Entropic Lattice Boltzmann Method

An implicit Large-Eddy Simulation?

G. Tauzin, L. Biferale, M. Sbragaglia,
A. Gupta, F. Toschi, A. Bartel, M. Ehrhardt

Università degli studi di Roma Tor Vergata
Bergische Universität Wuppertal

Erlangen - July 13, 2017

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No' 642069

Outline

Outline

ELBM: A search for LBGK stabilization

Can we use LBGK to study turbulent flows?
Instabilities arise when $\tau \rightarrow 0.5$ ($\nu \rightarrow 0$) making standard LBGK unadapted to the study of turbulent flows

ELBM: A search for LBGK stabilization

Can we use LBGK to study turbulent flows?
Instabilities arise when $\tau \to 0.5$ ($\nu \to 0$) making standard LBGK unadapted to the study of turbulent flows

Entropic Lattice Boltzmann (ELBM) **[Karlin *et al.*, 1999]**

- Introduced by I. Karlin to overcome instabilities issues at high Reynolds

ELBM: A search for LBGK stabilization

Can we use LBGK to study turbulent flows?
Instabilities arise when $\tau \to 0.5$ ($\nu \to 0$) making standard LBGK unadapted to the study of turbulent flows

Entropic Lattice Boltzmann (ELBM) **[Karlin *et al.*, 1999]**

- Introduced by I. Karlin to overcome instabilities issues at high Reynolds
- LBGK is built directly from the Boltzmann Equation and is not equipped with a Boltzmann H-theorem even if BGK collision operator implies irreversibility

ELBM: A search for LBGK stabilization

Can we use LBGK to study turbulent flows?
Instabilities arise when $\tau \to 0.5$ ($\nu \to 0$) making standard LBGK unadapted to the study of turbulent flows

Entropic Lattice Boltzmann (ELBM) **[Karlin *et al.*, 1999]**

- Introduced by I. Karlin to overcome instabilities issues at high Reynolds
- LBGK is built directly from the Boltzmann Equation and is not equipped with a Boltzmann H-theorem even if BGK collision operator implies irreversibility
- Stabilization of LBGK has been linked to the existence of an underlying Lyapunov functional in the form of a H-function

[Succi *et al.*, 2002]

ELBM: A search for LBGK stabilization

Can we use LBGK to study turbulent flows?
Instabilities arise when $\tau \rightarrow 0.5$ ($\nu \rightarrow 0$) making standard LBGK unadapted to the study of turbulent flows

Entropic Lattice Boltzmann (ELBM) **[Karlin *et al.*, 1999]**

- Introduced by I. Karlin to overcome instabilities issues at high Reynolds
- LBGK is built directly from the Boltzmann Equation and is not equipped with a Boltzmann H-theorem even if BGK collision operator implies irreversibility
- Stabilization of LBGK has been linked to the existence of an underlying Lyapunov functional in the form of a H-function **[Succi *et al.*, 2002]**

ELBM principle is to equip LBGK with an in-built H-theorem

ELBM: A LBGK with an in-built H-theorem

- $\mathbf{f^{eq}}$ is defined as the maxima of a convex H-function under the constraints of mass and momentum conservation:

$$H(\mathbf{f}) = \sum_{0}^{q-1} f_i \log\left(\frac{f_i}{\omega_i}\right), \qquad \rho = \sum_i f_i^{eq}, \quad \rho\vec{u} = \sum_i \vec{c}_i\, f_i^{eq}$$

LBGK Equation

$$f_i(\vec{x} + \vec{c}_i, t + 1) - f_i(\vec{x}, t) = -\frac{1}{\tau}\left[\, f_i(\vec{x}, t) - f_i^{eq}(\vec{x}, t)\right]$$

ELBM: A LBGK with an in-built H-theorem

- $\mathbf{f^{eq}}$ is defined as the maxima of a convex H-function under the constraints of mass and momentum conservation:

$$H(\mathbf{f}) = \sum_{0}^{q-1} f_i \log\left(\frac{f_i}{\omega_i}\right), \qquad \rho = \sum_i f_i^{eq}, \quad \rho\vec{u} = \sum_i \vec{c}_i\, f_i^{eq}$$

- ELBM introduces a fixed parameter β and a local one α ($\tau_{eff} = \frac{1}{\alpha\beta}$)

ELBM Equation [Karlin *et al.*, 1999]

$$f_i(x + c_i, t + 1) = f_i(x, t) + \alpha\beta\left[f_i^{eq}(x, t) - f_i(x, t)\right]$$

where $\beta = \frac{1}{2\tau}$ and LBGK is recovered if $\alpha \equiv 2$

ELBM: A LBGK with an in-built H-theorem

- $\mathbf{f^{eq}}$ is defined as the maxima of a convex H-function under the constraints of mass and momentum conservation:

$$H(\mathbf{f}) = \sum_{0}^{q-1} f_i \log\left(\frac{f_i}{\omega_i}\right), \qquad \rho = \sum_i f_i^{eq}, \quad \rho\vec{u} = \sum_i \vec{c}_i\, f_i^{eq}$$

- ELBM introduces a fixed parameter β and a local one α ($\tau_{eff} = \frac{1}{\alpha\beta}$)
- Setting $\mathbf{f^{mirror}} = \mathbf{f} - \alpha\,(\mathbf{f} - \mathbf{f^{eq}})$, we can rewrite the ELBM eq.

ELBM Equation **[Karlin *et al.*, 1999]**

$$f_i(x + c_i, t+1) = (1-\beta)\; f_i(x,t) + \beta\; f_i^{mirror}(x,t)$$

where $\beta = \frac{1}{2\tau}$, with $0 < \beta < 1$ as we have $0.5 < \tau < +\infty$

ELBM: A LBGK with an in-built H-theorem

- $\mathbf{f}^{\mathbf{eq}}$ is defined as the maxima of a convex H-function under the constraints of mass and momentum conservation:

$$H(\mathbf{f}) = \sum_{0}^{q-1} f_i \log\left(\frac{f_i}{\omega_i}\right), \qquad \rho = \sum_i f_i^{eq}, \quad \rho\vec{u} = \sum_i \vec{c}_i\, f_i^{eq}$$

- ELBM introduces a fixed parameter β and a local one α ($\tau_{eff} = \frac{1}{\alpha\beta}$)
- Setting $\mathbf{f}^{\mathbf{mirror}} = \mathbf{f} - \alpha\,(\mathbf{f} - \mathbf{f}^{\mathbf{eq}})$, we can rewrite the ELBM eq.
- α is calculated at each node and each time step as the solution of the following equation:

$$H(\mathbf{f}) = H\left(\mathbf{f}^{\mathbf{mirror}}(\alpha)\right)$$

ELBM Equation **[Karlin *et al.*, 1999]**

$$f_i(x + c_i, t + 1) = (1 - \beta)\; f_i(x, t) + \beta\; f_i^{mirror}(x, t)$$

where $\beta = \frac{1}{2\tau}$, with $0 < \beta < 1$ as we have $0.5 < \tau < +\infty$

Outline

Is ELBM a LBGK with an implicit SGS?

- The viscosity ν is allowed to fluctuate locally: **[Karlin *et al.*, 1999]**

$$\nu(\alpha) = c_s^2 \left(\frac{1}{\alpha\beta} - 0.5 \right)$$

Is ELBM a LBGK with an implicit SGS?

- The viscosity ν is allowed to fluctuate locally: **[Karlin *et al.*, 1999]**

$$\nu(\alpha) = c_s^2 \left(\frac{1}{\alpha\beta} - 0.5 \right)$$

[Karlin *et al.*, 2015]

- Whenever the simulation is resolved $\alpha = 2$ and the ELBM equations is equivalent to the standard LBGK equation ($\tau_{BGK} = \tau_\alpha(2)$)

Is ELBM a LBGK with an implicit SGS?

