Direct Numerical Simulations and models of Fluid and Magnetohydrodynamic(MHD) turbulence

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MHD equations:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = \nu \Delta \vec{u} - \nabla \bar{p} + (\vec{b} \cdot \nabla)\vec{b} + \vec{f}, \tag{1}$$

$$\frac{\partial \vec{b}}{\partial t} = \nabla \times (\vec{u} \times \vec{b}) + \eta \Delta \vec{b}, \tag{2}$$

Structure functions:

$$S_p(\ell) \equiv \langle [\delta u_{||}(\vec{r}, \ell)]^p \rangle, \tag{3}$$

where

$$\delta u_{||}(\vec{r}, \ell) \equiv [\vec{u}(\vec{r} + \vec{\ell}, t) - \vec{u}(\vec{r}, t)] \cdot \frac{\vec{\ell}}{\ell}. \tag{4}$$

K41 in fluid turbulence:

$$S_p(\ell) \sim (\varepsilon \ell)^{\zeta_p^{K41}}; \ \zeta_p^{K41} = p/3. \tag{5}$$

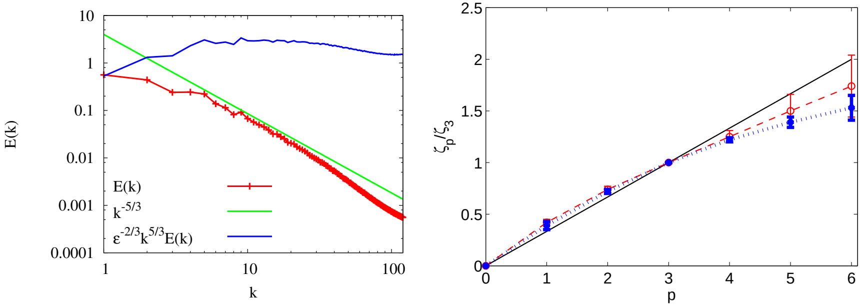


Figure : (left) Kinetic energy spectrum showing $k^{-5/3}$ power law in the inertial range. (right) Multiscaling in velocity and magntic fields. From simulations with 1024^3 collocation points.

Direct numerrical simulations

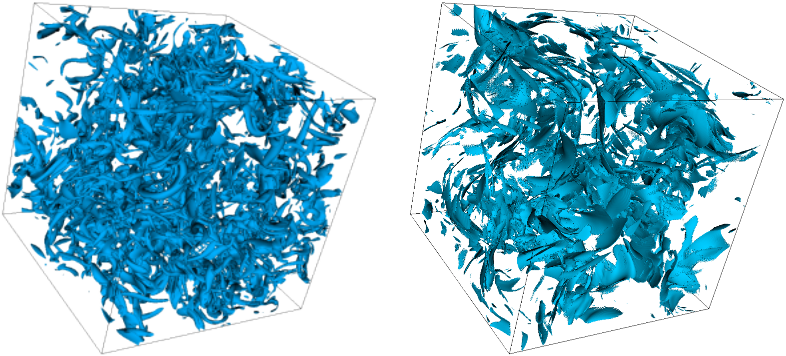


Figure : (Left) Vortex tubes in a NS simulation. (Right) Current sheets in a MHD simulation. Visualization using data from 1024^3 simulations.

- Efficient modular pseudospectral codes for NS and MHD equations.
- Higher order time integration scheme: Adams-Bashforth.
- Adaptive time stepping.
- Hybrid parallelisation using MPI and OpenMP.
- Large simulations (1024^3 grid points) on IBM Bluegene.
- Large size data management, analysis and visualization.
- Shell and Phython programming for workflow.
- Matlab, Matplotlib, Visit for data plotting and visualisation.

Dynamo Problem

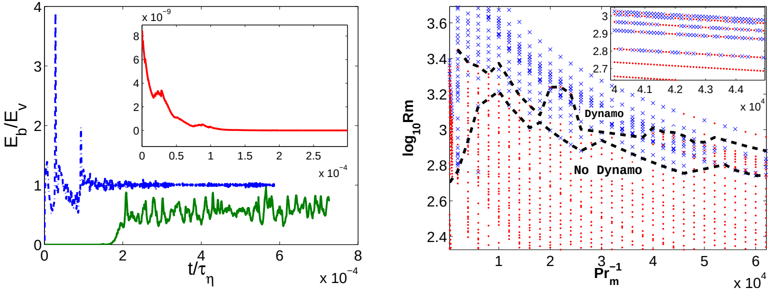
Shell model for MHD

$$\begin{aligned}\frac{du_n}{dt} + \nu k_n^2 u_n &= \imath[A_n(u_{n+1}u_{n+2} - b_{n+1}b_{n+2}) \\ &+ B_n(u_{n-1}u_{n+1} - b_{n-1}b_{n+1}) \\ &+ C_n(u_{n-2}u_{n-1} - b_{n-2}b_{n-1})]^*. \qquad (6)\end{aligned}$$

$$\begin{aligned}\frac{db_n}{dt} + \eta k_n^2 b_n &= \imath[D_n(u_{n+1}b_{n+2} - b_{n+1}u_{n+2}) \\ &+ E_n(u_{n-1}b_{n+1} - b_{n-1}u_{n+1}) \\ &+ F_n(u_{n-2}b_{n-1} - b_{n-2}u_{n-1})]^*. \qquad (7)\end{aligned}$$

- Dynamo onset: a nonequilibrium, first-order phase transition between two different turbulent, but statistically steady, states.
- The ratio of the magnetic and kinetic energies is a convenient order parameter for this transition.
- Stability diagram (or nonequilibrium phase diagram) for dynamo formation in the (Pr_m^{-1}, Re_m) plane for wide ranges of Pr_m and Re_m.
- This boundary appears to have a fractal character.
- The order parameter shows hysteretic behavior across this boundary.
- There are also suggestions of nucleation-type phenomena.

Dynamo Problem



(Left) Representative plots of the dynamo order parameter E_b/E_u versus time t/τ_η, in the dynamo region (blue), near the dynamo boundary (green), and in the no-dynamo regime (red, inset).

(Right) The dynamo stability diagram. The boundary between the two regions shows an intricate, interleaved pattern of fine, dynamo (blue cross) and no-dynamo (red dot) regimes. Inset shows a detailed view.

Thank you!

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