European Research Council

Università di Roma

Tor Vergata

On the role of the helicity in the energy transfer in three-dimensional turbulence

Ganapati Sahoo

with Fabio Bonaccorso and Luca Biferale

University of Rome Tor Vergata, Italy

15th European Turbulence Conference, August 25-28, 2015, Delft, The Netherlands

Supported by European Research Council Advanced Grant "Newturb"

Introduction

- Energy and enstrophy are conserved in 2D Navier-Stokes equations.

$$\text{Energy } E = \int d^2r\ \vec{u}\cdot\vec{u} \text{ and Enstrophy } \Omega = \int d^2r\ \vec{\omega}\cdot\vec{\omega}$$

- Forward cascade of energy is blocked, since enstrophy is positive and definite. (Ray PRL 2011, Boffetta, Ann. Rev. Fluid Mech 2012)

- Energy and Helicity are invariants of 3D Navier-Stokes equations.

$$\text{Energy } E = \int d^3r\ \vec{u}\cdot\vec{u} \text{ and Helicity } H = \int d^3r\ \vec{u}\cdot\vec{\omega}$$

- Both cascade forward, from large scales to small scales. (Chen, Phys. Fluids 2003)

- Helicity could be positive or negative.

- Each Fourier mode of velocity could be decomposed into positive and negative helical modes.

What happens when we change the relative weight of the positive and the negative helicity modes?

Helical decomposition

- Following Waleffe, Phys. Fluids (1992)

$$\begin{aligned} \mathbf{u}(\mathbf{k},t) &= \mathbf{u}^{+}(\mathbf{k},t) + \mathbf{u}^{-}(\mathbf{k},t), \\ \mathbf{u}^{\pm}(\mathbf{k},t) &= u^{\pm}(\mathbf{k},t)\mathbf{h}^{\pm}(\mathbf{k}) \end{aligned}$$

where $\mathbf{h}^{\pm}(\mathbf{k})$ are the eigenvectors of the curl operator $i\mathbf{k} \times \mathbf{h}^{\pm}(\mathbf{k}) = \pm k\mathbf{h}^{\pm}(\mathbf{k})$, $u^{\pm}(\mathbf{k},t)$ are the time-dependent scalar co-efficients.

- Projection operator:

$$\mathcal{P}^{\pm}(\mathbf{k}) \equiv \frac{\mathbf{h}^{\pm}(\mathbf{k}) \otimes \mathbf{h}^{\pm}(\mathbf{k})^{*}}{\mathbf{h}^{\pm}(\mathbf{k})^{*} \cdot \mathbf{h}^{\pm}(\mathbf{k})}$$

$$\mathbf{u}^{\pm}(\mathbf{k},t) = \mathcal{P}^{\pm}(\mathbf{k})\mathbf{u}(\mathbf{k},t)$$

- Decimated Navier-Stokes equations in Fourier space:

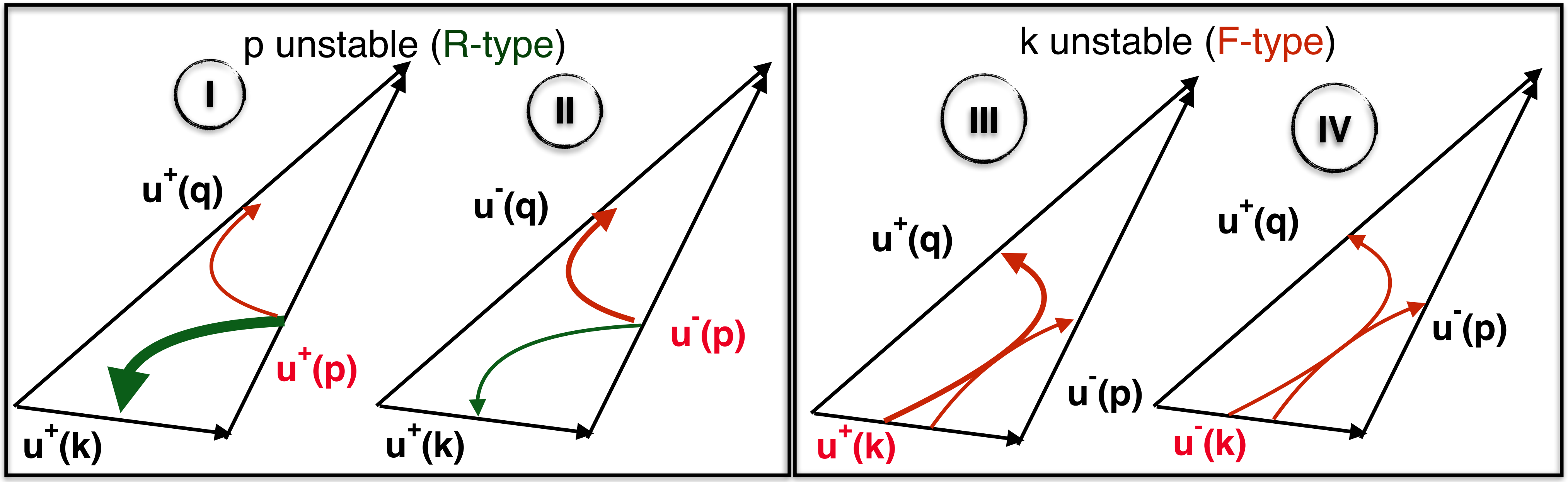
$$\partial_t \mathbf{u}^{\pm}(\mathbf{k},t) = \mathcal{P}^{\pm}(\mathbf{k})\mathbf{N}_{u^{\pm}}(\mathbf{k},t) + \nu k^2 \mathbf{u}^{\pm}(\mathbf{k},t) + \mathbf{f}^{\pm}(\mathbf{k},t)$$

