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# Intermittency in the Fractal Fourier Burgers Equation

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**Motivation:**

Robustness of statistical properties as the number of degrees of freedom decreases

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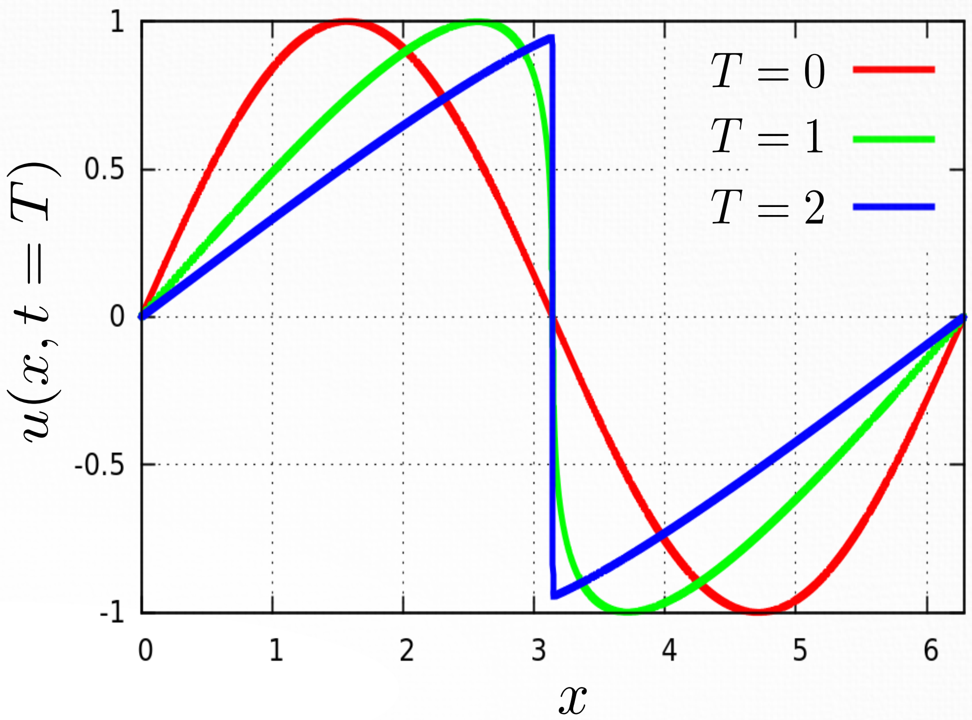
# Burgers' equation:

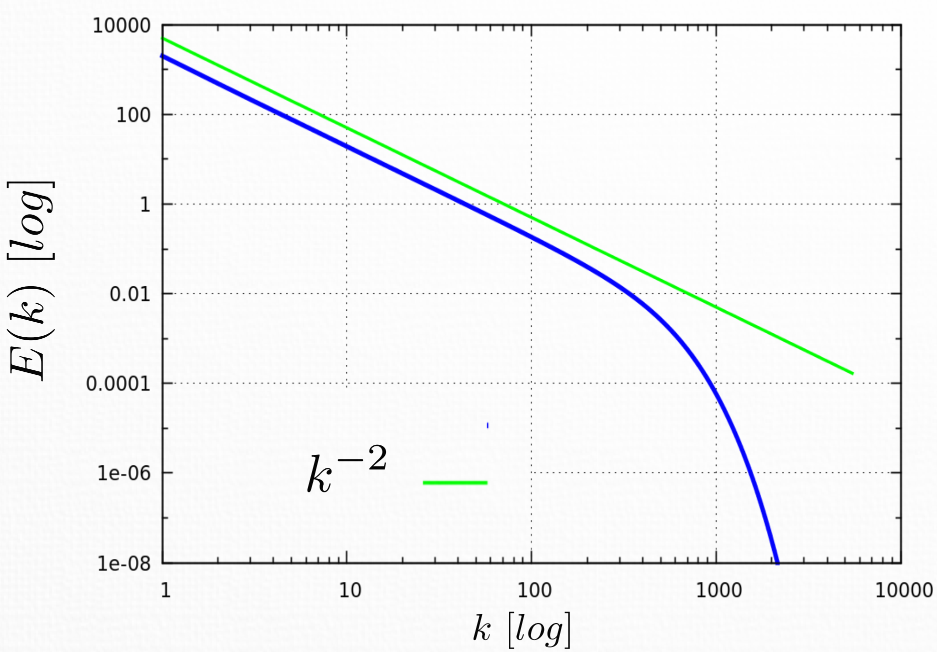
$$\boxed{\frac{\partial u(x,t)}{\partial t} + u\frac{\partial u(x,t)}{\partial x} = \nu\frac{\partial^2 u(x,t)}{\partial x^2}}$$

