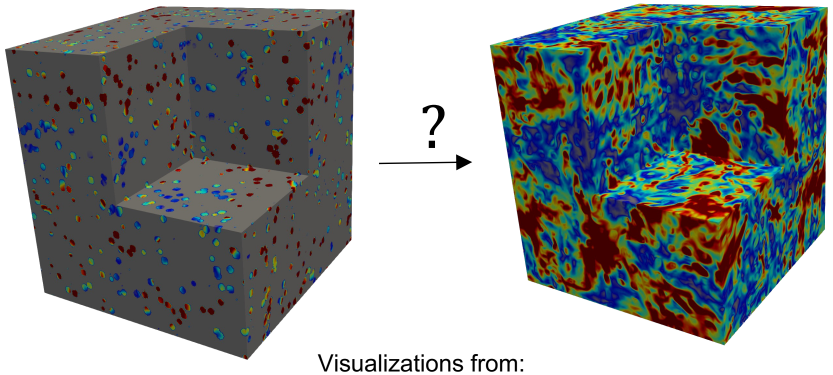
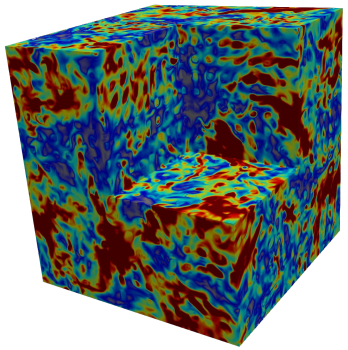
INFERRING TURBULENT PARAMETERS VIA MACHINE LEARNING

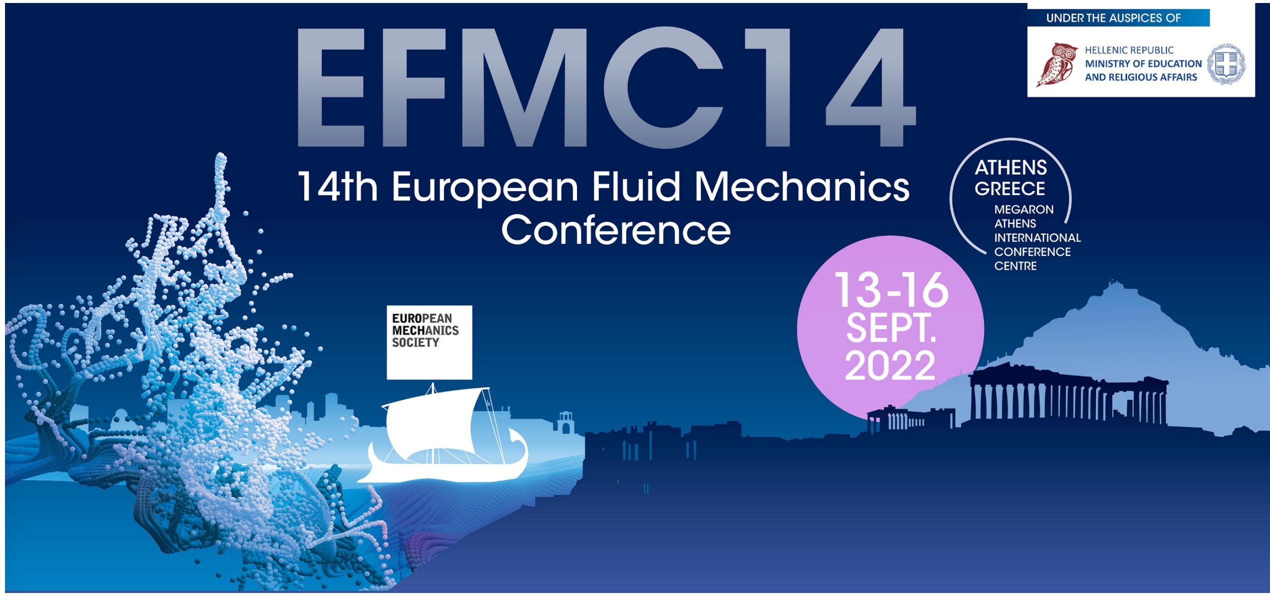


Visualizations from:

Di Leoni, P. C., Mazzino, A., & Biferale, L. (2020). Physical Review X, 10(1), 011023.



? $\longrightarrow$ $\dfrac{d\,\boldsymbol{u}}{dt}$



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CREDITS: L. Biferale (Uni. Tor Vergata, IT), **F. Bonaccorso** (Uni. Tor Vergata, IT), **P. Clark di Leoni** (Uni. of Buenos Aires, ARG)

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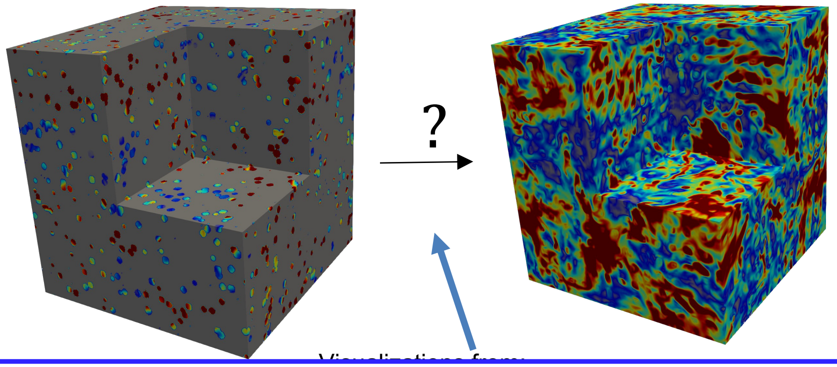
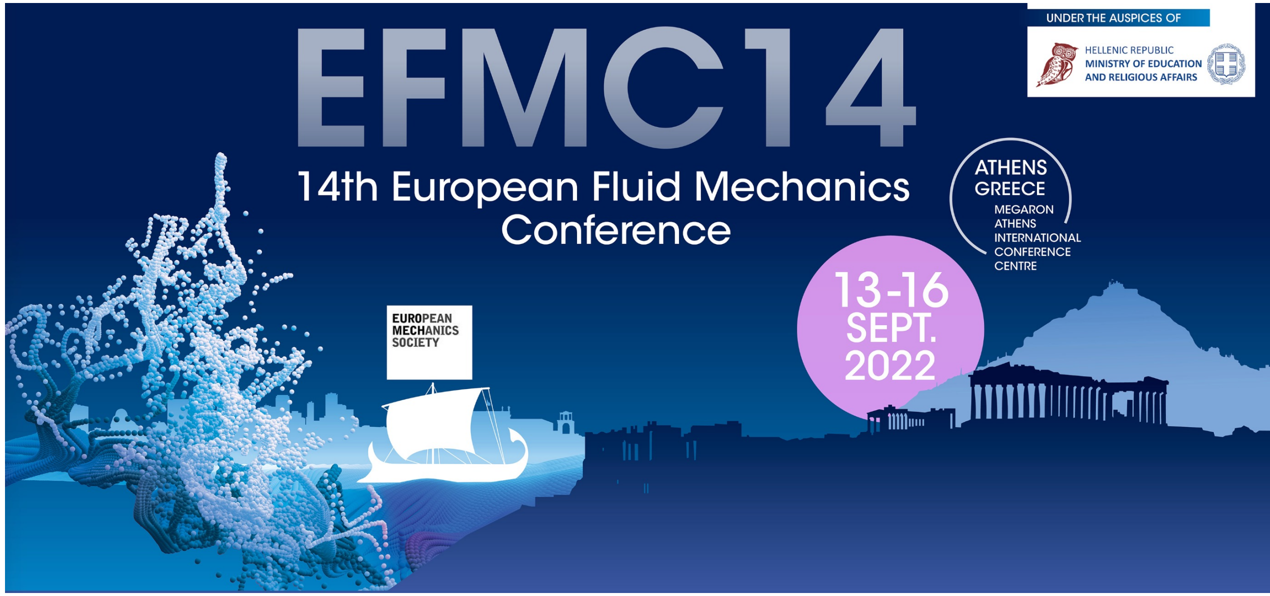
INFERRING TURBULENT PARAMETERS VIA MACHINE LEARNING





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Tor Vergata, IT),
ata, IT), **P. Clark di**
Aires, ARG)

Data reconstruction of turbulent flows with Gappy POD and Generative Adversarial Networks T. Li, M. Buzzicotti, F. Bonaccorso, L. Biferale, M. Wan and S. Chen (Submitted to JOT)

Reconstructing Rayleigh–Bénard flows out of temperature-only measurements using nudging L Agasthya, P Clark Di Leoni, L Biferale Physics of Fluids 34 (1), 015128

Reconstruction of turbulent data with deep generative models for semantic inpainting from TURB-Rot database M Buzzicotti, F Bonaccorso, PC Di Leoni, L Biferale Physical Review Fluids 6 (5), 050503

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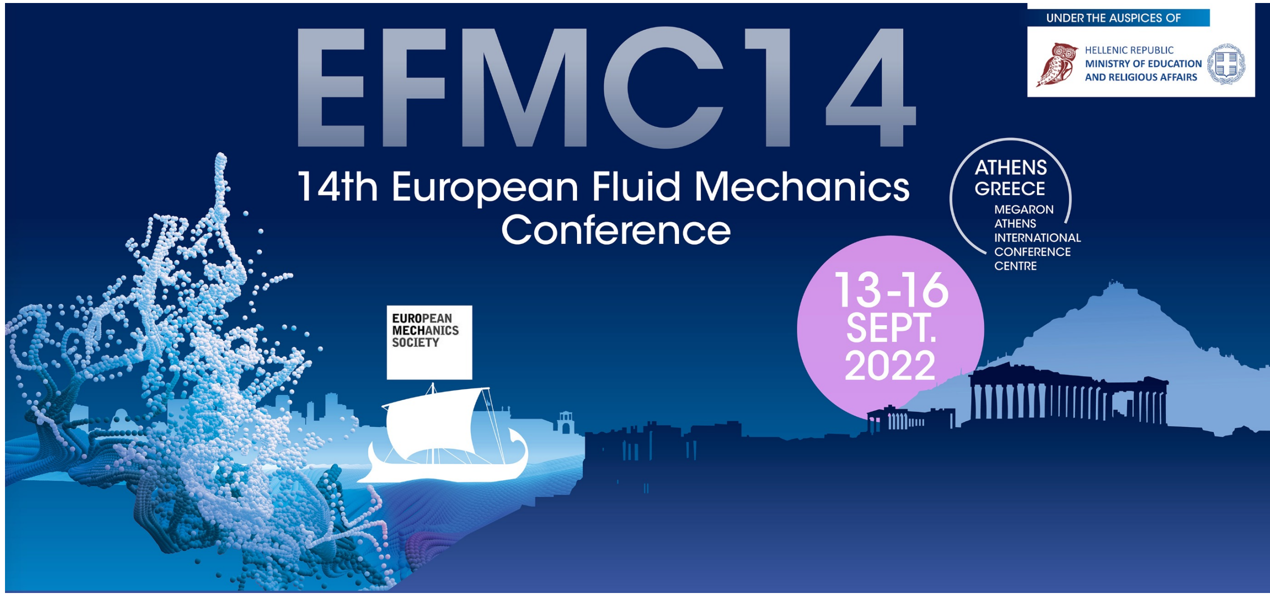