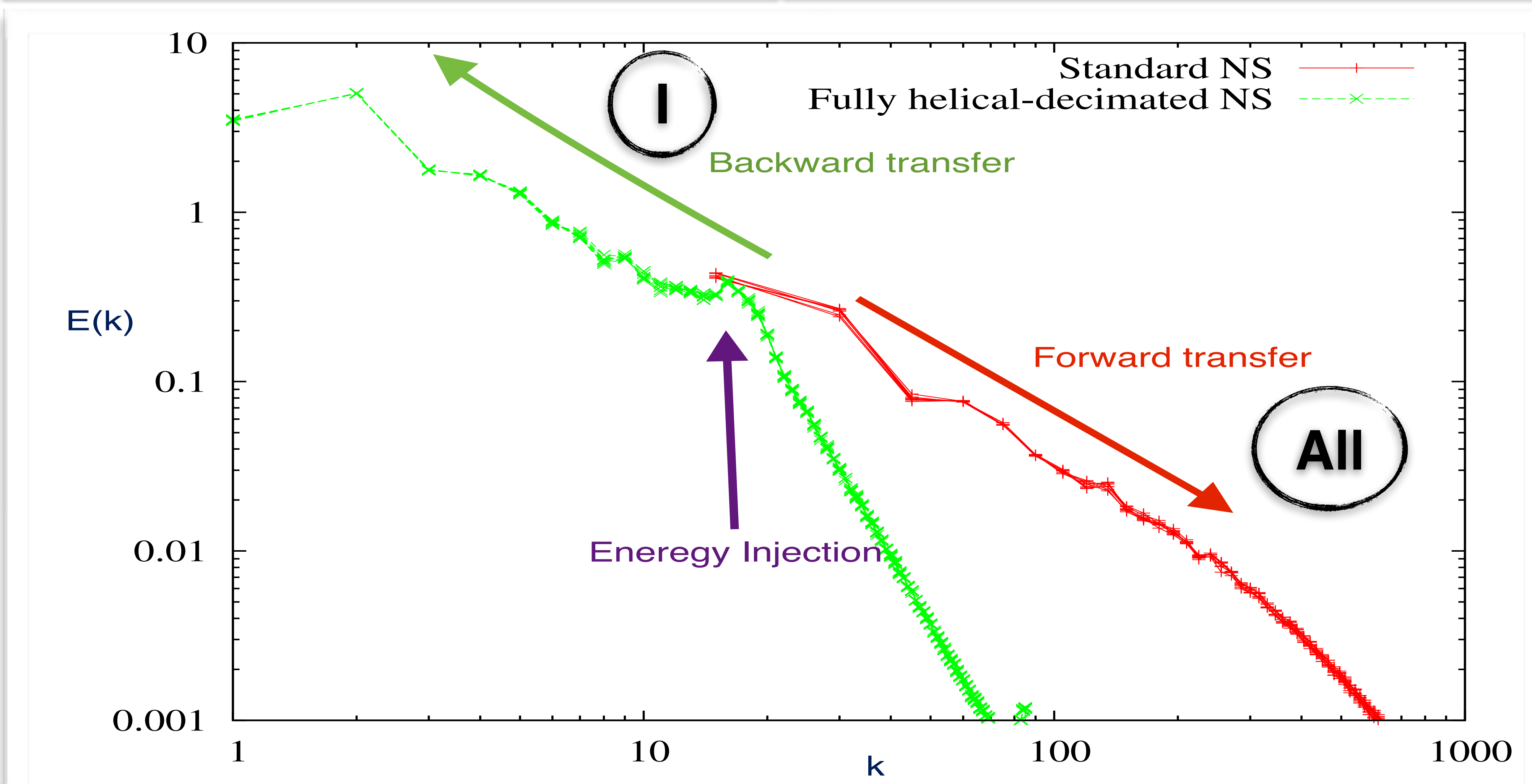
where ν is kinematic viscosity and $\mathbf{f}$ is external forcing.

- The non-linear term $\mathbf{N}_{u^{\pm}}(\mathbf{k},t) = \mathcal{FT}(\mathbf{u}^{\pm} \cdot \nabla \mathbf{u}^{\pm} - \nabla p)$, contains 8 possible triadic interactions $\mathbf{q} = \mathbf{k} + \mathbf{p}$ which fall into four classes.

Classes of triadic interactions in NS equations

$$\mathbf{N}_{\mathbf{u}^\pm}(\mathbf{q}) = \mathcal{F}\mathcal{T}\left[\mathbf{u}^\pm(\mathbf{k})\cdot\nabla\mathbf{u}^\pm(\mathbf{p})\right];\ \mathbf{q}=\mathbf{k}+\mathbf{p};\ k\le p\le q$$





European Research Council

Partial Helical-decimation

What happens in between??

when we give different weights to different class of triads…

- Modified projection operator:

