Taming turbulence via spectral nudging

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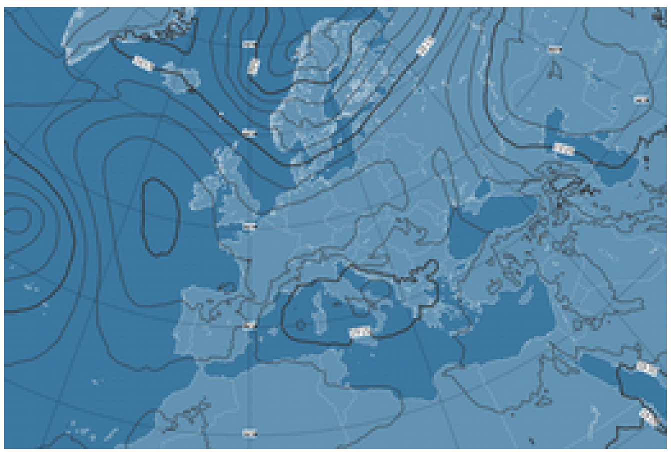
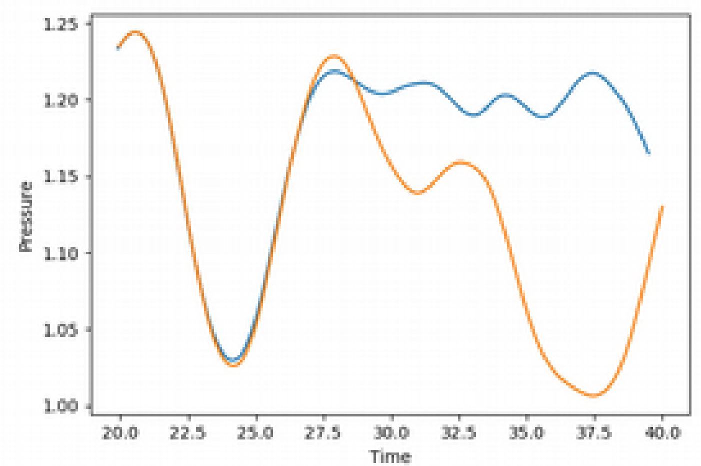
Data assimilation

Arises out of the need to control the evolution of a chaotic system that starts with imperfect initial conditions

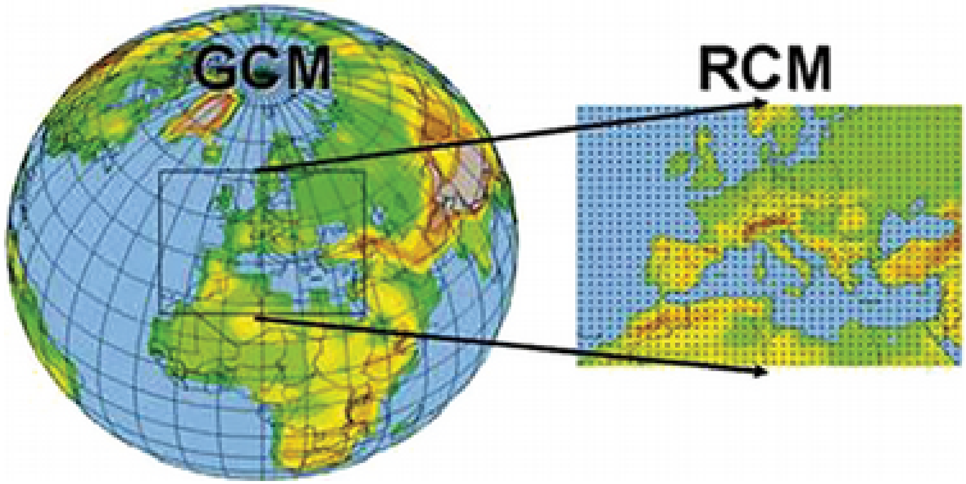




The idea is to incorporate data (measurements, observations…) into the dynamical evolution in order to “correct” it

Spectral nudging

Technique used to go from Global to Regional model [Waldron et al, (1996)] or to incorporate data from measurements [Azouani et al (2013)].



Global Circulation Model

Regional Climate Model

Consists of controlling the evolution of certain scales in the system by introducing spatially and temporally filtered information.