- The viscosity ν is allowed to fluctuate locally: **[Karlin *et al.*, 1999]**

$$\nu(\alpha) = c_s^2 \left(\frac{1}{\alpha\beta} - 0.5 \right)$$

[Karlin *et al.*, 2015]

- Whenever the simulation is resolved $\alpha = 2$ and the ELBM equations is equivalent to the standard LBGK equation ($\tau_{BGK} = \tau_\alpha(2)$)

[Malaspinas *et al.*, 2013]

- Chapman-Enskog expansion was performed for $\alpha \approx 2$ and an additional term of the form $\nu_r S_{ij}$ appeared with:

$$\nu_r = -\frac{c_s^2 \Delta t}{3(2\beta)^2} \frac{S_{\theta\kappa} S_{\kappa\gamma} S_{\gamma\theta}}{S_{\lambda\mu} S_{\lambda\mu}}$$

Smagorinsky-like SGS model

Is ELBM a LBGK with an implicit SGS?

- The viscosity ν is allowed to fluctuate locally: **[Karlin *et al.*, 1999]**

$$\nu(\alpha) = c_s^2 \left(\frac{1}{\alpha\beta} - 0.5 \right)$$

[Karlin *et al.*, 2015]

- Whenever the simulation is resolved $\alpha = 2$ and the ELBM equations is equivalent to the standard LBGK equation ($\tau_{BGK} = \tau_\alpha(2)$)

[Malaspinas *et al.*, 2013]

- Chapman-Enskog expansion was performed for $\alpha \approx 2$ and an additional term of the form $\nu_r S_{ij}$ appeared with:

$$\nu_r = -\frac{c_s^2 \Delta t}{3(2\beta)^2} \frac{S_{\theta\kappa} S_{\kappa\gamma} S_{\gamma\theta}}{S_{\lambda\mu} S_{\lambda\mu}}$$

Smagorinsky-like SGS model

Objective: Numerically check the existence of an implied SGS

Outline

Check of the hydrodynamic balance

Through Chapman-Enskog expansion, we know that LBGK approximates the weekly compressible Navier-Stokes equations.

Check of the hydrodynamic balance

Through Chapman-Enskog expansion, we know that LBGK approximates the weekly compressible Navier-Stokes equations.

- Kinetic energy $E = \frac{\rho \vec{u}^2}{2}$ balance equation:

$$\partial_t \frac{\rho \vec{u}^2}{2} = - c_s^2 u_i \partial_i \rho - \nu \rho \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i + u_i F_i$$
$$+ \partial_j \left[-\frac{\rho \vec{u}^2}{2} u_j + \nu \rho u_i \left(\partial_j u_i + \partial_i u_j \right) \right]$$

Check of the hydrodynamic balance

Through Chapman-Enskog expansion, we know that LBGK approximates the weekly compressible Navier-Stokes equations.

- Kinetic energy $E = \frac{\rho \vec{u}^2}{2}$ balance equation:

$$\partial_t \frac{\rho \vec{u}^2}{2} = - c_s^2 u_i \partial_i \rho - \nu \rho \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i + u_i F_i + \partial_j \left[- \frac{\rho \vec{u}^2}{2} u_j + \nu \rho u_i \left(\partial_j u_i + \partial_i u_j \right) \right]$$

- Enstrophy $\Omega = \frac{\vec{\omega}^2}{2}$ balance equation:

$$\partial_t \frac{\vec{\omega}^2}{2} = - \frac{\vec{\omega}^2}{2} \partial_j u_j + \omega_i \omega_j \partial_j u_i + \nu \vec{H} \cdot \left(\vec{\nabla} \times \vec{\omega} \right) + \vec{\omega} \cdot \left(\vec{\nabla} \times \frac{1}{\rho} \vec{F} \right) + \partial_j \left[- \frac{\vec{\omega}^2}{2} u_j + \nu \epsilon_{ijk} \omega_i H_k \right], \text{ where } \vec{H} = \frac{1}{\rho} \vec{\nabla} \cdot \left[\rho \left(\vec{\nabla} \vec{u} + \left(\vec{\nabla} \vec{u} \right)^T \right) \right]$$

Averaging of balance equations over a sub-volume

We will focus on 2D flows and 2D sub-volumes in this presentation

Averaging of balance equations over a sub-volume

We will focus on 2D flows and 2D sub-volumes in this presentation

- Average of the Kinetic energy $E = \frac{\rho \vec{u}^2}{2}$ balance equation:

$$\partial_t \left\langle \frac{\rho \vec{u}^2}{2} \right\rangle = - c_s^2 \left\langle u_i \partial_i \rho \right\rangle - \nu \left\langle \rho \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i \right\rangle + \left\langle u_i F_i \right\rangle$$
$$- \left\langle \partial_j \frac{\rho \vec{u}^2}{2} u_j \partial_j u_i \right\rangle + \nu \left\langle \partial_j \rho u_i \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i \right\rangle$$

where $\langle \dots \rangle$ denotes average over a sub-volume V

Averaging of balance equations over a sub-volume

We will focus on 2D flows and 2D sub-volumes in this presentation

- Average of the Kinetic energy $E = \frac{\rho \vec{u}^2}{2}$ balance equation:

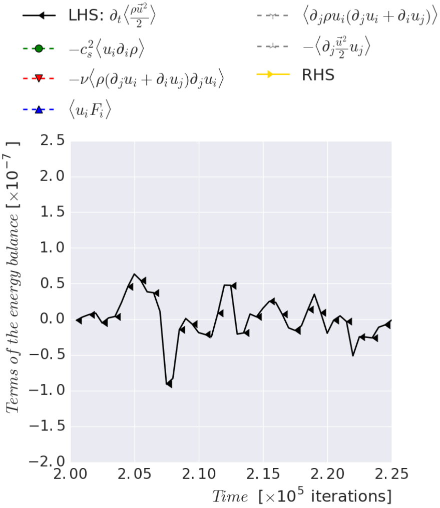
$$\partial_t \Big\langle \frac{\rho \vec{u}^2}{2} \Big\rangle = - c_s^2 \big\langle u_i \partial_i \rho \big\rangle - \nu \big\langle \rho \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i \big\rangle + \big\langle u_i F_i \big\rangle - \Big\langle \partial_j \frac{\rho \vec{u}^2}{2} u_j \partial_j u_i \Big\rangle + \nu \big\langle \partial_j \rho u_i \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i \big\rangle$$

- Average of the enstrophy $\Omega = \frac{\vec{\omega}^2}{2}$ balance equation:

$$\partial_t \Big\langle \frac{\omega^2}{2} \Big\rangle = - \Big\langle \frac{\omega^2}{2} \partial_j u_j \Big\rangle + \nu \big\langle H_x \partial_y \omega - H_y \partial_x \omega \big\rangle + \Big\langle \omega \left(\partial_x \frac{F_y}{\rho} - \partial_y \frac{F_x}{\rho} \right) \Big\rangle - \Big\langle \partial_j \frac{\vec{\omega}^2}{2} u_j \Big\rangle + \nu \big\langle \epsilon_{ijk} \omega_i H_k \big\rangle, \text{ where } \vec{H} = \frac{1}{\rho} \partial_j [\rho \left(\partial_j u_i + \partial_i u_j \right)]$$

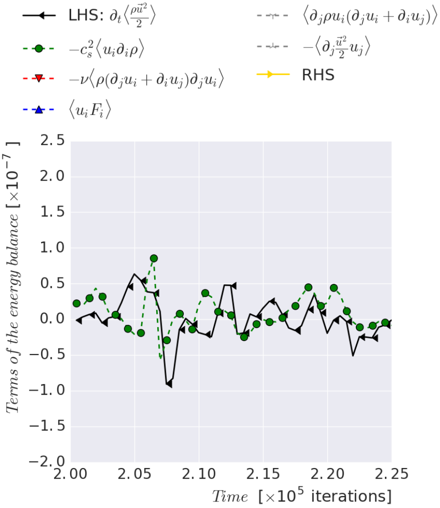
where $\langle \dots \rangle$ denotes average over a sub-volume V