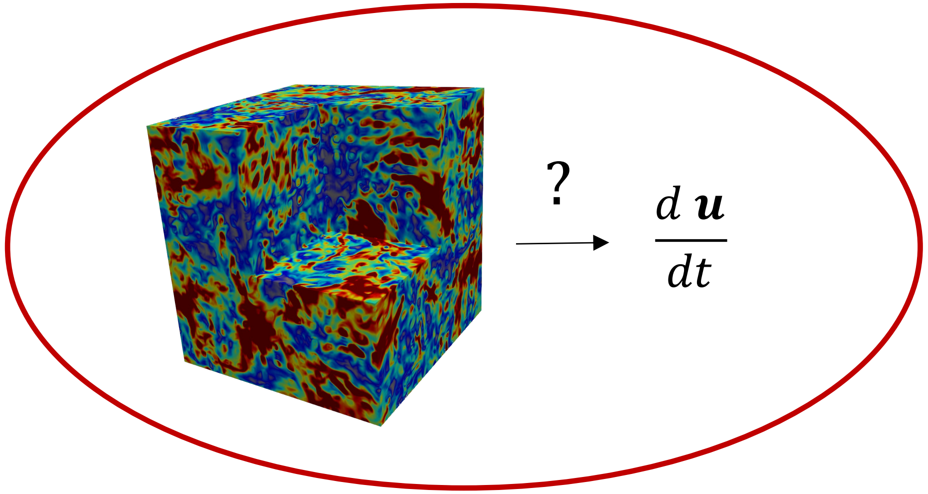

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Can we improve our data-analysis capability to refine our physical knowledge of complex systems?

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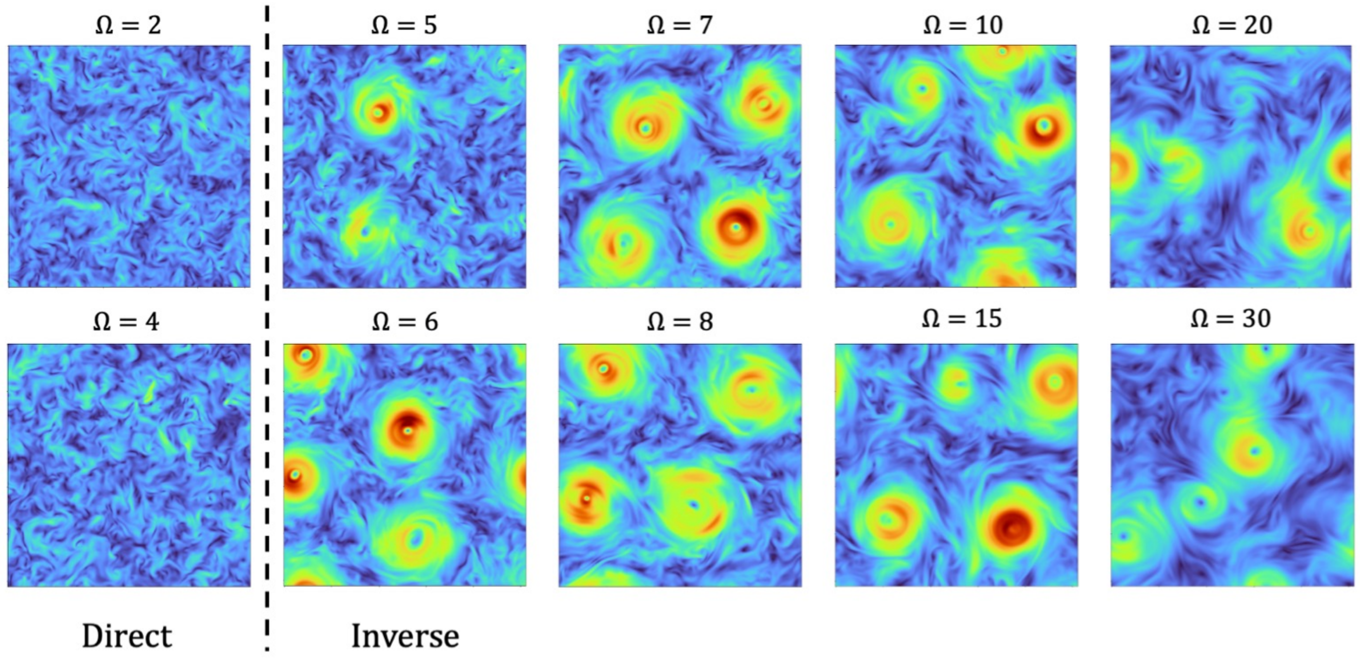
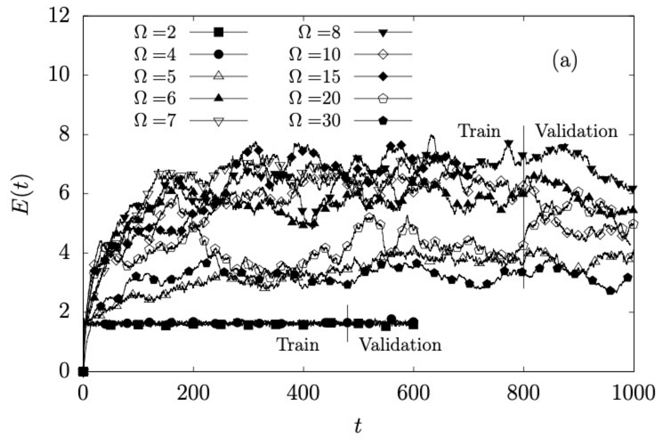
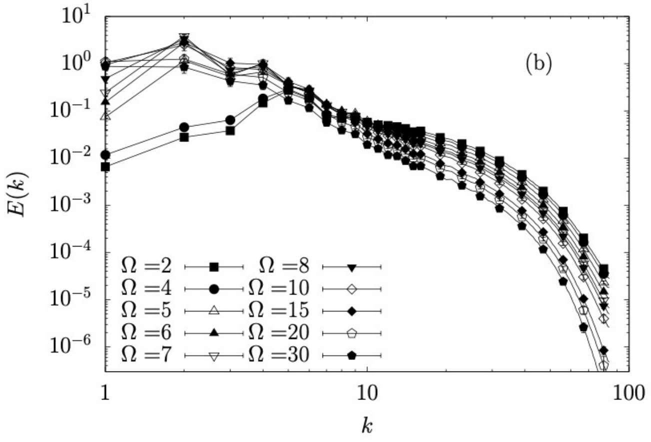
Problem: Inferring the rotation rate

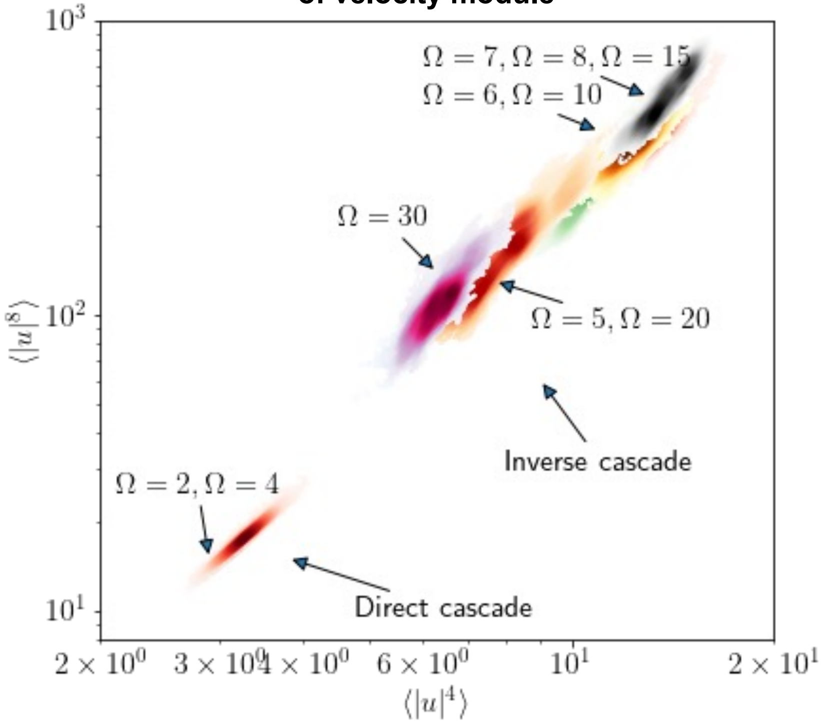
$$\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \partial_x \boldsymbol{v} + \partial_x P - \nu \Delta \boldsymbol{v} = 2\,\boldsymbol{v} \times \boldsymbol{\Omega} + \mathbf{f}$$







Scatterplot of **8th vs 4th order moments of velocity module**



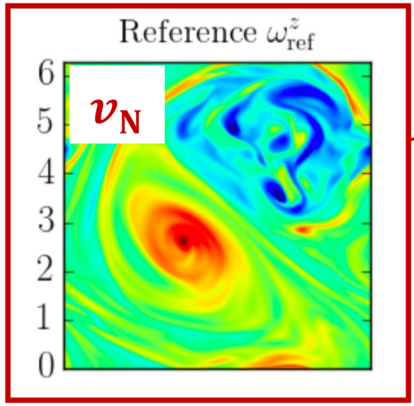
How can we extract useful information from one single velocity plane?

-] Alexakis, A., & Biferale, L. (2018). Cascades and transitions in turbulent flows. *Physics Reports*, *767*, 1-101.

-] Di Leoni, P. Clark, Alexandros Alexakis, L. Biferale, and MB. "Phase transitions and flux-loop metastable states in rotating turbulence." *Physical Review Fluids* 5, no. 10 (2020): 104603.