Spectral nudging experiment

Reference simulation

$$\frac{\partial u_{\text{ref}}}{\partial t} + (u_{\text{ref}} \cdot \nabla) u_{\text{ref}} = -\nabla p + \nu \nabla^2 u_{\text{ref}} + f$$

Constant in time, isotropic, and acting on $k = [1, 2]$

Nudged simulation

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\nabla p + \nu \nabla^2 u - \alpha (u^< - u_{\text{ref}}^{<,\tau})$$

Easy to implement in a pseudo-spectral code with periodic boundary conditions and $k_{\max} = N/3$

$$u(x) = \sum_{-k_{\max}}^{k_{\max}} \hat{u}(k)^{-ik\cdot x} \quad u^<(x) = \sum_{-k_c}^{k_c} u_k e^{-ik\cdot x}, \; k_c < k_{\max}$$

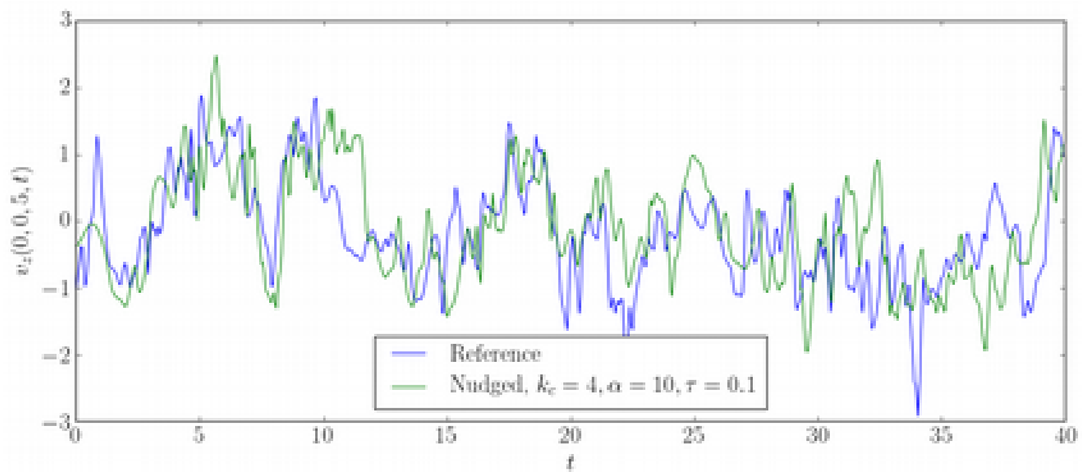
Spatial filtering

$$u_{\text{ref}}^{\tau} = \frac{\tau - t}{\tau} u_1 + \frac{t}{\tau} u_2$$

