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Clustering of vertically constrained passive particles in homogeneous, isotropic turbulence

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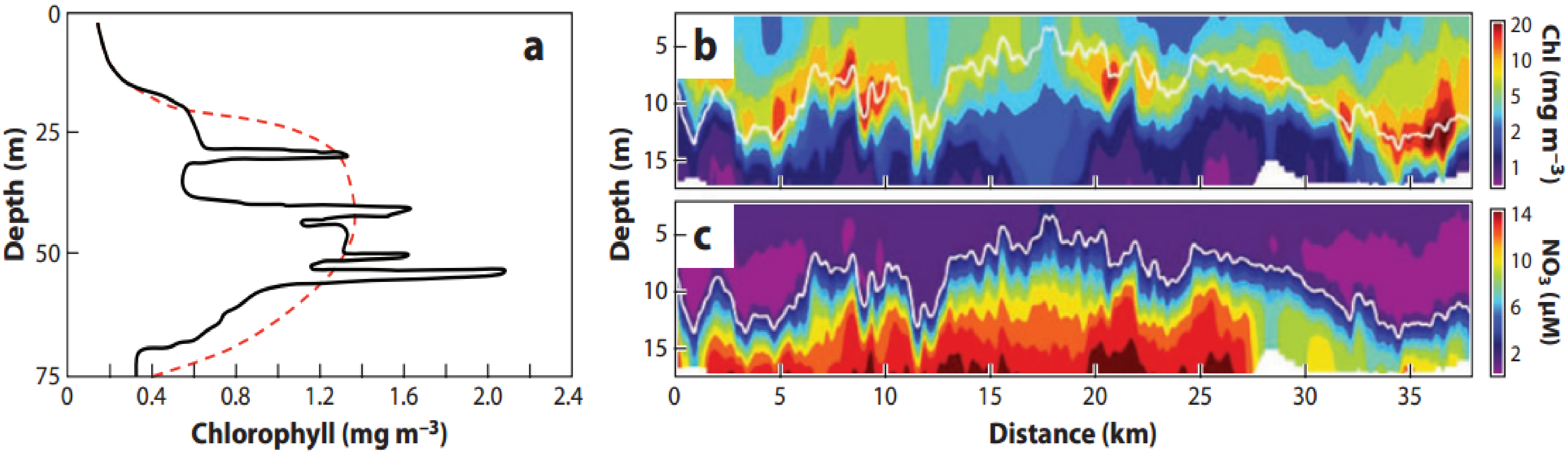
Plankton Layers

Plankton is often found to be distributed in thin layers a few meters under the sea surface.

Characteristics:

- Width/Depth ratio >> 1 (~ 10Km/10m);
- Concentration of “particles” ~1 order of magnitude bigger respect to backgroung;
- temporal persistence: hours to weeks.

There are several hypotheses on the mechanisms of formation / persistence.



Durham, W. M. & Stocker, R. Thin phytoplankton layers: characteristics, mechanisms, and consequences. Ann Rev Mar Sci 4, 177–207 (2012).

Particles in turbulent flow

- Simplified Maxey-Riley equations, for buoyancy driven particles, with 2 parameters:

$$\frac{d\mathbf{u}}{dt} = \beta \frac{D\mathbf{v}}{Dt} - \frac{\mathbf{u} - \mathbf{v}}{\tau_s} + (1 - \beta)\,\mathbf{g}$$

  - Density ratio $\beta = 3\rho_f / (2\rho_p + \rho_f)$
  - Stokes time $\tau_p = a^2 / (3\beta\nu)$

- Preferential concentration (Maxey centrifuge) when $\beta \neq 1$

Objective

- What happens when particles are constrained to stay at a specific depth in a turbulent flow?
  - Situation that usually happens with acquatic microorganisms
- We want to quantify the horizontal clustering of particles
  - Because clustering can have severe consequences on life and biology of these microorganisms

Our model

- Underlying fluid: homogeneous and isotropic turbulence
  - Simulated with DNS pseudo-spectral method. Parameters:
    - Domain: $2\pi^3$; resolution: 128^3
    - $\nu = 0.01$; $\epsilon = 2.39$; $\eta \sim \Delta X/2$
    - $Re \sim 830$
- Passive, pointlike, non inertial particles.

