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Isotropic and anisotropic scaling in homogeneous turbulence

Frontiers in Turbulence
KRS70 at Denver
Symposium

Credits: **K. Iyer** (NYU), **F. Bonaccorso** (U. Tor Vergata), **F. Toschi** (TuE)

INFN Istituto Nazionale di Fisica Nucleare

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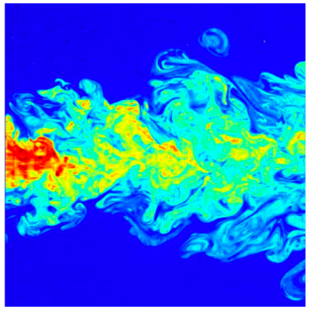
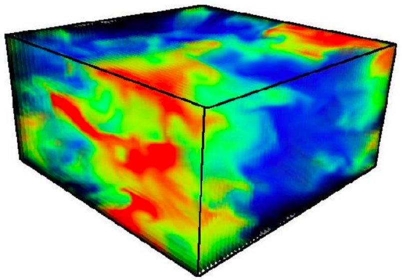
CINECA

- UNIVERSALITY OF SMALL-SCALES FLUCTUATIONS IN TURBULENCE
- RETURN-TO-ISOTROPY IN THE PRESENCE OF LARGE-SCALE SHEAR
- TOOLS: SO(3) DECOMPOSITION
- NUMERICAL AND EXPERIMENTAL RESULTS FROM LATE 90'S
- WHERE WE ARE NOW
- OPEN QUESTIONS

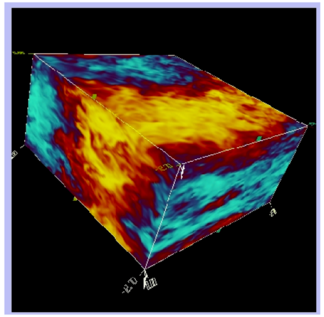
$$\begin{cases} \partial_t \mathbf{v} + (\mathbf{v}\cdot\partial)\mathbf{v} = -\partial P + \nu\Delta\mathbf{v} + \mathbf{F} \\ \partial\cdot\mathbf{v} = 0 \\ + \; Boundary \; Conditions \end{cases}$$



Turbulent jet



3d Convective Cell



Shear Flow

UNIVERSAL OR NOT UNIVERSAL?

$$S^{(n)}(\mathbf{r}) \equiv \left\langle \left[(\mathbf{v}(\mathbf{x}+\mathbf{r}) - \mathbf{v}(\mathbf{x})) \cdot \hat{\mathbf{r}} \right]^n \right\rangle$$

rotational invariant operators

$$\partial_t S^{(2)}(\mathbf{r}) + \Gamma^{(3)} S^{(3)}(\mathbf{r}) - 2\nu\nabla^2 S^{(2)}(\mathbf{r}) = f^{(2)}(\mathbf{r})$$

$r \ll L_f$

$$\partial_t S^{(2)}(\mathbf{r}) + \Gamma^{(3)} S^{(3)}(\mathbf{r}) - 2\nu\nabla^2 S^{(2)}(r) \sim 0$$

$+$ SO(3)

Weak Anisotropy

$$S_j^{(2)}(\mathbf{r}) = \sum_{m=-j}^{m=+j} Y_{jm}(\hat{\mathbf{r}}) \int S_{jm}^{(2)}(r) Y_{jm}(\hat{\mathbf{r}}) d\hat{\mathbf{r}}$$

$$\partial_t S_j^{(2)}(\mathbf{r}) + \Gamma_j^{(3)} S_j^{(3)}(\mathbf{r}) - 2\nu\nabla^2 S_j^{(2)}(\mathbf{r}) \sim 0 \qquad j = 0, 1, 2, \ldots$$

FOLIATION

$$S^{(n)}(\mathbf{r}) = \sum_{j=0}^{\infty} \sum_{m=-j}^{m=j} S_{jm}^{(n)}(r) Y_{jm}(\hat{\mathbf{r}})$$

Working Hypothesis

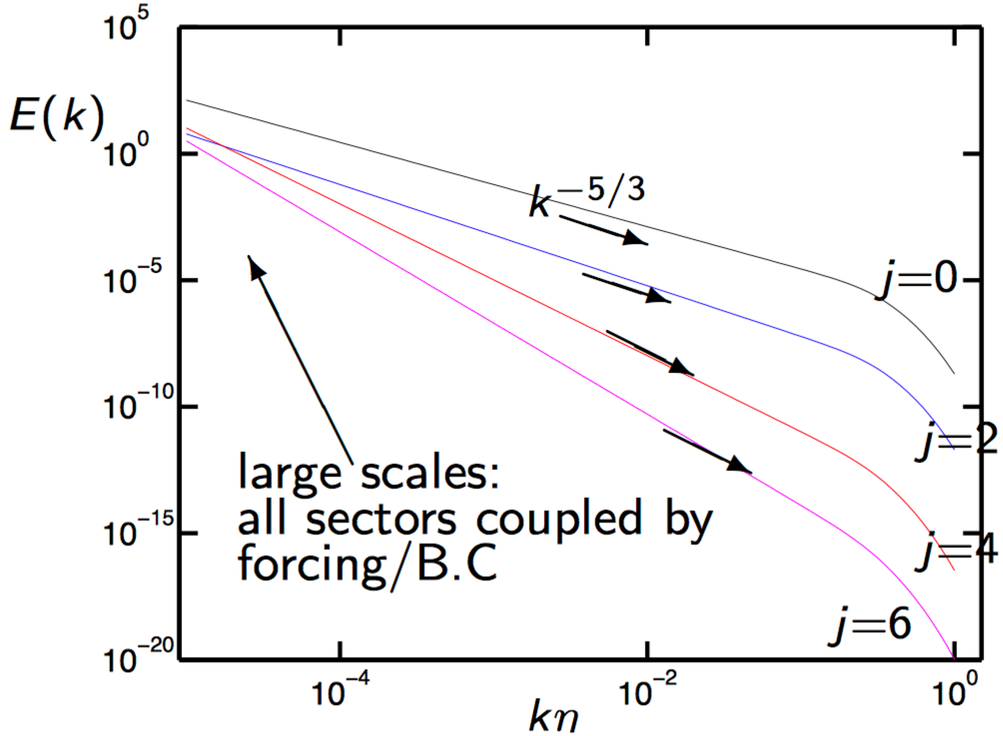
$$S_{jm}^{(n)}(r) = A_{jm}^{(n)} \left(\frac{r}{L}\right)^{\varsigma_j^n}$$

- Projection on sector-j has universal scaling exponent ς_j^n in inertial range depending on that sector **only**
- Power law behavior **only** in each separated sector
- Prefactors depend on large scale physics

UNIVERSAL

$$S^{(n)}(\mathbf{r}) \sim A_0 \left(\frac{r}{L}\right)^{\varsigma_0^n} + A_1 \left(\frac{r}{L}\right)^{\varsigma_1^n} + A_2 \left(\frac{r}{L}\right)^{\varsigma_2^n} + + \dots$$