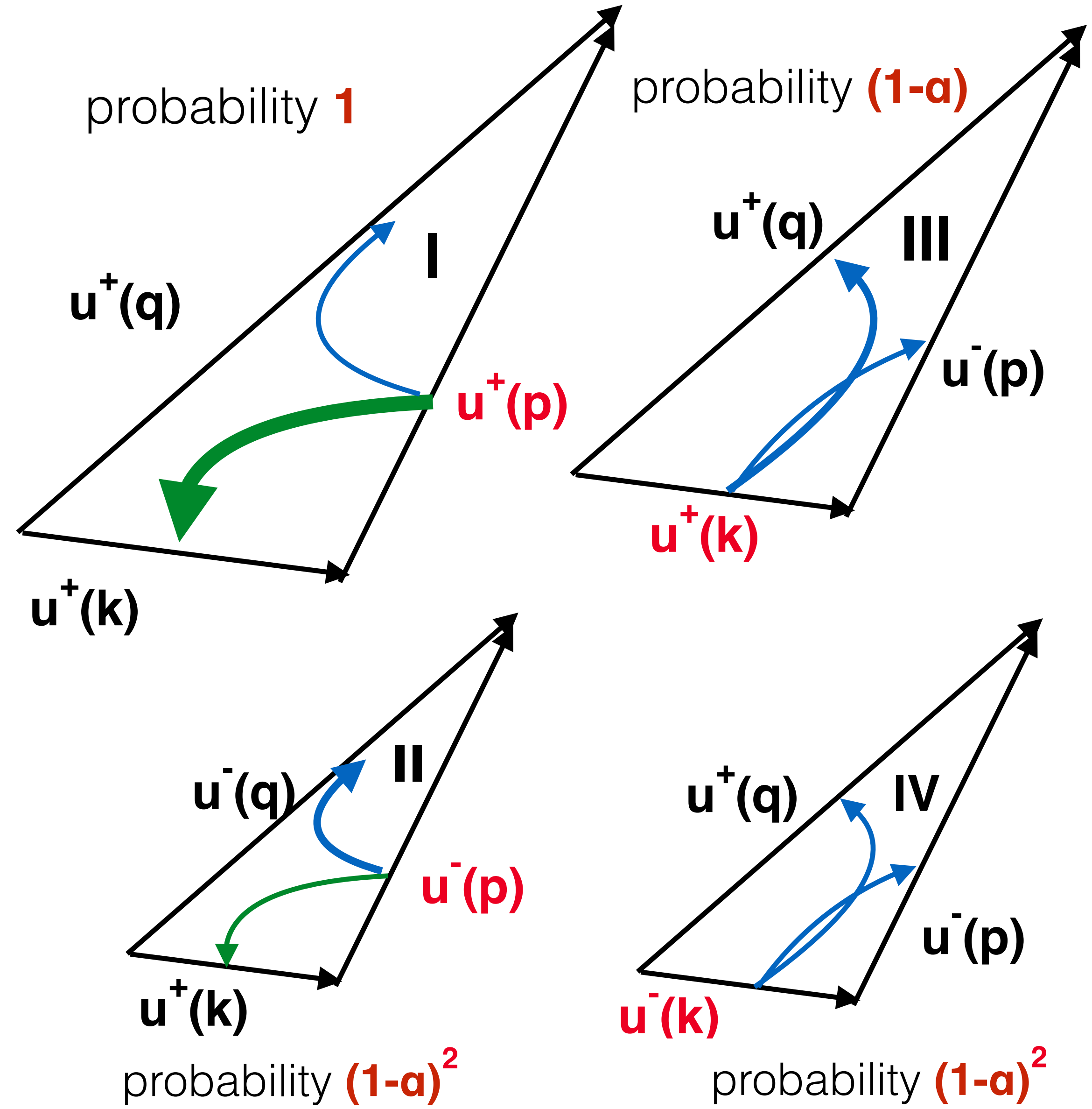
$$\mathcal{P}^{+}_{\alpha}(\mathbf{k})\mathbf{u}(\mathbf{k},t) = \mathbf{u}^{+}(\mathbf{k},t) + \theta_{\alpha}(\mathbf{k})\mathbf{u}^{-}(\mathbf{k},t)$$

  where $\theta_{\alpha}(\mathbf{k})$ is 0 with probability α and is 1 with probability $1-\alpha$.

- We consider triads of Class-I with probability 1, Class-III with probability $1-\alpha$ and Class-II and Class-IV with probability $(1-\alpha)^2$.

- $\alpha = 0 \rightarrow$ Standard Navier-Stokes.
  $\alpha = 1 \rightarrow$ Fully helical-decimated NS.

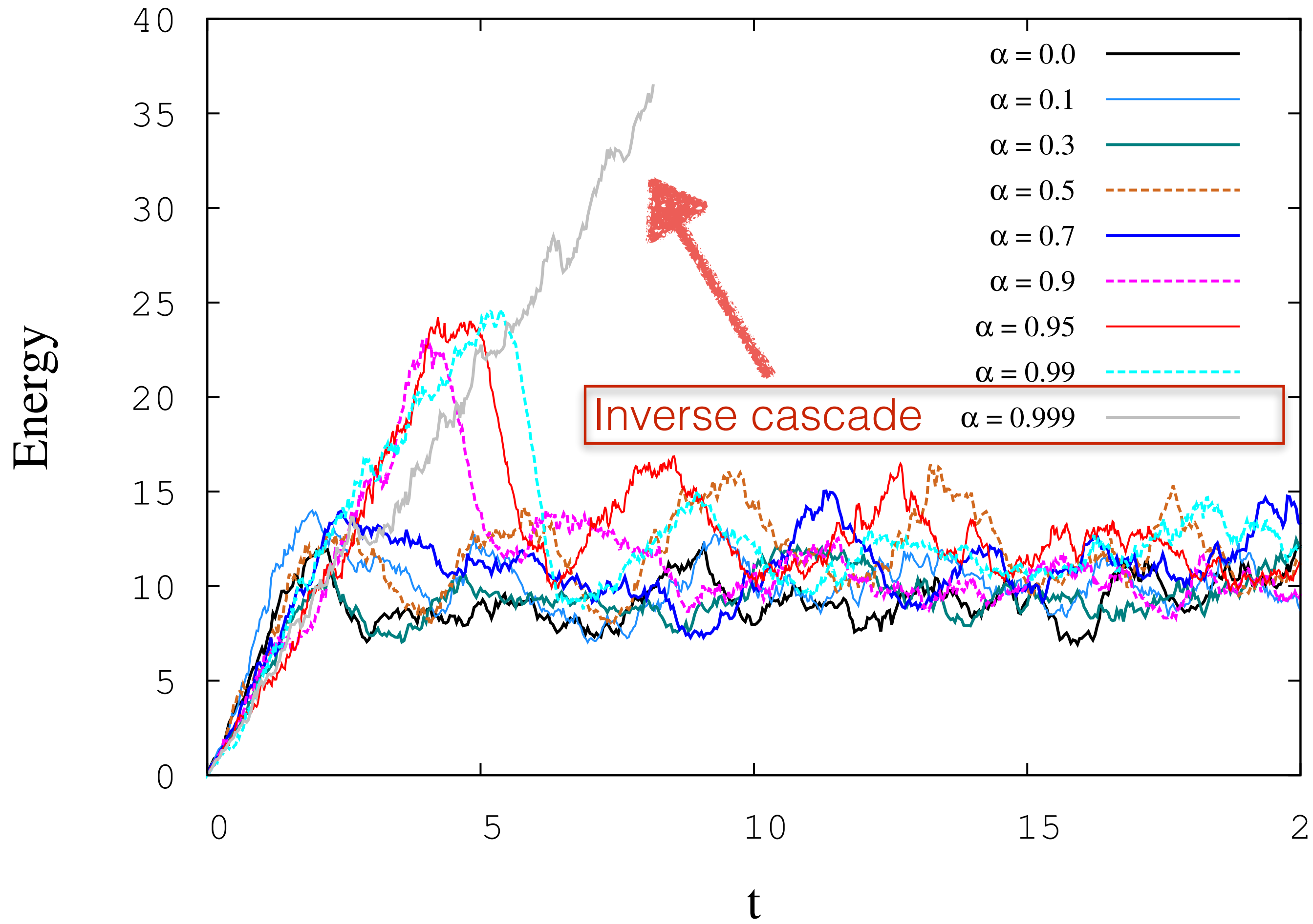
- Critical value of α at which forward cascade of energy stops?
  alternatively, inverse cascade of energy stops if forced at small scales.