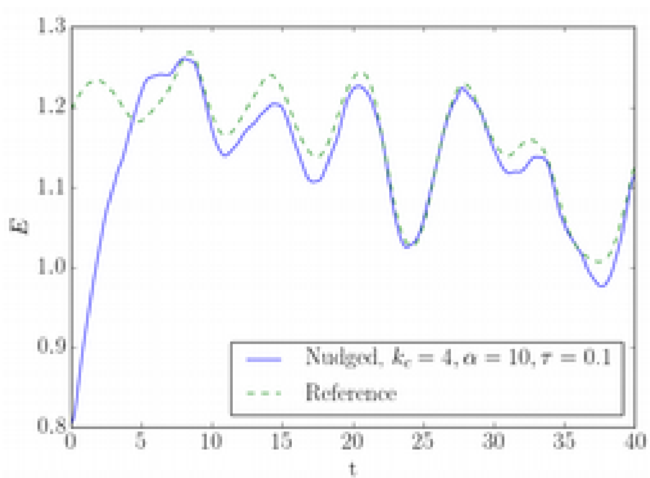
Temporal interpolation

How nudging looks like

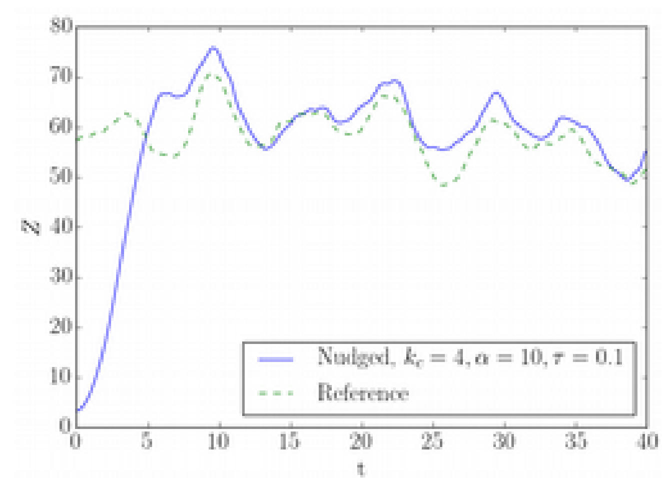
Velocity evolution



Energy



Enstrophy

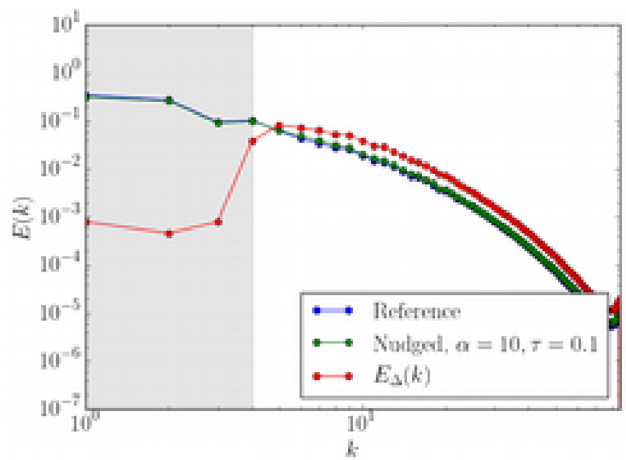


Easier to control the larger scales (energy) than the smaller ones (enstrophy)

Comparing the simulations

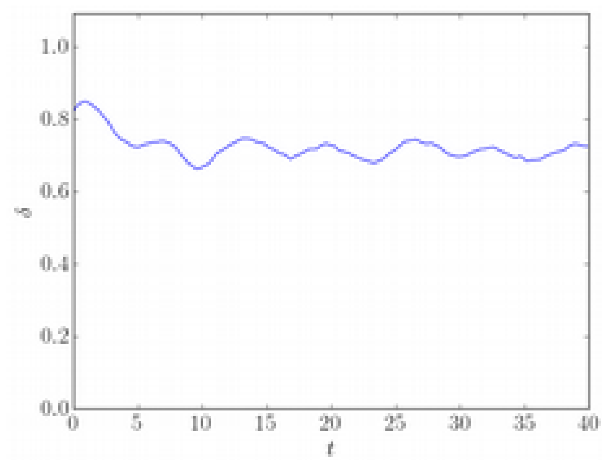
In our numerical set-up we can access the smaller scales of the reference flow. This way we can compare with the “truth”.

Spectrum of differences



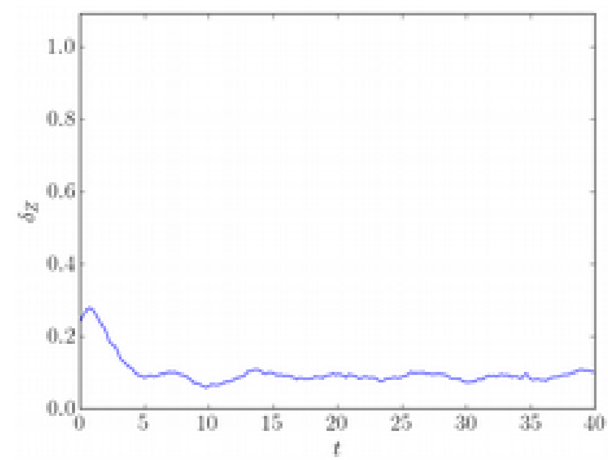
$$E_\Delta(k) = \sum_{|k|=k} |\hat{u} - \hat{u}_{\mathrm{ref}}|^2(k)$$

Velocity correlations



$$\delta = \frac{\langle u \cdot u_{\mathrm{ref}} \rangle}{\sqrt{\langle u^2 \rangle \langle u_{\mathrm{ref}}^2 \rangle}}$$