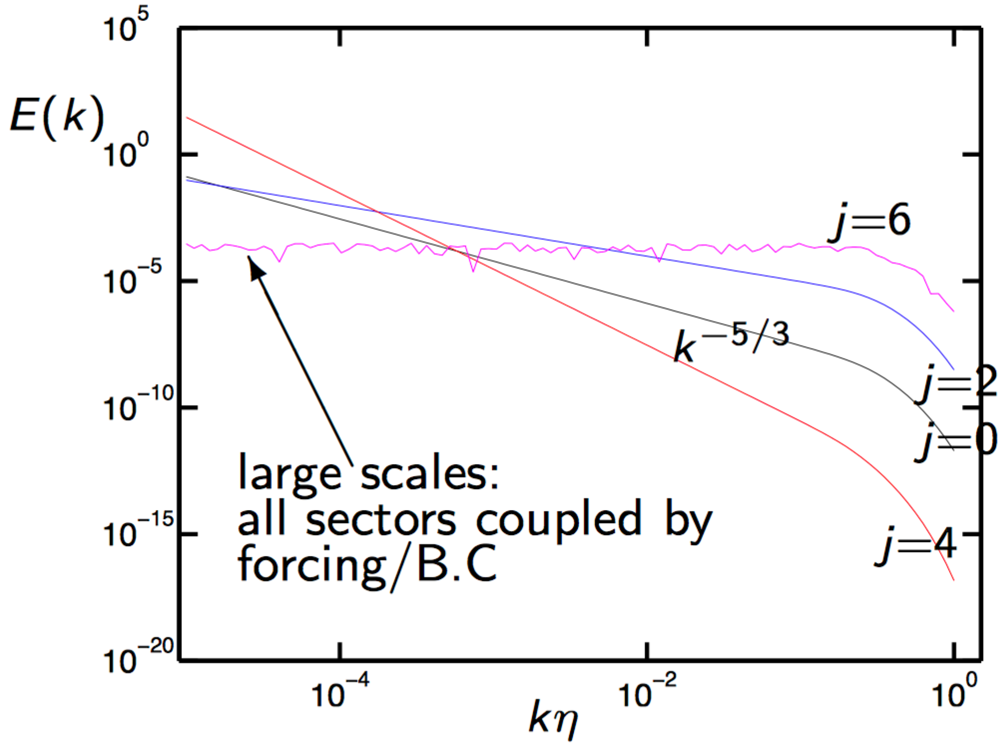
NON UNIVERSAL



Univerality: leading isotropic sector

return-to-isotropy

$$\zeta^n_{j=0} < \zeta^n_{j=2} < \zeta^n_{j=4} \cdots$$



NO Univerality:
sub-leading isotropic sector

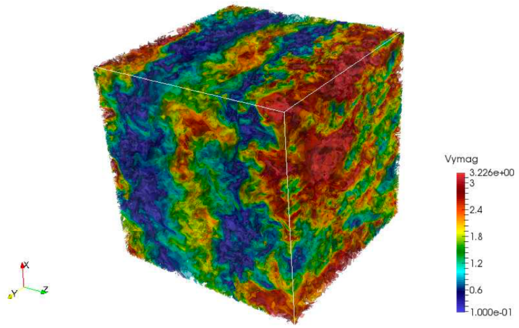
Isotropy not recovered at small-scales

$$\zeta^n_{j=0} < \zeta^n_{j=2} < \zeta^n_{j=4} \cdots$$

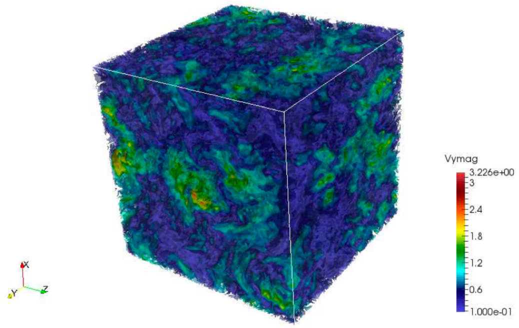
Random Kolmogorov Flow (Biferale, Toschi PRL 01)

- RKF is stationary, homogeneous on average and anisotropic

$1024^3, R_\lambda = 280$



t/T_E=1



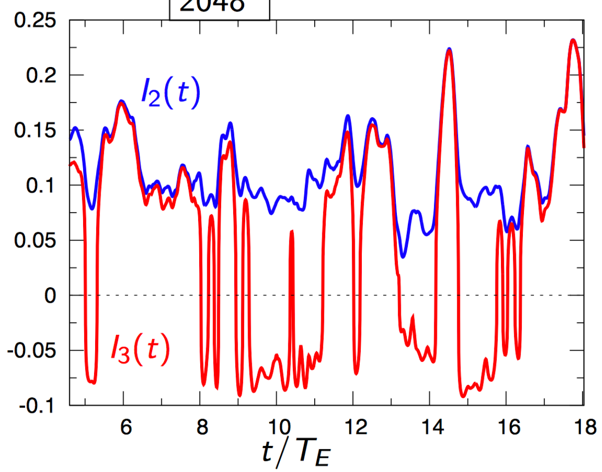
t/T_E=3

- Anisotropic forcing:

$$\boxed{f_i(\mathbf{k}_{1,2}) = \delta_{i,2} f_{1,2}(t) e^{i\theta_{1,2}(t)}}\ , \mathbf{k}_1 = (1,0,0), \mathbf{k}_2 = (2,0,0)$$

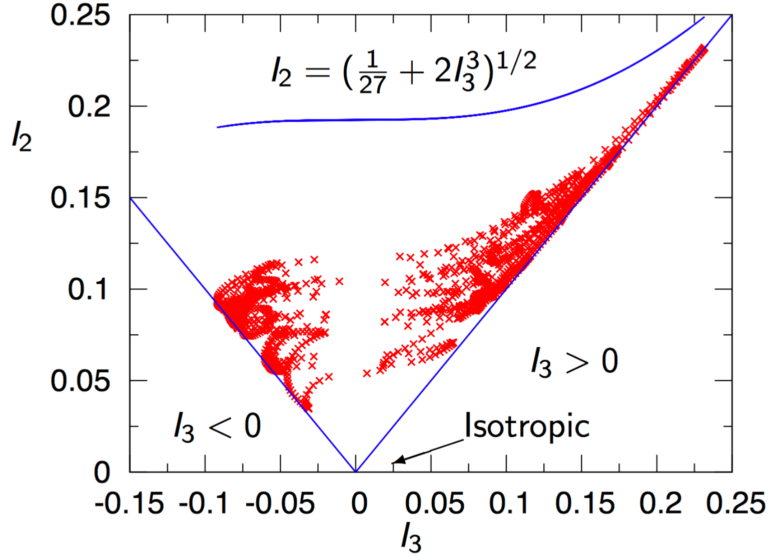
- Reynolds Stress Tensor $\boxed{b_{ij} \equiv \langle u_i u_j \rangle / \langle u_k u_k \rangle - \delta_{ij}/3}$
- $\boxed{I_2 \equiv b_{ii}^2/6}$ $\boxed{I_3 \equiv (b_{ij} b_{jk} b_{ki})^{1/3}/6}$