NUDGING: AN **EQUATION-INFORMED** TOOL TO INFER THE PHYSICS

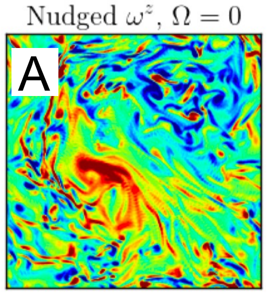
Reference DNS:

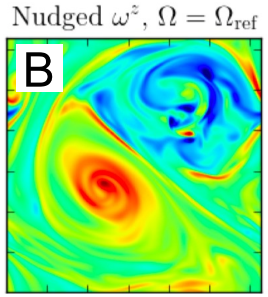


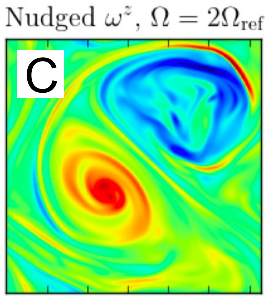
Nudging Simulation:

DNS with the addition of a drag term against partial field measurements

$$\partial_t \boldsymbol{v} + \boldsymbol{v}\cdot\partial_x \boldsymbol{v} + \partial_x P - \nu\Delta\boldsymbol{v} = 2\,\boldsymbol{v}\times\boldsymbol{\Omega} + \mathbf{f} - N(\boldsymbol{v}_N - \boldsymbol{v})$$

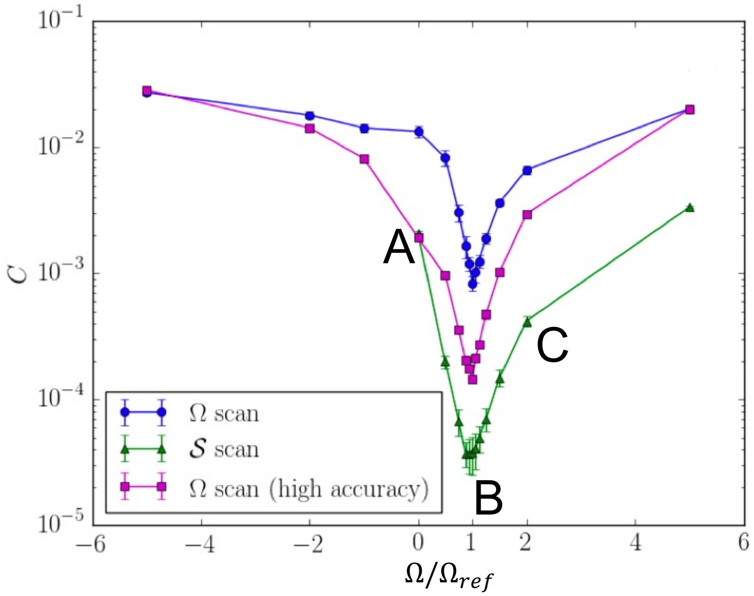






$$E_\Delta(k) = \int_{|\boldsymbol{k}|=k} |\hat{u}(\boldsymbol{k}) - \hat{u}_N(\boldsymbol{k})|^2\, d\boldsymbol{k}$$

$$C = \int_0^{k_c} E_\Delta(k)dk$$



Shows strong sensitivity to the physical parameter used in the nudged simulation

Di Leoni, P. C., Mazzino, A., & Biferale, L. (2018). Inferring flow parameters and turbulent configuration with physics-informed data assimilation and spectral nudging. *Physical Review Fluids*, *3*(10), 104604.

NUDGING: TO OPTIMIZE SUBGRID SCALE MODELS

Nudging the Large Eddy Simulation:

Filtered solution from fully resolved DNS:

$$\partial_t \widetilde{\boldsymbol{v}} + \widetilde{\boldsymbol{v}} \cdot \nabla \widetilde{\boldsymbol{v}} = -\boldsymbol{\nabla} \tilde{p} + \nu \Delta \widetilde{\boldsymbol{v}} - \nabla \cdot \boldsymbol{\tau} + \mathbf{f} - N(\widetilde{\boldsymbol{v}}_N - \widetilde{\boldsymbol{v}})$$

$$\tilde{v}_i = \mathcal{F}(v_i) \qquad \tilde{p} = \mathcal{F}(p) \qquad \tau_{ij} = \mathcal{F}(v_i v_j) - \mathcal{F}(v_i)\,\mathcal{F}(v_j)$$

Filtered fields

Smagorinsky closure

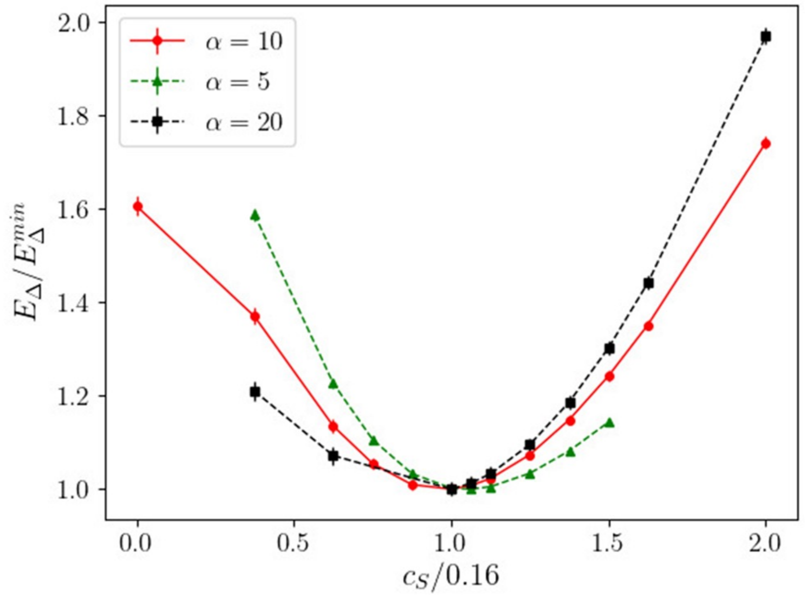
$-2\nu_S\, S_{ij}$, with:

$$S_{ij} = \frac{1}{2}\left(\partial_j \tilde{v}_i + \partial_i \tilde{v}_j\right) \qquad \nu_S = (c_s \Delta)^2 \sqrt{2\, S_{kl} S_{kl}}$$

Is there an optimal value of c_s ?

We can optimize the Smagorinsky free-parameter, by minimizing the error between the reference DNS and the nudged LES:

$$E_\Delta = \int_0^{k_c} \int_{|\boldsymbol{k}|=k} |\hat{u}(\boldsymbol{k}) - \hat{u}_N(\boldsymbol{k})|^2 \, d\boldsymbol{k}$$



$c_s = 0.16$ Is found to minimize the average relative error.

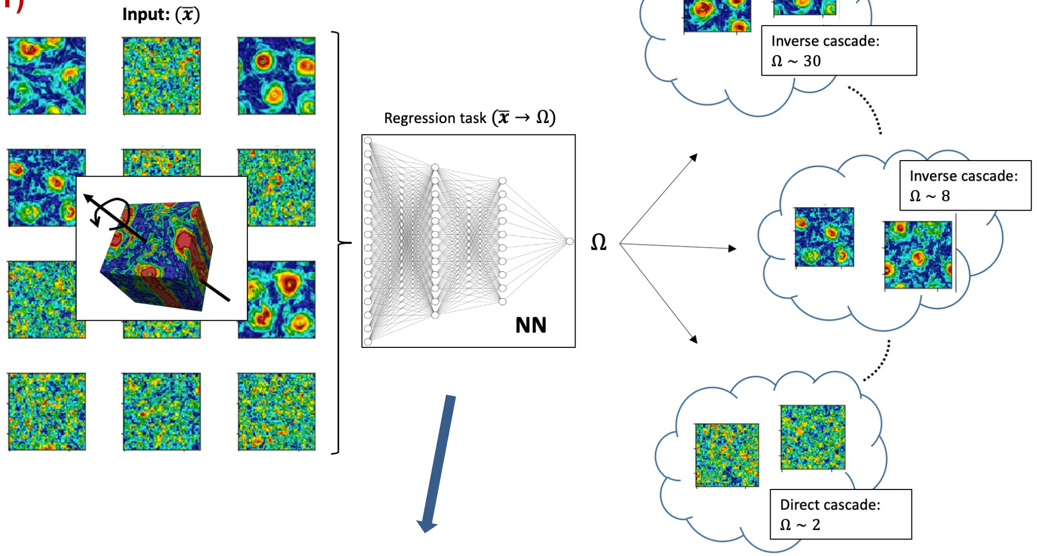
Important result to optimize new model and/or modeling in different setups (i.e. non-homogenous flows)

MB, & Clark Di Leoni, P. (2020). Synchronizing subgrid scale models of turbulence to data. Physics of Fluids, 32(12), 125116.

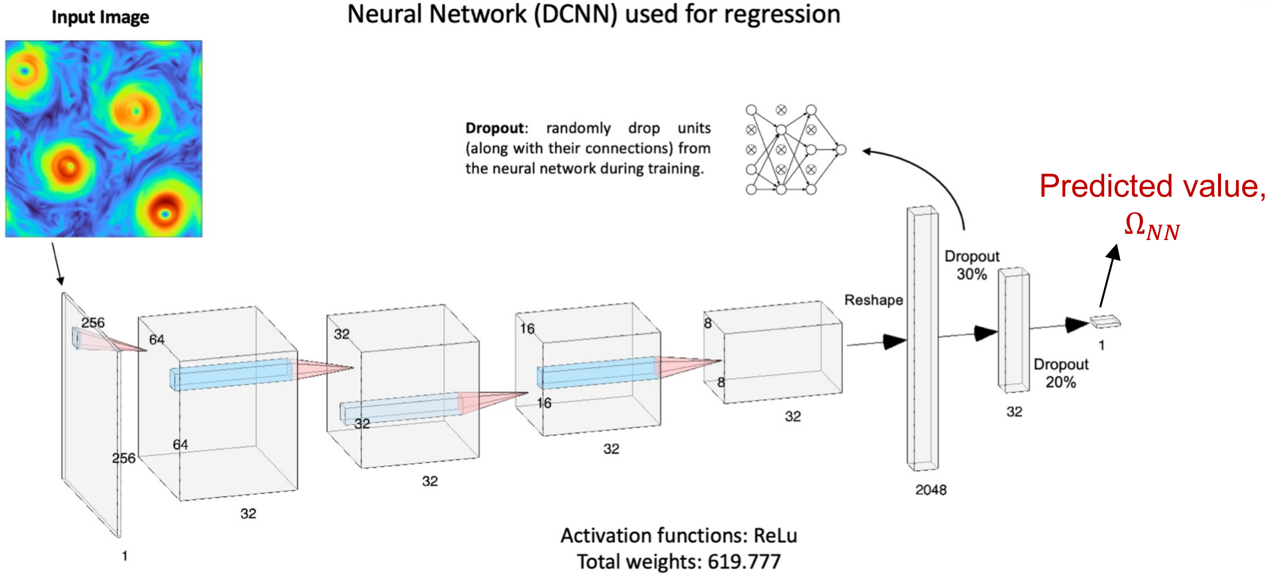
MACHINE LEARNING: A **DATA-DRIVEN** TOOL TO INFER THE PHYSICS

PROBLEM SETUP & DATASET

- 400k 2D planes (TRAINING SET)
- 100k 2D planes (TEST SET)
- 10 Ω values
- Velocity module
- No time information