Example of a term by term balance of E and Ω



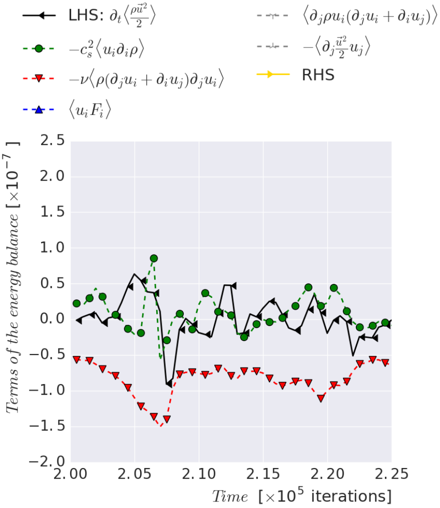
From energy balance

Example of a term by term balance of E and Ω



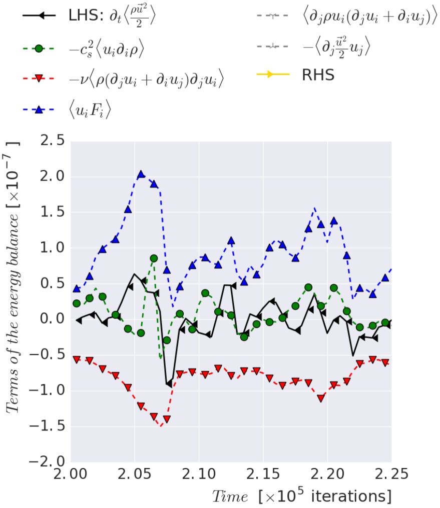
From energy balance

Example of a term by term balance of E and Ω



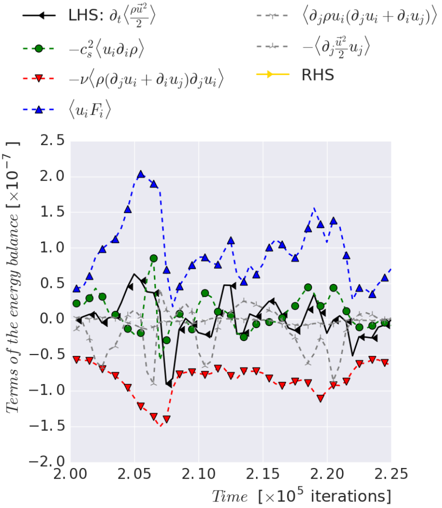
From energy balance

Example of a term by term balance of E and Ω



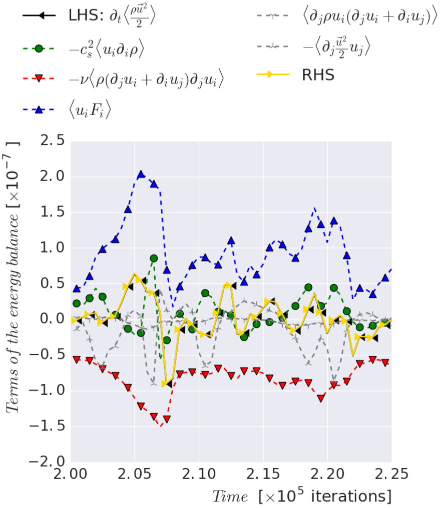
From energy balance

Example of a term by term balance of E and Ω



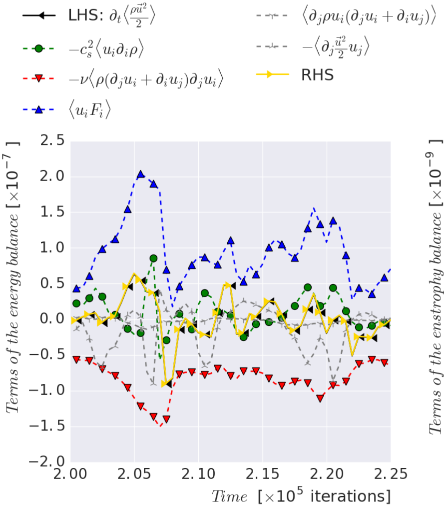
From energy balance

Example of a term by term balance of E and Ω

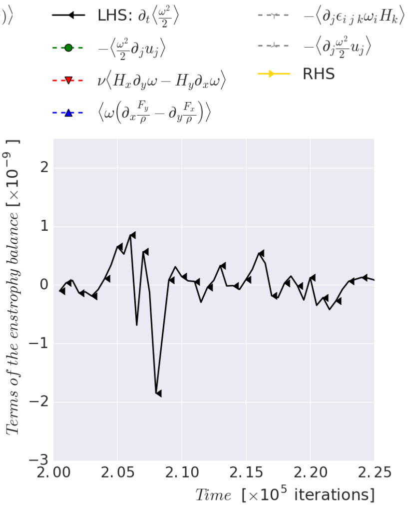


From energy balance

Example of a term by term balance of E and Ω

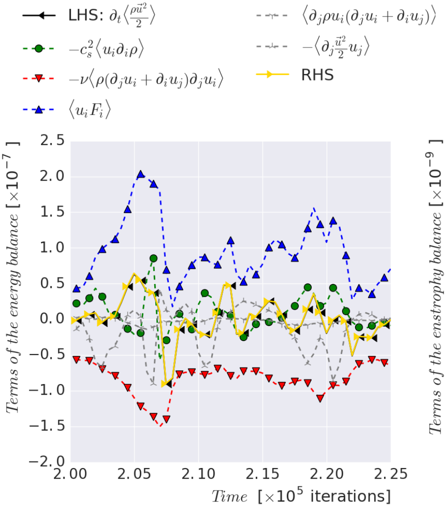


From energy balance

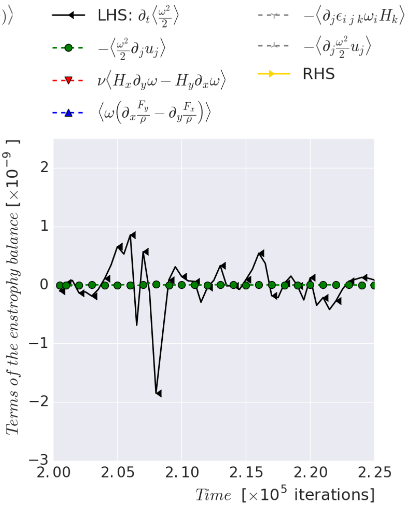


From enstrophy balance

Example of a term by term balance of E and Ω

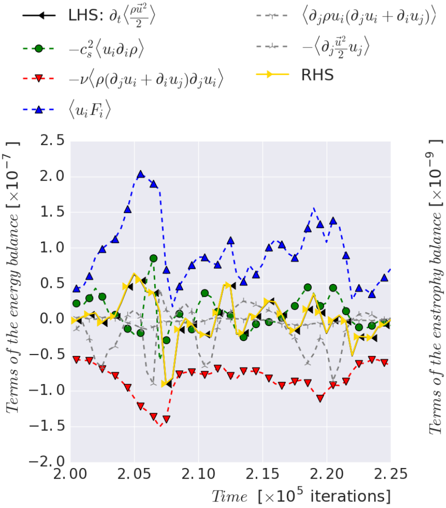


From energy balance

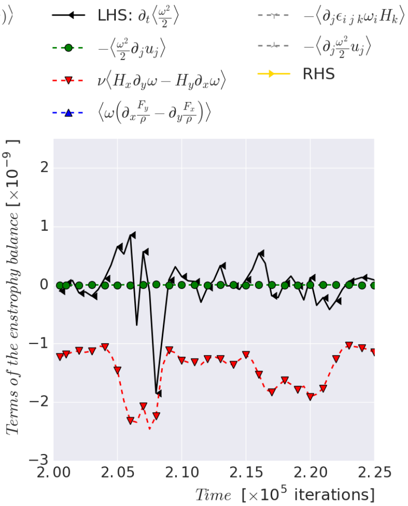


From enstrophy balance

Example of a term by term balance of E and Ω