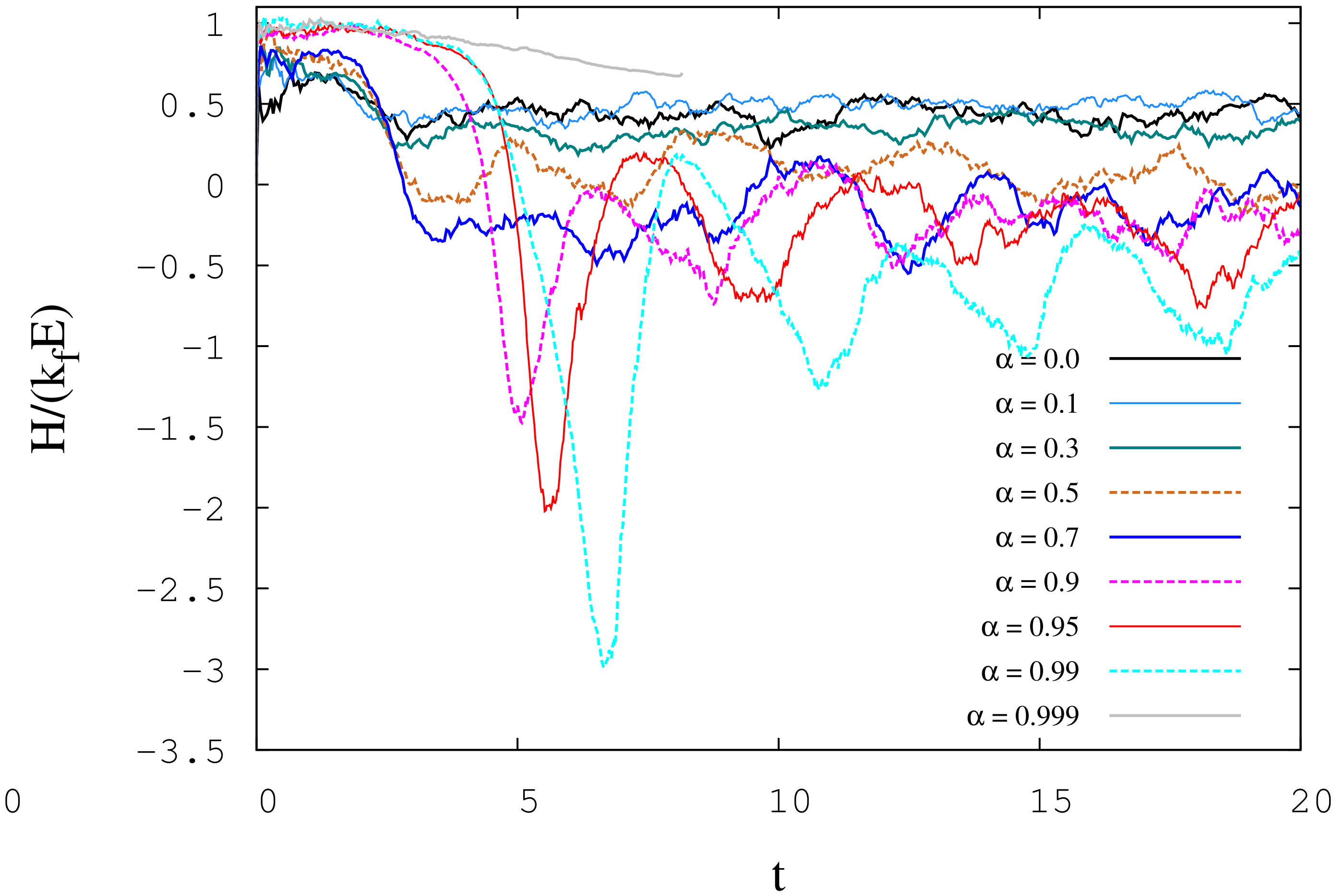


$$\mathbf{N}_{\mathbf{u}^{\pm}}(\mathbf{q}) = \mathcal{FT}\left[\mathbf{u}^{\pm}(\mathbf{k})\cdot\nabla\mathbf{u}^{\pm}(\mathbf{p})\right];\ \mathbf{q}=\mathbf{k}+\mathbf{p};\ k\le p\le q$$

Evolution of Energy and helicity

- Pseudo-spectral DNS on a triply periodic cubic domain of size $L = 2\pi$ with resolutions up to 1024^3 collocation points.