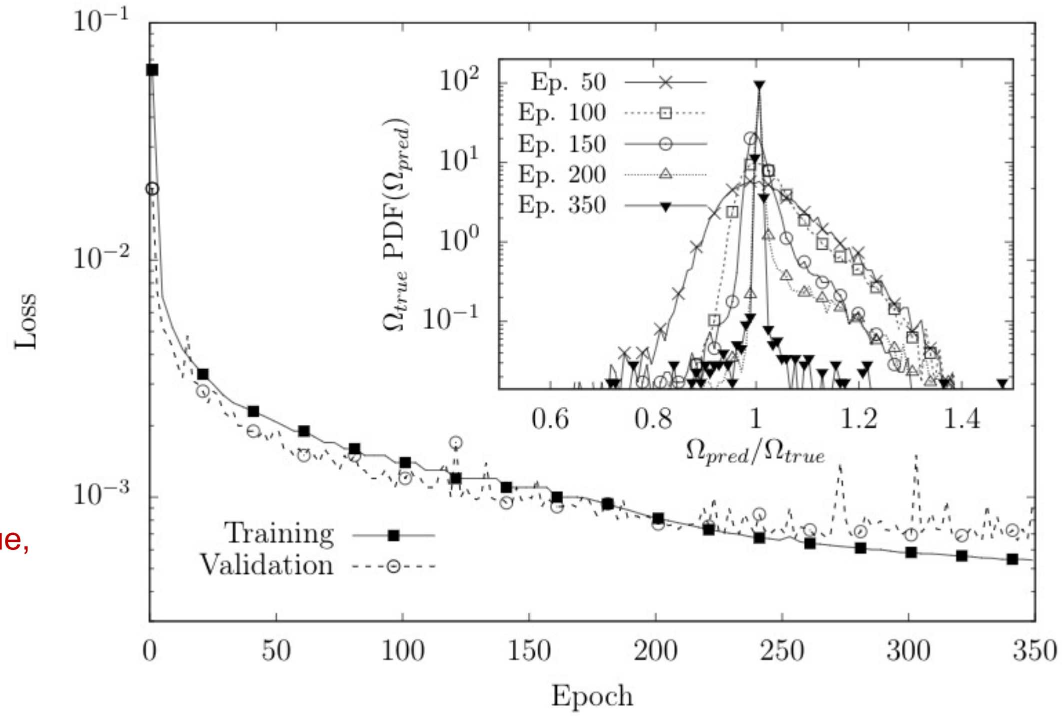
Neural Network (DCNN) used for regression



Activation functions: ReLu
Total weights: 619.777

Loss Function for the Ω regression task:

$$\mathcal{L} = \mathbb{E}\{ (\Omega_{true} - \Omega_{NN})^2 \}$$



Training details: Minibatch 256 planes, Adam optimizer with Nesterov momentum, learning rate 10^{-5}, ReLu activation function

MB and Fabio Bonaccorso. "Classifying Turbulent Environments via Machine Learning" *arXiv preprint arXiv:2201.00732* (2022) (Submitted to EPJE).

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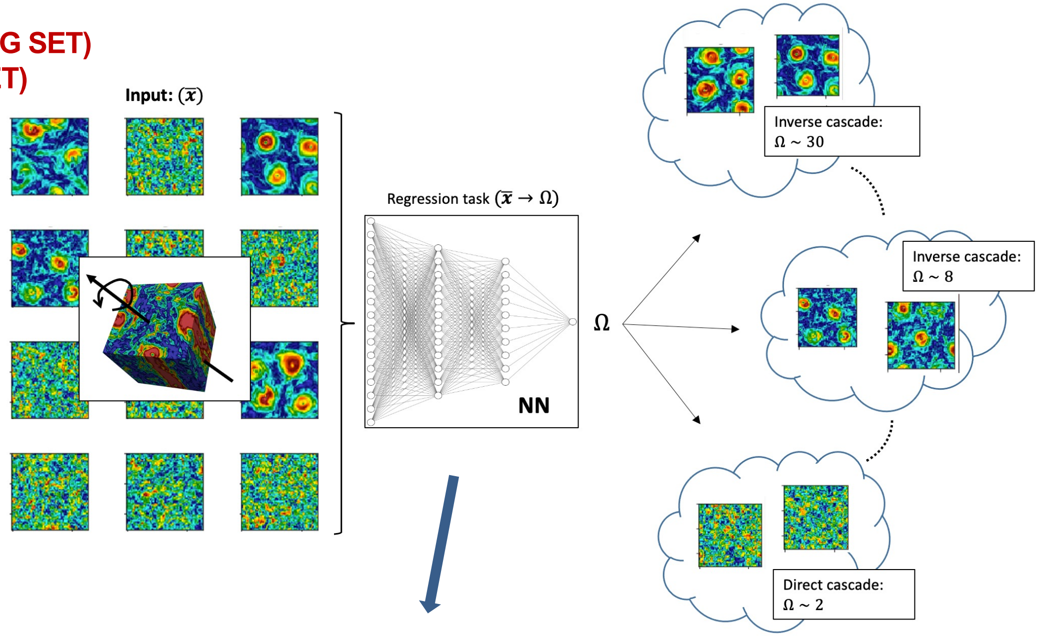

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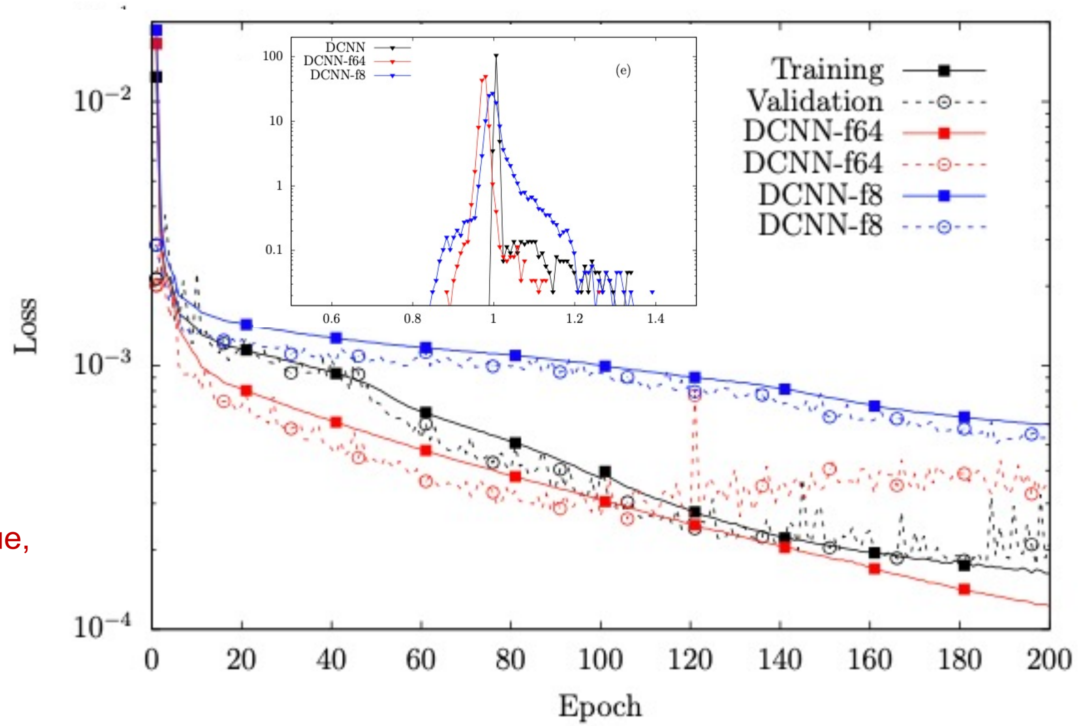

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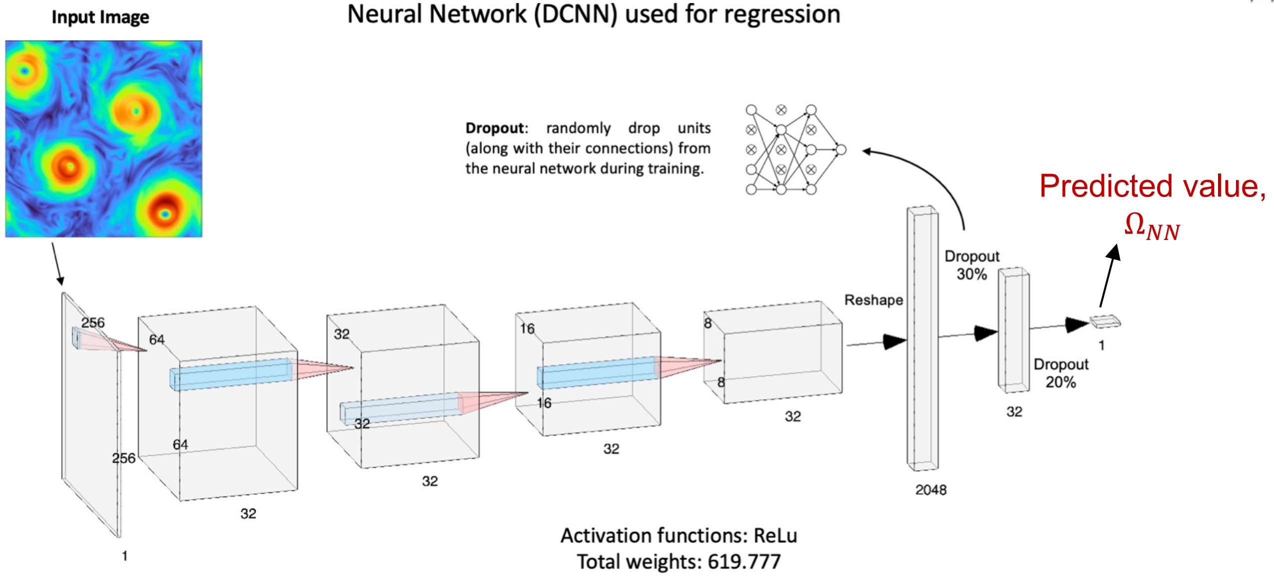

