Role of the helicity in the energy transfer in three dimensional turbulence

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3D hydrodynamic turbulence

- Navier-Stoke's equations for incompressible flow

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{u} + \mathbf{f},$$

$$\nabla\cdot\mathbf{u} = 0.$$

- Energy, positive definite,

$$E = \int \mathbf{u}(\mathbf{x})\cdot\mathbf{u}(\mathbf{x})d^3x$$

- Helicity, pseudoscalar, not sign-definite (Betchov 1961; Moffatt 1969)

$$H = \int \mathbf{u}(\mathbf{x})\cdot\omega(\mathbf{x})d^3x$$

- Energy cascades from large scales to small scales

- For helical flows, there is joint forward cascade of energy and helicity (Chen 2003).

See

- The degree of knottiness of tangled vortex lines, **H. K. Moffatt**, J. Fluid Mech. **35,** 117 (1969).
- Helicity in laminar and turbulent flow, **H. K. Moffatt and A. Tsinober**, Annu. Rev. Fluid Mech. **24,** 281 (1992).

Inverse energy cascade in 3D

- Making the helicity sign-definite, we observe inverse cascade of energy.

PRL **108,** 164501 (2012) PHYSICAL REVIEW LETTERS week ending 20 APRIL 2012

Inverse Energy Cascade in Three-Dimensional Isotropic Turbulence

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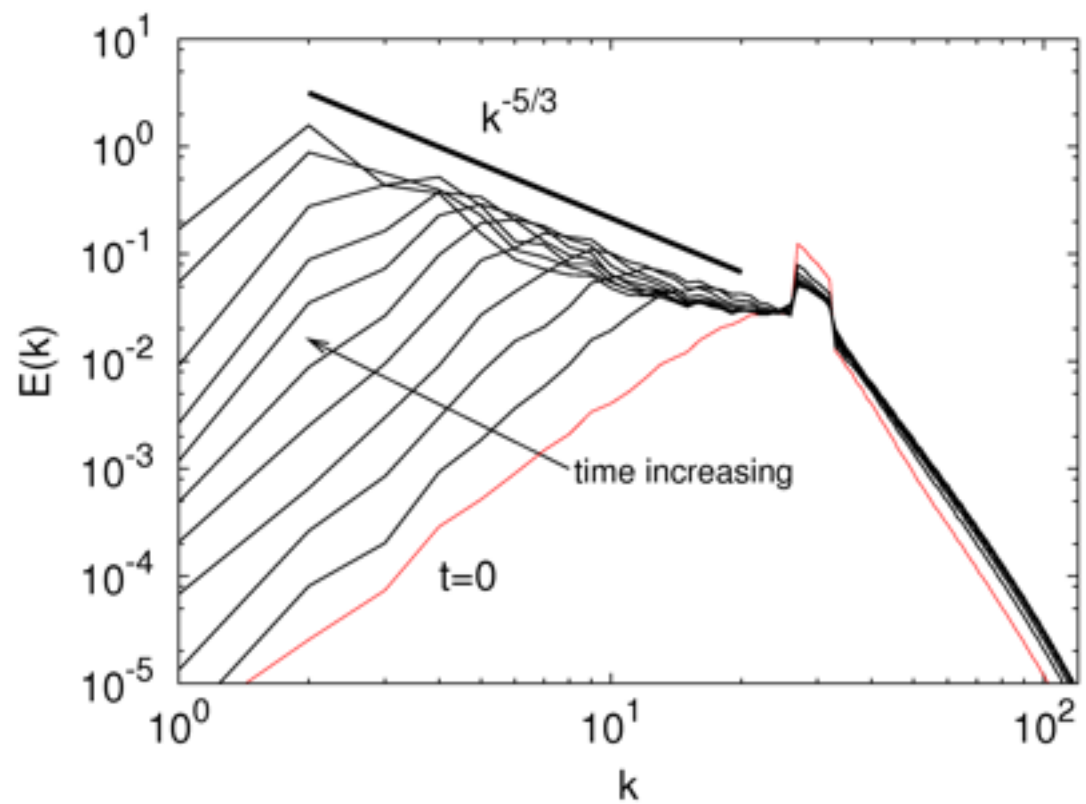

- By projecting the velocity field on helically polarised waves of with one sign of helicity, helicity could be made sign-definite.

Navier-Stokes equations

- We performed DNS of 3D Navier-Stokes equations in Fourier-space

$$\dot{u}_i(k) + \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) N_j(k) = -\nu k^2 u_i(k),$$
$$\text{where} \quad N_i(q) = \sum_{\mathbf{q}=\mathbf{k}+\mathbf{p}} i k_j u_i(k) u_j(p)$$

- For incompressible flows, $\boldsymbol{u}(\boldsymbol{k})$ has two degrees of freedom, since $\boldsymbol{k} \, . \, \boldsymbol{u}(\boldsymbol{k}) = 0$.

- Following Waleffe (1992) we write $\boldsymbol{u}(\boldsymbol{k})$ as sum of projections on the orthonormal helical waves with definite sign of helicity.

$$\mathbf{u}(\mathbf{k},t) = a^+(\mathbf{k},t)\mathbf{h}^+(\mathbf{k}) + a^-(\mathbf{k},t)\mathbf{h}^-(\mathbf{k})$$
$$\equiv \mathbf{u}^+(\mathbf{k},t) + \mathbf{u}^-(\mathbf{k},t)$$

▸ Decimated Navier-Stokes equations in Fourier space:

$$\partial_t \mathbf{u}^\pm(\mathbf{k},t) = \mathcal{P}^\pm(\mathbf{k})\mathbf{N}_{u^\pm}(\mathbf{k},t) + \nu k^2 \mathbf{u}^\pm(\mathbf{k},t) + \mathbf{f}^\pm(\mathbf{k},t)$$

where ν is kinematic viscosity and $\mathbf{f}$ is external forcing.