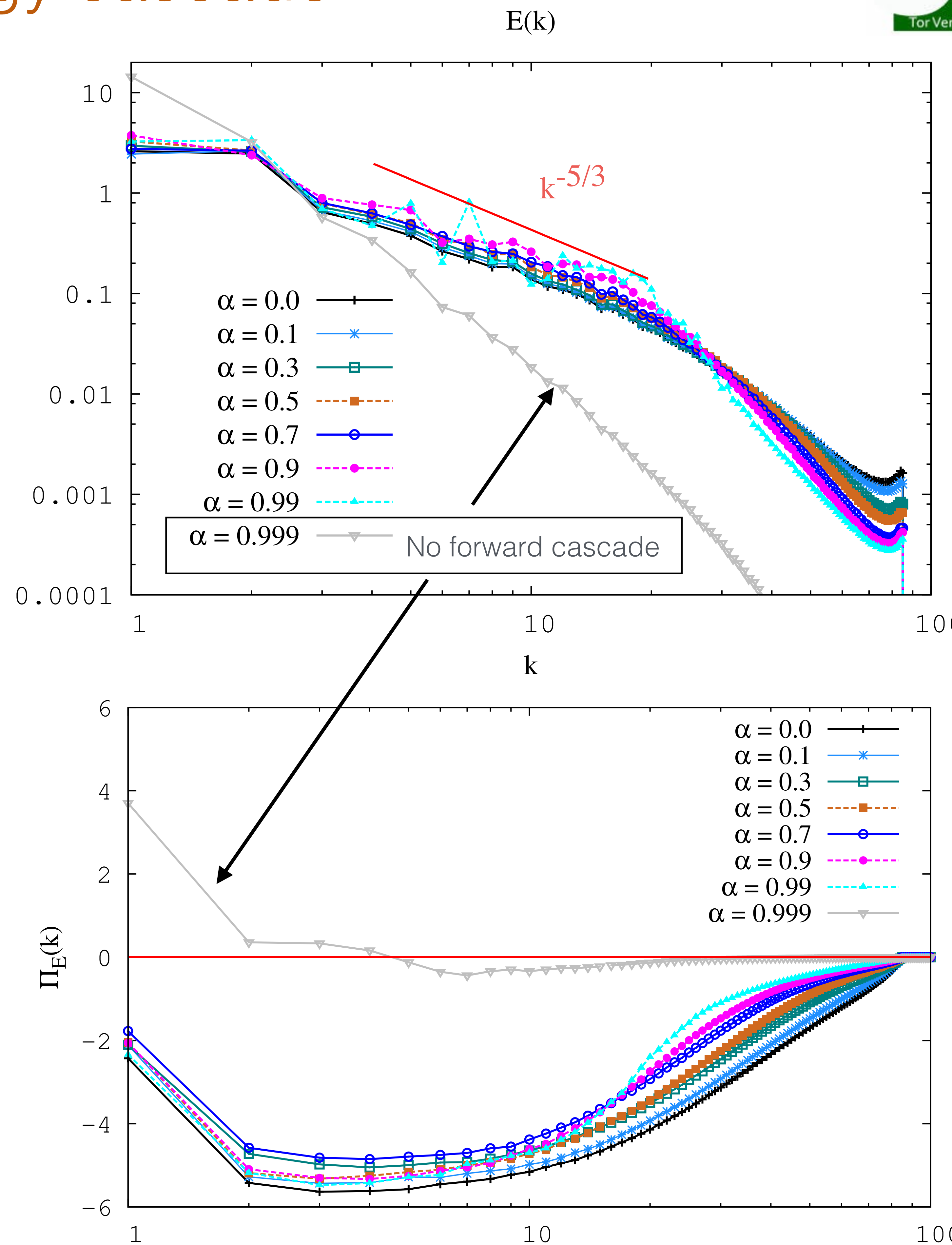




- The peaks suggest the building up of the energy at forced large scales before being able to transfer to the small scales.
- The cascade of energy starts only when helicity becomes active, i.e., modes with negative helicity becomes energetic.
- With increase in α the peak grows, a signature of inverse cascade.

Robustness of energy cascade

- Spectra for all values of α showing $k^{-5/3}$ suggest the forward cascade of to be strongly robust.
- Unless we kill almost all the modes of one helicity-type energy always finds a way to reach small scales.
- The energy flux also remains unaffected by the decimation until α is very close to 1.
- **Critical value of α is ~ 1 !**