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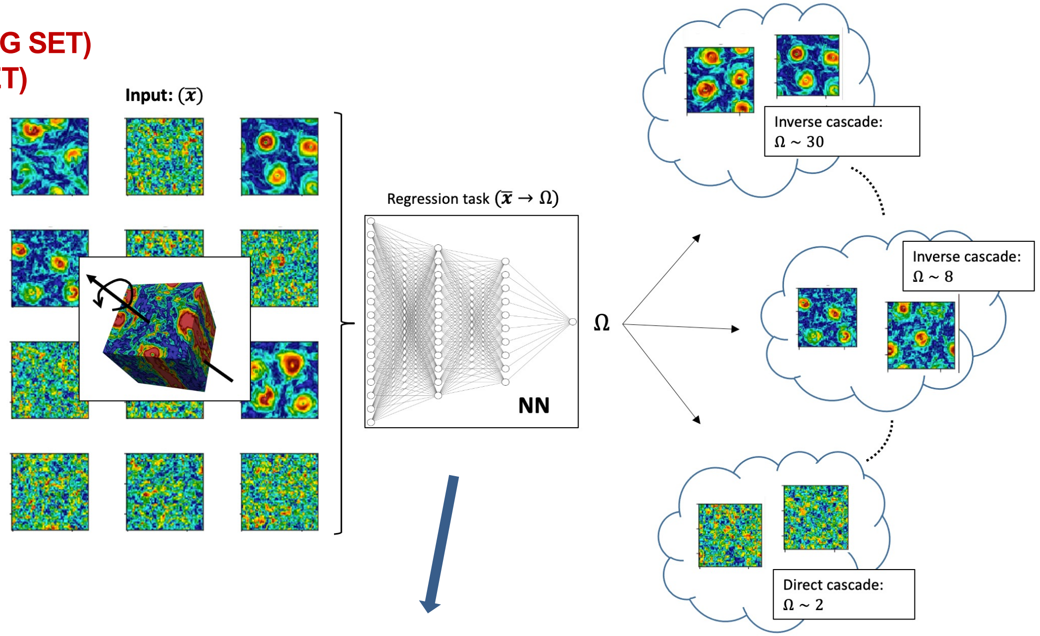

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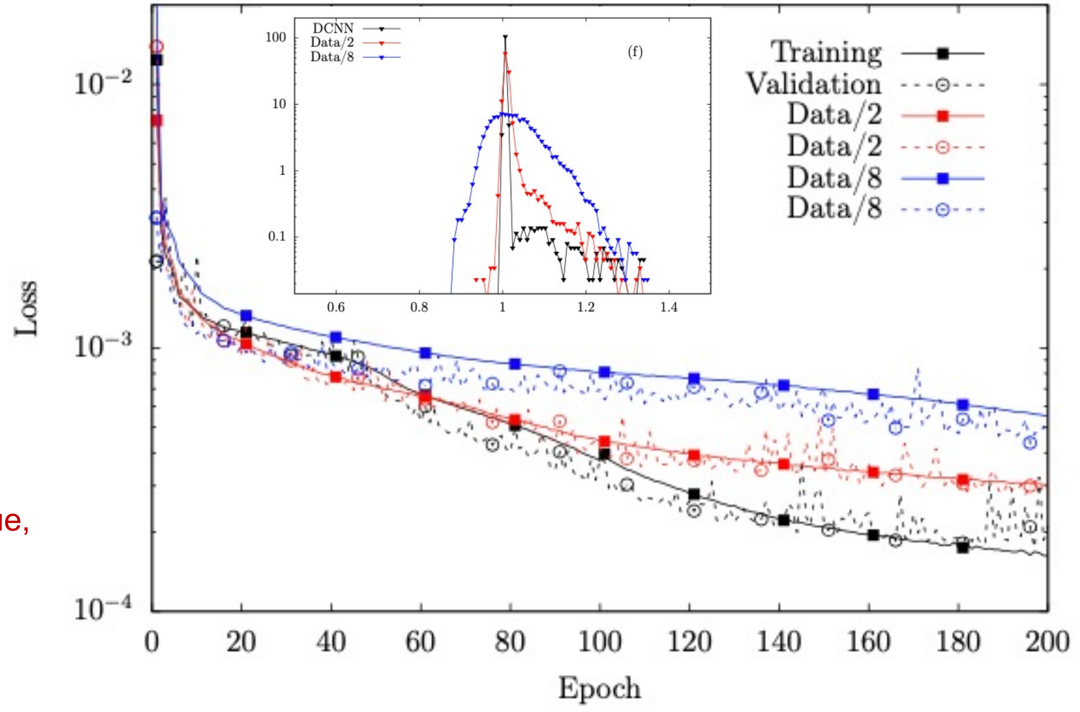

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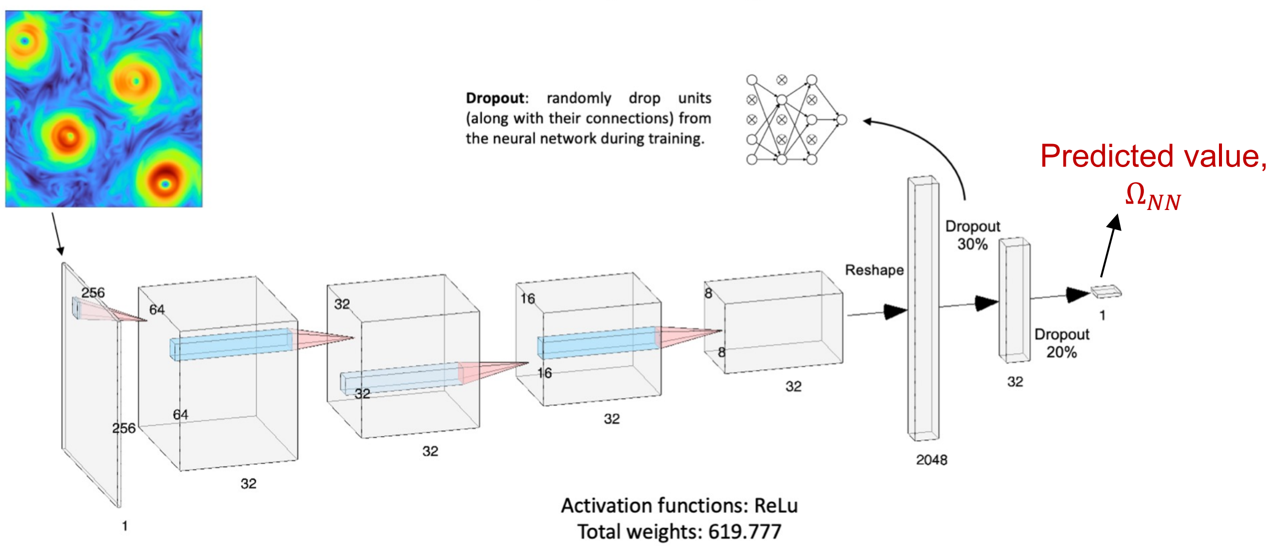

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DATA-DRIVEN BAYESIAN INFERENCE

$$\Omega_{Bay}(\mathcal{O}) = \max_{\Omega_i} P(\Omega_i|\mathcal{O})$$

We measure **mean squared velocity** and **mean squared velocity gradients** :

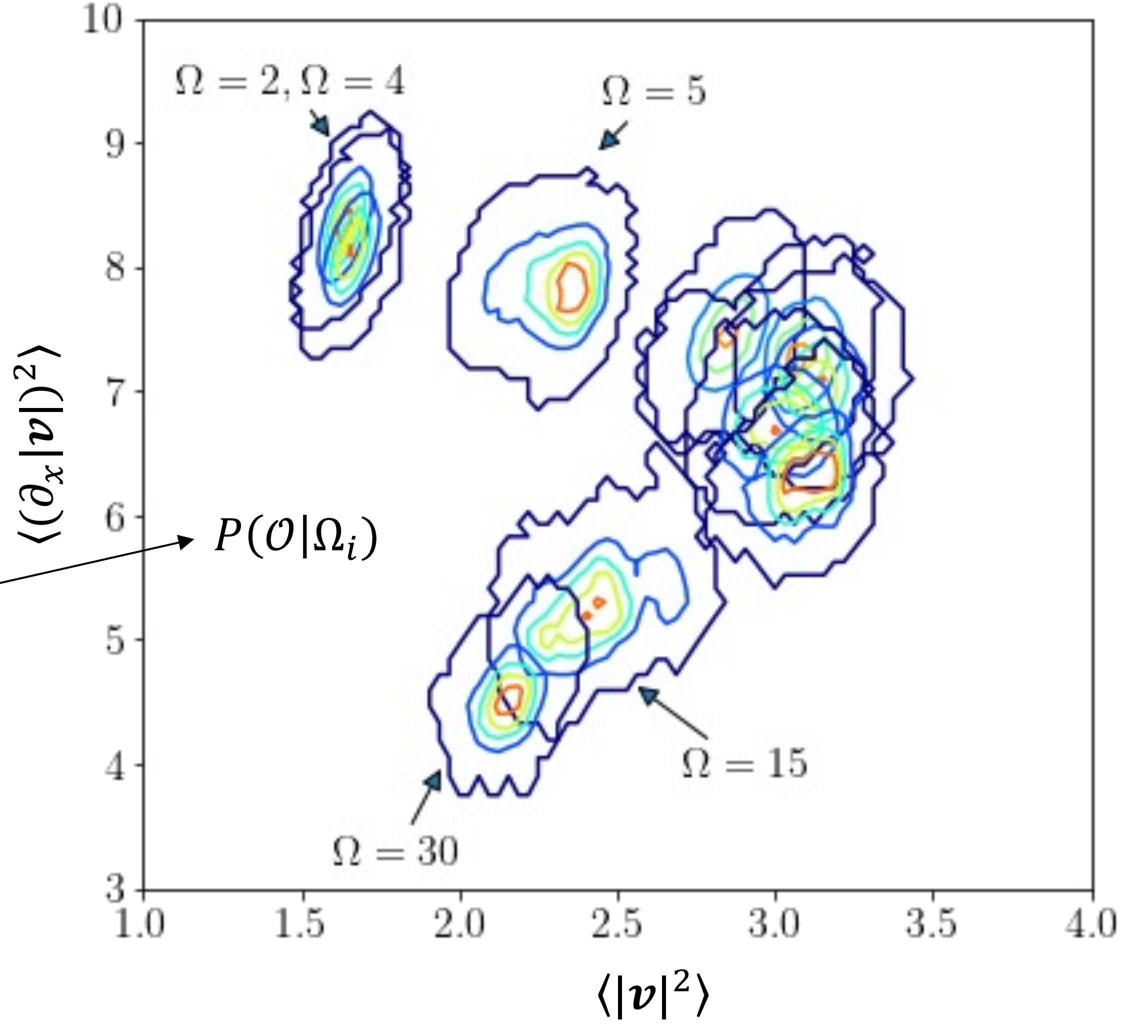
$$\mathcal{O} = (\langle|\boldsymbol{v}|^2\rangle, \langle(\partial_x|\boldsymbol{v}|)^2\rangle)$$

$\langle..\rangle$ mean on space, over the 2D plane

We can obtain the joint probability using Bayes Theorem:

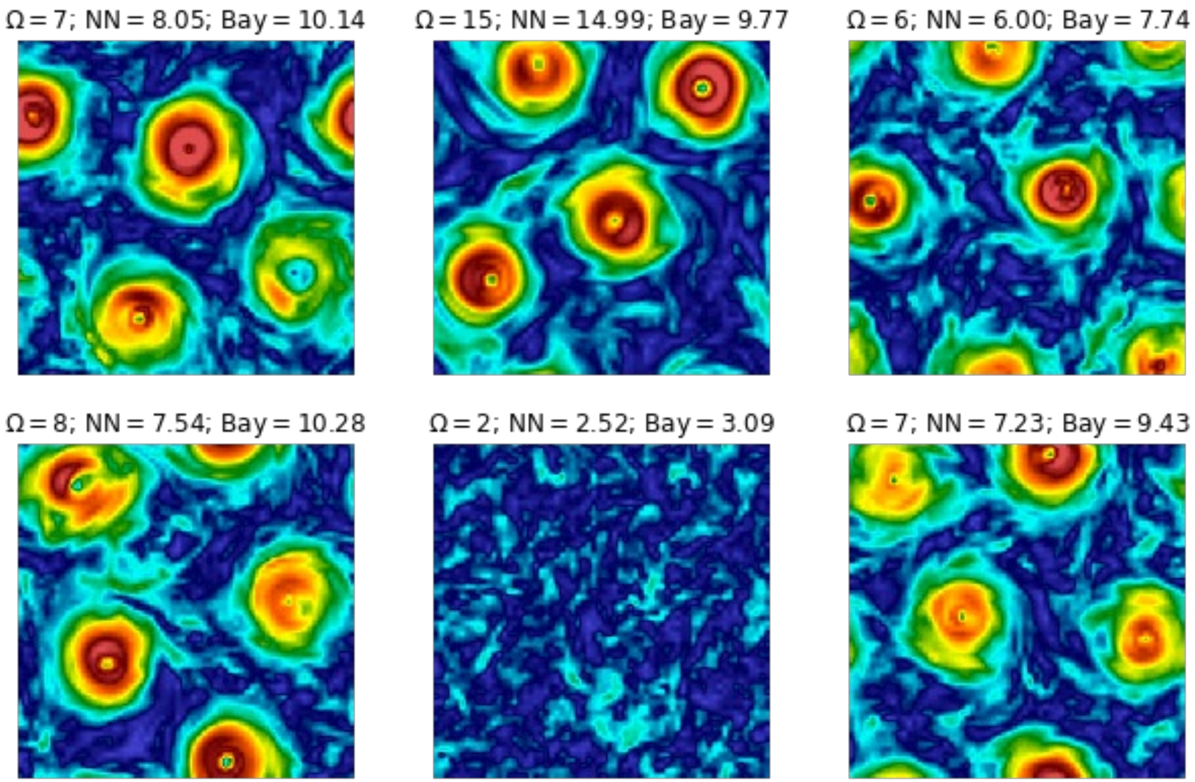
$$P(\Omega_i|\mathcal{O}) = \frac{P(\mathcal{O}|\Omega_i)P(\Omega_i)}{P(\mathcal{O})} = \frac{P(\mathcal{O}|\Omega_i)P(\Omega_i)}{\sum_i P(\mathcal{O}|\Omega_i)P(\Omega_i)}$$

Joint-PDF of **mean squared velocity** and **mean squared velocity gradients**
(estimated over the 400k planes of the training set)



MB and Fabio Bonaccorso. "Classifying Turbulent Environments via Machine Learning" *arXiv preprint arXiv:2201.00732* (2022) (Submitted to EPJE).

comparison: BAYESIAN INFERENCE vs ML



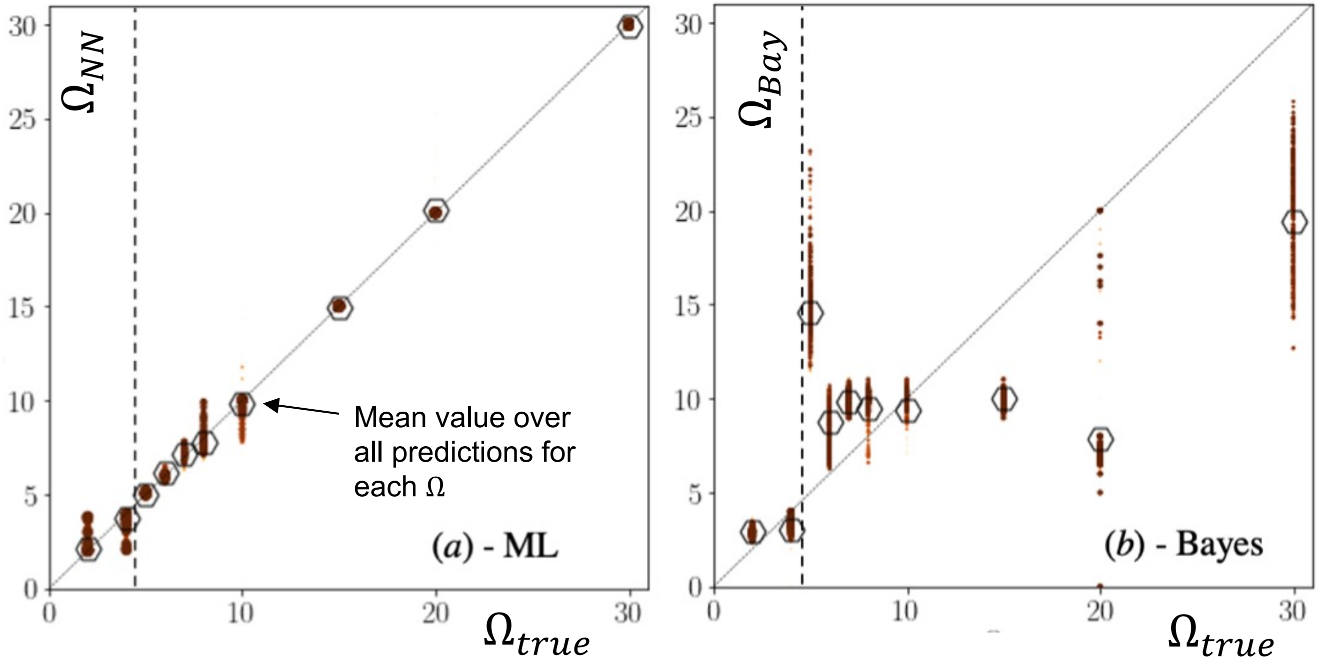
Examples of Ω inferring by Neural Network and Bayesian Inference

Scatterplot of Prediction vs True Ω values

Results obtained on the TEST SET (100k planes)

Bayes infers on: velocity square module

$$P(\Omega_i | \langle |\boldsymbol{v}|^2 \rangle)$$



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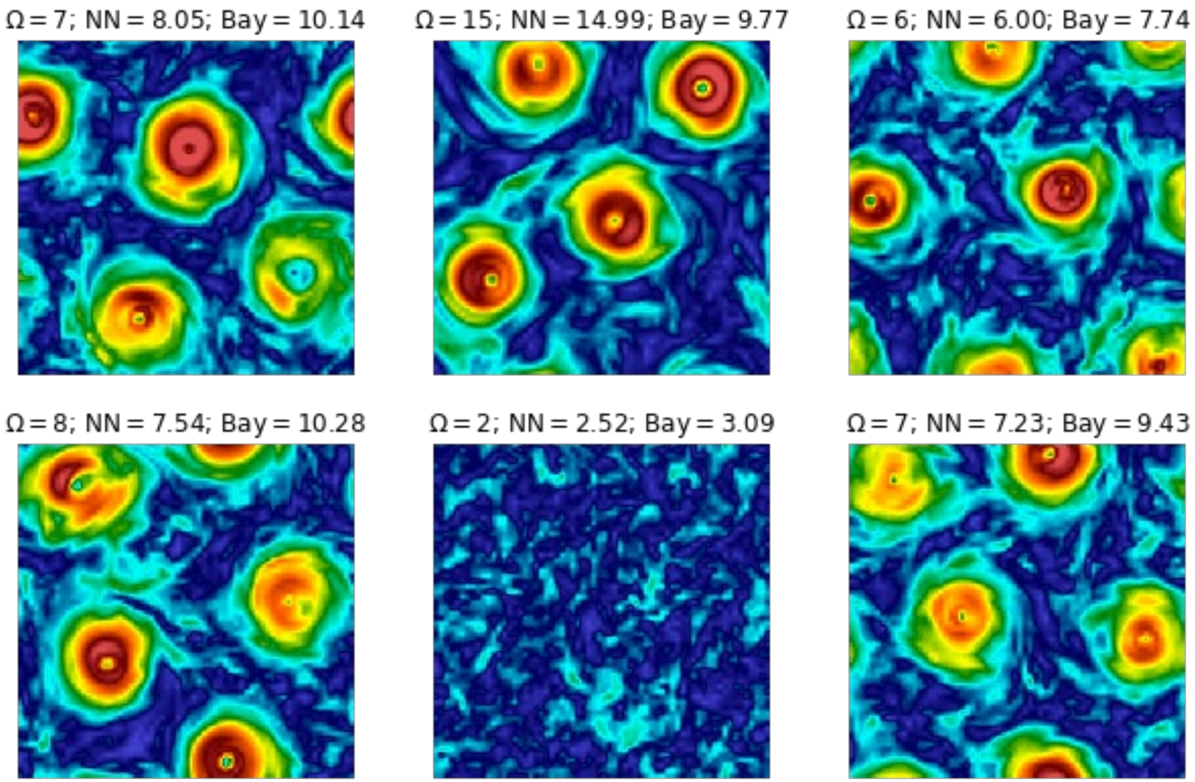