2048^3

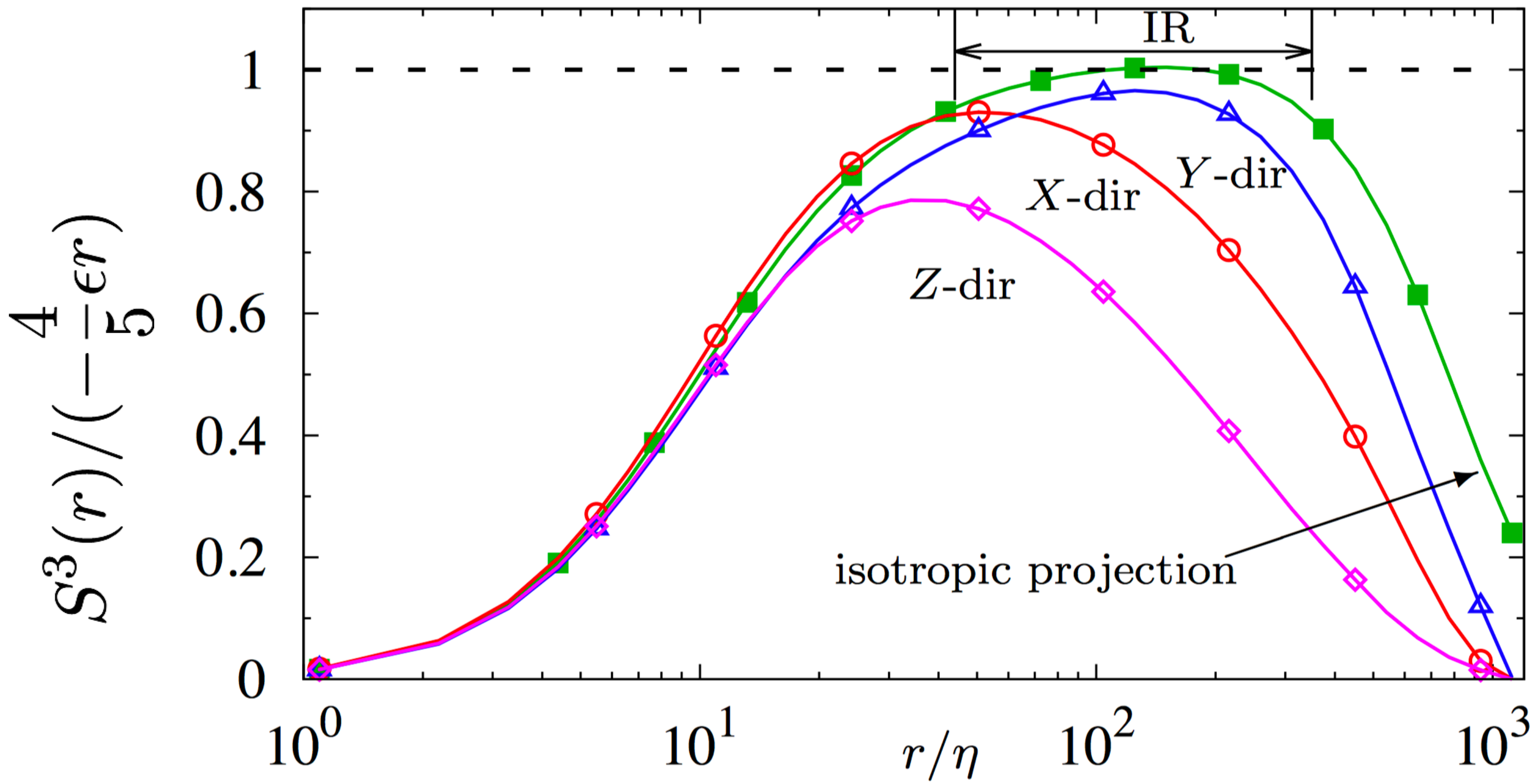


Lumley Triangle

2048^3



IF ISOTROPIC: $$\partial_r S^3(r) = -\frac{4}{5}\epsilon r$$

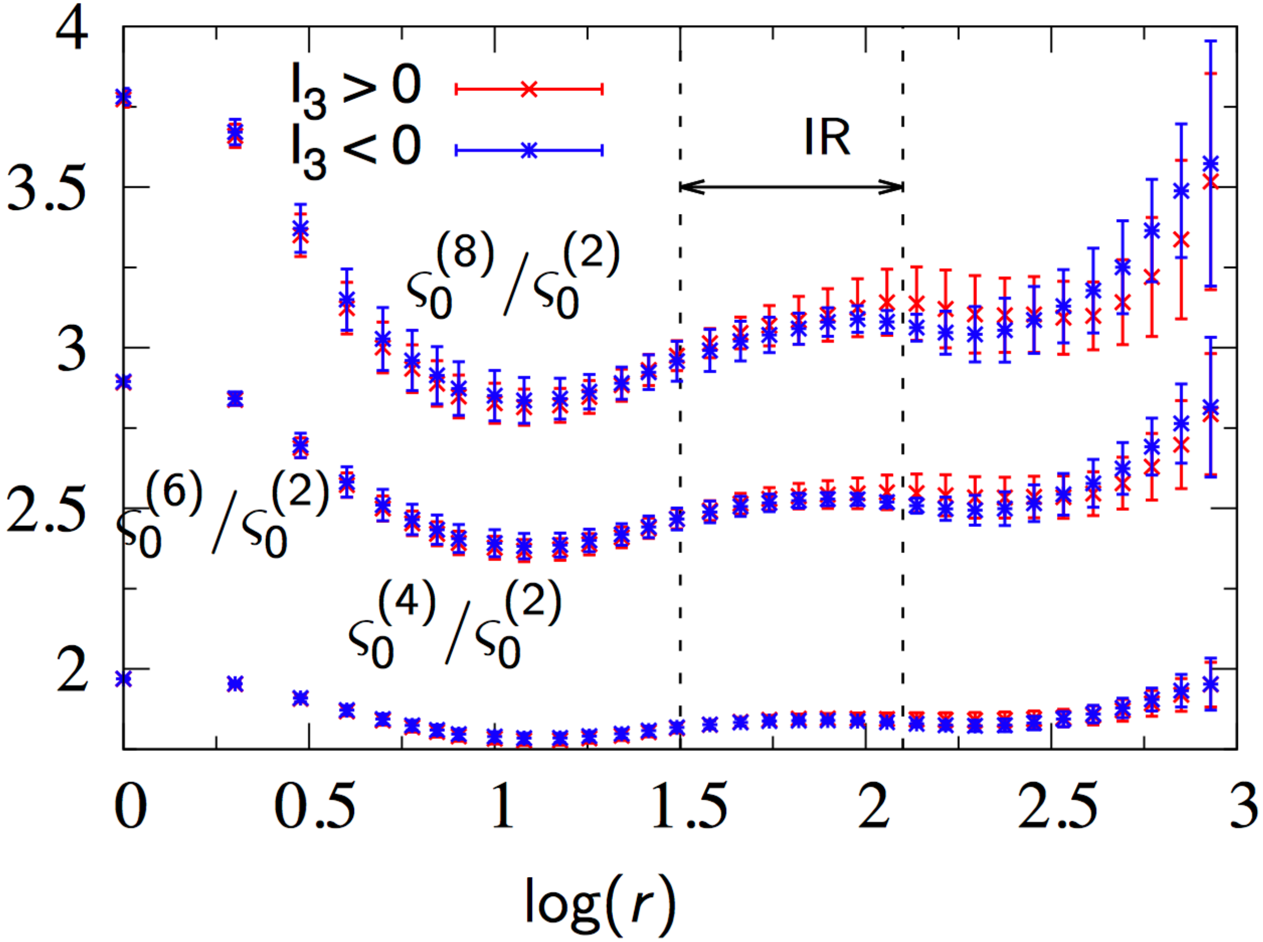


EFFECTS OF ANISOTROPY ON THE 4/5 LAW

$$S^3(r) = -\frac{4}{5}r + A_2(\theta,\phi)r^{\zeta(2)} + A_4(\theta,\phi)r^{\zeta(4)} + \cdots$$

Universality of scaling exponent in isotropic sector