▸ The nonlinear term containing all triadic interactions

$$\mathbf{N}_{u^\pm}(\mathbf{k},t) = \mathcal{F}T(\mathbf{u}^\pm \cdot \nabla \mathbf{u}^\pm - \nabla p)$$

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Classes of triadic interactions in NS equations

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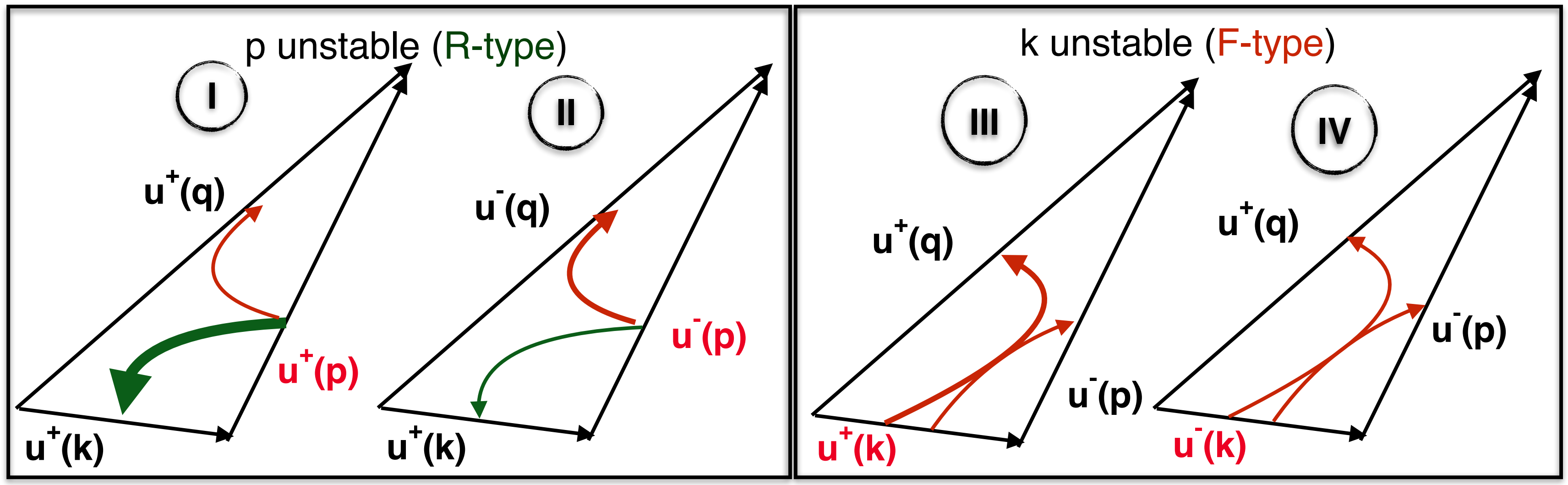
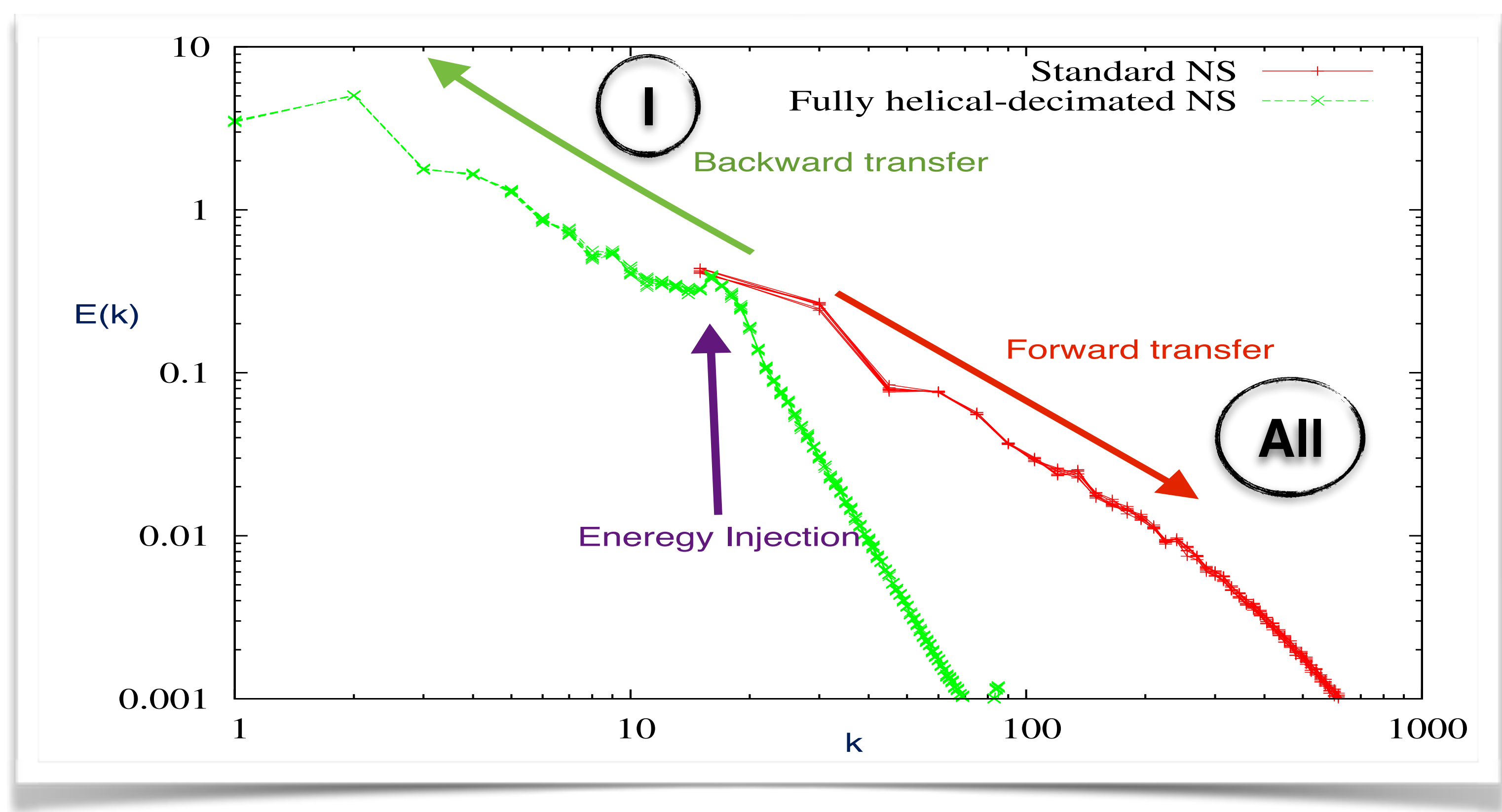
$$\mathbf{N}_{\mathbf{u}^{\pm}}(\mathbf{q}) = \mathcal{FT}\left[\mathbf{u}^{\pm}(\mathbf{k}) \cdot \nabla \mathbf{u}^{\pm}(\mathbf{p})\right]; \mathbf{q} = \mathbf{k} + \mathbf{p}; k \leq p \leq q$$



- There are 8 possible triads divided into 4 classes.
- Energy and helicity are conserved for each individual triad. Waleffe PoF A 4 (2) (1992), Moffat JFM 741, R3 (2014).
- The Class-I of triads made of helical modes of same sign results in inverse energy transfer.