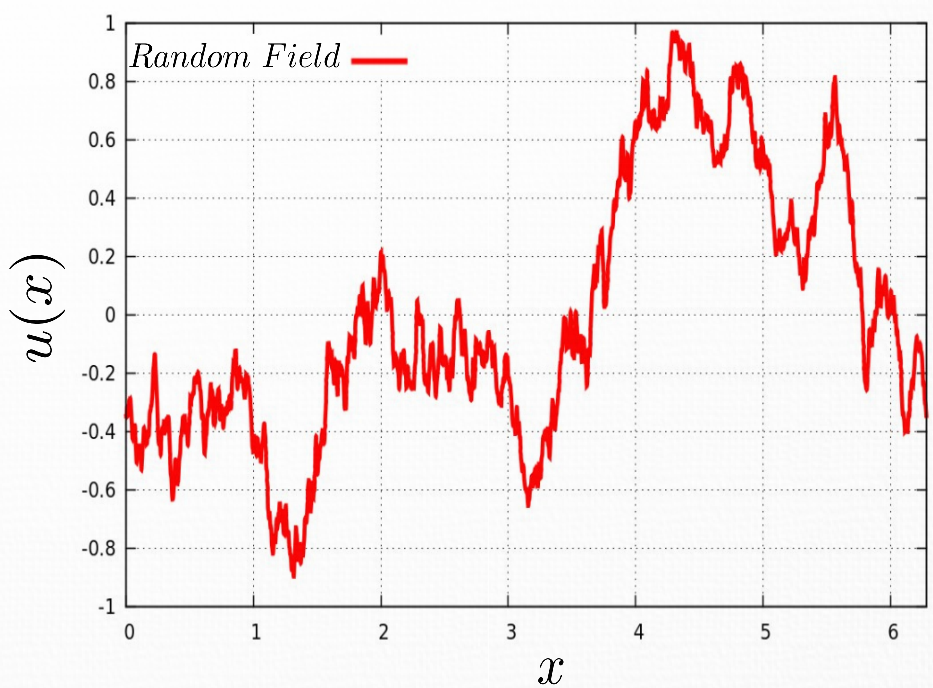



Burgers' evolution **produces a singularity**, (**shock**)

$$\partial_t u(k,t) = -ik\int u(p,t)u(p-k,t)dp - \nu k^2 u(k,t)$$

$$E(k) = u(k)u^*(k)$$



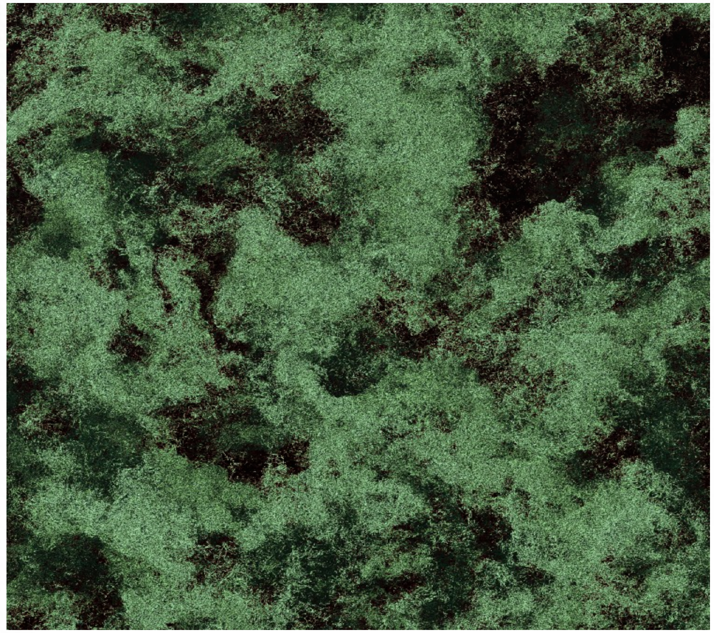
# ..Burgers equation + Random Forcing:

$$\frac{\partial u(x,t)}{\partial t} + u\frac{\partial u(x,t)}{\partial x} =$$

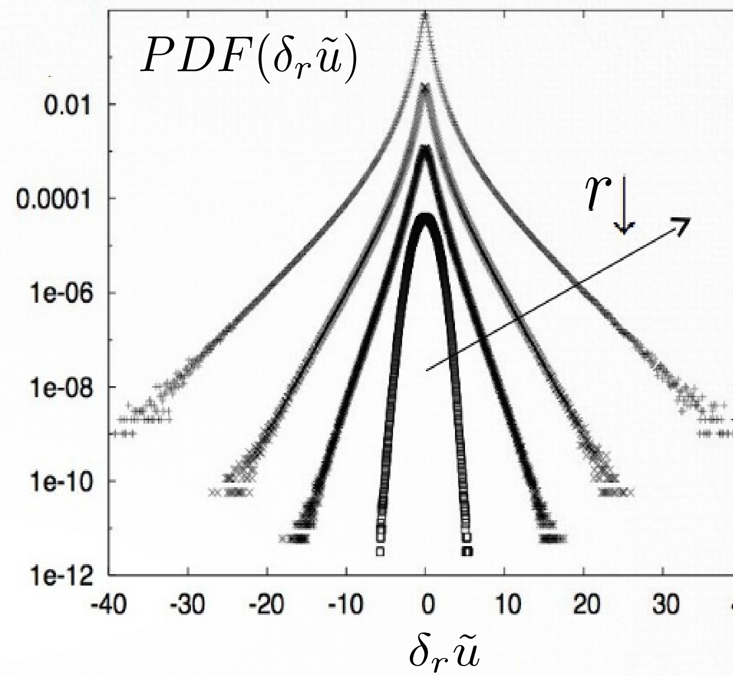
$$= \nu\frac{\partial^2 u(x,t)}{\partial x^2} + \boxed{f_s(x,t)}$$