$$S^{(n)}(\mathbf{r}) = \boxed{A_0\left(\frac{r}{L}\right)^{\zeta_0^{(n)}}} + A_1\left(\frac{r}{L}\right)^{\zeta_n^{(1)}} + \ldots \implies \text{Is } \zeta_0^{(n)} \text{ universal?}$$

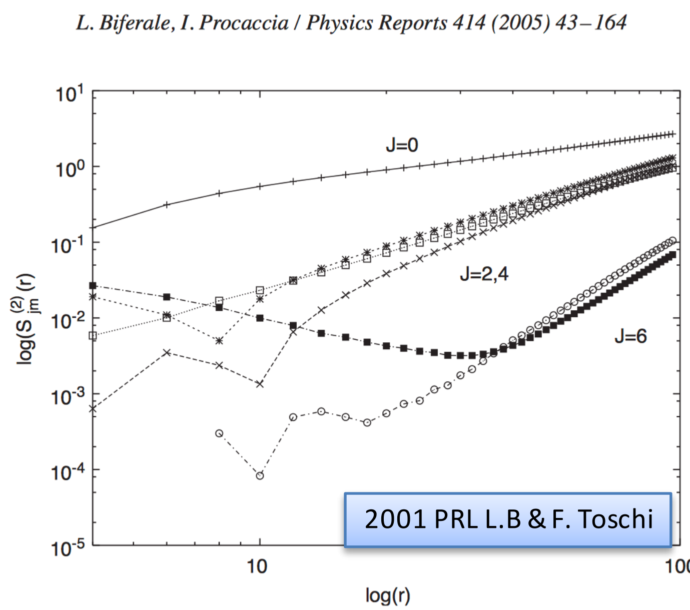


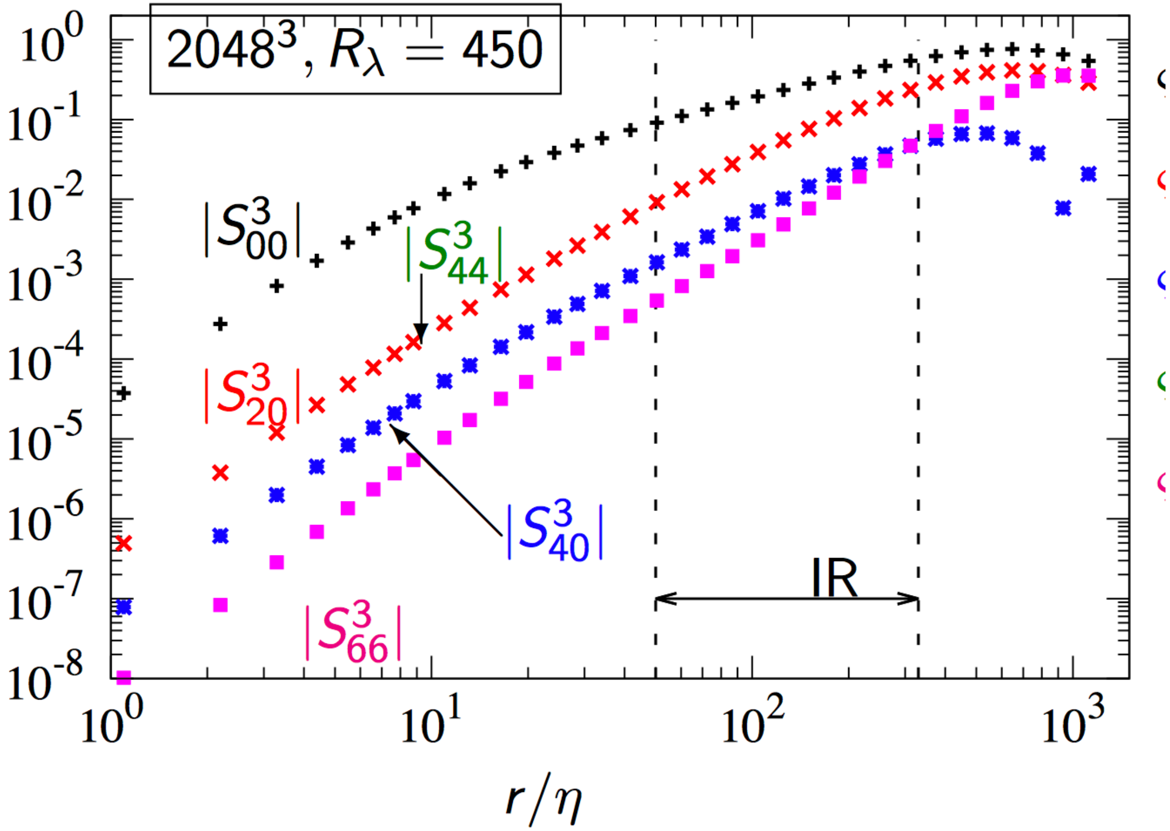
2048^3, $R_\lambda = 450$

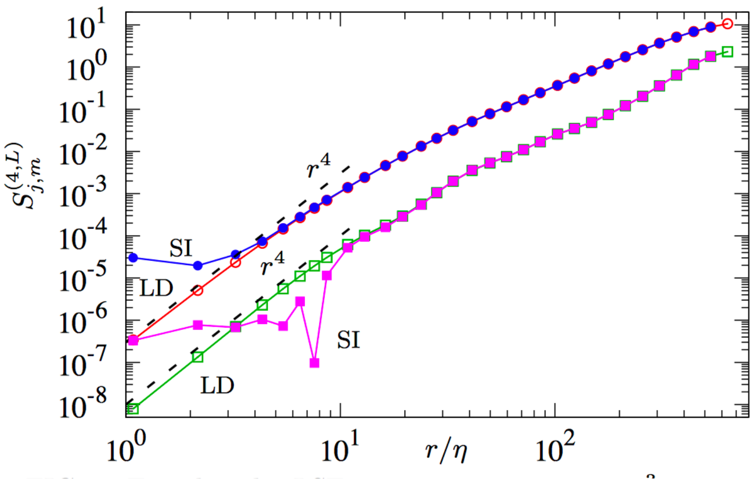
- Different I2-I3 confs in Lumley triangle have similar Inertial Range scaling at least up to order 8