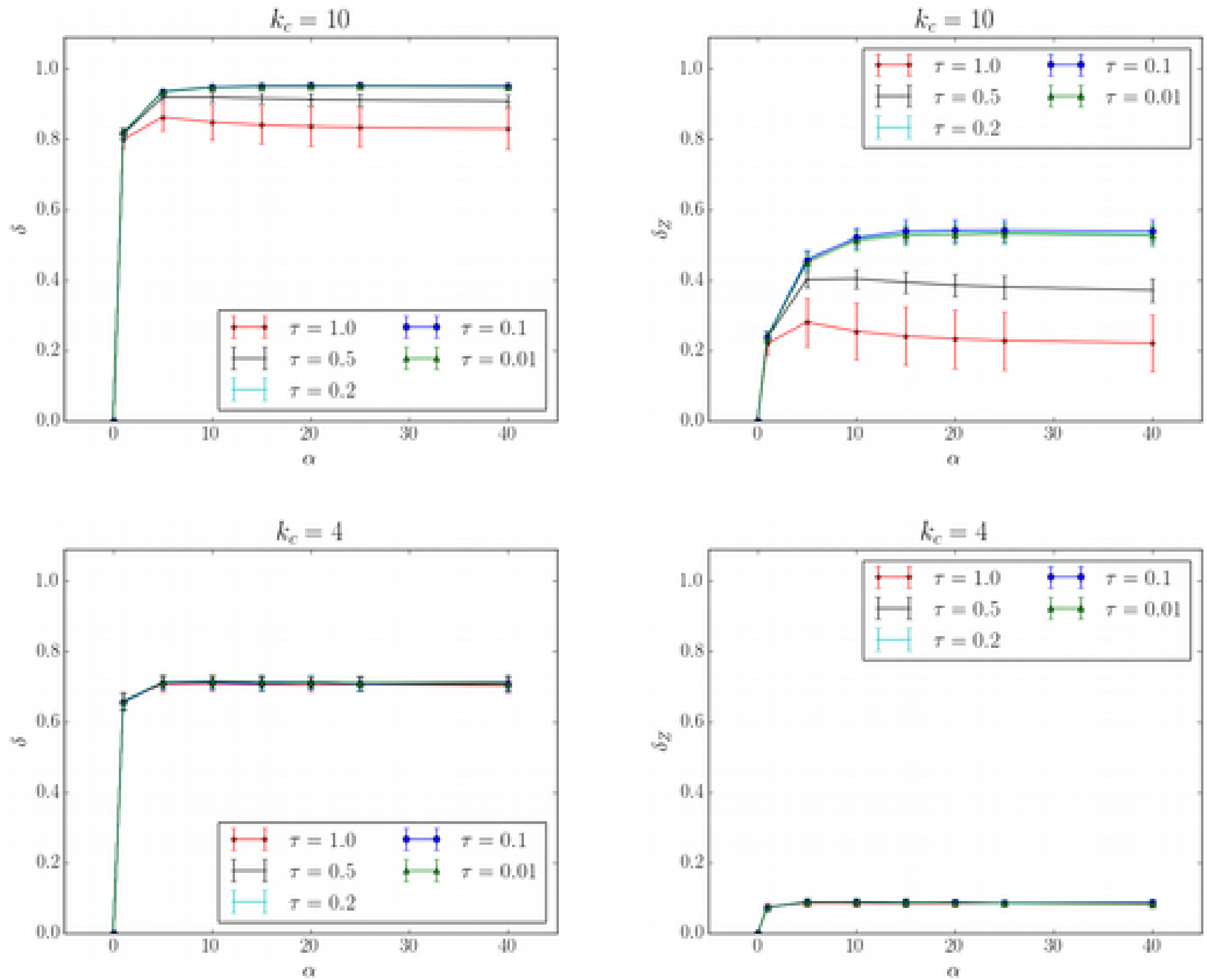
Vorticity correlations



$$\delta_Z = \frac{\langle \omega \cdot \omega_{\mathrm{ref}} \rangle}{\sqrt{\langle \omega_u^2 \rangle \langle \omega_{\mathrm{ref}}^2 \rangle}}$$

Parameter scan

Effects of varying parameters in simulations with N=256, $k_{\max} = 85, \ \mathrm{Re} \approx 400$