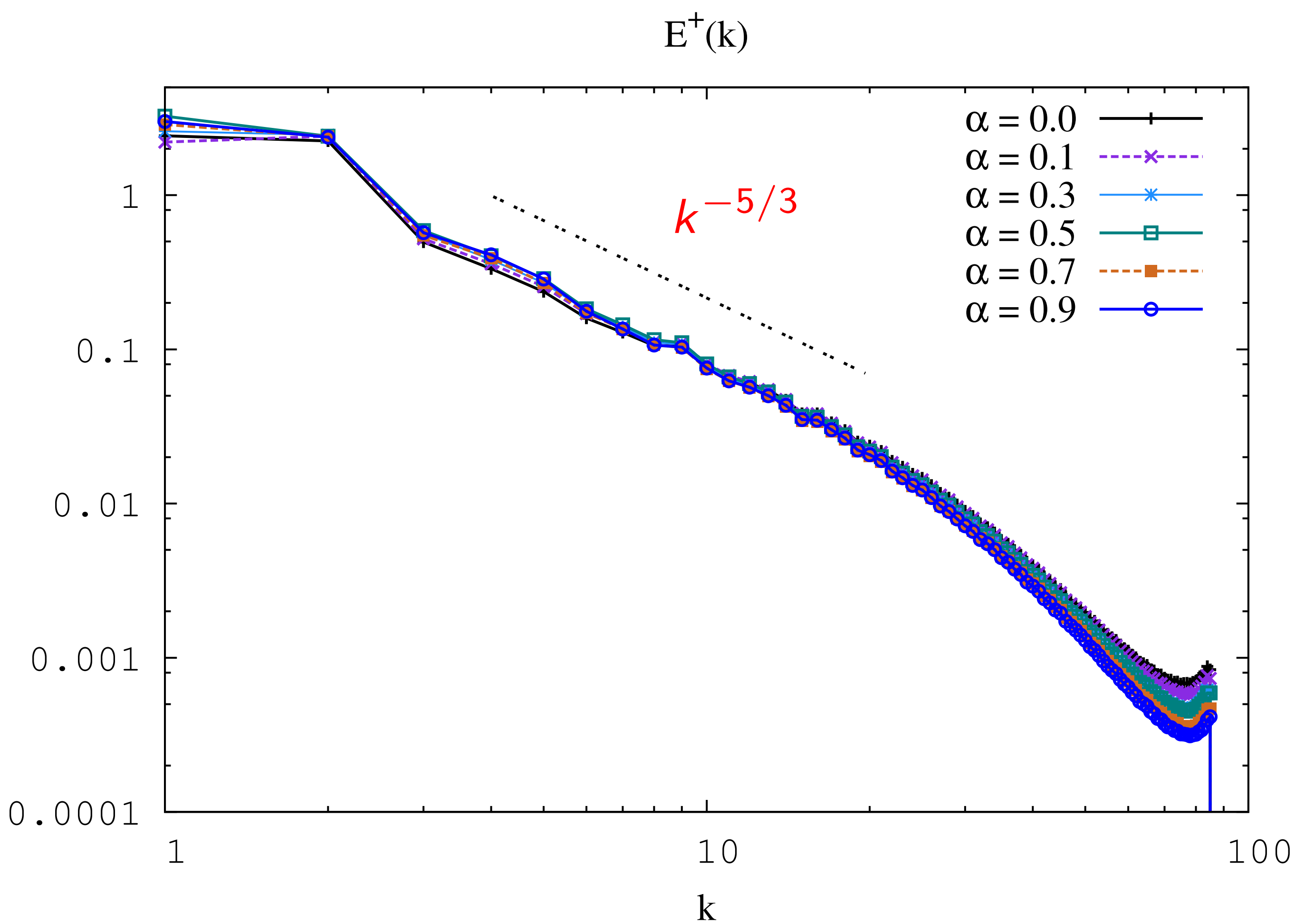


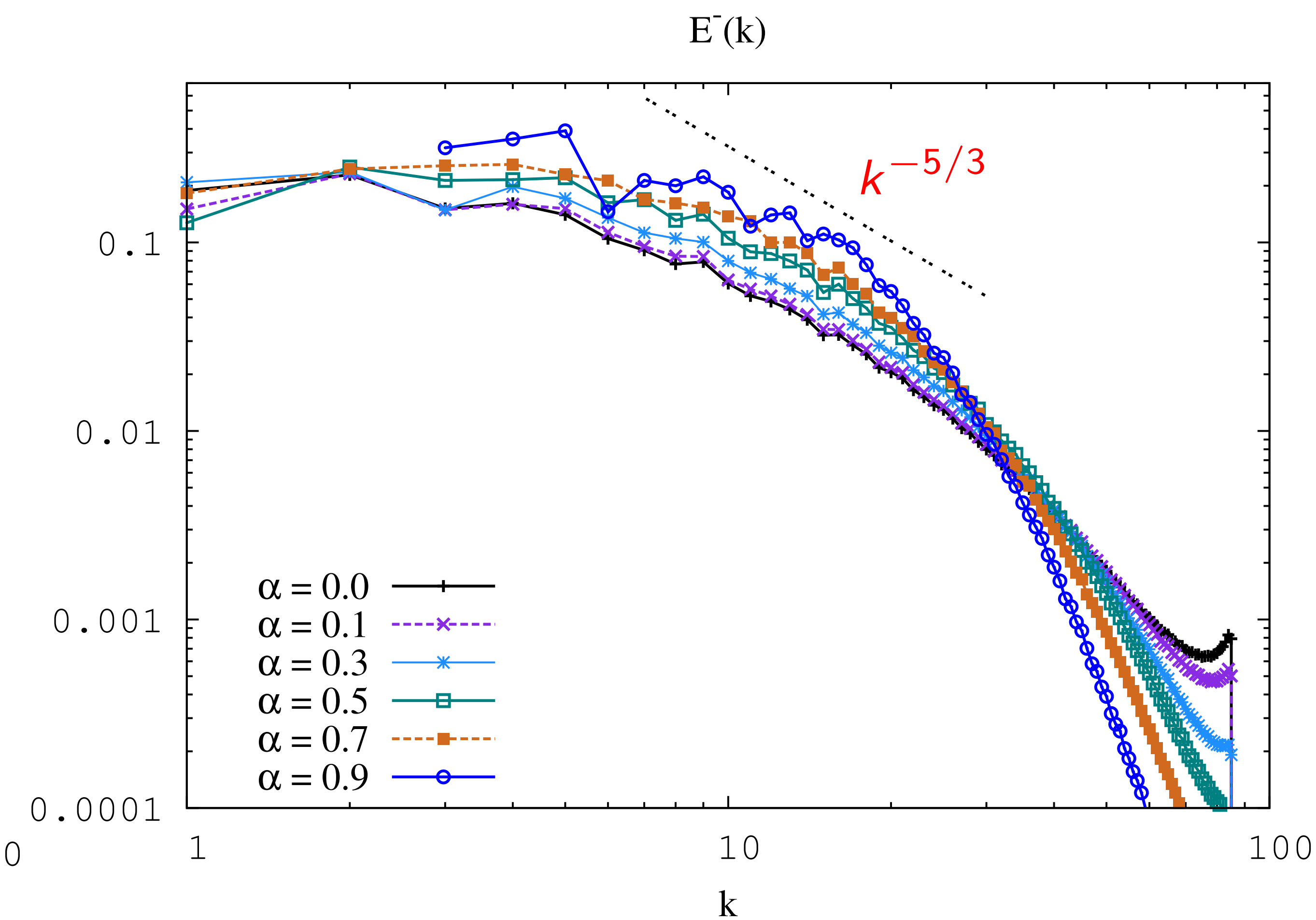
Reaction of negative modes

Chen, Phys. Fluids 2003

$$E^{\pm}(k) \sim C_1 \epsilon_E^{2/3} k^{-5/3} \left[1 \pm C_2 \left(\frac{\epsilon_H}{\epsilon_E}\right) k^{-1}\right],$$

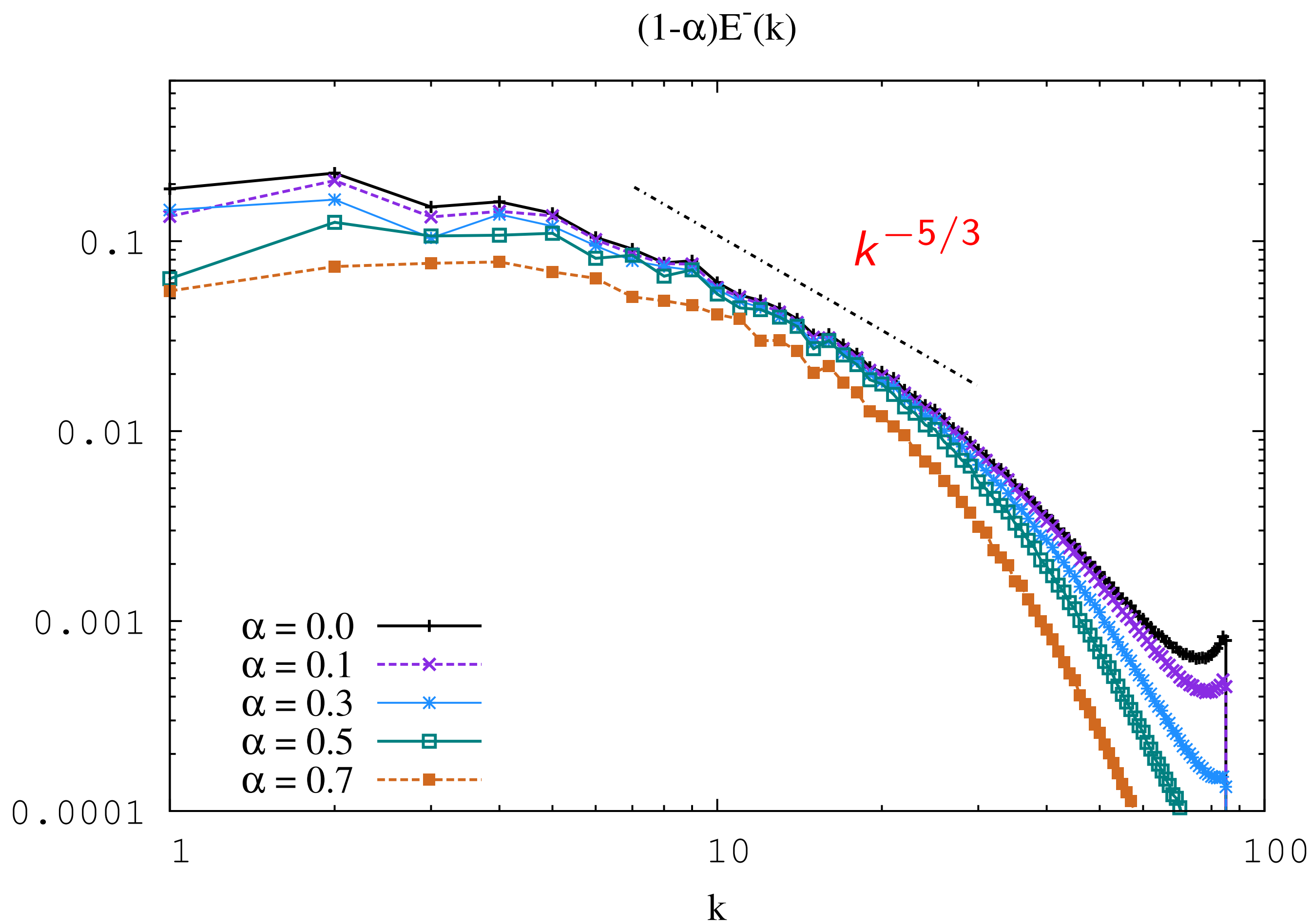
where ϵ_E is the mean energy dissipation rate
and ϵ_H is the mean helicity dissipation rate.





- $E^+(k)$ does not change with decimation.
- $E^-(k)$ contains more energy in the inertial range of scales and less in the dissipative scales.
- Invariance of parity is restored through scaling of $E^-(k)$ by the factor (1-α).