LEBEDEV QUADRATURE

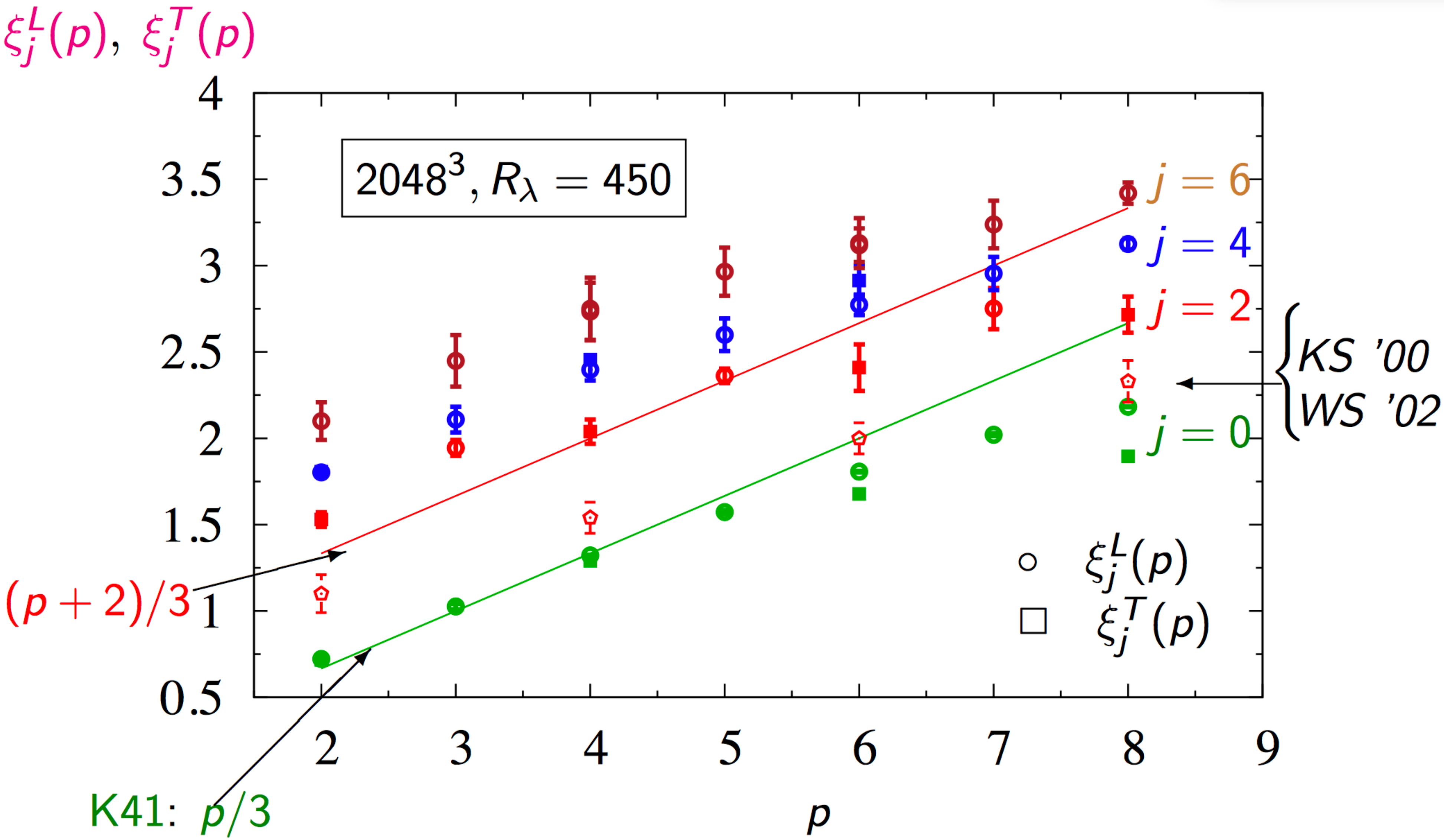
$$S^{(p,L)}_{j,m}(r) = \frac{1}{N_d}\sum_{i=1}^{N_d} w_i S^{(p,L)}(r\hat{\mathbf{r}}_i) Y_{jm}(\hat{\mathbf{r}}_i)\ , \qquad (3)$$