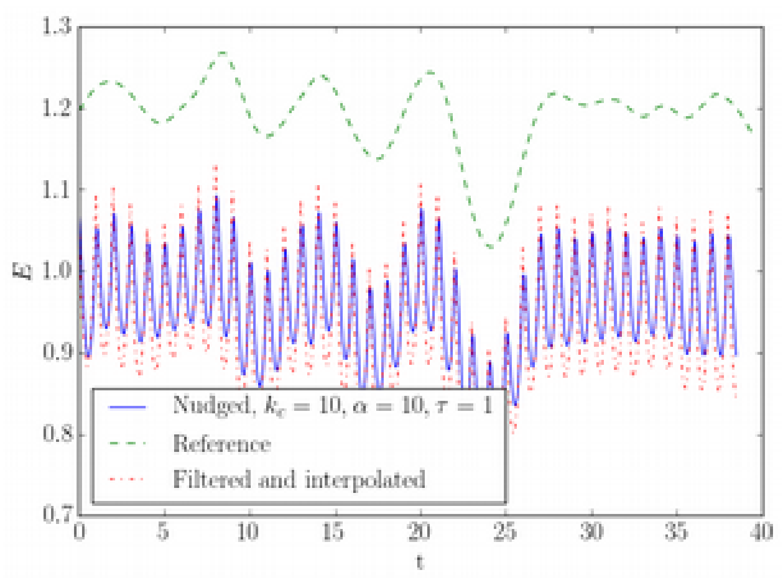


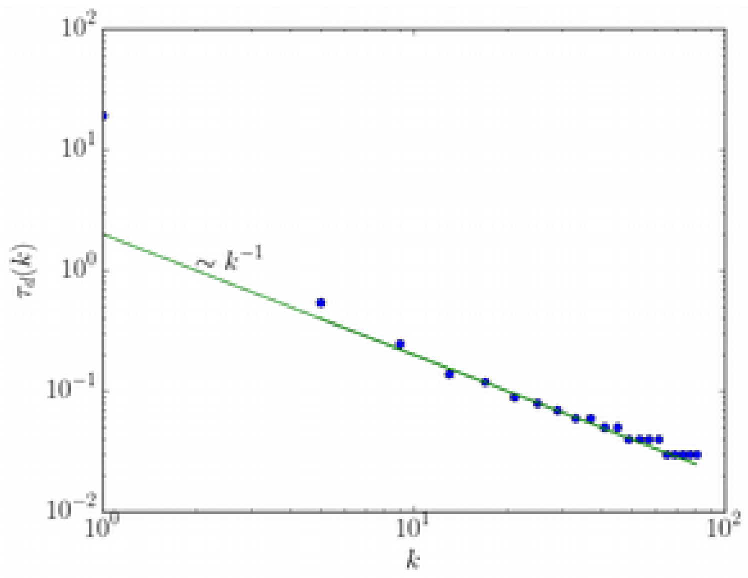
There's a critical τ that coincides with decorrelation time of the mode $\hat{u}(k = 10)$