Our model

- Vertical confinement by means of a linear restoring force:

$$\begin{cases} u_i = v_i & (i = x, y) \\ u_z = v_z - K\,(z - z_0) \end{cases}$$

- The equation can formally be obtained in the case of particle motion driven by buoyancy:

$$\frac{d\mathbf{u}}{dt} = \beta \frac{D\mathbf{v}}{Dt} - \frac{\mathbf{u} - \mathbf{v}}{\tau_s} + (1 - \beta)\,\mathbf{g}$$

$$\beta = 3\rho_f/(2\rho_p + \rho_f)$$
$$\tau_s = a^2/(3\beta\nu)$$

if we assume:

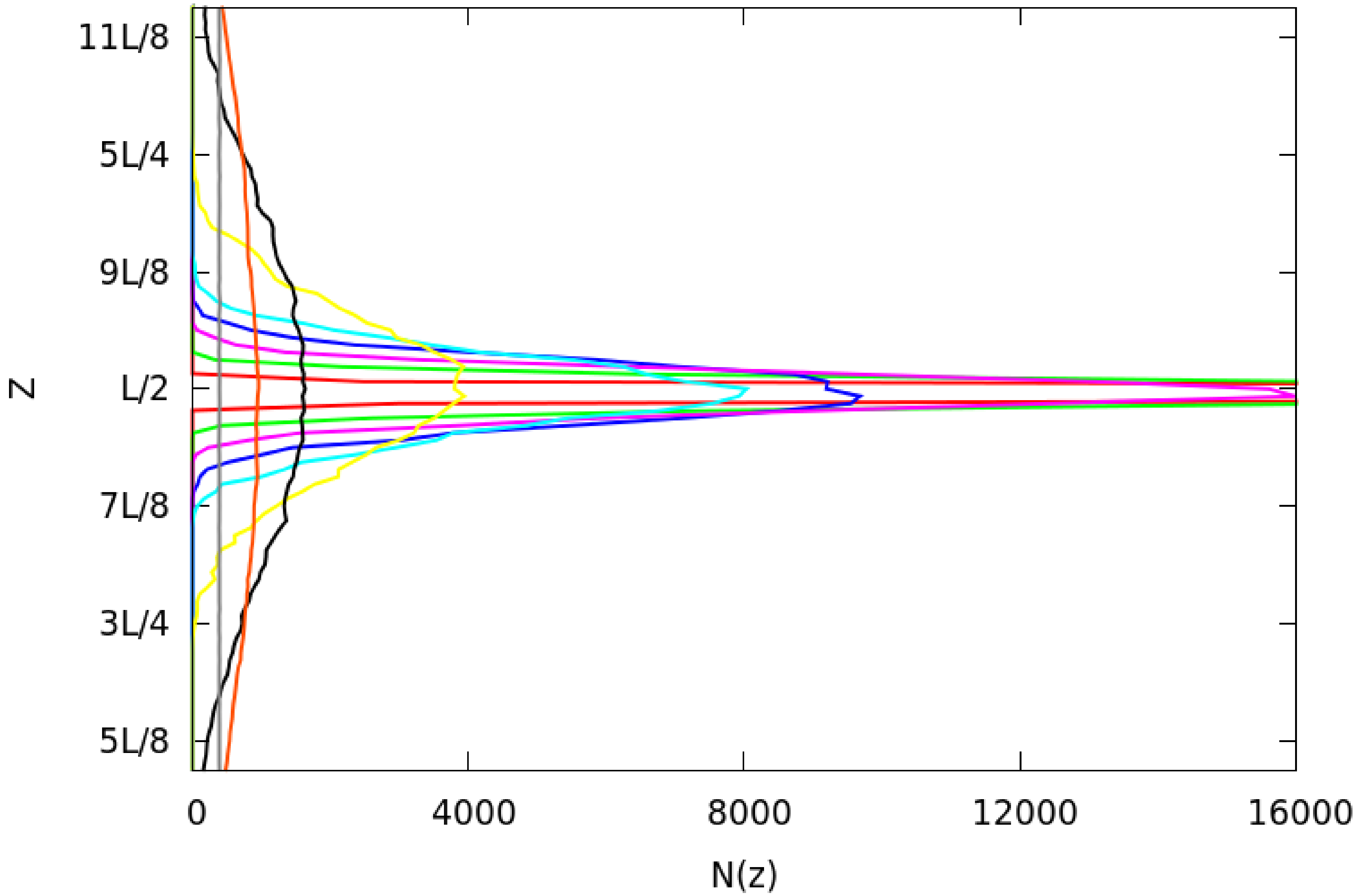
(1) $\tau_s \ll 1 \rightarrow Dv/Dt = du/dt + O(\tau_s)$