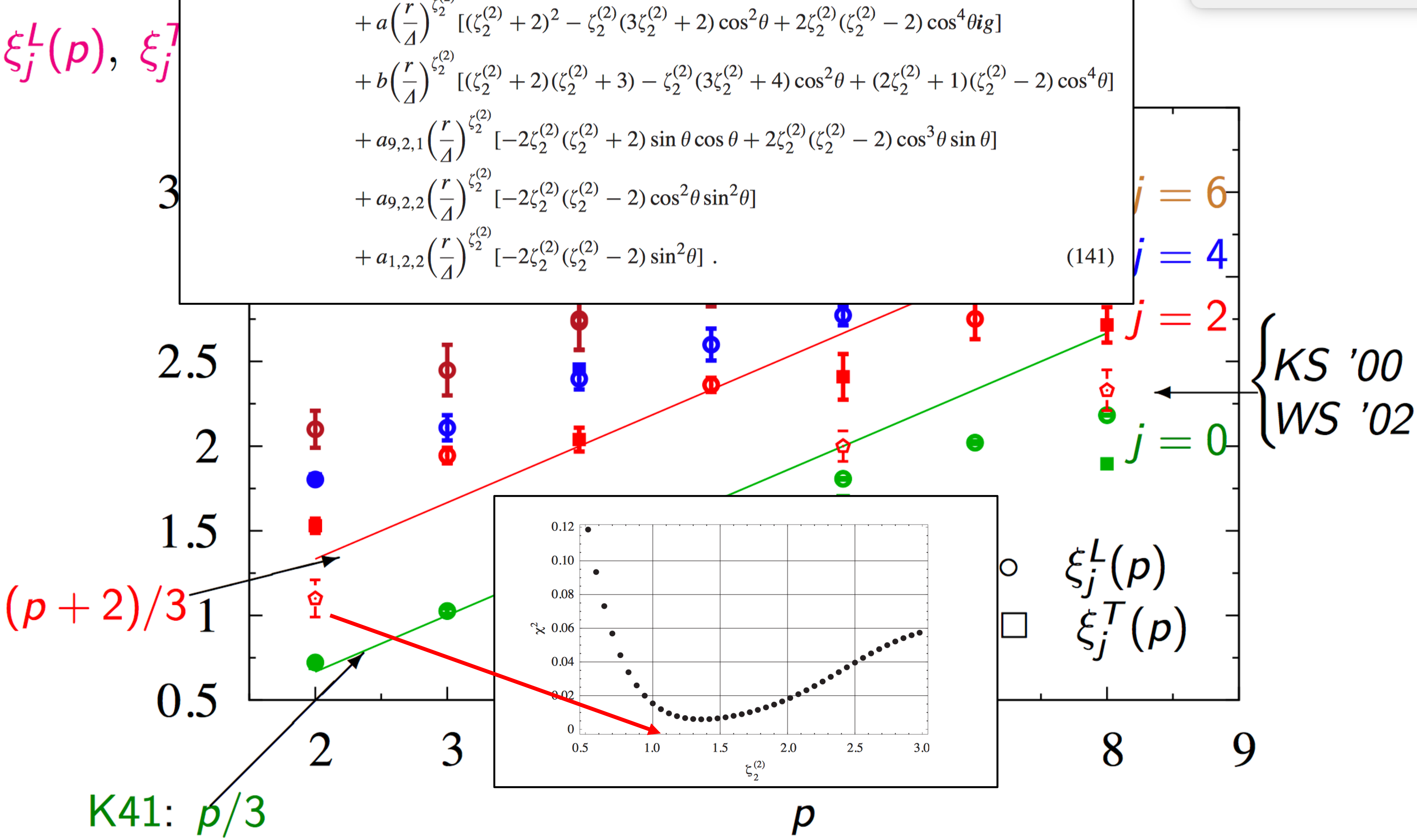


2016 PRF K. Iyer, L. B., F. Bonaccorso & F. Toschi



$\xi_j^L(p)$, $\xi_j^T(p)$
2048^3, $R_\lambda = 450$
$j = 6$
$j = 4$
$j = 2$
$j = 0$
KS '00
WS '02
$(p+2)/3$
K41: $p/3$
$\xi_j^L(p)$
$\xi_j^T(p)$
p
4
3.5
3
2.5
2
1.5
1
0.5
2
3
4
5
6
7
8
9

$$
\begin{aligned}
S^{33}(r,\theta) = S^{33}_{j=0}(r,\theta) + S^{33}_{j=2}(r,\theta) &= c_0\left(\frac{r}{\Delta}\right)^{\zeta_0^{(2)}}[2+\zeta_0^{(2)}-\zeta_0^{(2)}\cos^2\theta] \\
&+ a\left(\frac{r}{\Delta}\right)^{\zeta_2^{(2)}}[(\zeta_2^{(2)}+2)^2-\zeta_2^{(2)}(3\zeta_2^{(2)}+2)\cos^2\theta+2\zeta_2^{(2)}(\zeta_2^{(2)}-2)\cos^4\theta ig] \\
&+ b\left(\frac{r}{\Delta}\right)^{\zeta_2^{(2)}}[(\zeta_2^{(2)}+2)(\zeta_2^{(2)}+3)-\zeta_2^{(2)}(3\zeta_2^{(2)}+4)\cos^2\theta+(2\zeta_2^{(2)}+1)(\zeta_2^{(2)}-2)\cos^4\theta] \\
&+ a_{9,2,1}\left(\frac{r}{\Delta}\right)^{\zeta_2^{(2)}}[-2\zeta_2^{(2)}(\zeta_2^{(2)}+2)\sin\theta\cos\theta+2\zeta_2^{(2)}(\zeta_2^{(2)}-2)\cos^3\theta\sin\theta] \\
&+ a_{9,2,2}\left(\frac{r}{\Delta}\right)^{\zeta_2^{(2)}}[-2\zeta_2^{(2)}(\zeta_2^{(2)}-2)\cos^2\theta\sin^2\theta] \\
&+ a_{1,2,2}\left(\frac{r}{\Delta}\right)^{\zeta_2^{(2)}}[-2\zeta_2^{(2)}(\zeta_2^{(2)}-2)\sin^2\theta]\ . \qquad (141)
\end{aligned}
$$