Can direct and inverse cascade of energy co-exist?

What happens in between??

when we give different weights to different class of triads…

- Modified projection operator:

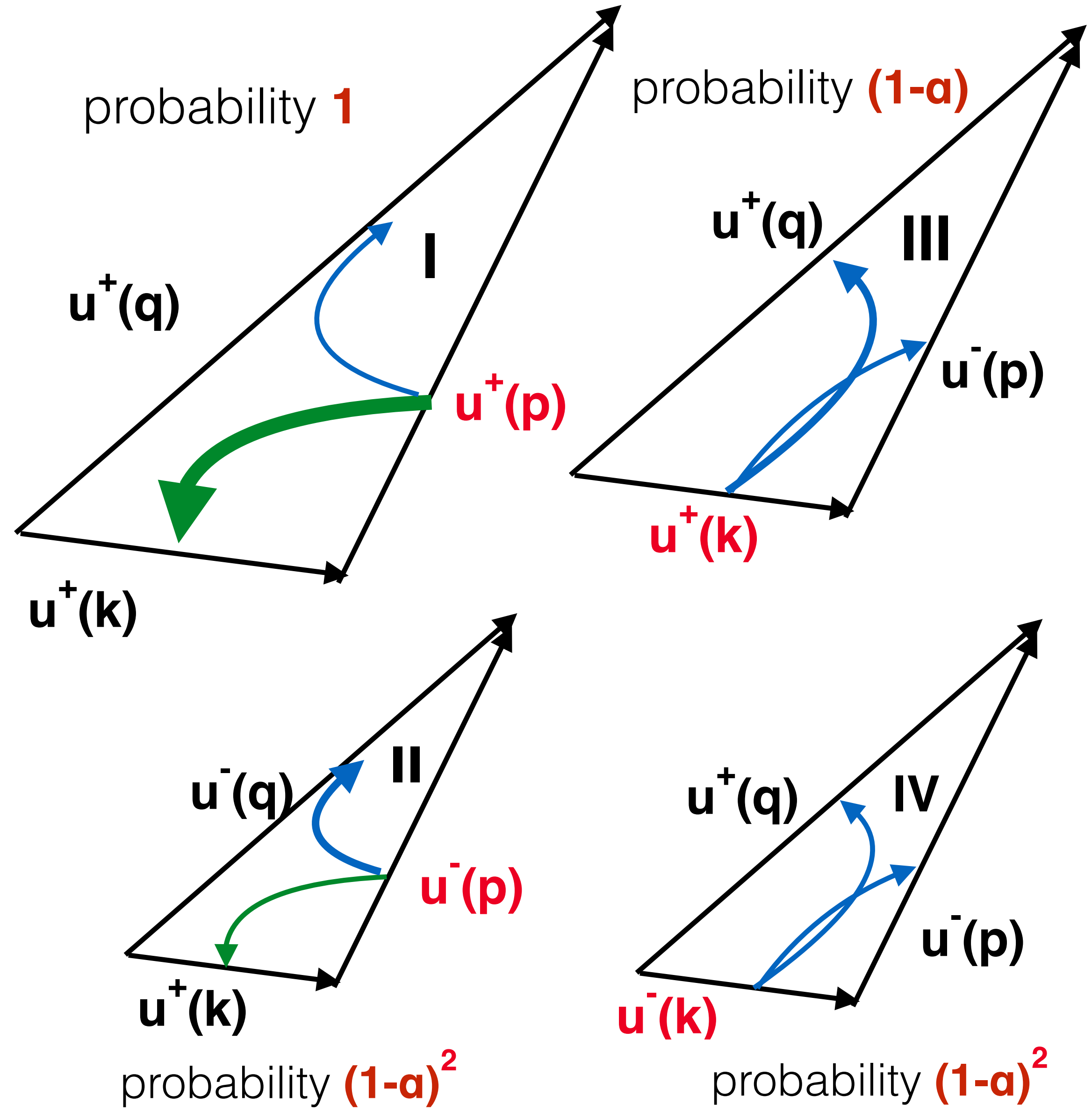
$$\mathcal{P}^+_\alpha(\mathbf{k})\mathbf{u}(\mathbf{k},t) = \mathbf{u}^+(\mathbf{k},t) + \theta_\alpha(\mathbf{k})\mathbf{u}^-(\mathbf{k},t)$$

  where $\theta_\alpha(\mathbf{k})$ is 0 with probability α and is 1 with probability $1-\alpha$.

- We consider triads of Class-I with probability 1, Class-III with probability $1-\alpha$ and Class-II and Class-IV with probability $(1-\alpha)^2$.

- $\alpha = 0 \rightarrow$ Standard Navier-Stokes.
  $\alpha = 1 \rightarrow$ Fully helical-decimated NS.

- Critical value of α at which forward cascade of energy stops?
  alternatively, inverse cascade of energy stops if forced at small scales.