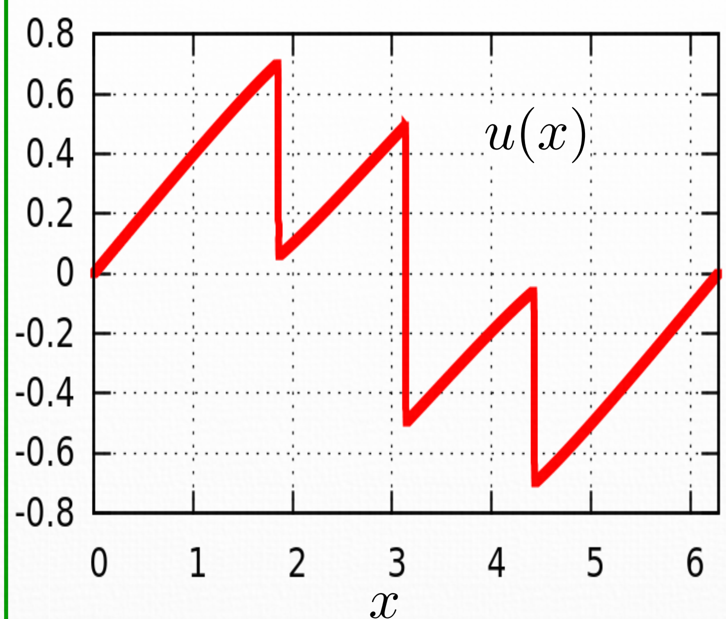
$$\delta_r u = u(x+r) - u(x)$$

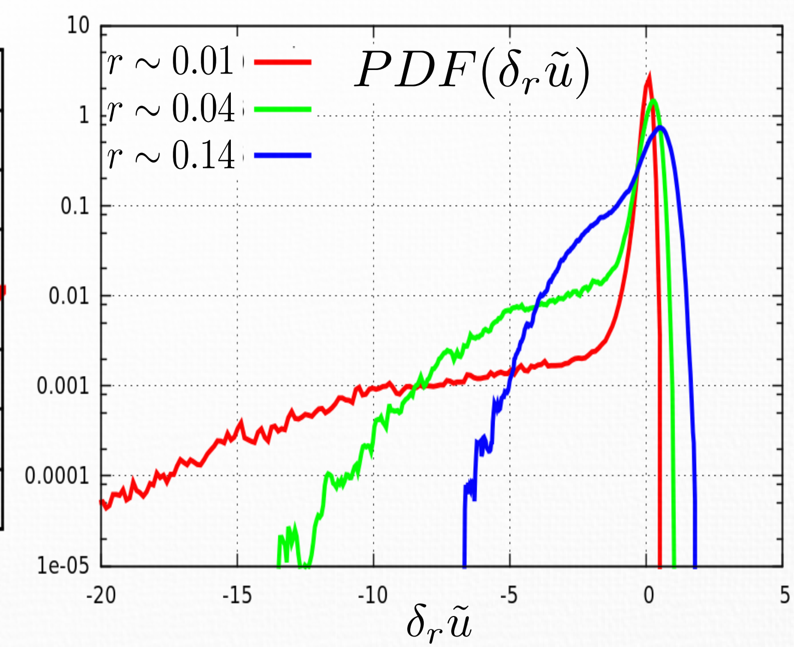
$$\delta_r \tilde{u} = \frac{\delta_r u}{< (\delta_r u)^2 >^{1/2}}$$



Kaneda et al., ARFM (2011).



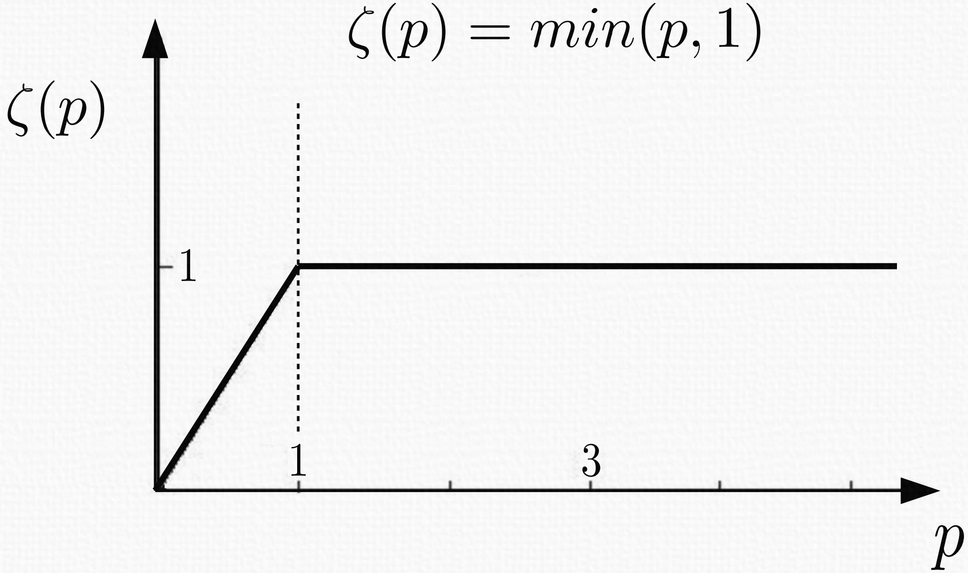

As vortices in real 3D turbulence, **shock produces a non-trivial statistics in the Burgers' velocity field**.

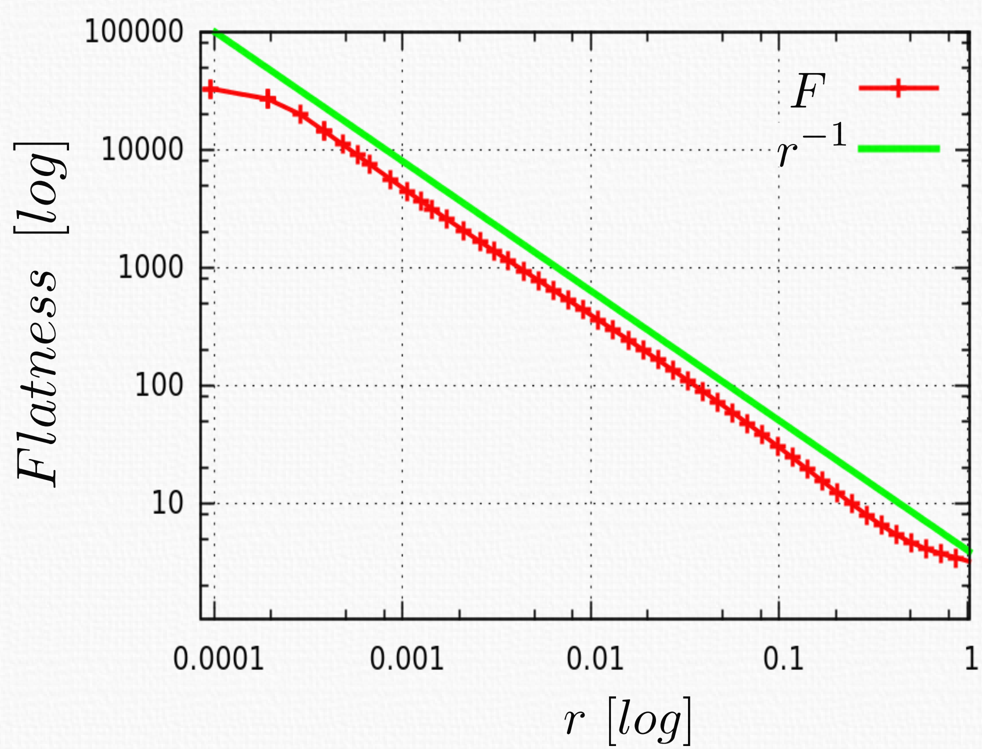
# ..Burgers equation + Random Forcing:

$$S_p(r) =< (\delta_r u)^p > \sim r^{\zeta(p)}$$

$$F = \frac{< (\delta_r u)^4 >_{x,t}}{< (\delta_r u)^2 >_{x,t}^2}$$

$$\zeta(p) = min(p, 1)$$

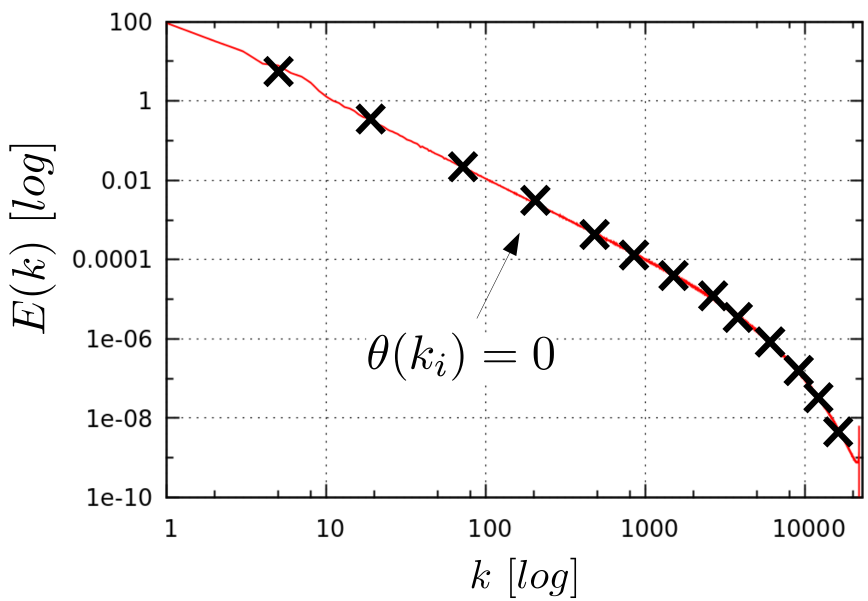



.. **Bifractal** system

- **How many degrees of freedom are related to the coherent structure and shock formation?**

## Fractal Fourier decimation:




**Numerical approach:**

$$v(x,t) = \sum_{k\in Z} e^{ikx}\theta(k)u(k,t)$$

$$\theta(k) = \begin{cases} 1 & with \quad probability \quad h_k \sim (k/k_0)^{D_f-1} \\ 0 & with \quad probability \quad 1-h_k \end{cases}$$