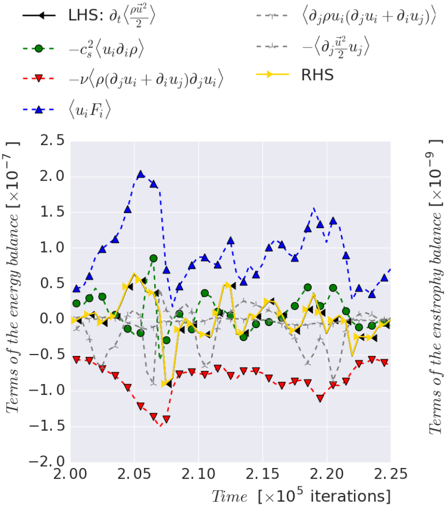


From energy balance

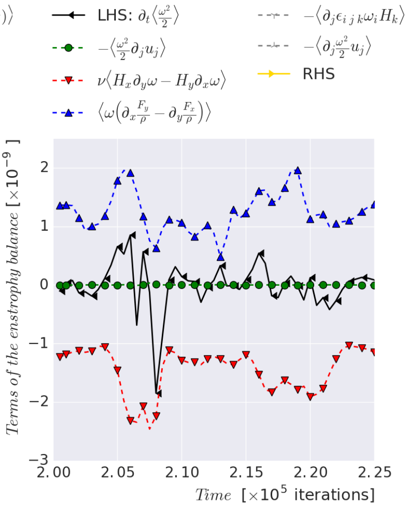


From enstrophy balance

Example of a term by term balance of E and Ω

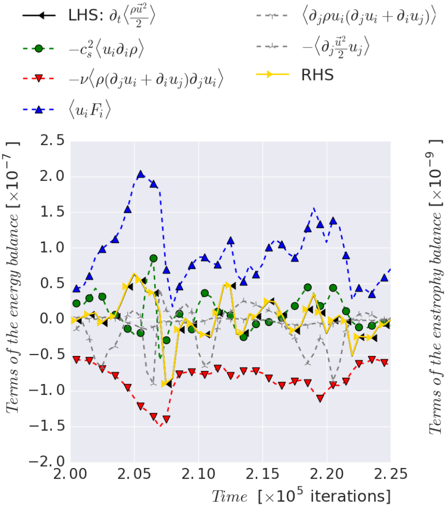


From energy balance

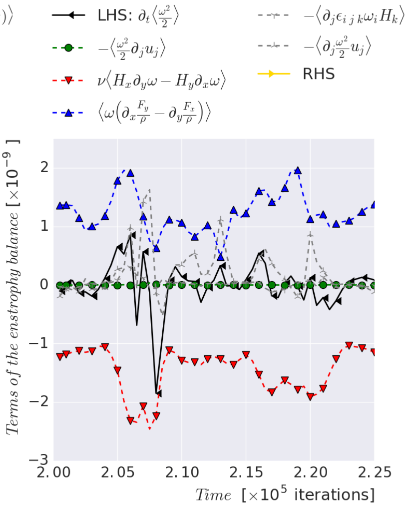


From enstrophy balance

Example of a term by term balance of E and Ω

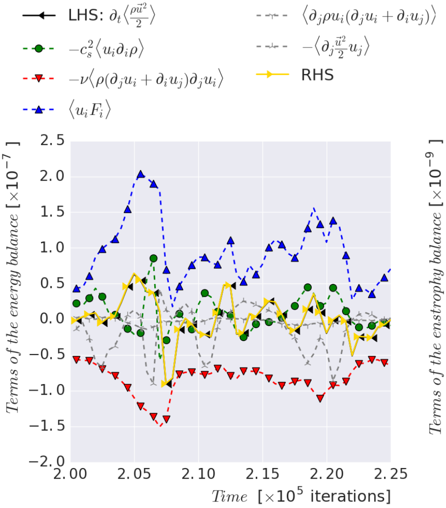


From energy balance

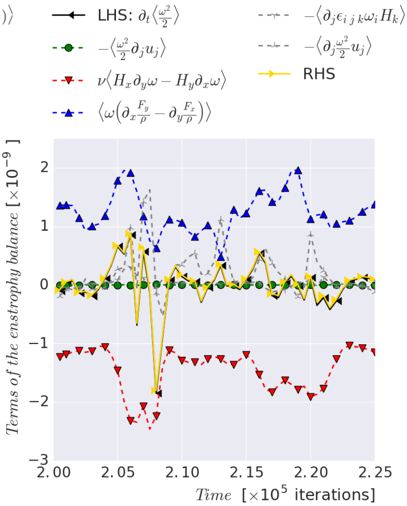


From enstrophy balance

Example of a term by term balance of E and Ω

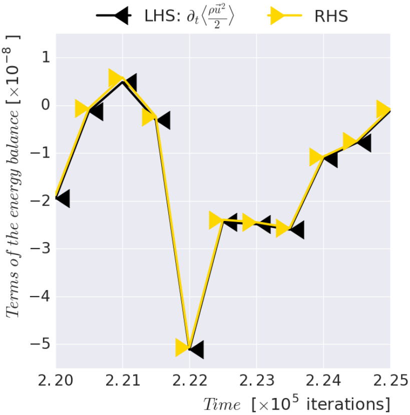


From energy balance

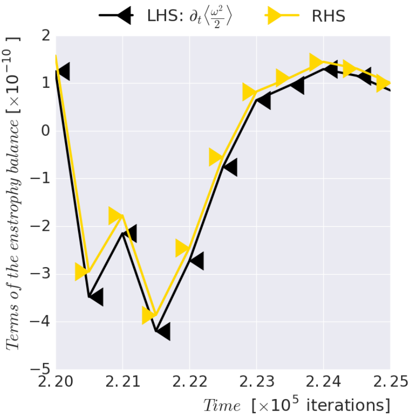


From enstrophy balance

Example of a term by term balance of E and Ω

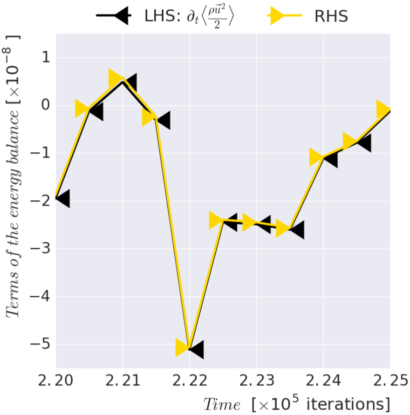


From energy balance

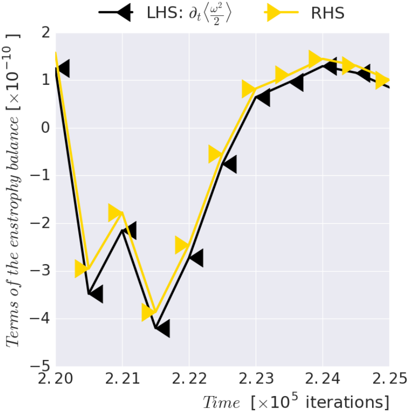


From enstrophy balance

Example of a term by term balance of E and Ω



From energy balance



From enstrophy balance

What is the accuracy with which LBGK/ELBM can recover the hydrodynamic balance equations?