(2) $\rho_f = \rho_0 + \dfrac{d\rho_f}{dz}(z - z_0) + O((z - z_0)^2)$

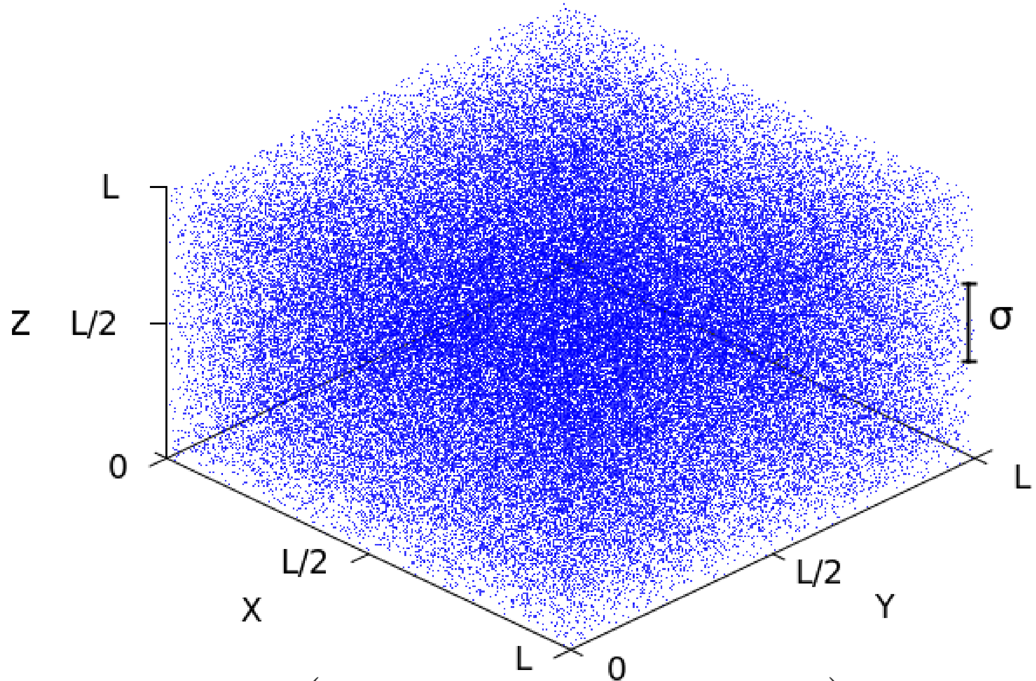
(3) $\rho_p \simeq \rho_0 \rightarrow \beta \simeq 1 + \dfrac{2N^2}{3g}(z - z_0)$