Increasing α to values larger than $1/\tau$ doesn't affect much

The general behaviour is replicated at different k_c

Choosing a bad interpolation time





Comes from doing $u^2 = (au_1 + bu_2)^2$ with $u_1 \cdot u_2 = 0$

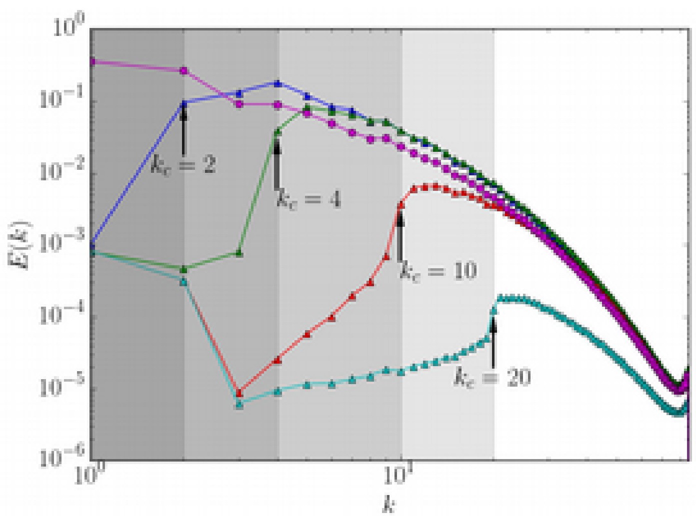
$$a + b = 1$$

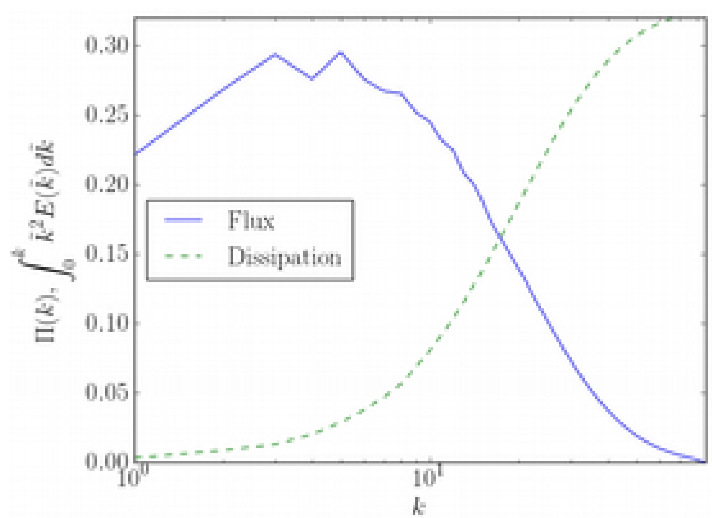
Decorrelation times for each fourier mode obtained from the autocorrelation functions $\langle \hat{u}^*(k, t + t') \cdot \hat{u}(k, t) \rangle_t$

It is important to choose an interpolation time smaller than the decorrelation time of the smallest scale you are nudging

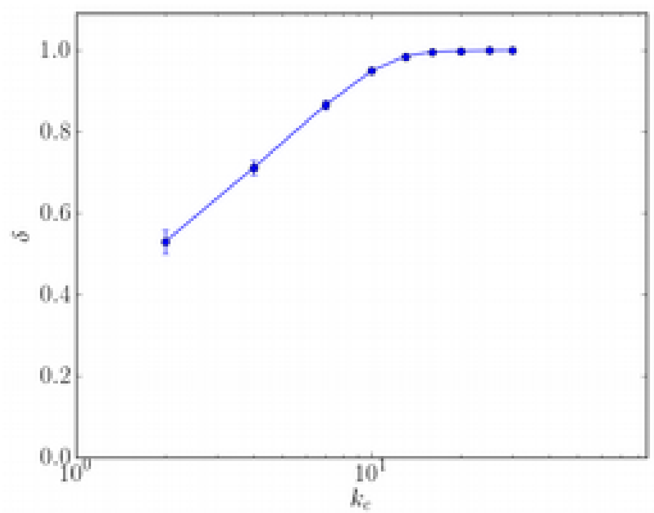
The decorrelation time in homogeneous isotropic turbulence is determined by the sweeping time [Chen & Kraichnan (1989)]

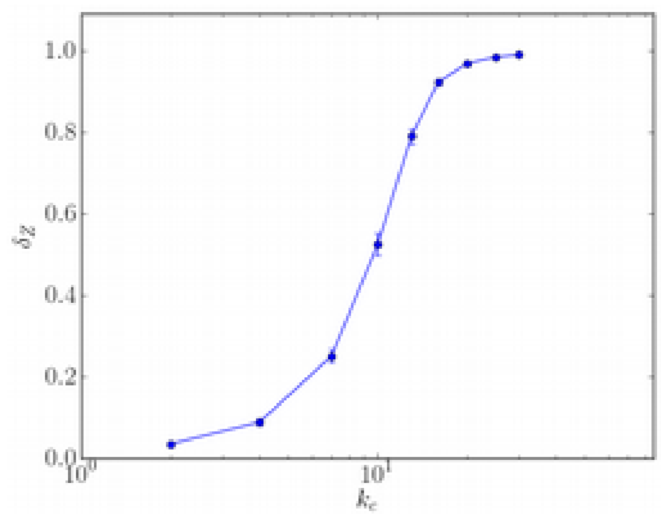
Nudging at different scales





In order to achieve good correlation in the large and the small scales one needs to nudge up to the scales when dissipation starts becoming important





Note that the number of nudged modes is still smal compared to the total

$$\left(\frac{17}{81}\right)^3 \approx 0.01$$

Spectral nudging as a physics based parameter estimator

Extracting information from a turbulent flow, or picking parameters for a closure model is not an easy task and a variety of tools are available to do this.

Spectral nudging tries to force correlations between the reference flow and the nudged flow. This can be enhanced or diminished in the presence of extra terms. Can we use this to find out the reference flow's parameters?