Measurement of the relative effective viscosity

In order to evaluate the inaccuracy of the recovery of the balance equation averaged over a sub-volume, we can define an effective viscosity:

Measurement of the relative effective viscosity

In order to evaluate the inaccuracy of the recovery of the balance equation averaged over a sub-volume, we can define an effective viscosity:

- From kinetic energy balance:

$$\nu_{eff}^{E} = \frac{\partial_t \left\langle \frac{\rho \vec{u}^2}{2} \right\rangle + c_s^2 \left\langle u_i \partial_i \rho \right\rangle - \left\langle u_i F_i \right\rangle + \left\langle \partial_j \frac{\rho \vec{u}^2}{2} u_j \partial_j u_i \right\rangle}{-\left\langle \rho \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i \right\rangle + \left\langle \partial_j \rho u_i \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i \right\rangle}$$

Measurement of the relative effective viscosity

In order to evaluate the inaccuracy of the recovery of the balance equation averaged over a sub-volume, we can define an effective viscosity:

- From kinetic energy balance:

$$\nu_{eff}^{E} = \frac{\partial_t \left\langle \frac{\rho \vec{u}^2}{2} \right\rangle + c_s^2 \left\langle u_i \partial_i \rho \right\rangle - \left\langle u_i F_i \right\rangle + \left\langle \partial_j \frac{\rho \vec{u}^2}{2} u_j \partial_j u_i \right\rangle}{-\left\langle \rho \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i \right\rangle + \left\langle \partial_j \rho u_i \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i \right\rangle}$$

- From enstrophy balance:

$$\nu_{eff}^{\Omega} = \frac{\partial_t \left\langle \frac{\omega^2}{2} \right\rangle + \left\langle \frac{\omega^2}{2} \partial_j u_j \right\rangle - \left\langle \omega \left(\partial_x \frac{F_y}{\rho} - \partial_y \frac{F_x}{\rho} \right) \right\rangle + \left\langle \partial_j \frac{\vec{\omega}^2}{2} u_j \right\rangle}{\left\langle H_x \partial_y \omega - H_y \partial_x \omega \right\rangle + \left\langle \epsilon_{ijk} \omega_i H_k \right\rangle}$$

Measurement of the relative effective viscosity

In order to evaluate the inaccuracy of the recovery of the balance equation averaged over a sub-volume, we can define an effective viscosity:

- From kinetic energy balance:

$$\nu_{eff}^{E} = \frac{\partial_t \left\langle \frac{\rho \vec{u}^2}{2} \right\rangle + c_s^2 \left\langle u_i \partial_i \rho \right\rangle - \left\langle u_i F_i \right\rangle + \left\langle \partial_j \frac{\rho \vec{u}^2}{2} u_j \partial_j u_i \right\rangle}{-\left\langle \rho \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i \right\rangle + \left\langle \partial_j \rho u_i \left(\partial_j u_i + \partial_i u_j \right) \partial_j u_i \right\rangle}$$

- From enstrophy balance:

$$\nu_{eff}^{\Omega} = \frac{\partial_t \left\langle \frac{\omega^2}{2} \right\rangle + \left\langle \frac{\omega^2}{2} \partial_j u_j \right\rangle - \left\langle \omega \left(\partial_x \frac{F_y}{\rho} - \partial_y \frac{F_x}{\rho} \right) \right\rangle + \left\langle \partial_j \frac{\vec{\omega}^2}{2} u_j \right\rangle}{\left\langle H_x \partial_y \omega - H_y \partial_x \omega \right\rangle + \left\langle \epsilon_{ijk} \omega_i H_k \right\rangle}$$

Relative effective viscosity

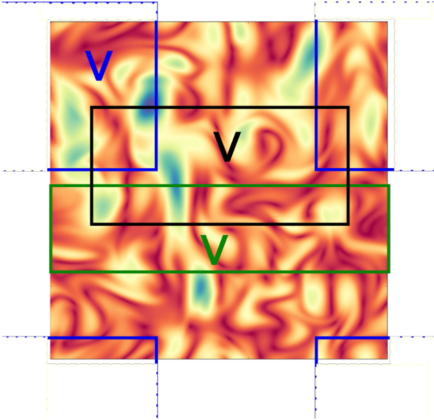
$$\frac{\nu_{eff}^{E,\Omega}}{\nu} = \frac{\nu_{eff}^{E,\Omega}}{c_s^2 \left(\tau - \frac{1}{2} \right)} \approx 1$$

Outline

Statistical analysis of $\frac{\nu_eff}{\nu}$

- Calculation on random sub-volumes of $\frac{\nu_eff}{\nu}$ for both kinetic energy and enstrophy balance equations.

- Sorting the results based on L, characteristic length of the sub-volume V defined as the square root of its volume V.

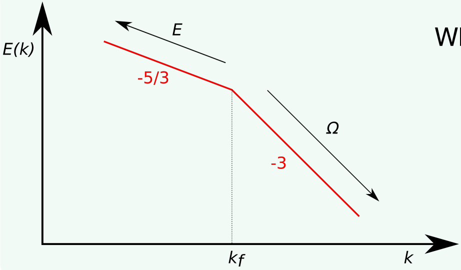


Numerical apparatus: 2D homogeneous turbulence

D2Q9 forced simulation on a periodic 256×256 grid.

Numerical apparatus: 2D homogeneous turbulence

D2Q9 forced simulation on a periodic 256 $\times$ 256 grid.

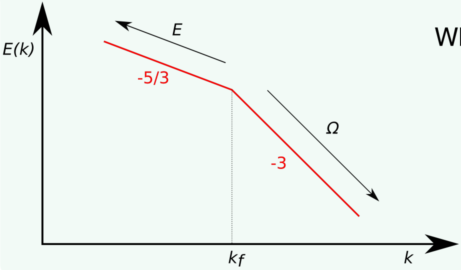


When forcing at k_f, we have:

- a direct energy cascade to large scales
- an indirect enstrophy cascade to small scales

Numerical apparatus: 2D homogeneous turbulence

D2Q9 forced simulation on a periodic 256×256 grid.



When forcing at k_f, we have:

- a direct energy cascade to large scales
- an indirect enstrophy cascade to small scales

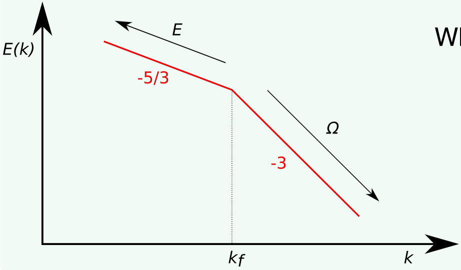
Forcing on a shell of wavenumber

$$F_\Psi^T = F_0^T \sum_{||\vec{k}||=5}^{7} \cos\left(\frac{2\pi}{L}\vec{k}.\vec{x} + \phi\right)$$

where ϕ is an arbitrary constant

Numerical apparatus: 2D homogeneous turbulence

D2Q9 forced simulation on a periodic 256 $\times$ 256 grid.



When forcing at k_f, we have:

- a direct energy cascade to large scales
- an indirect enstrophy cascade to small scales

Forcing on a shell of wavenumber

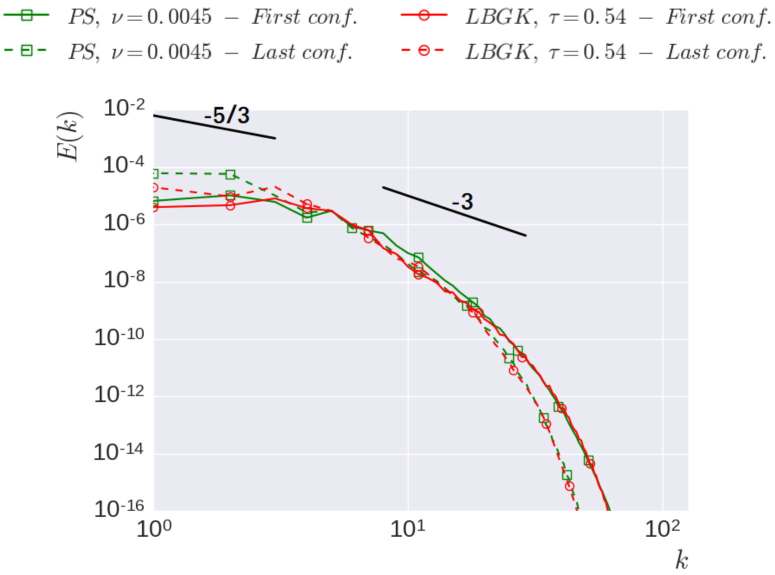
$$F_\Psi^T = F_0^T \sum_{||\vec{k}||=5}^{7} \cos\left(\frac{2\pi}{L}\vec{k}.\vec{x} + \phi\right)$$

where ϕ is an arbitrary constant

Energy removal at large scale

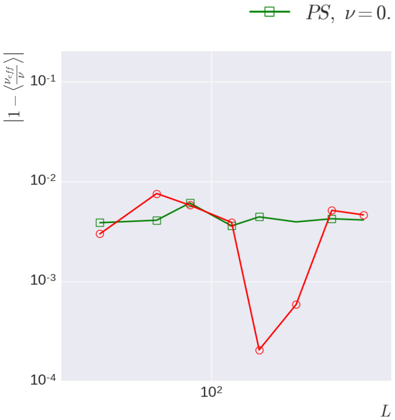
$$\vec{F^E}(\vec{x}, t) = -F_0^E \sum_{||\vec{k}||=1}^{2} \hat{\vec{u}}(\vec{k}, t) e^{\frac{2\pi}{L}\vec{k}\cdot\vec{x}}$$