(4) $\dfrac{du}{dt} \ll g$

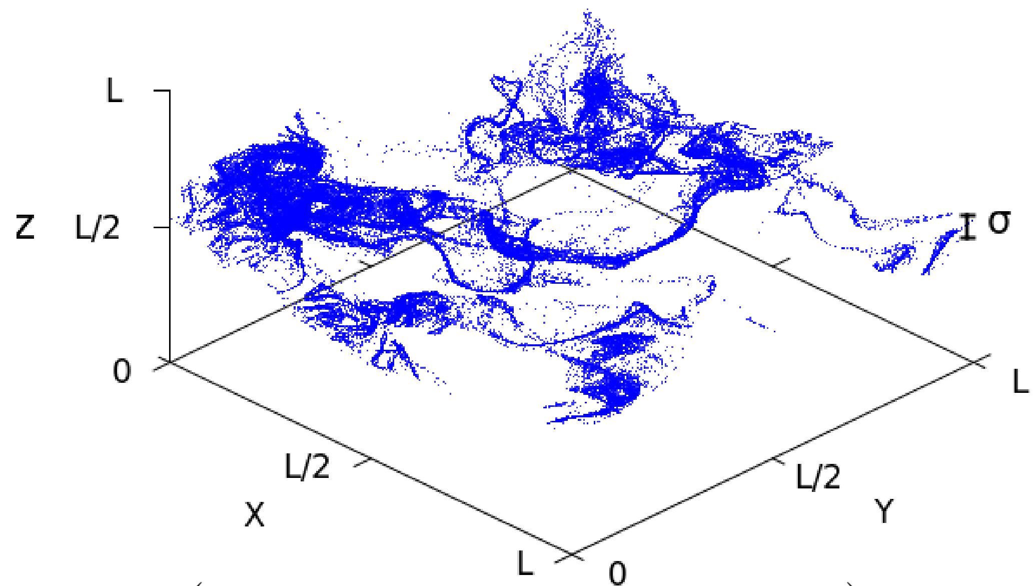
Particles – Vertical distribution



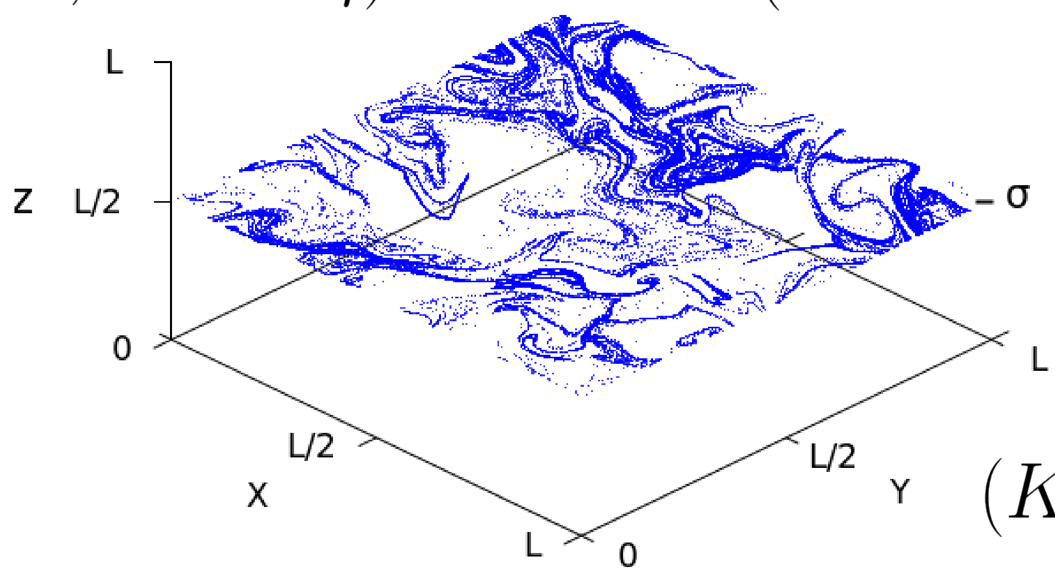
Particles – 3D distribution



$(K = 0, \sigma \sim 72\eta)$



$(K = 0.125, \sigma \sim 21\eta)$



$(K = 6, \sigma \sim 0.46\eta)$

Clustering - definitions