$$0 < D_f \leq 1$$

### Decimated Burgers equation:

$$\partial_t v(x,t) + P_{D_f}\left[v\partial_x v(x,t)\right] =$$

$$= \nu\partial^2_{xx}v(x,t) + F_{D_f}$$

### Decimation Main Properties:

**1)** The space **dimension** can **change continuously**

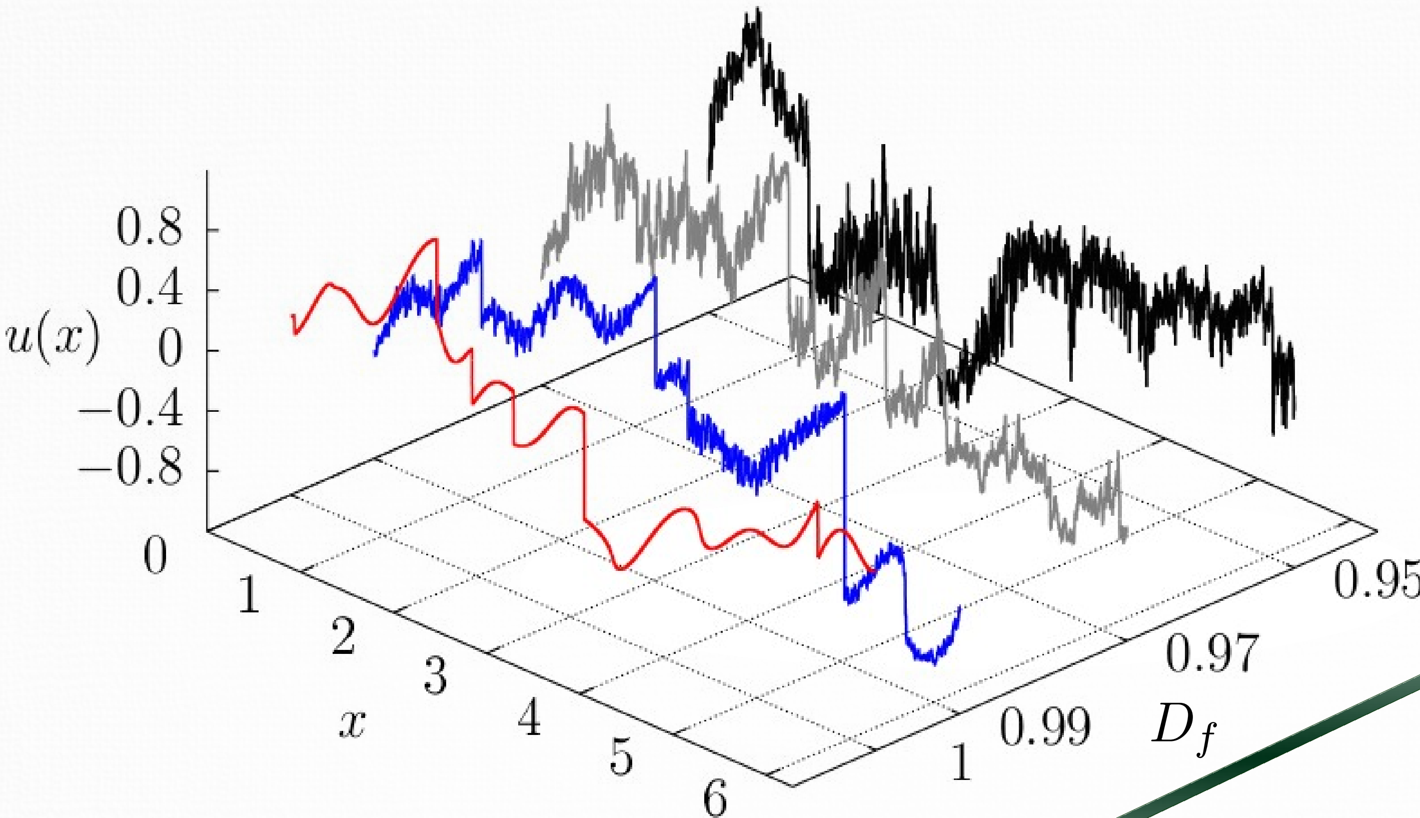
**2)** The original **symmetries** of the system are kept

**3)** It acts as a **Galerkin Truncation** without the introduction of any characteristic scale

**4)** The numerical evolution can be obtained via a **psedu-specral code**

**Frisch, Pomyalov, Procaccia, and Ray.** *Turbulence in non-integer dimensions by fractal Fourier decimation.* **Phys. Rev. Lett. 108, (2012).**