Validation: LBGK vs. Pseudo-spectral - Decaying spectrum

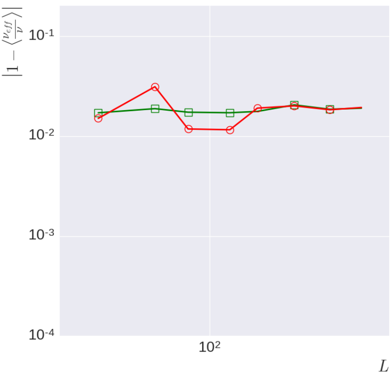


Validation: LBGK vs. Pseudo-spectral - Error in $\left\langle \frac{\nu_eff}{\nu} \right\rangle$

$\left| 1 - \left\langle \frac{\nu_eff}{\nu} \right\rangle \right|$ against sub-volume characteristic length L



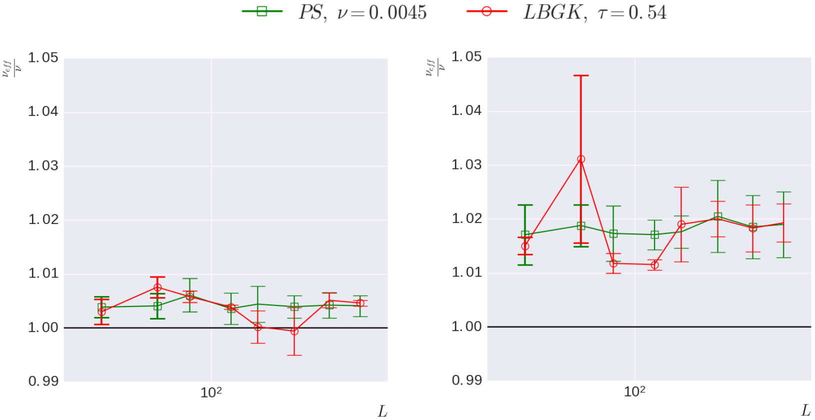
From energy balance



From enstrophy balance

Validation: LBGK vs. Pseudo-spectral - Variation of $\frac{\nu_{eff}}{\nu}$

$\frac{\nu_{eff}}{\nu}$ against sub-volume characteristic length L

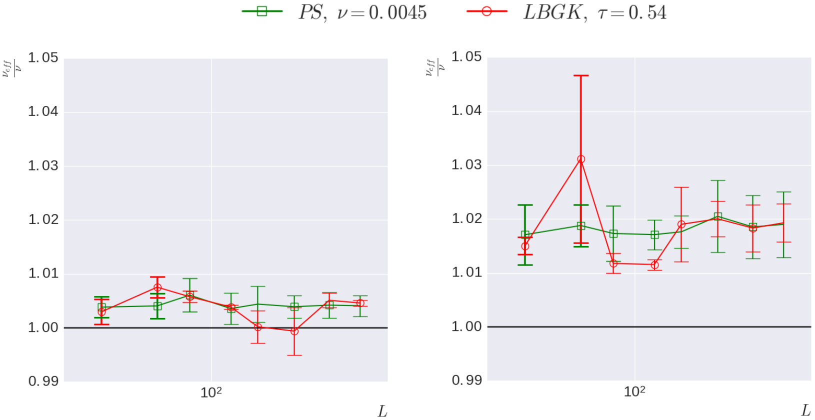


From energy balance

From enstrophy balance

Validation: LBGK vs. Pseudo-spectral - Variation of $\frac{\nu_{eff}}{\nu}$

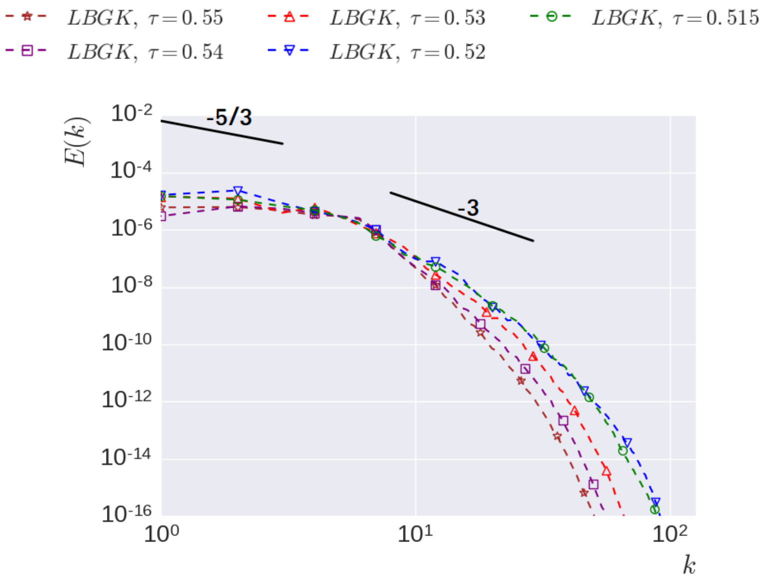
$\frac{\nu_{eff}}{\nu}$ against sub-volume characteristic length L



From energy balance — From enstrophy balance

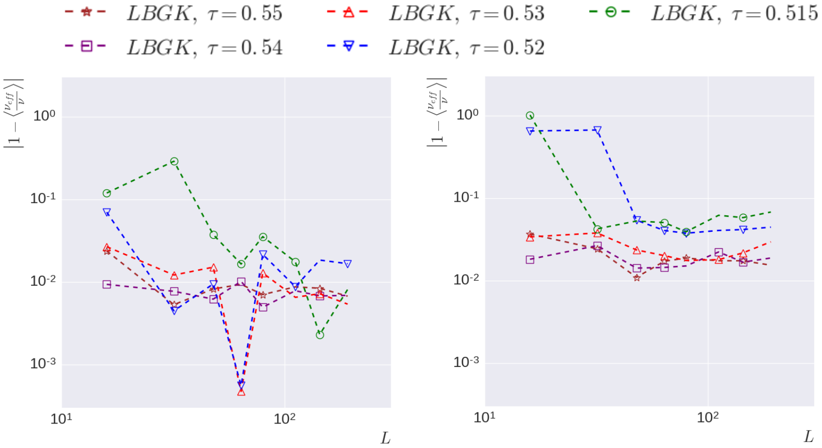
Good agreement for both Pseudo-Spectral and LBGK

Benchmark: Forced LBGK - Superposed spectrum



Benchmark: Forced LBGK - Error in $\left\langle \frac{\nu_{eff}}{\nu} \right\rangle$

$\left| 1 - \left\langle \frac{\nu_{eff}}{\nu} \right\rangle \right|$ against sub-volume characteristic length L