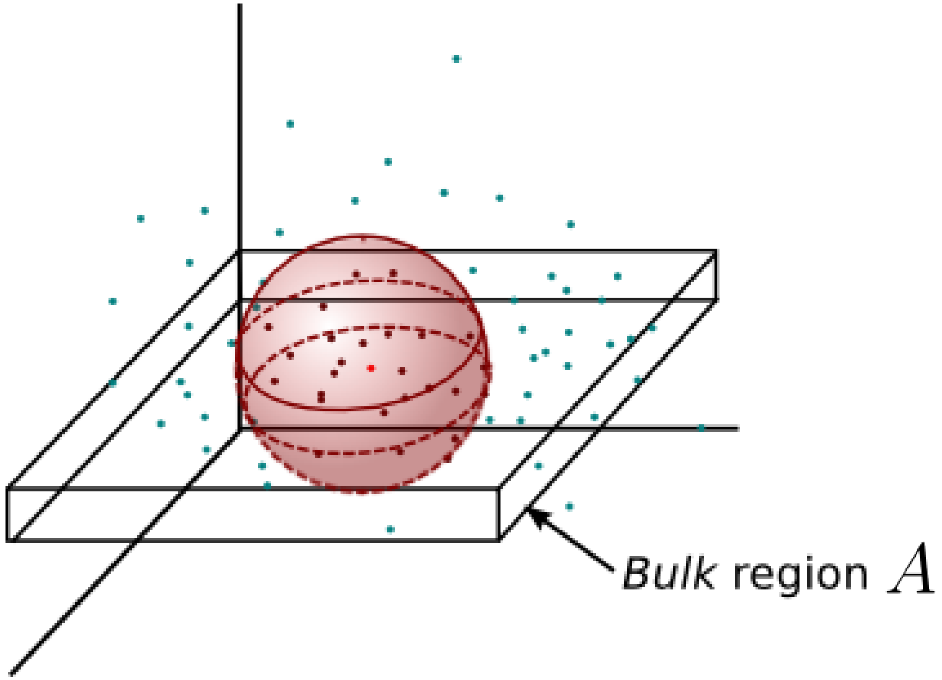
Standard Correlation Integral:

$$C_2(r) = \frac{1}{N_p(N_p-1)} \sum_{i}^{N_p} \sum_{j>i}^{N_p} \Theta(r - |x_i - x_j|)$$

Scaling: $\lim_{r \to 0} C_2(r) \propto r^{\epsilon}$

Local scaling exponent:

$$\epsilon(r) = \frac{d \log\left(C_2(r')\right)}{d \log\left(r'\right)}\Bigg|_{r'=r}$$