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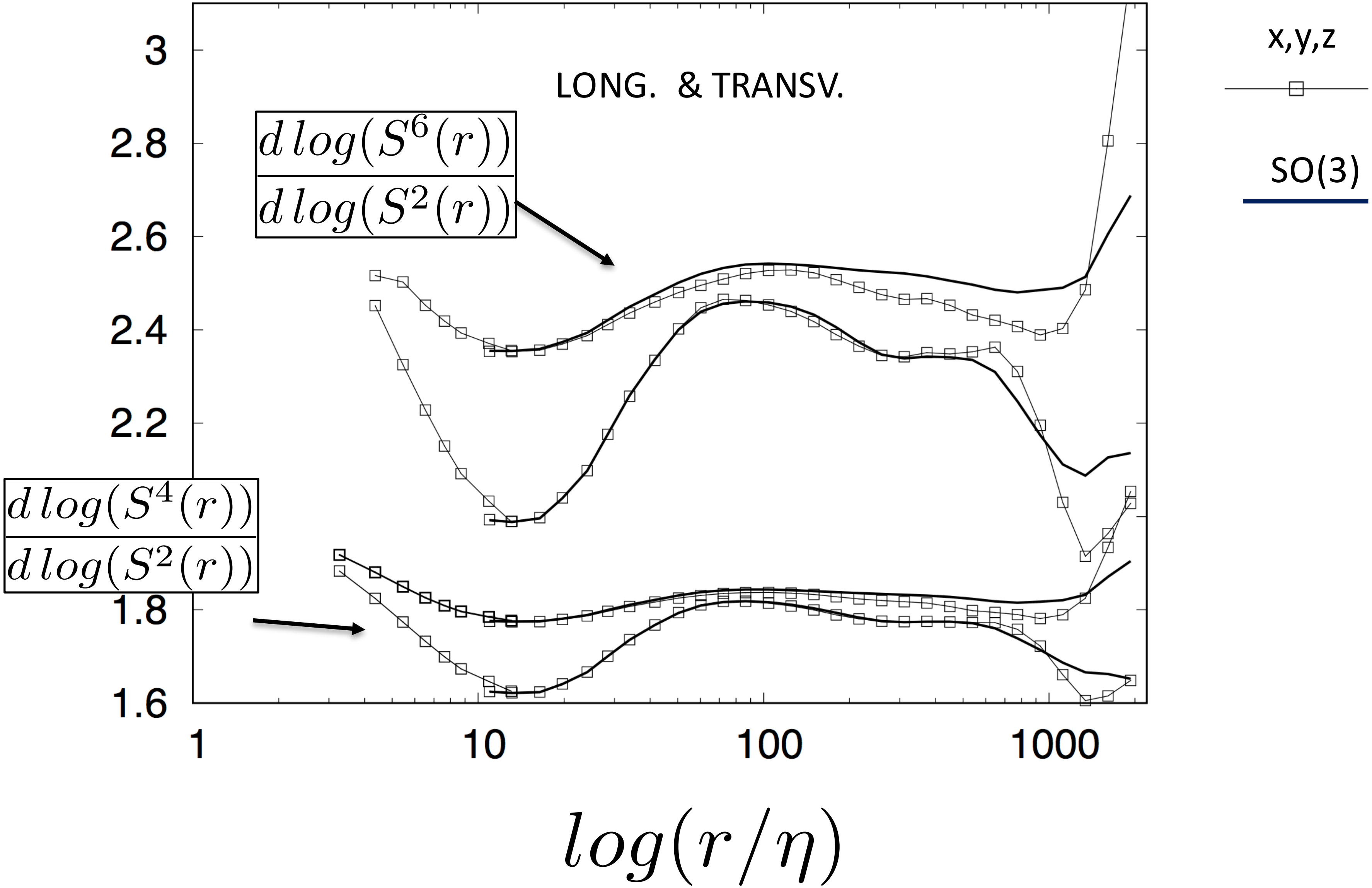
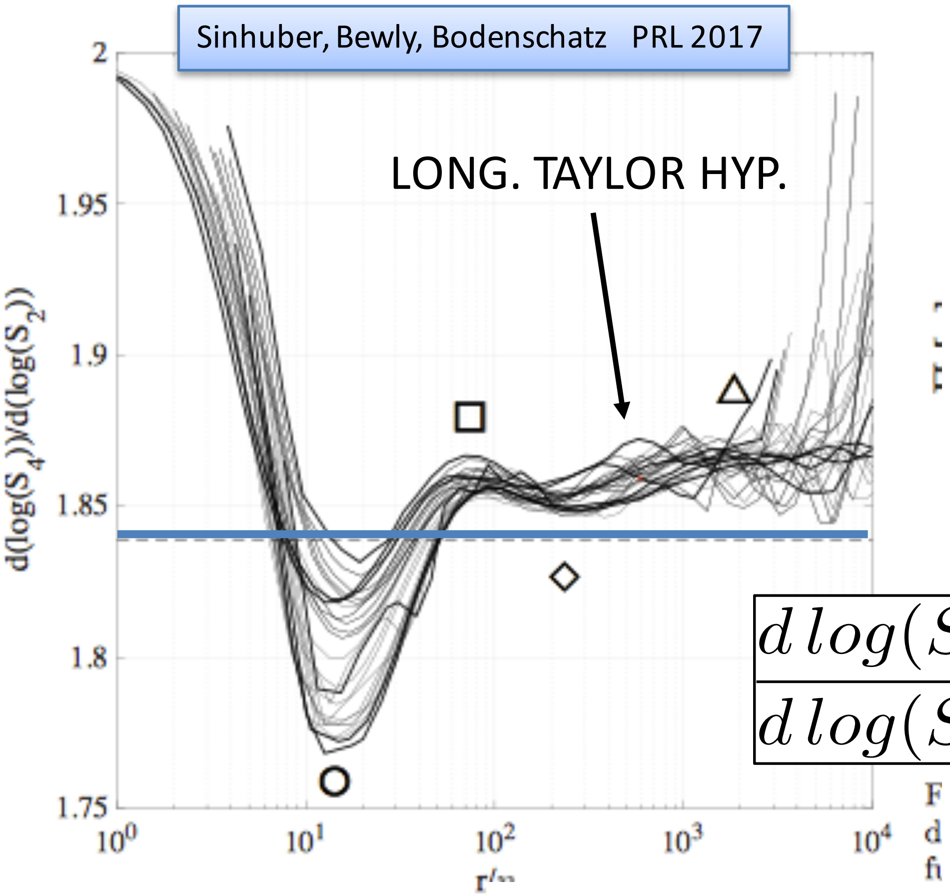
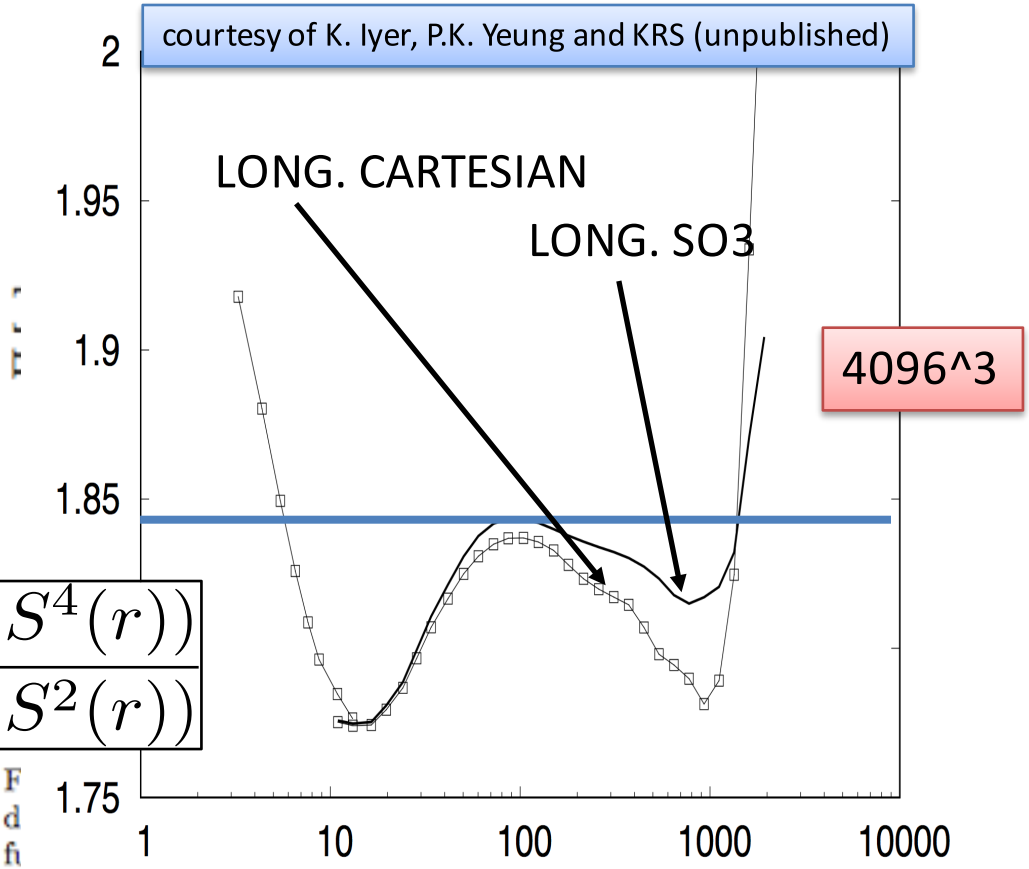
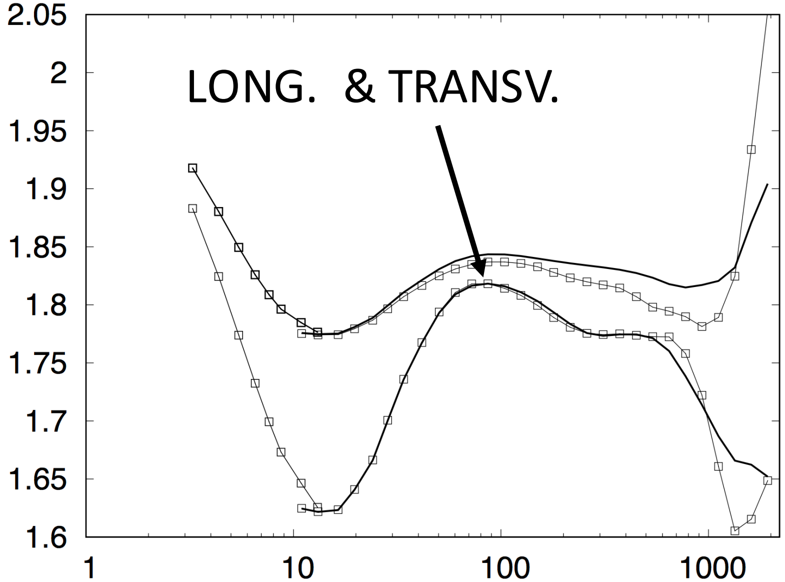
IMPACT OF SO(3) ON HIT !!!