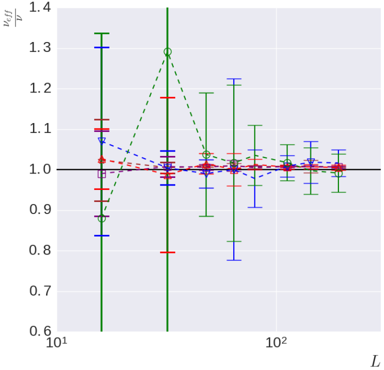
From energy balance

From enstrophy balance

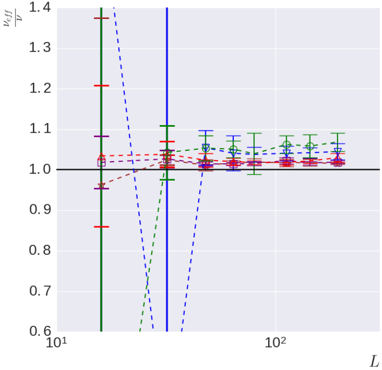
Benchmark: Forced LBGK - Variation of $\frac{\nu_{eff}}{\nu}$

$\frac{\nu_{eff}}{\nu}$ against sub-volume characteristic length L

$LBGK,\ \tau = 0.55$ $LBGK,\ \tau = 0.53$ $LBGK,\ \tau = 0.515$ $LBGK,\ \tau = 0.54$ $LBGK,\ \tau = 0.52$



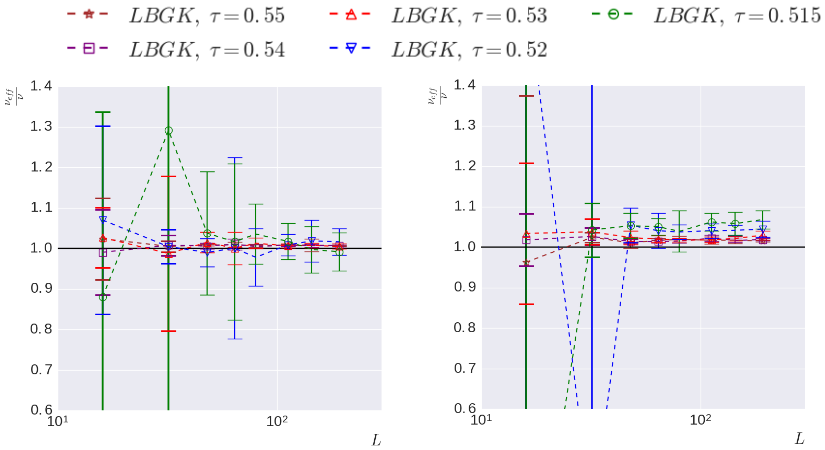
From energy balance



From enstrophy balance

Benchmark: Forced LBGK - Variation of $\frac{\nu_{eff}}{\nu}$

$\frac{\nu_{eff}}{\nu}$ against sub-volume characteristic length L

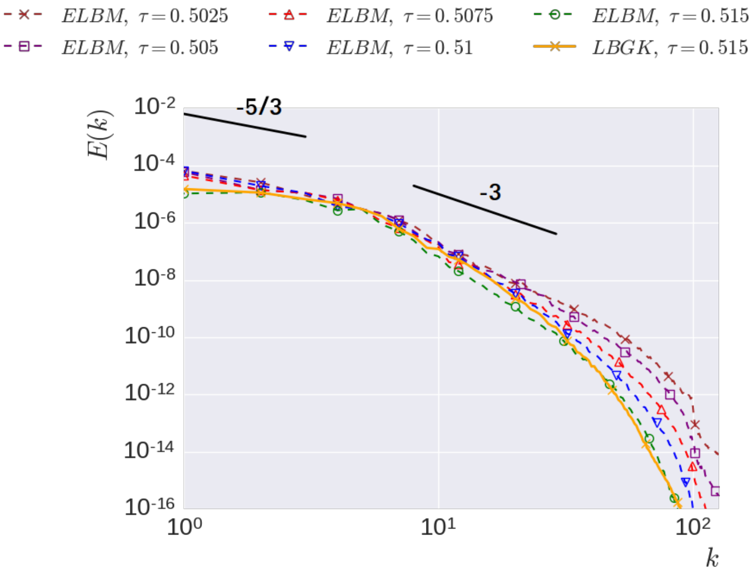


From energy balance

From enstrophy balance

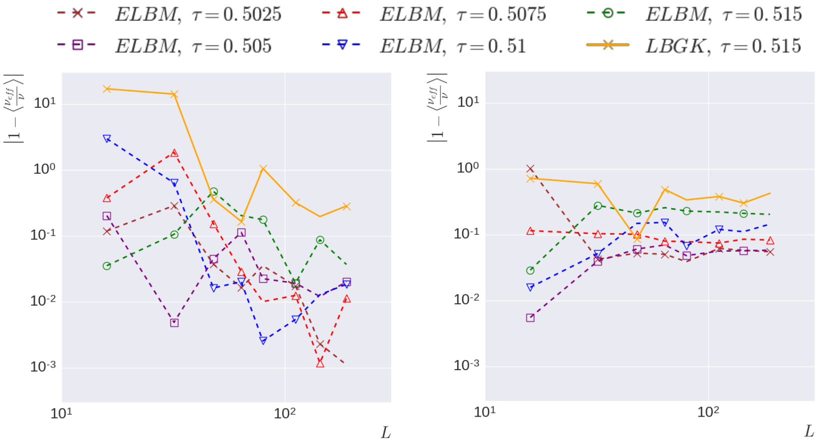
Reynolds number effect + proximity to $\tau_{critical}$?

Benchmark: Forced ELBM - Superposed spectrum



Benchmark: Forced ELBM - Error in $\left\langle \frac{\nu_{eff}}{\nu} \right\rangle$

$\left| 1 - \left\langle \frac{\nu_{eff}}{\nu} \right\rangle \right|$ against sub-volume characteristic length L