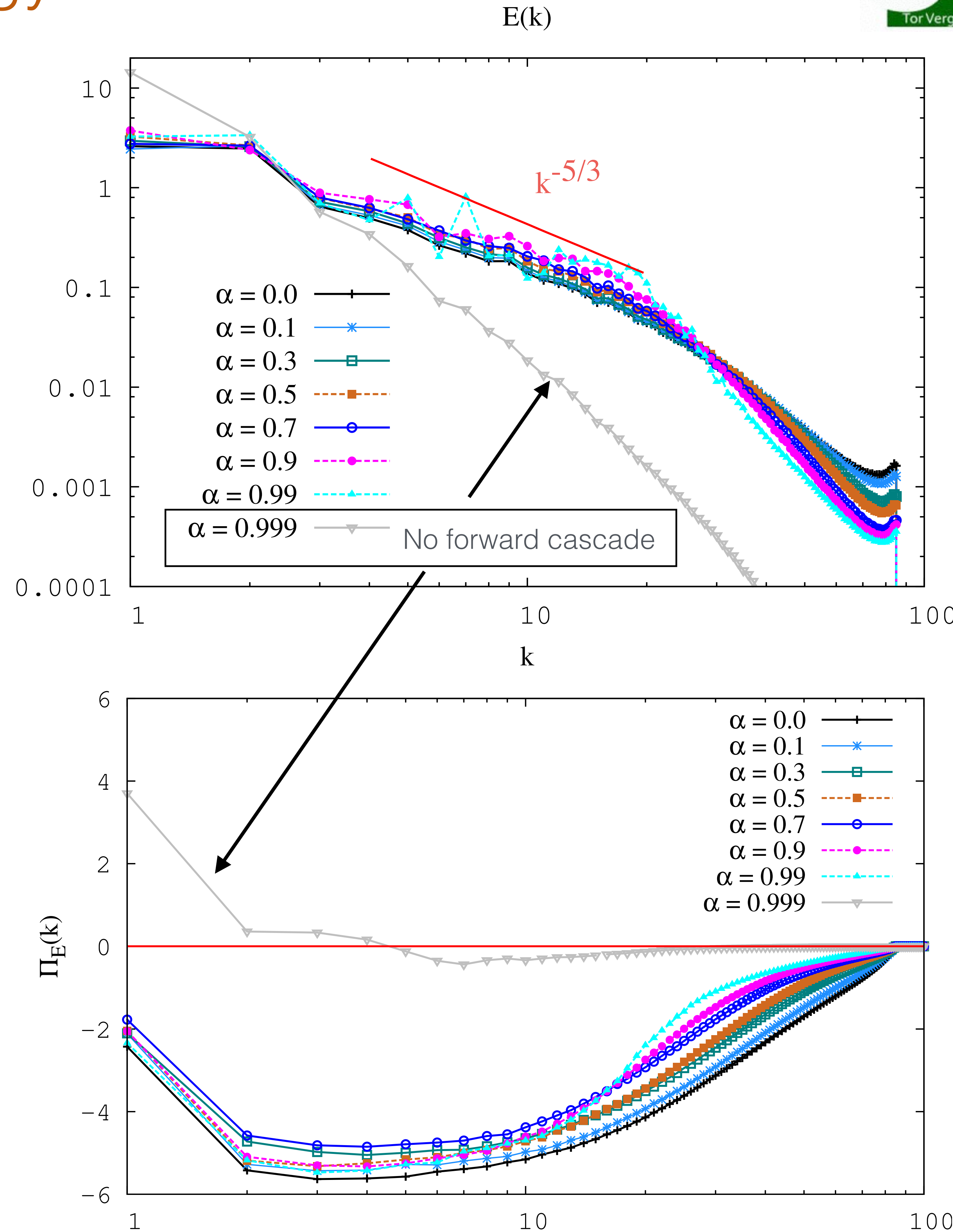
$$\mathbf{N}_{\mathbf{u}^\pm}(\mathbf{q}) = \mathcal{FT}\left[\mathbf{u}^\pm(\mathbf{k})\cdot\nabla\mathbf{u}^\pm(\mathbf{p})\right];\ \mathbf{q}=\mathbf{k}+\mathbf{p};\ k\le p\le q$$

Note: We still have infinitely many triads, in the limit of infinite Re.

Robustness of energy cascade

- Spectra for all values of α showing $k^{-5/3}$ suggest the forward cascade of to be strongly robust.
- Unless we kill almost all the modes of one helicity-type energy always finds a way to reach small scales.
- The energy flux also remains unaffected by the decimation until α is very close to 1.
- **Critical value of α is ~ 1 !**
- Modes with opposite helicity act like catalysers for the forward energy transfer.

Sahoo, Bonaccorso, and Biferale,
Phys. Rev. E 92, 051002 (2015) (Rapid Comm.).