Restoration of parity invariance

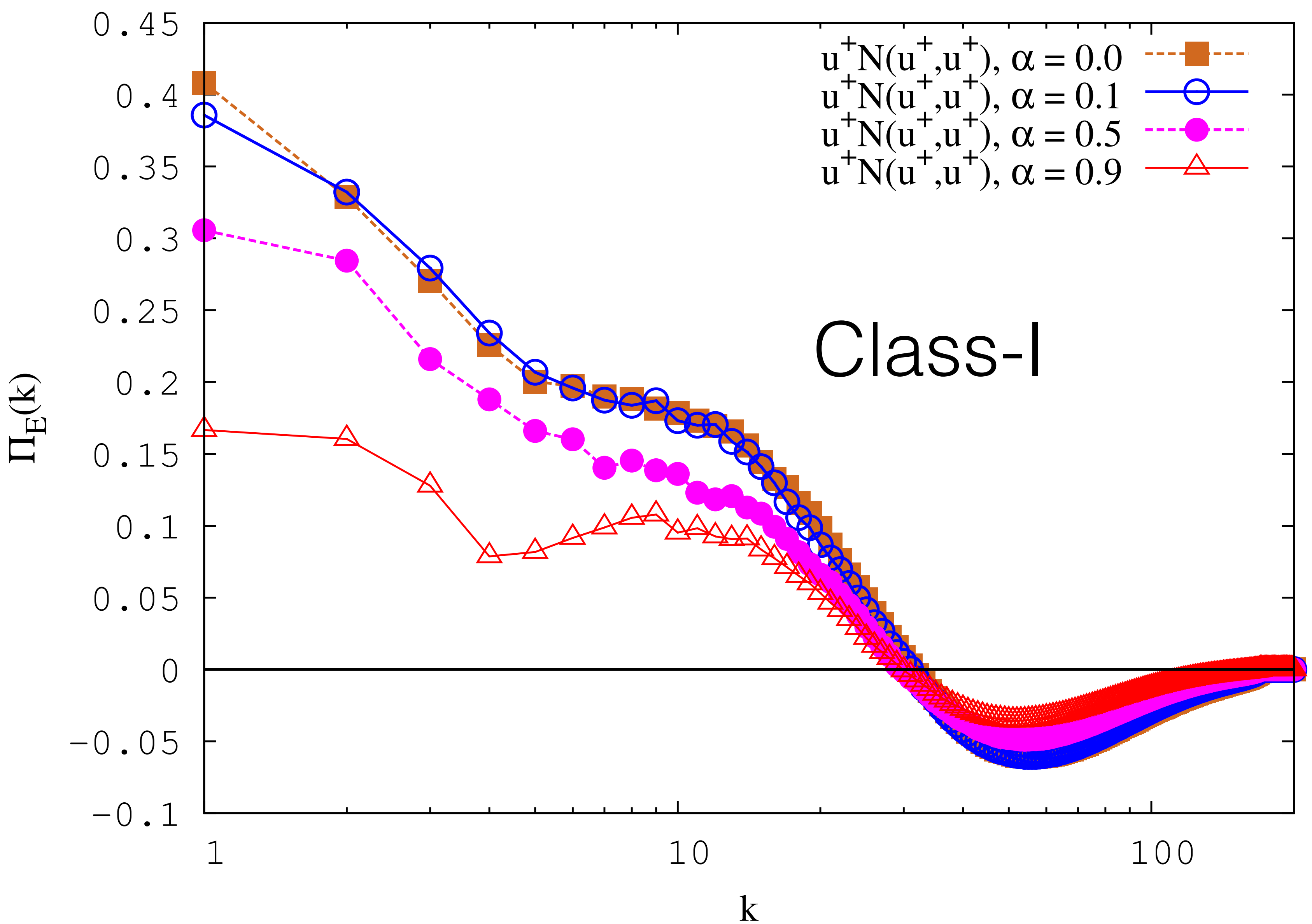


- The forward cascade of energy is though the triads of class-III where two large wavenumber modes have opposite sign of helicity.

- To maintain the constant flux, $u^-(k)$ must be rescaled by (1-α). since $u^-(k)$ exists with probability (1-α)

- Invariance of parity is restored through scaling of $E^-(k)$ by the factor (1-α).

Flux for different class of triads



- Using helical decomposition it is also possible to analyze in the importance of different triadic interactions.
- The contribution to the flux coming from the triads of class-I is always 'backward', even in FULL Navier-Stokes equations.

European Research Council

Summary

Università di Roma Tor Vergata

- As we increase decimation of the modes with negative helicity (α), the contribution of triads leading to inverse energy cascade grows.
- The forward cascade of energy is very robust in 3D turbulence. It requires only a few negative modes to act as catalyst to transfer energy forward.
- Only when α is very close to 1, i.e., we decimate almost all modes of one helical sign, inverse energy cascade takes over the forward cascade.
- We observe a strong tendency to recover parity invariance even in the presence of an explicit parity-invariance symmetry breaking ($\alpha > 0$).

- What about abrupt symmetry breaking at some k_c?
  - can we stop the cascade by killing all negatives modes from $k > k_c$?
  - or can we start it at our wish (killing all modes up to k_c)?
- What about intermittency in the forward cascade regime at changing α?

Thank you!

For more look at

- *On the role of helicity for large-and small-scales turbulent fluctuations*, G Sahoo, F Bonaccorso, L Biferale - arXiv preprint arXiv:1506.04906 (2015).

- *Inverse energy cascade in three-dimensional isotropic turbulence,* L Biferale, S Musacchio, F Toschi, Phys. Rev. Lett. 108, 164501 (2012).