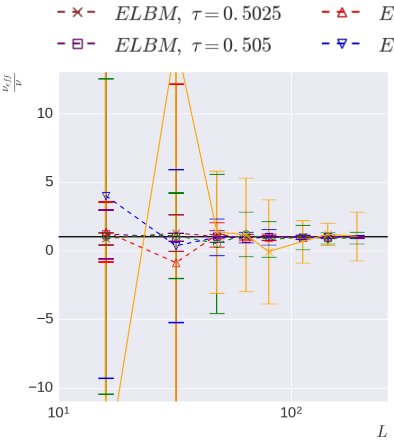


From energy balance

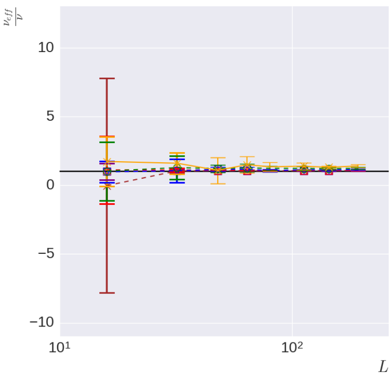
From enstrophy balance

Benchmark: Forced ELBM - Variation of $\frac{\nu_eff}{\nu}$

$\frac{\nu_eff}{\nu}$ against sub-volume characteristic length L



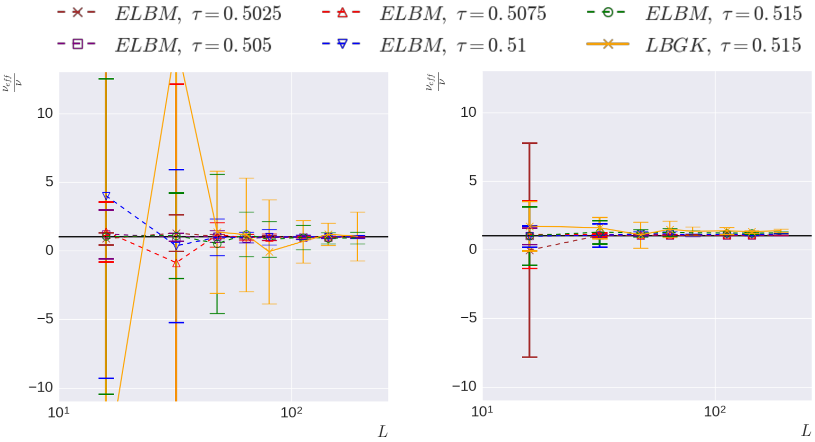
From energy balance



From enstrophy balance

Benchmark: Forced ELBM - Variation of $\frac{\nu_{eff}}{\nu}$

$\frac{\nu_{eff}}{\nu}$ against sub-volume characteristic length L



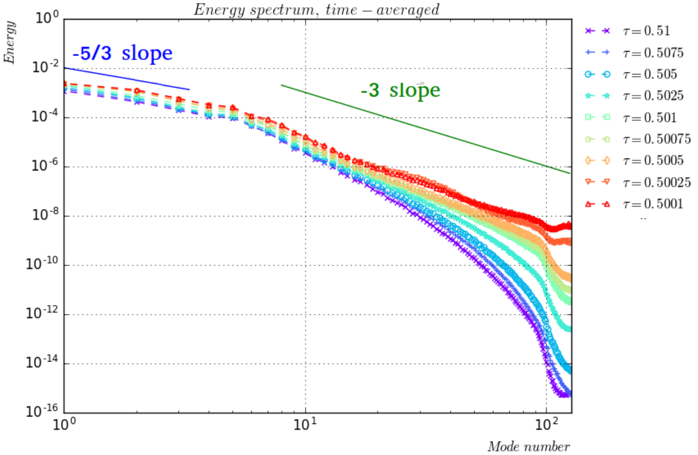
From energy balance

From enstrophy balance

No agreement expected for ELBM: An extra term in the balance eqs?

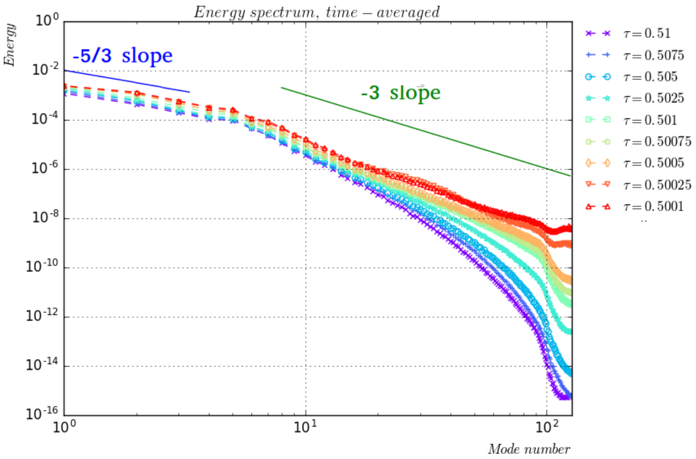
Benchmark: Forced ELBM - Dissipative properties

Going further to $\tau \to 0.5$, we observe an extension of the inertial range



Benchmark: Forced ELBM - Dissipative properties

Going further to $\tau \to 0.5$, we observe an extension of the inertial range



ELBM has the dissipative properties expected from a LES

Outline

Conclusion & Outlook

Conclusions:

- Developped a tool to check the balance equations and validated it on configurations obtained from a Pseudo-Spectral code.
- LBGK's recovery of hydrodynamics gets broken as the critically stable τ is approached.
- ELBM's effective visocisty ν_{eff} as $\tau \to 0.5$ cannot be represented by a simple renormalization of the input viscosity ν:
  Presence of an extra SGS to be taken into account in the balance equations?
- ELBM dissipative properties as $\tau \to 0.5$ are as expected for a LES.

Conclusion & Outlook

Conclusions:

- Developped a tool to check the balance equations and validated it on configurations obtained from a Pseudo-Spectral code.
- LBGK's recovery of hydrodynamics gets broken as the critically stable τ is approached.
- ELBM's effective visocisty ν_{eff} as $\tau \to 0.5$ cannot be represented by a simple renormalization of the input viscosity ν:
  Presence of an extra SGS to be taken into account in the balance equations?
- ELBM dissipative properties as $\tau \to 0.5$ are as expected for a LES.

Outlook:

- Validate Malaspinas' suggested implicit SGS to the balance equations on ELBM simulations
- Switch to 3D turbulent simulations.

References

Ilya V. Karlin, Fabian Bösch, Shyam Chikatamarla, and Sauro Succi, *Entropy-Assisted Computing of Low-Dissipative Systems*, Entropy **17** (2015), no. 12, 8099–8110.

Iliya V. Karlin, Antonio Ferrante, and Hans Christian Öttinger, *Perfect entropy functions of the Lattice Boltzmann method*, Europhysics Letters **47** (1999), no. 2, 182–188.

Orestis Malaspinas, Michel Deville, and Bastien Chopard, *Towards a physical interpretation of the entropic lattice Boltzmann method*, Physical Review E **78** (2008), no. 6, 066705.

Sauro Succi, Iliya V. Karlin, and Hudong Chen, *Role of the H theorem in lattice Boltzmann hydrodynamic simulations*, Review of Modern Physics **74** (2002), no. 4, 1203–1220.

Sauro Succi, *The Lattice Boltzmann Equation for Fluid Dynamics and Beyond*, Oxford University Press, 2001.

Thank you for your attention!

Any questions?

UNIVERSITA' degli STUDI di ROMA TOR VERGATA

HPC-LEAP EUROPEAN JOINT DOCTORATES

BERGISCHE UNIVERSITÄT WUPPERTAL

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No' 642069

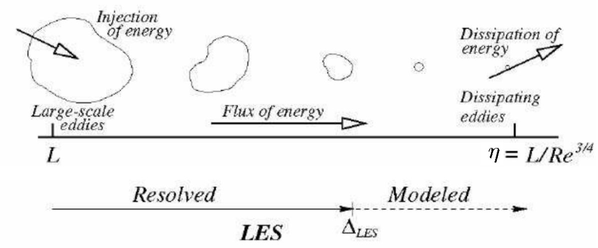
What is Large Eddy Simulation (LES)?

- Reduces the number of degrees of freedom by resolving scales only up to a cutoff scale and modeling the remaining smaller scales
- Enables cost-effective high Reynolds turbulent flow simulations

LES equation: Filtered Navier-Stokes + SGS model

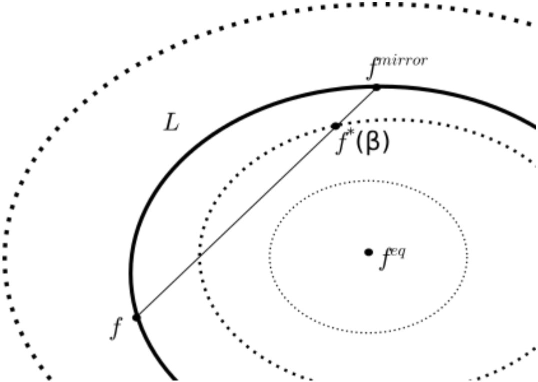
$$\partial_t \overline{u}_i + \partial_j \left(\overline{u}_i \, \overline{u}_j \right) = -\frac{1}{\rho} \partial_i p + \nu \, \partial_j \overline{S}_{ij} - \partial_j \tau_{ij}, \text{ where } \overline{S}_{ij} = \left(\partial_j \overline{u}_i + \partial_i \overline{u}_j \right)$$

$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \, \overline{u}_j$ must be modeled using a Sub-Grid Scale (SGS) Model



ELBM: Perspective from H-functional hypersurface

Calculation of α and convexity of H insure monotonic decreases of H



Solving the Entropic step equation

Entropic step Equation

$$H(\mathbf{f}) = H(\mathbf{f} - \alpha\,(\mathbf{f} - \mathbf{f^{eq}}))$$

with $H(\mathbf{f}) = \sum_0^{q-1} f_i \log\left(\frac{f_i}{\omega_i}\right)$

- Nont-trivial: typically solved using Newton-Raphson in 6-8 iterations for a tolerance of 10^{-5}
- When Newton-Raphson does not converge, 2, the LBGK's value of α is used