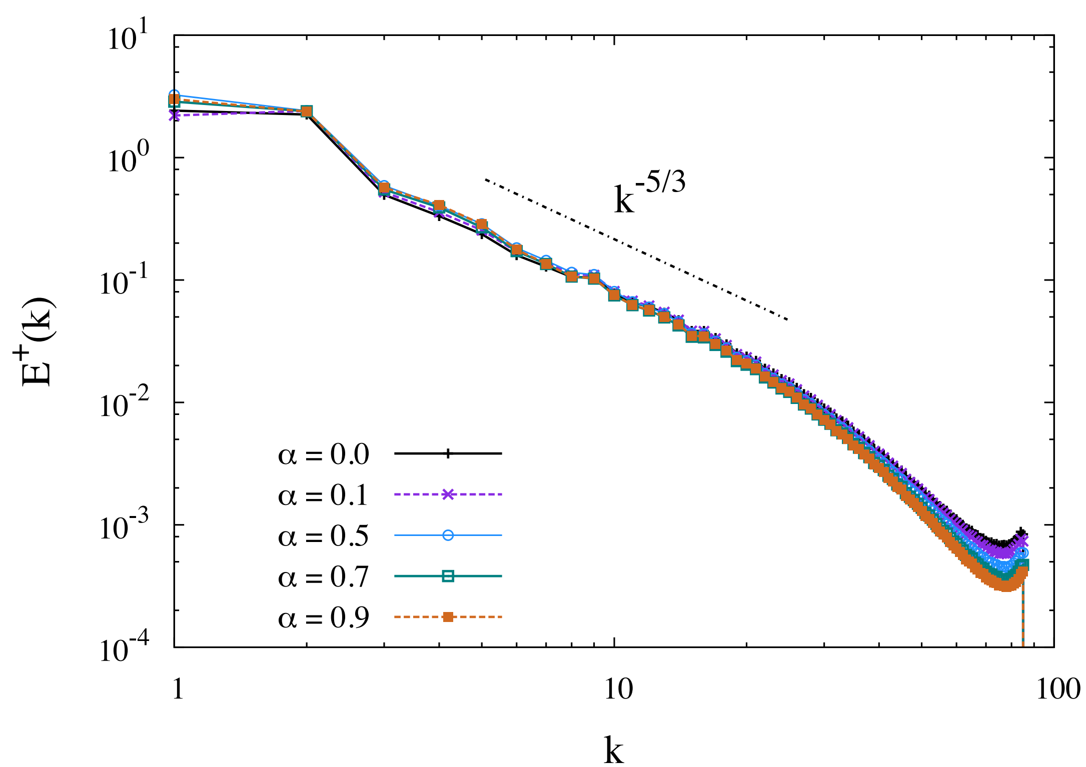
Restoration of parity invariance

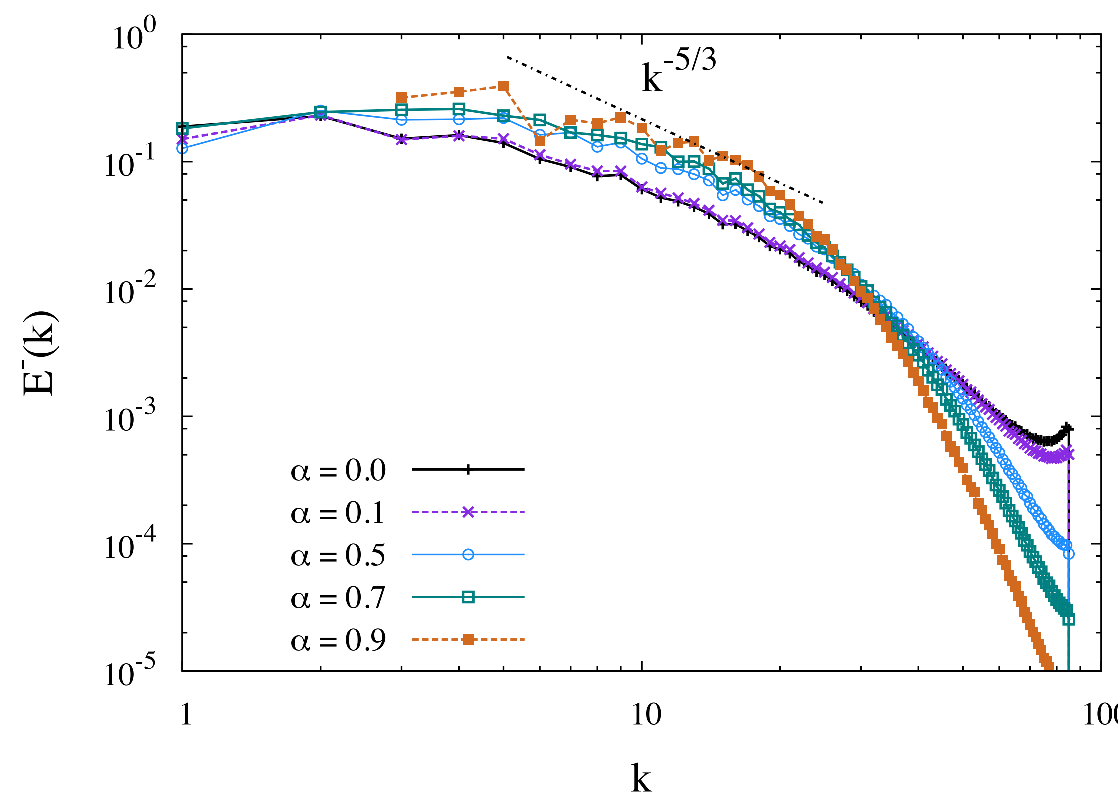
Chen, Phys. Fluids 2003

$$E^{\pm}(k) \sim C_1 \epsilon_E^{2/3} k^{-5/3} \left[1 \pm C_2 \left(\frac{\epsilon_H}{\epsilon_E} \right) k^{-1} \right],$$

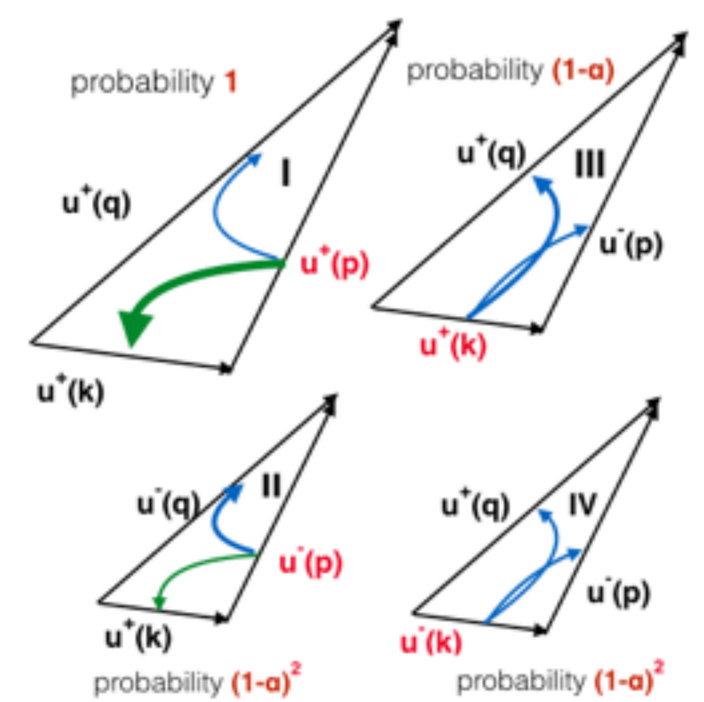
where ϵ_E is the mean energy dissipation rate and ϵ_H is the mean helicity dissipation rate.

Sahoo, Bonaccorso, and Biferale, Phys. Rev. E 92, 051002 (2015) (Rapid Comm.).