# Real space evolution at changing of fractal dimension:



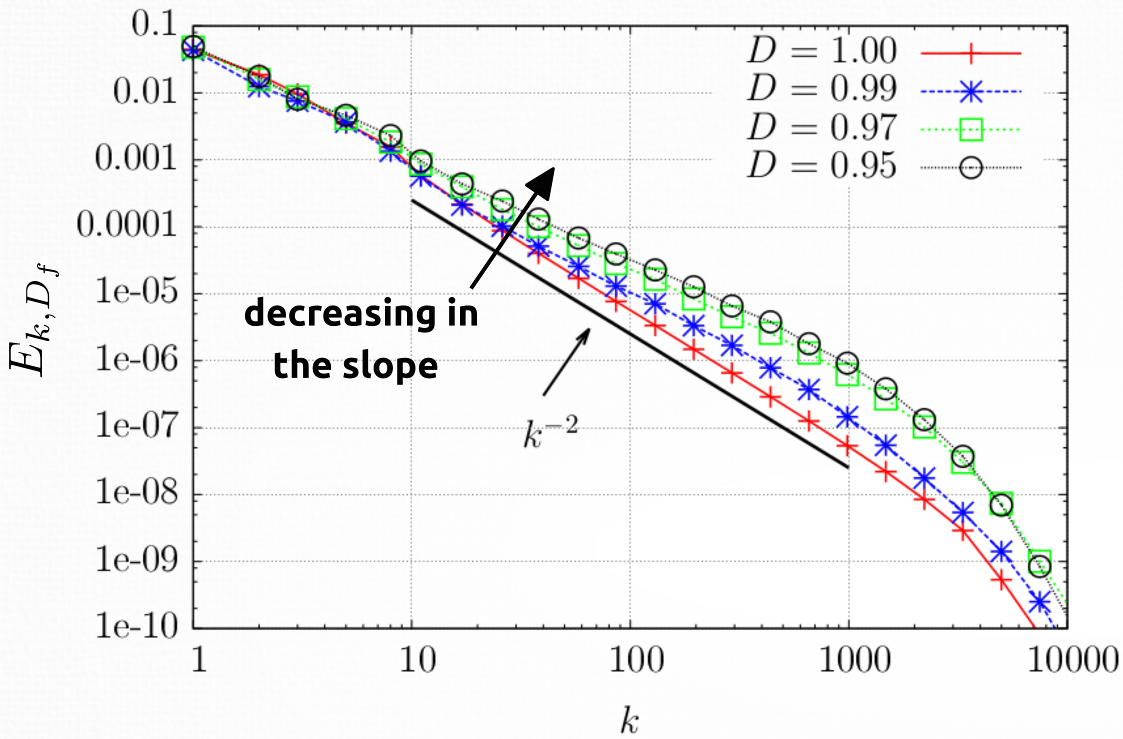

$D_f = 0.95$

Random small scales structures

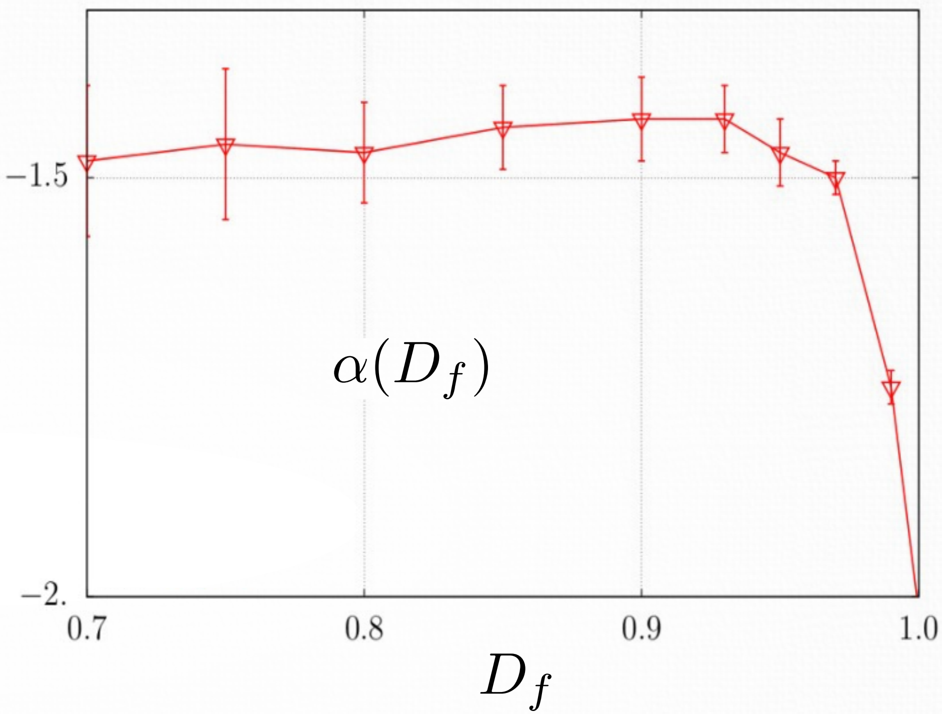
$D_f = 1$

Shock evolution in energy decaying

# Energy Spectra:




Energy Spectra scaling exponents as a function of Fractal dimension:

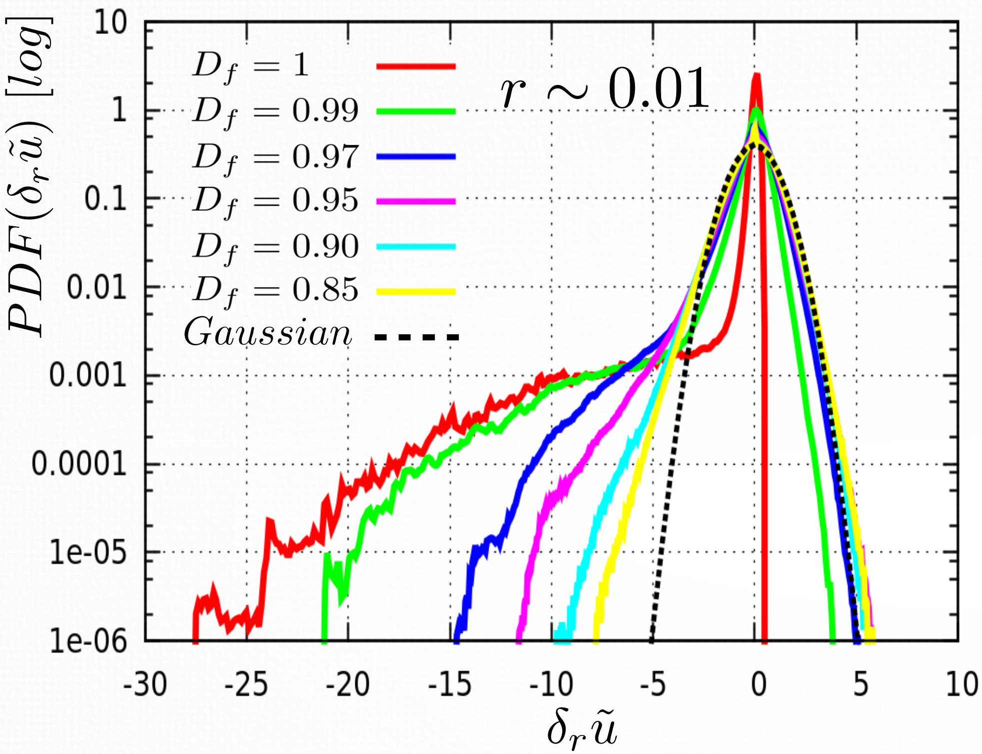


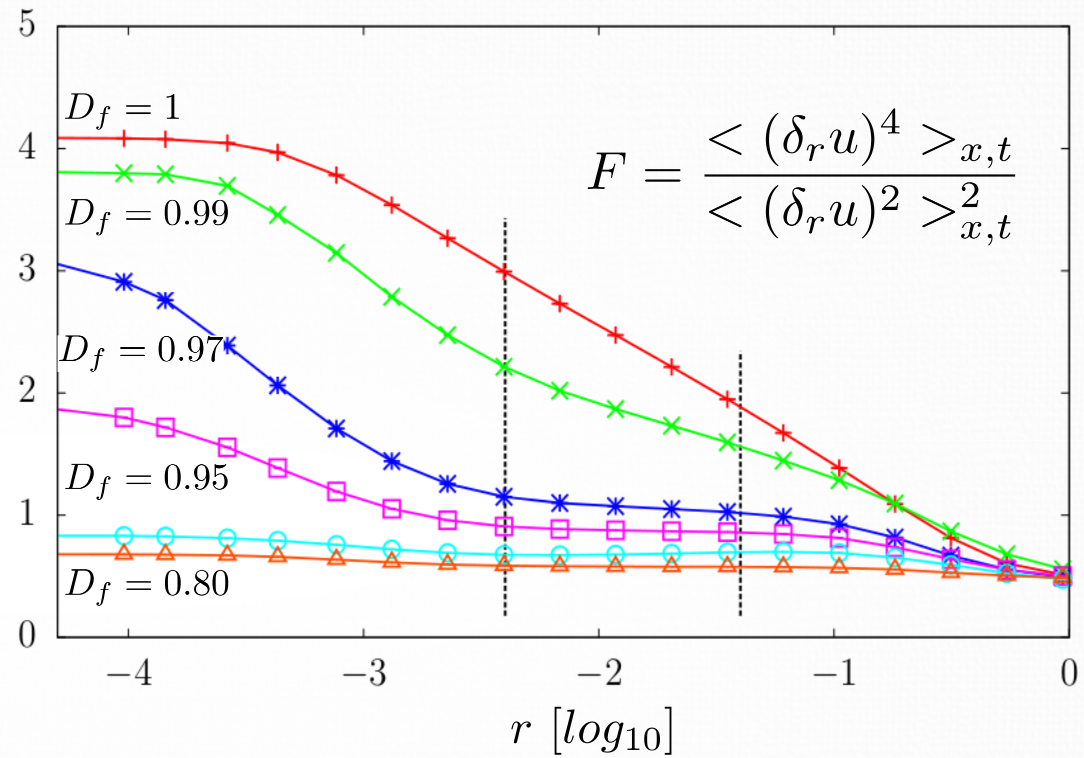

$$E_{k,D_f=1} = \langle \hat{u}_k^2 \rangle \sim k^{-2} \longrightarrow E_{k,D_f} = \langle \hat{v}_k^2 \rangle \sim k^{\alpha(D_f)}$$

Results show a **dependence only on the dimension** and not on the random masks.

The energy spectrum **slope decreases** with the dimension up to $D_f \sim 0.95$

# Statistical properties at different Fractal dimensions:




Self-similar fluctuations are introduced by decimation

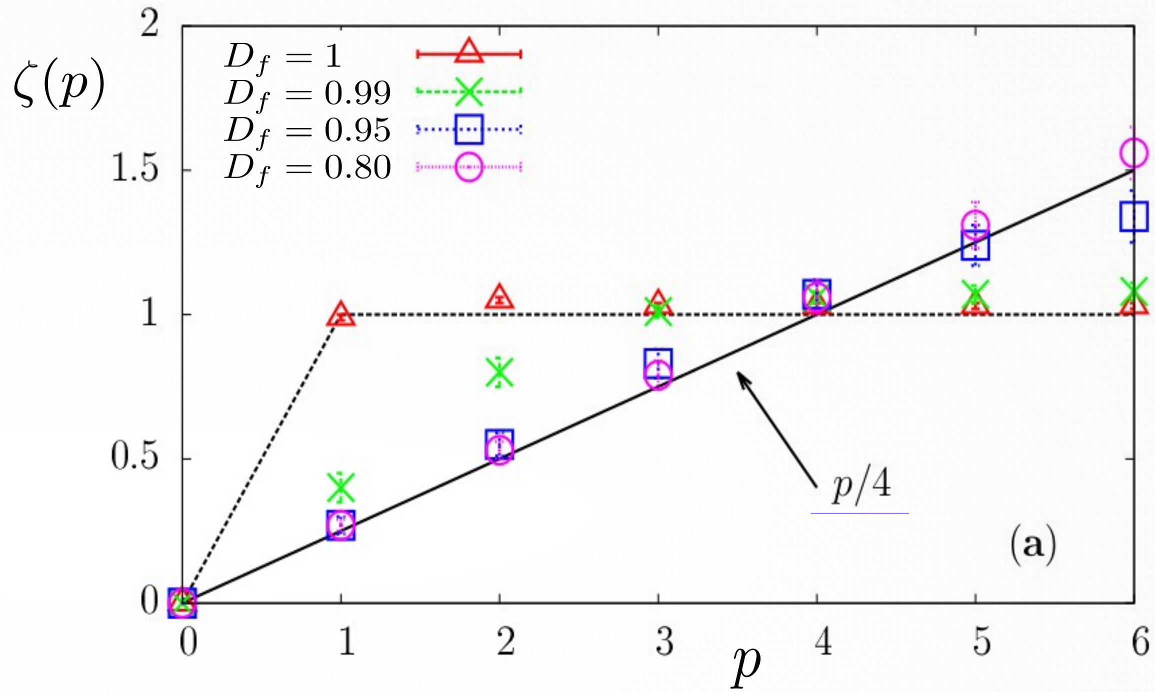
Intermittency is strongly reduced.

# Statistical properties at different Fractal dimensions:

$$S_p(r) =< (\delta_r u)^p > \sim r^{\zeta(p)}$$

Structure function scaling exponents becomes linear in p:

$$\zeta(p) \sim p/4$$




..A possible explanation:

The introduction of an "extra decorrelation time"

$$\tau_{dec} \propto 1/k$$

Following the same approach used in the case of **Alfvén waves** in MHD:

$$\Pi(k) = const \rightarrow E(k) \sim k^{-3/2}$$

$$S_2(r) =< (\delta_r u)^2 > \sim r^{1/2}$$

$$S_p(r) \sim r^{p/4}$$

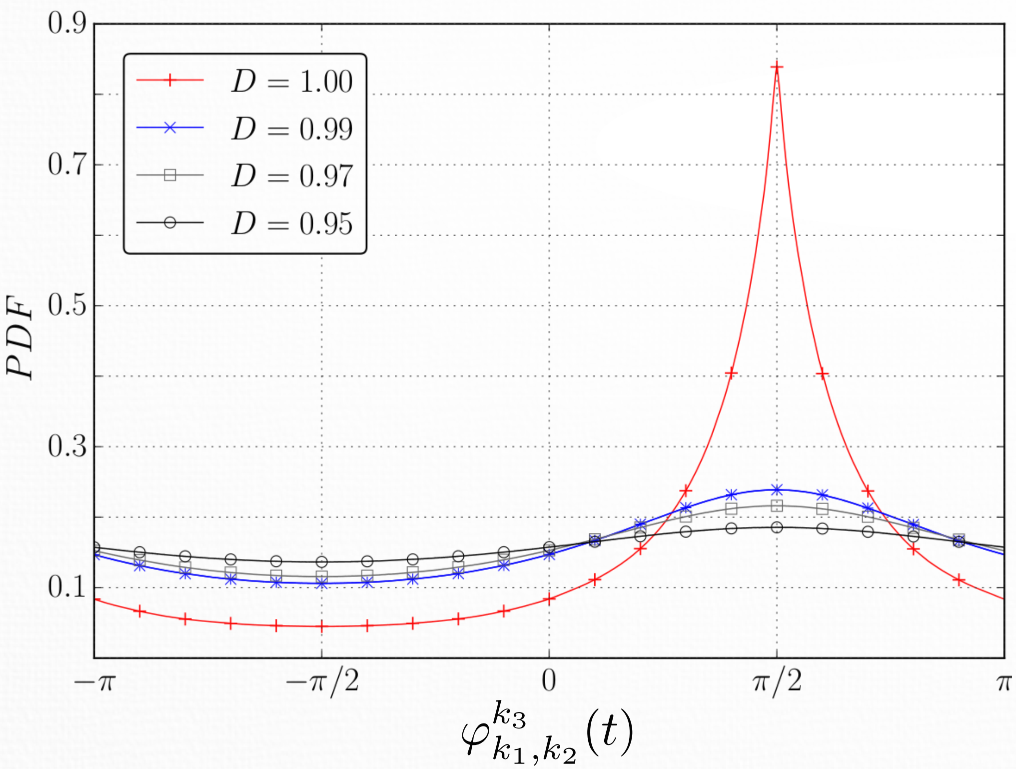
# Energy flux:

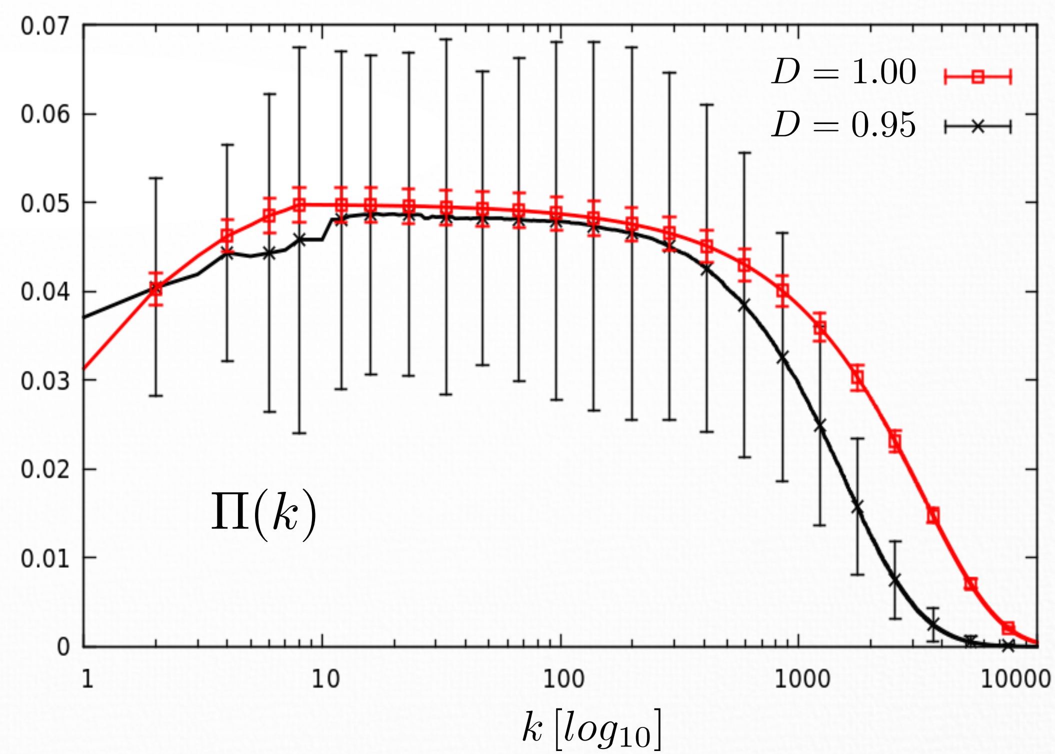
$$\Pi(k) = \sum_{k_1=1}^{k} \sum_{k_2=-\infty}^{\infty} k_1 \Im\left\{\langle \hat{u}_{k_1} \hat{u}_{k_2} \hat{u}^*_{k_1+k_2} \rangle\right\}$$

$$\hat{u}_k(t) = a_k(t) e^{i\phi_k(t)}$$

$$\Pi(k) = \sum_{k_1=1}^{k} \sum_{k_2=-\infty}^{\infty} \langle a_{k_1} a_{k_2} a_{k_3} \sin(\varphi^{k_3}_{k_1,k_2}) \rangle \times k_1 \theta_{k_1} \theta_{k_2} \theta_{k_3}$$

$$\varphi^{k_3}_{k_1,k_2}(t) = \phi_{k_1}(t) + \phi_{k_2}(t) - \phi_{k_3}(t)$$



# Conclusions

1) We performed a random decimation of **Burgers' equation in Fourier space**.

2) We found the presence of shocks like singularities also after the decimation of many degrees of freedom, but at the same time self-similar fluctuations are introduced, affecting heavily the velocity field statistics.

3) Alongside the **intermittency reduction** we observe the development of a much noisier energy flux which nevertheless continues to be constant in the inertial range.

We are still interested in the observation of possible decorrelation in the Fourier space triads evolution.

We do not know if the decimated equations possess a well posed behavior for $\nu \to 0$ and if the system is able to develop a dissipative anomaly leading to a stationary behavior for all D.