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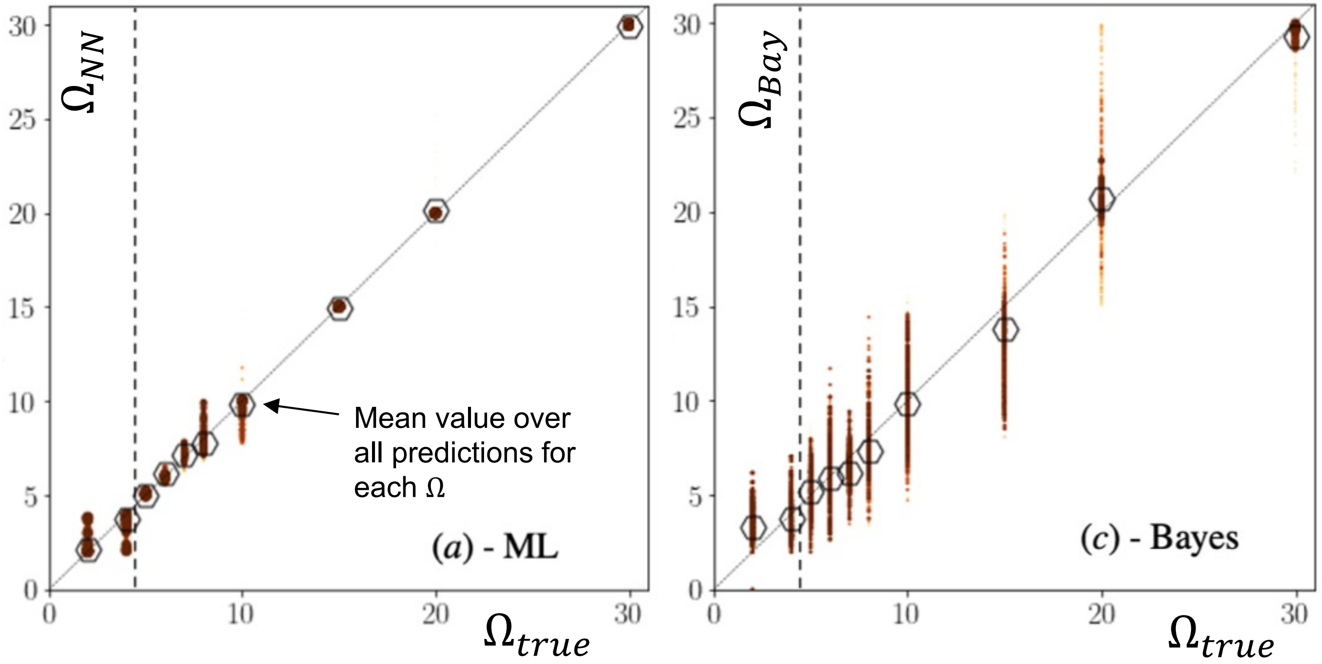

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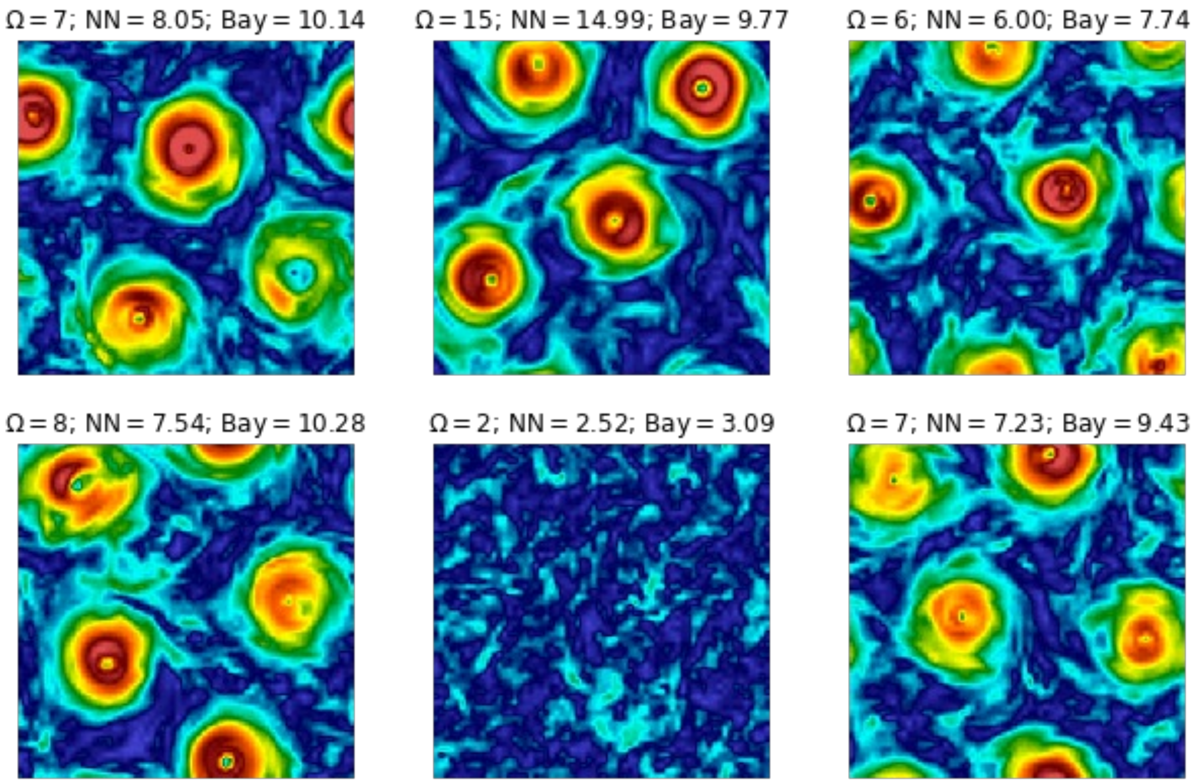

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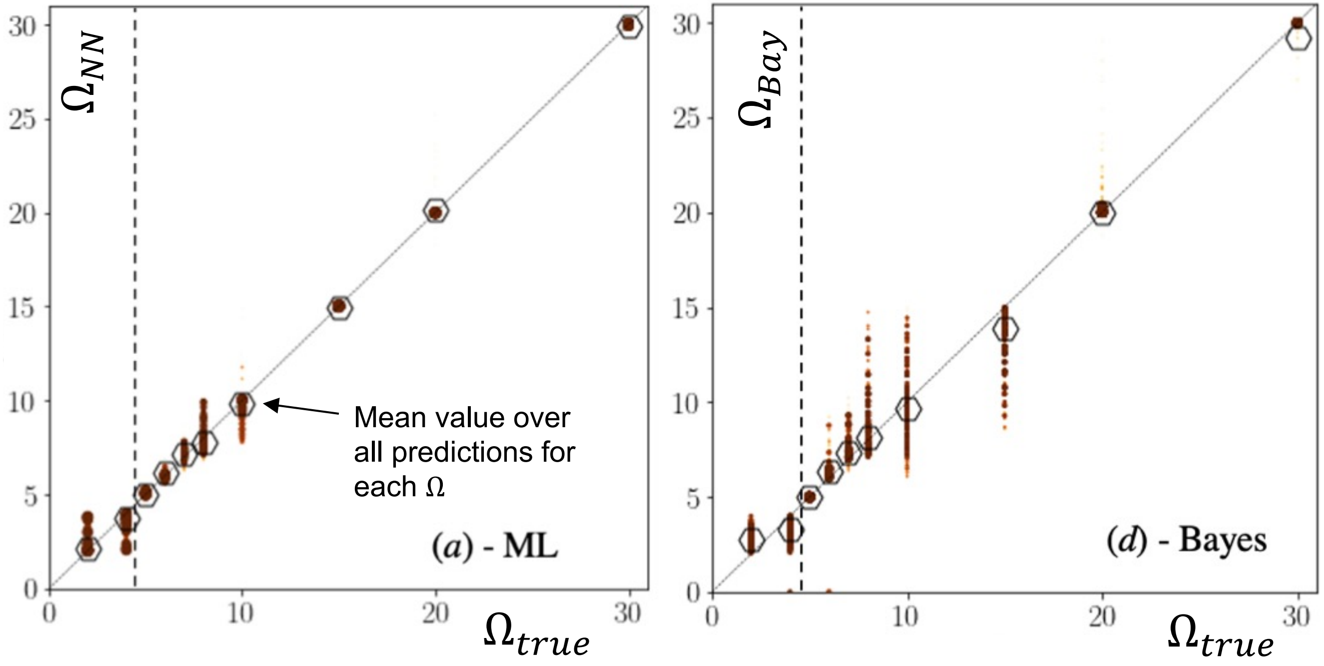

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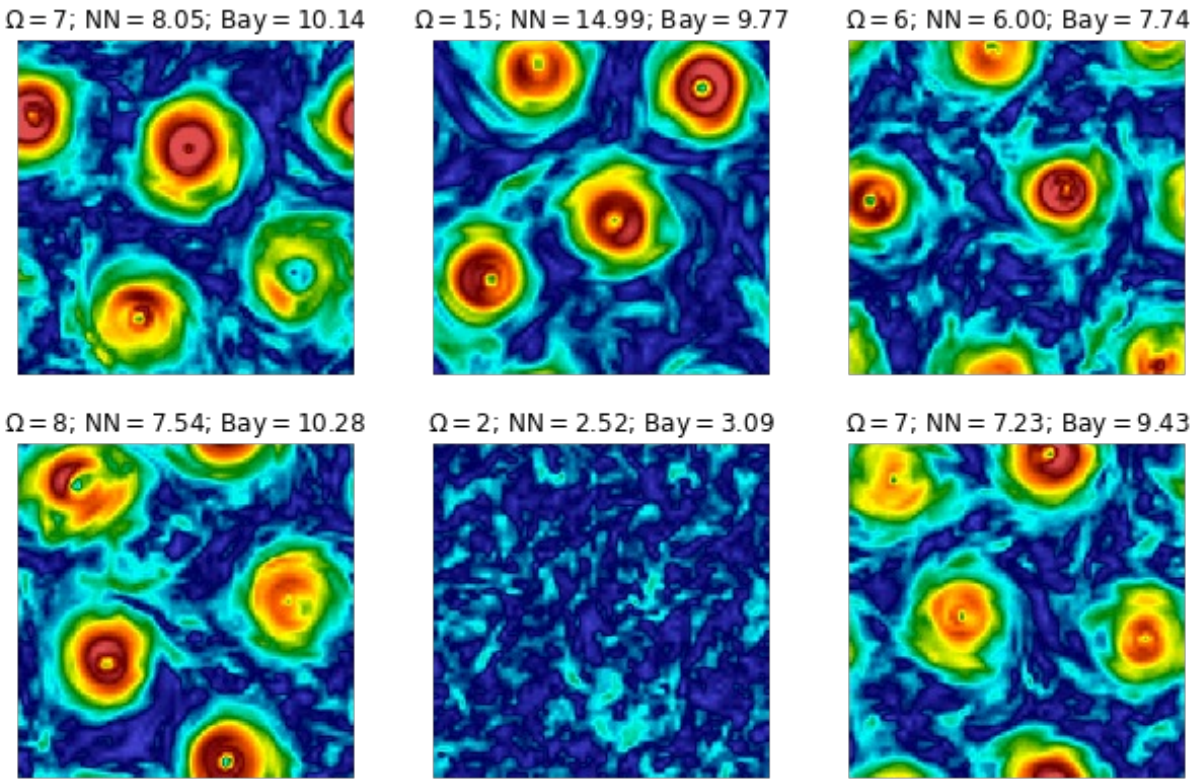
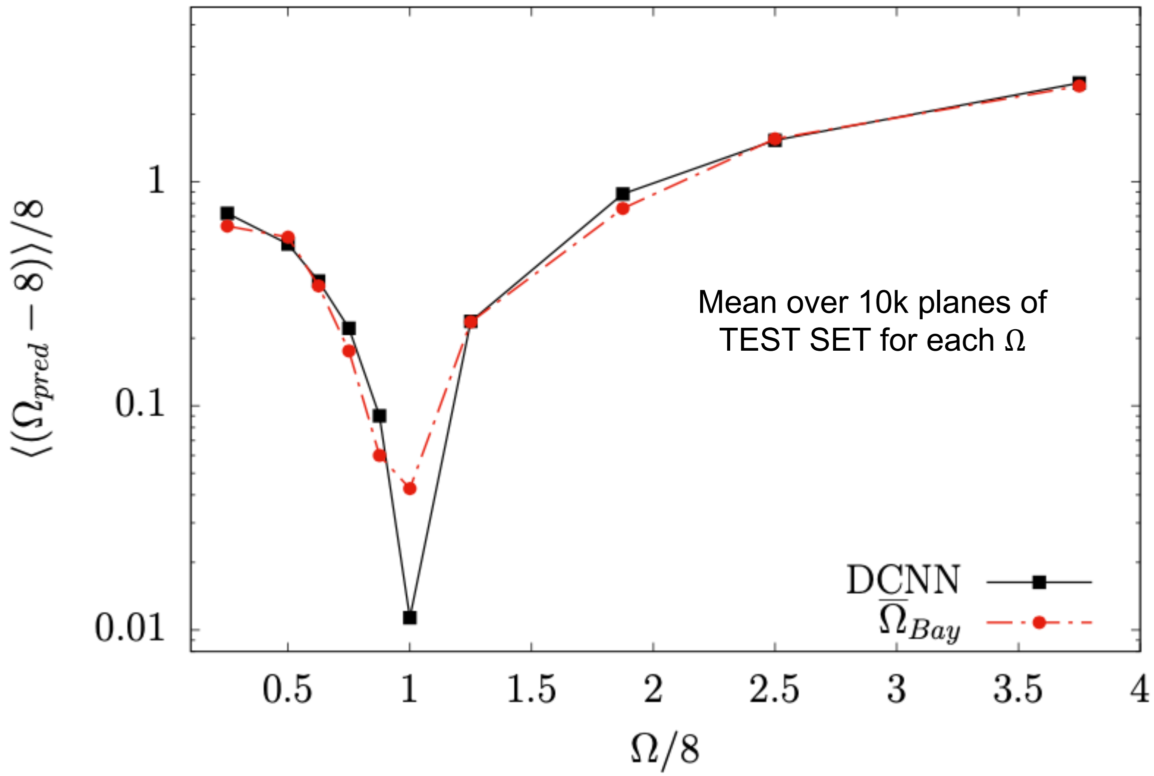