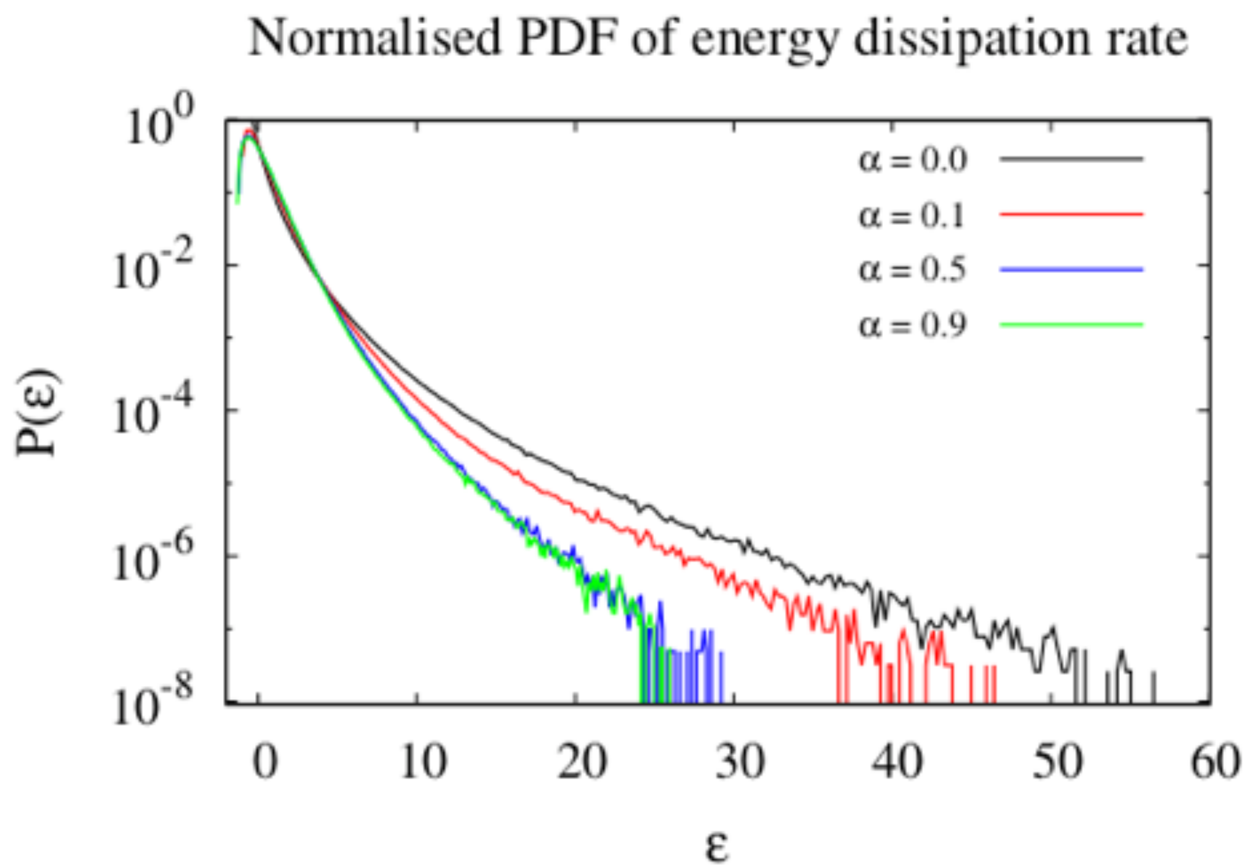


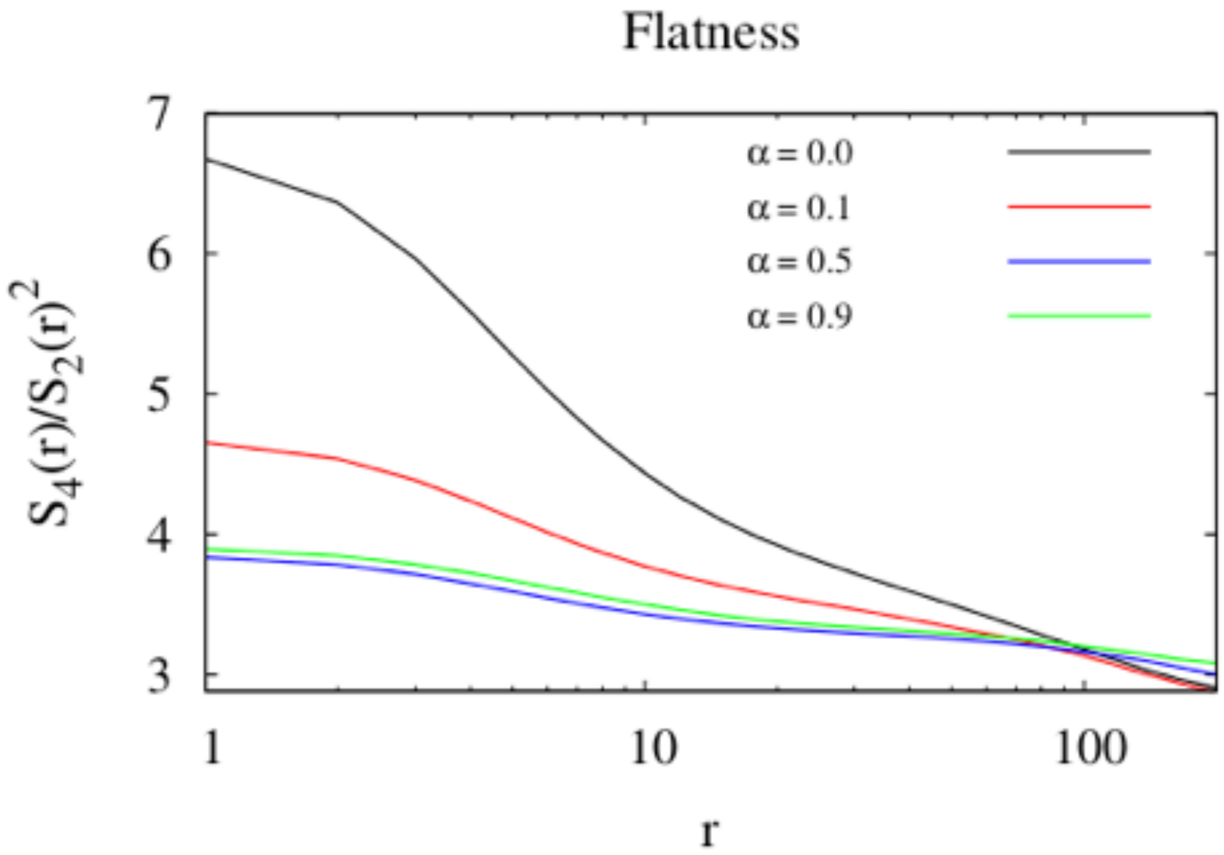


- The $E^+(k)$ does not change with decimation.
- Fewer the negative helical modes, more energetic they are, to restore the explicitly broken parity invariance!
- Triads of Class-III and Class-IV are more effective than Class-I.

