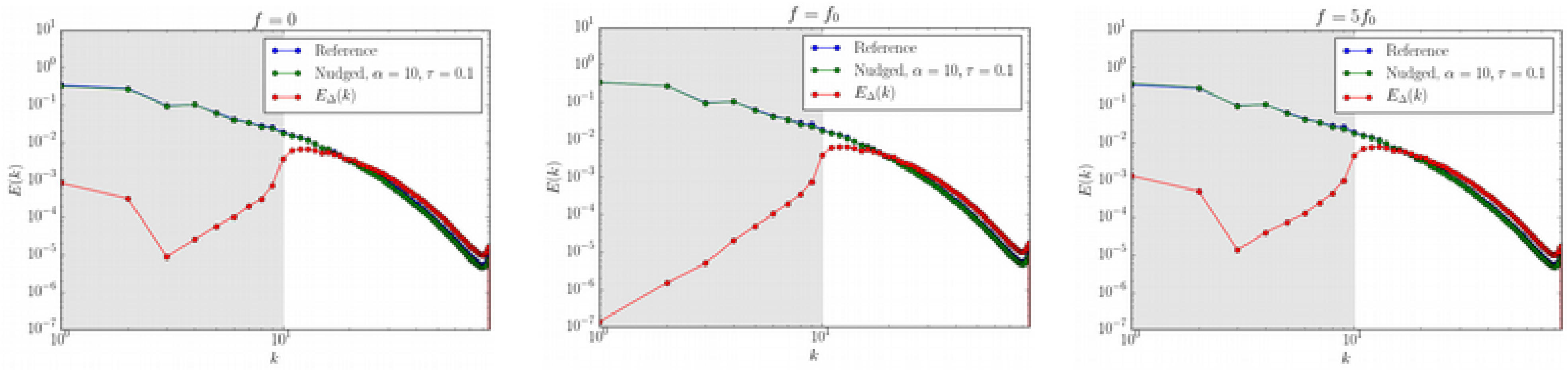
Nudging on bands



Although the total number of nudged modes is high, only those modes are properly affected.

Matching the equations

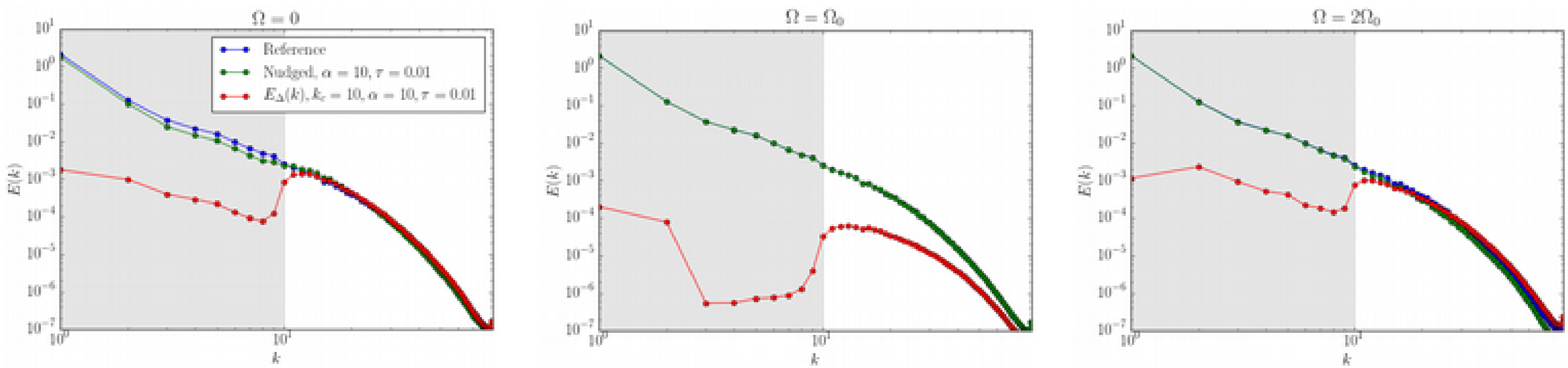
We now add a forcing term to the nudged equations of the same form as the forcing of the refence, but with varying amplitude

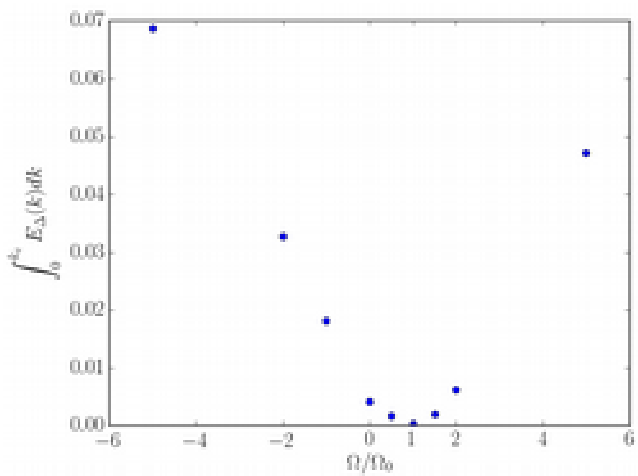


The results are sensitve to the correct value of the forcing amplitude

Finding out parameters

We now add a Coriolis term $\Omega \times u$ to both the reference and the nudged





We can us the nudging to find Ω_0, the rotation frequency of the refence simulation

This can be easily applied to other kind of terms and forces

Conclusions

- We explored how the spectral nudging technique can be applied to fully developed three dimensional turbulence
- We presented criteria for the relevant parameters based on physical arguments
- Controlling multiscale turbulent flows requires a lot of information, but not all of it
- The nudging algorithm can be used to find out parameters from the data

Thank you!