Examples of Ω inferring by Neural Network and Bayesian Inference

Sensitivity of Data-Driven Tools

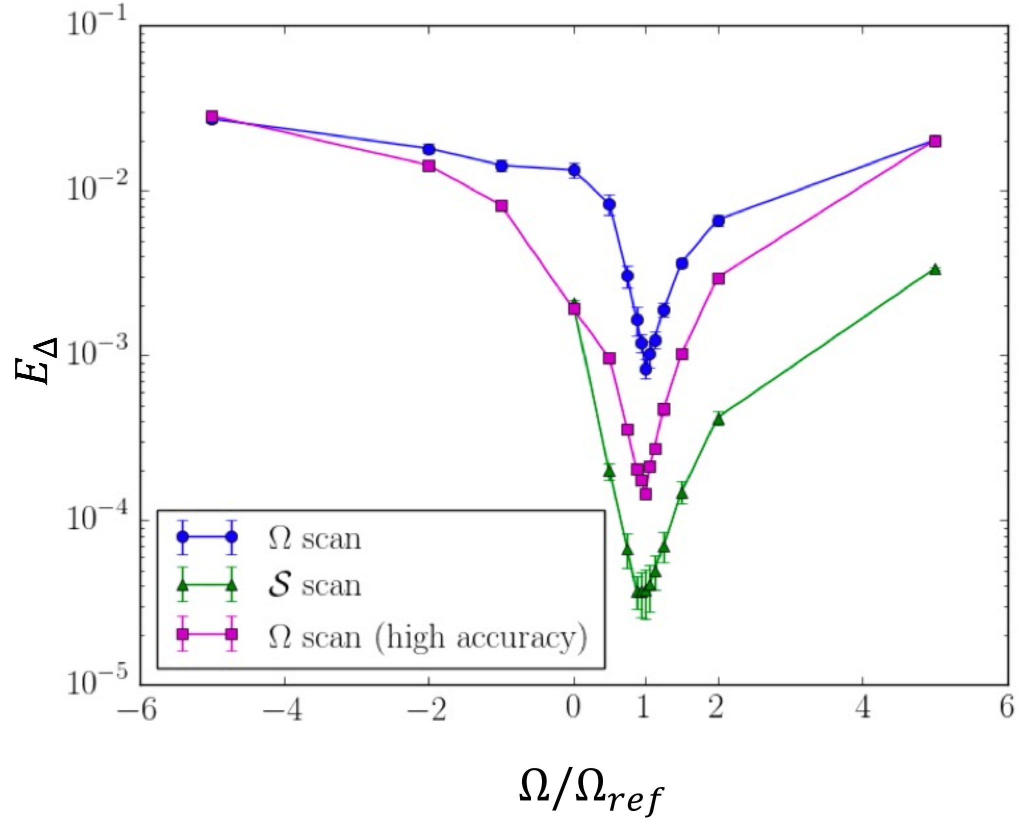


$$\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \partial_{\boldsymbol{x}} \boldsymbol{v} + \partial_{\boldsymbol{x}} P - \nu \Delta \boldsymbol{v} = 2\, \boldsymbol{v} \times \Omega\, \hat{z} + \mathbf{f}$$

MB and Fabio Bonaccorso. "Classifying Turbulent Environments via Machine Learning" *arXiv preprint arXiv:2201.00732* (2022) (Submitted to EPJE).

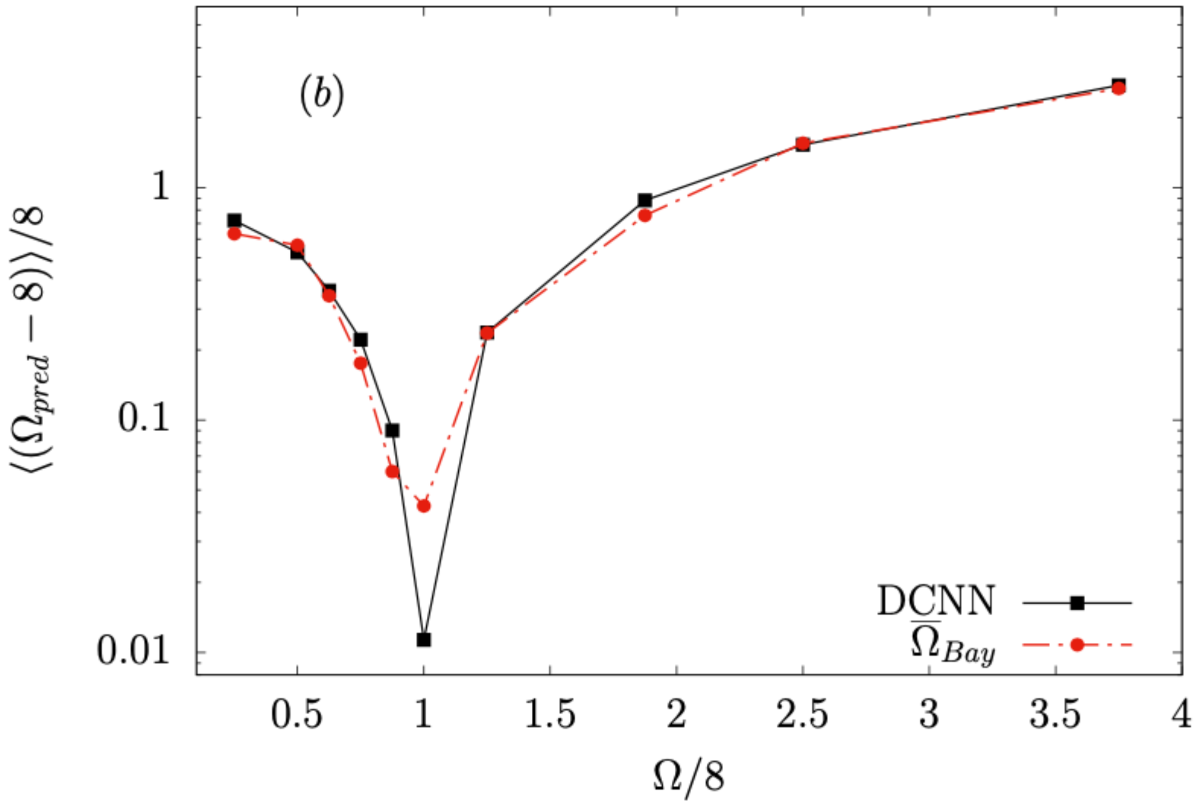
Contributions

NUDGING: **EQUATION-INFORMED**



+ Strong Sensitivity to the physical parameter
+ No need of Ensemble/Training Dataset
- It requires complete knowledge of the Equations
- Heavy in computation – it requires DNS

DATA-DRIVEN TOOLS



+ Strong Sensitivity to the physical parameter
+ No need of Eq. of Motion
+ Fast in computation after the training
- Need of large Training Dataset

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References

- MB and Fabio Bonaccorso. “Classifying Turbulent Environments via Machine Learning" ***arXiv preprint arXiv:2201.00732*** **(2022)**.
- MB, Bonaccorso, Clark Di Leoni & Biferale. **Phys. Rev. Fluids 6, 050503 (2021).** Reconstruction of turbulent data with deep generative models for semantic inpainting from TURB-Rot database.
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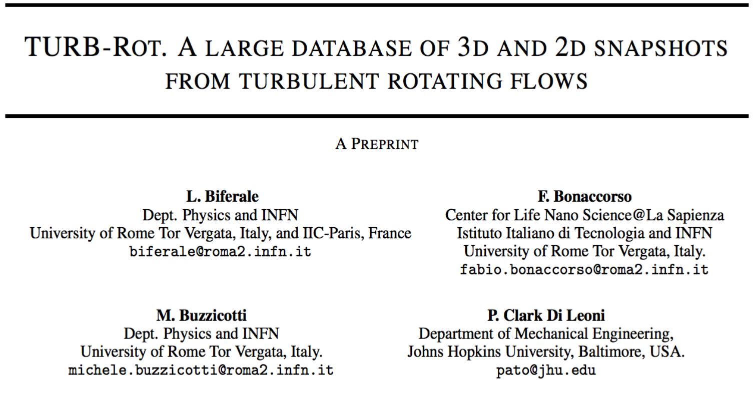
TURB-ROT. A LARGE DATABASE OF 3D AND 2D SNAPSHOTS FROM TURBULENT ROTATING FLOWS

A PREPRINT

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