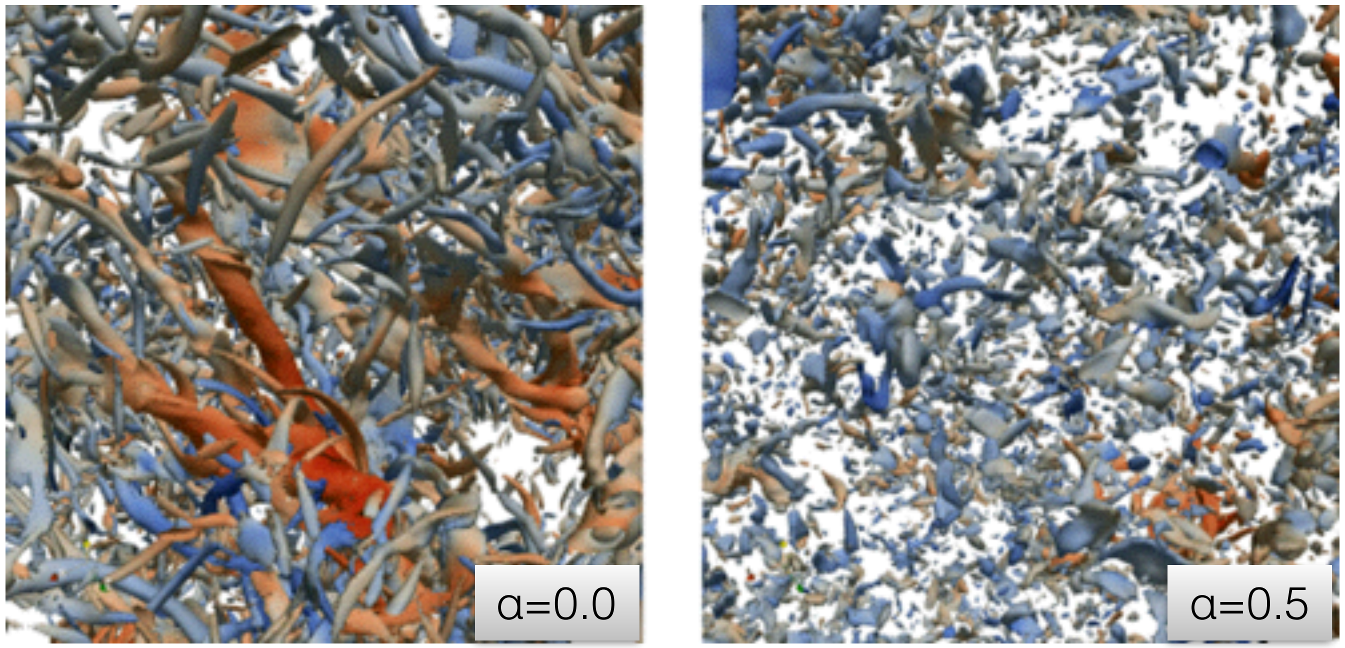


Intermittency





- Intermittency reduces significantly even when 10% of *u*- modes are removed.
- Extreme events of dissipation and the vortex tubes disappear.
- It reduces further and seems to be saturated with increasing α.
- Presence of all helical modes is crucial for intermittency.