$$\frac{d\,log(S^4(r))}{d\,log(S^2(r))}$$



$$\frac{d\,log(S^4(r))}{d\,log(S^2(r))}$$

SCALING EXPONENTS ARE (ALMOST) UNIVERSAL IN ISOTROPIC AND ANISOTROPIC SECTORS (after 20 years of great experimental and numerical work we have brought this statment to a "within 1 % of accuracy")

RATE-OF-RETURN TO ISOTROPY (E.G. RATE-OF-RECOVERY-OF UNIVERSALITY) DEPENDS ON THE GAP AMONG SCALING EXPONENTS -> DEPENDS ON THE INTENSITY OF THE FLUCTUATIONS

LONGITUDINAL AND TRANSVERSE ISOTROPIC EXPONENTS ARE (ALMOST) EQUAL

TOO MANY "ALMOST" !!!!

SCALING EXPONENTS ARE (ALMOST) UNIVERSAL IN ISOTROPIC AND ANISOTROPIC SECTORS (after 20 years of great experimental and numerical work we have brought this statment to a “within 1 % of accuracy”)

RATE-OF-RETURN TO ISOTROPY (E.G. RATE-OF-RECOVERY-OF UNIVERSALITY) IS FASTER THAN PREVIOUSLY BELIEVED

LONGITUDINAL AND TRANSVERSE ISOTROPIC EXPONENTS ARE (ALMOST) EQUAL

TOO MANY “ALMOST” !!!!

WE NEED SREENI’S INPUTS FOR TWENTY YEARS MORE (AT LEAST)!

Multifractality in the Statistics of the Velocity Gradients in Turbulence

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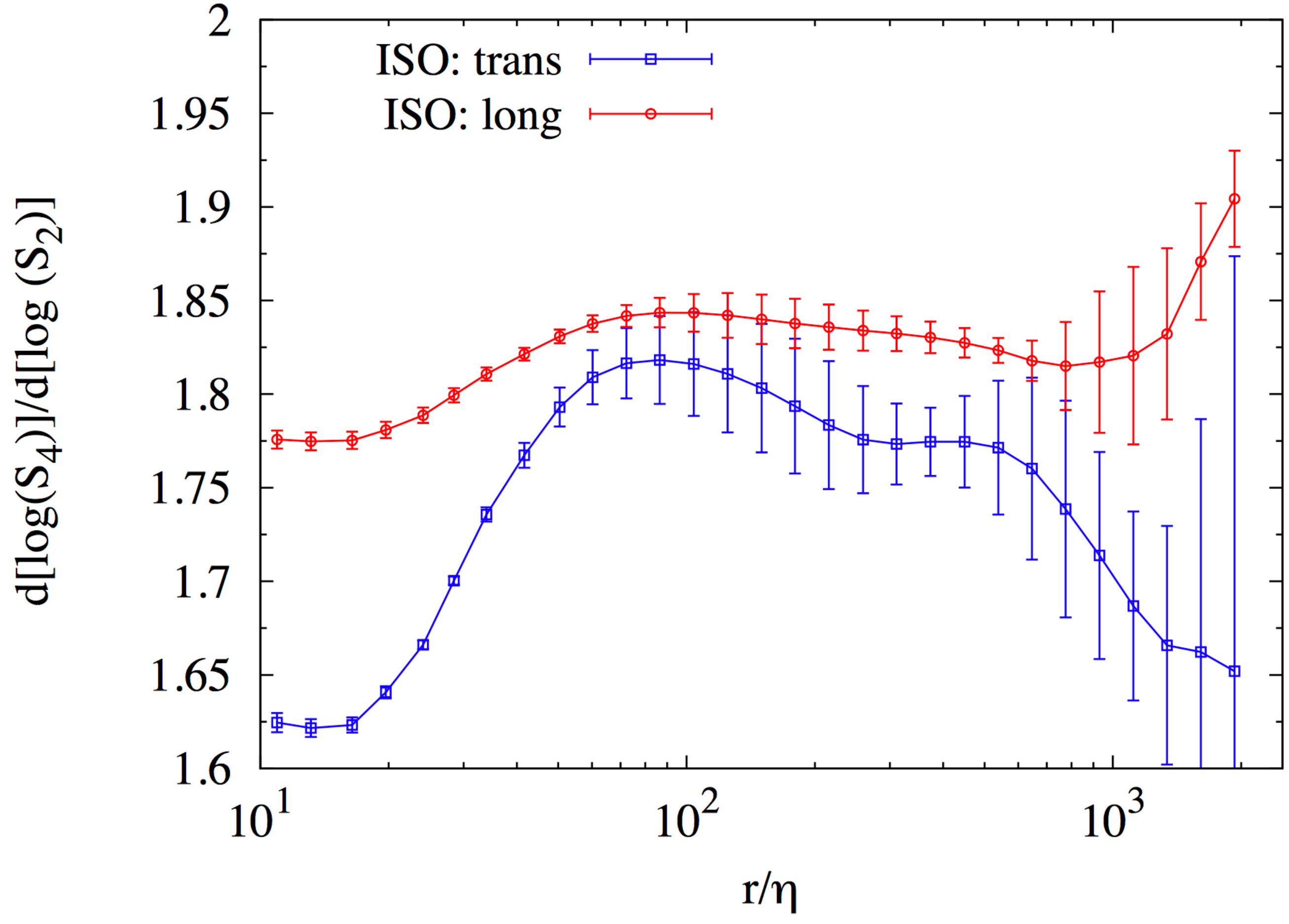
(Received 28 June 1991)

Using the multifractal approach, we derive the probability distribution function (PDF) of the velocity gradients in fully developed turbulence. The PDF is given by a nontrivial superposition of stretched exponentials, corresponding to the various singularity exponents. The form of the distribution is explicitly dependent on the Reynolds number. The experimental data are in good agreement with the PDF predicted by the same random beta model used to fit the scaling of the velocity structure functions.

PACS numbers: 47.25.−c



ISO: trans
ISO: long
d[log(S4)]/d[log (S2)]
r/η
2
1.95
1.9
1.85
1.8
1.75
1.7
1.65
1.6
10^1
10^2
10^3

- Scaling exponents universal in isotropic and anisotropic sectors
- Rate-of-return to isotropy FASTER than previously believed at least at lower orders
- Longitudinal and transverse exponents are (almost) equal in different anisotropic sectors
- Isotropic sector: higher order long/trans exponents differ at $R_\lambda \sim 450$