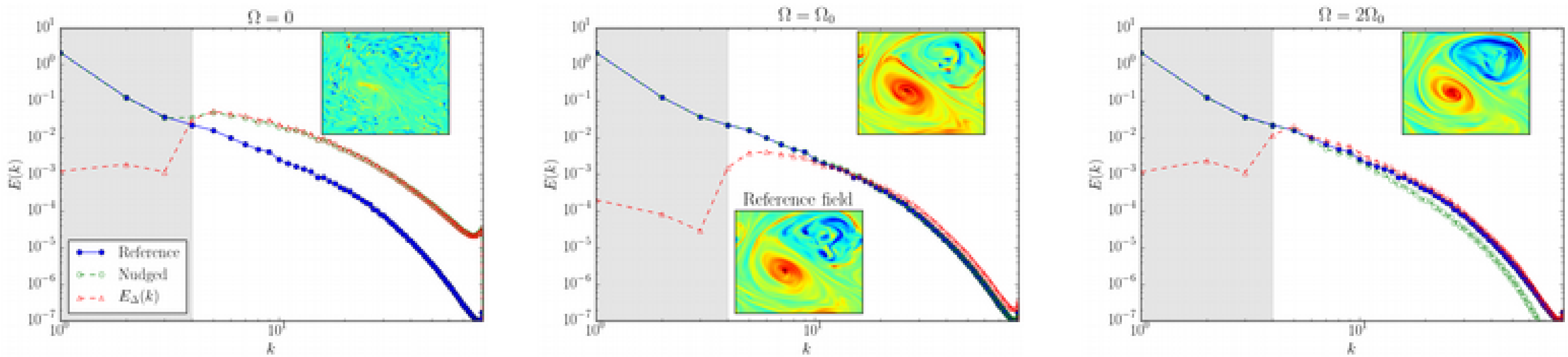
We put this idea to test with a rotating turbulence flow:

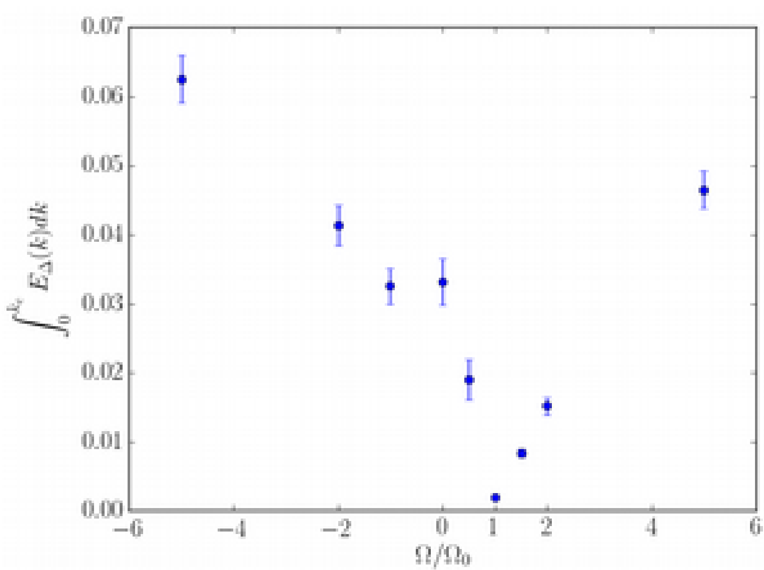
$$\frac{\partial u_{\mathrm{ref}}}{\partial t} + (u_{\mathrm{ref}} \cdot \nabla) u_{\mathrm{ref}} + 2\Omega_0 \times u_{\mathrm{ref}} = -\nabla p + \nu \nabla^2 u_{\mathrm{ref}} + f$$

We can run different nudged simulations varying the rotation frequency Ω

Finding out parameters

Nudged simulations done varying the rotation frequency Ω
The reference has a frequency of Ω_0





We can us the nudging to find Ω_0, the rotation frequency of the refence simulation

This can be easily applied to other kind of terms and forces

Conclusions

- We explored how the spectral nudging technique can be applied to fully developed three dimensional turbulence
- We presented criteria for the relevant parameters based on physical arguments
- Controlling multiscale turbulent flows requires a lot of information, but not all of it
- The nudging algorithm can be used to find out parameters from the reference data. The method is physics based and easily applicable to different flows.

Thank you!