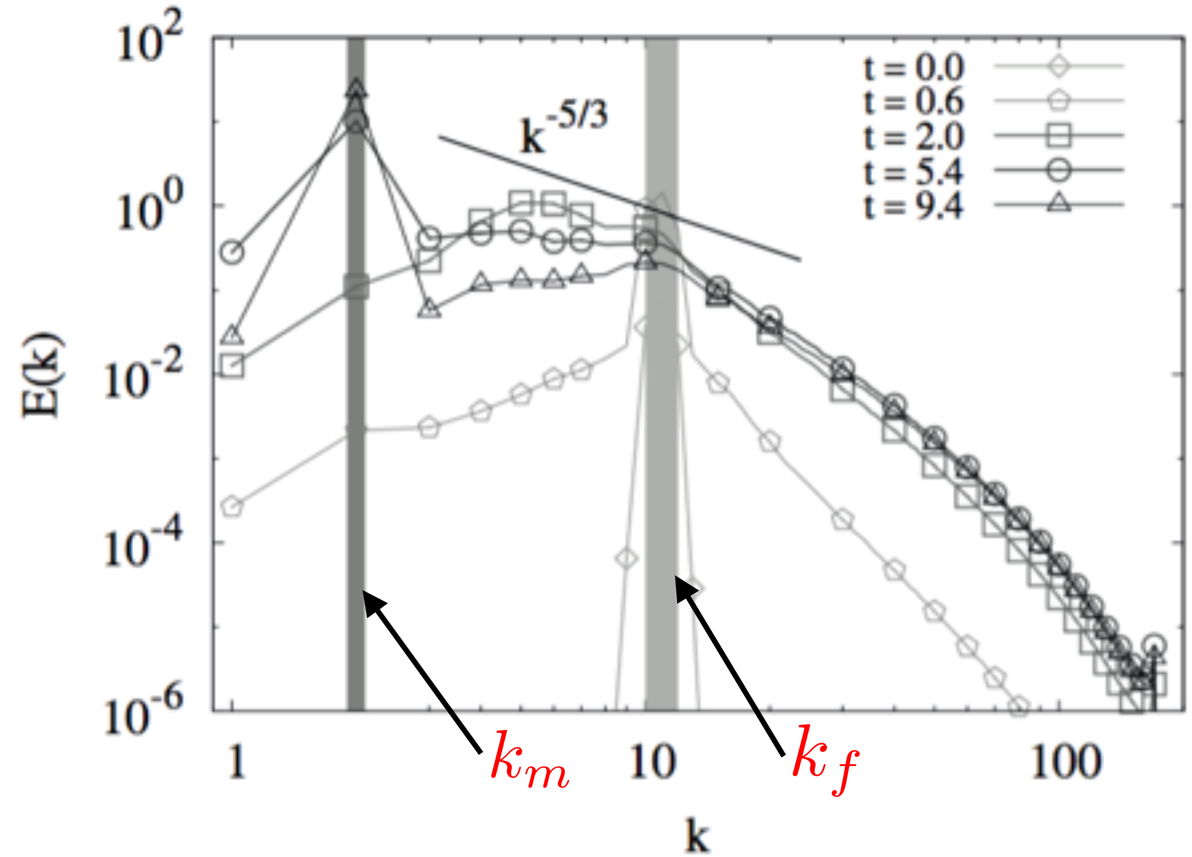


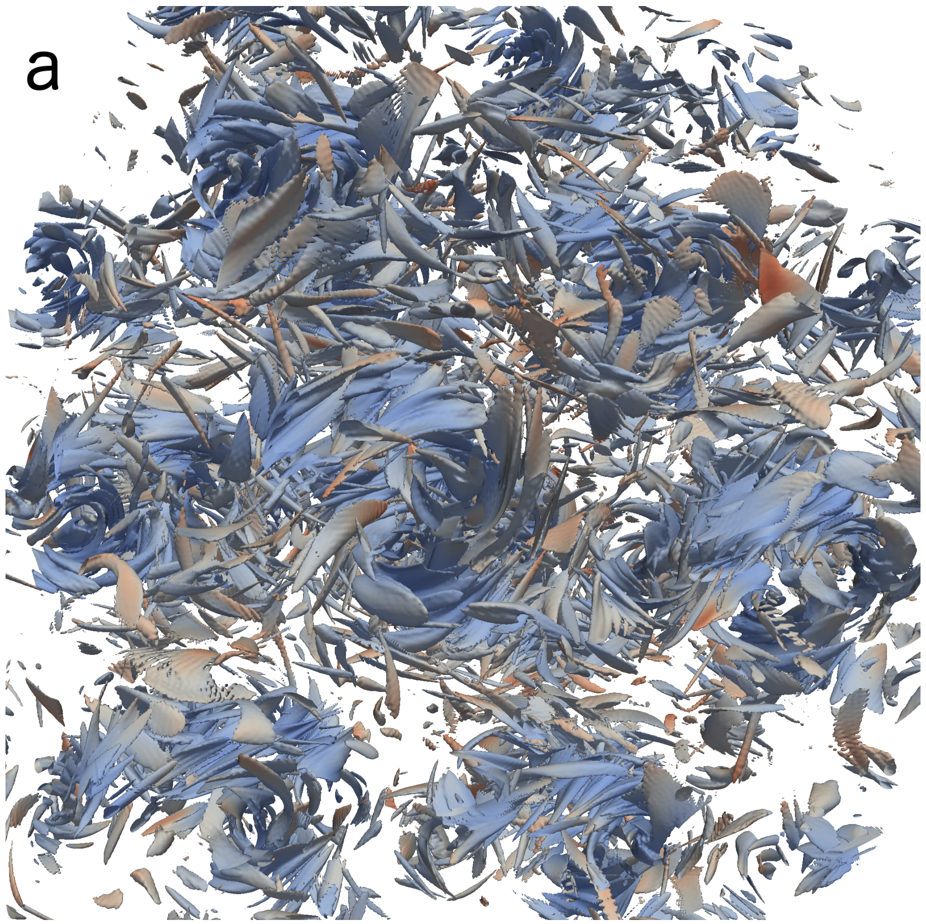
Isovorticity surfaces

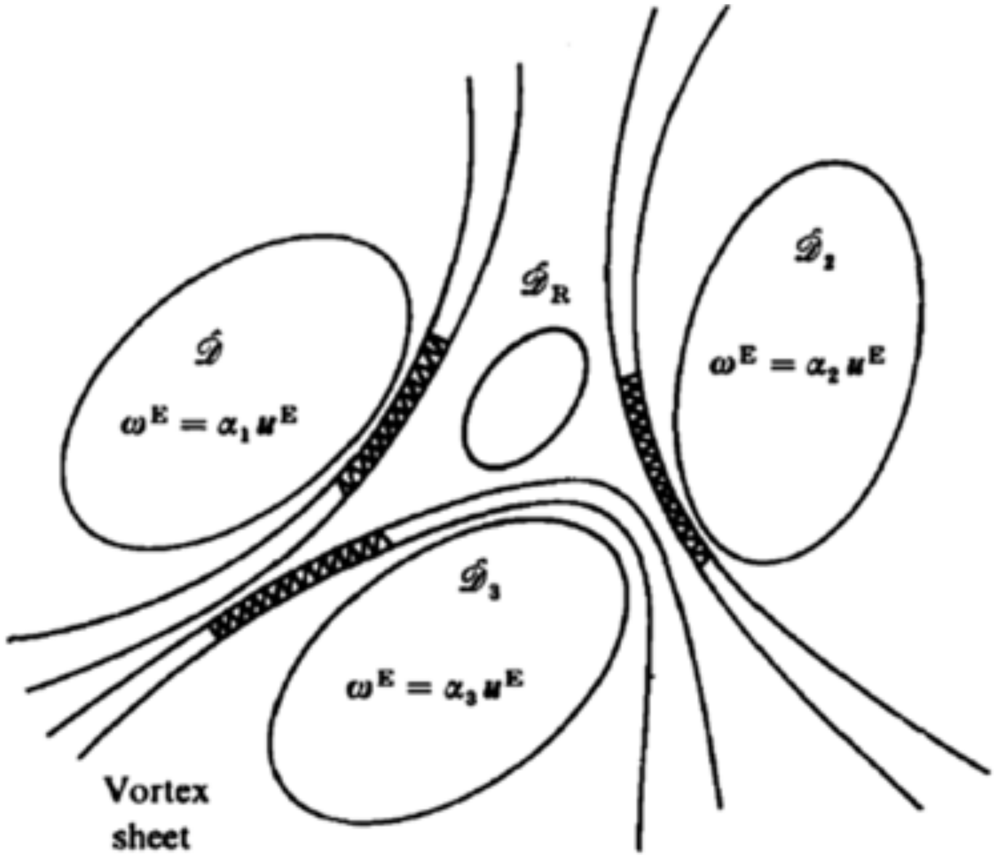
Helical condensates

- When the u^- modes are present only around a wavenumber k_m helical condensed are formed, realising a sort of Beltrami-patches.

- These coherent structures are similar to the ones suggested by Moffatt as ideal structures near maximal helicity,







Sahoo and Biferale
Eur. Phys. J. E 38, 114 (2015)

H. K. Moffatt
Proc. Natl. Acad. Sci.111, 3663 (2014)

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Conclusions

- The forward cascade of energy is very robust in 3D turbulence. It requires only a few negative modes to act as catalyst to transfer energy forward.
- Only when α is very close to 1, i.e., we decimate almost all modes of one helical sign, inverse energy cascade takes over the forward cascade.
- We observe a strong tendency to recover parity invariance even in the presence of an explicit parity-invariance symmetry breaking ($\alpha > 0$).
- There is drastic reduction of intermittency with decimation. Vortex tubes usually associated with extreme events of energy dissipation disappear.
- Most importantly, only removal of helical modes dynamically, make this difference.
- Helical condensates are observed at the scales where oppositely signed helical modes in minority are present.

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Thank you!

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- *Role of helicity for large-and small-scales turbulent fluctuations,* G Sahoo, F Bonaccorso, and L Biferale. Phys. Rev. E 92, 051002 (R) (2015).
- *Disentangling the triadic interactions in Navier-Stokes equations*, G Sahoo and L Biferale. Eur. Phys. J. E 38, 114 (2015).
- *Inverse energy cascade in three-dimensional isotropic turbulence*, L Biferale, S Musacchio, and F Toschi. Phys. Rev. Lett. 108, 164501 (2012).