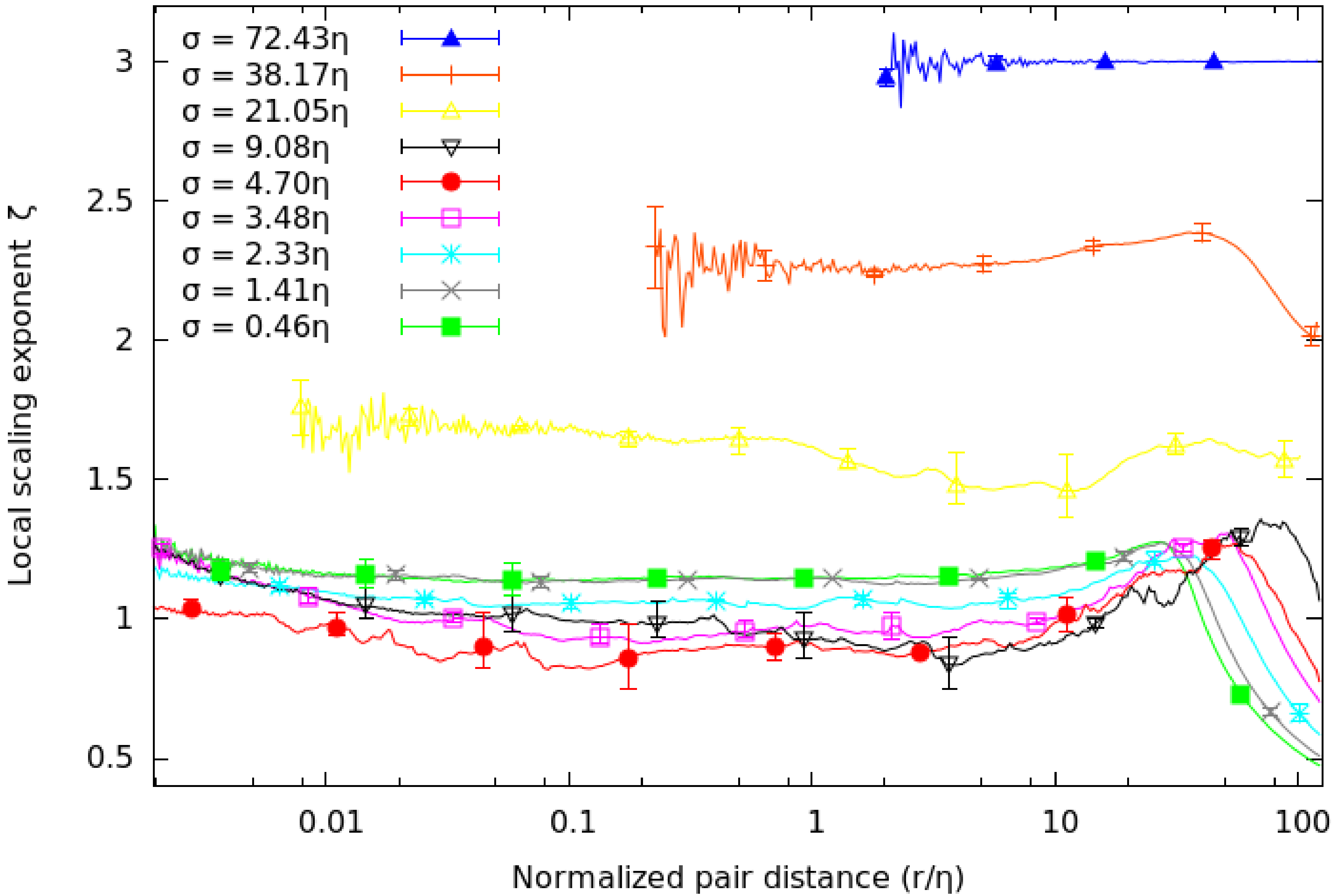


Our Correlation Integral:

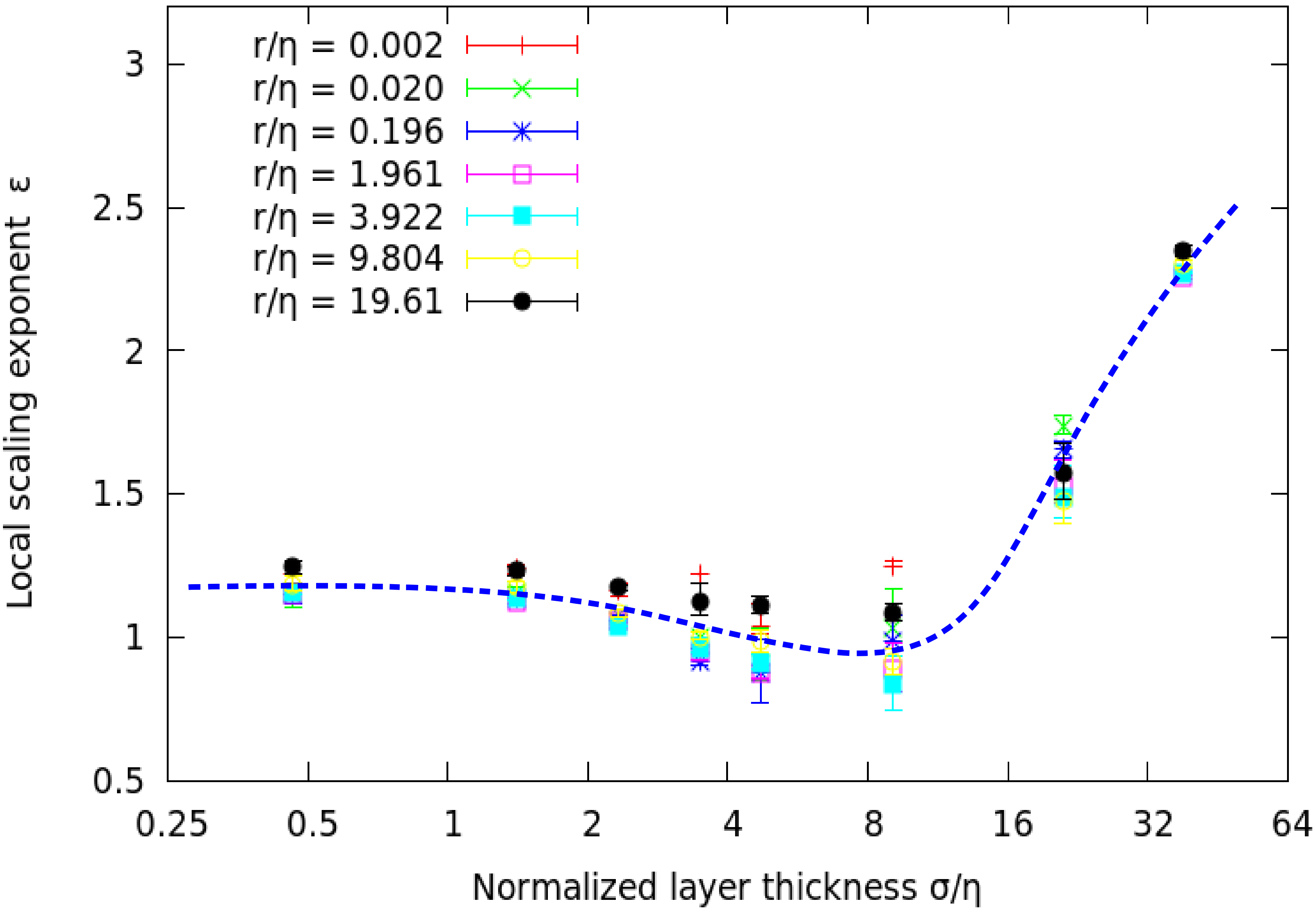
$$C_2(r) = \frac{1}{N_A(N_p-1)} \sum_{i}^{N_A} \sum_{j \neq i}^{N_p} \Theta(r - |x_i - x_j|)$$

$$A = \{i | z_i \in [L/2 - 0.25\eta; L/2 + 0.25\eta]\}$$

Local scaling exponent vs. r



Local scaling exponent vs. σ



Conclusions

- Plankton has been modelized as passive, pointlike, non inertial particles.
- Vertical confinement obtained using a linear restoring force.
- Underlying flow: omogeneous and isotropic turbulence.
- The spatial (horizontal) distribution of particles has been investigated. The degree of clustering was quantified using the correlation integral.
- We found it exists an optimal value for the restoring force constant, that maximizes the clustering of particles; this value of the constant corresponds to a thickness of the particle layer around a few Kolmogorov lengths η.

Future directions:

- Add interactions between particles, to model plankton biology
  - Also populations with different equilibrium depth
- Add effects induced by swimming, to model those species of plankton that are able to swim

More on the equations of motion...

- Maxey-Riley with only stokes drag and buoyancy:

$$\frac{d\mathbf{u}}{dt} = \beta \frac{D\mathbf{v}}{Dt} - \frac{\mathbf{u} - \mathbf{v}}{\tau_s} + (1 - \beta)\,\mathbf{g}$$

$$\beta = 3\rho_f / (2\rho_p + \rho_f)$$
$$\tau_s = a^2 / (3\beta\nu)$$

- Assuming:

(1) $\tau_s \ll 1 \rightarrow Dv/Dt = du/dt + O(\tau_s)$

(2) $\rho_f = \rho_0 + \frac{d\rho_f}{dz}(z - z_0) + O((z - z_0)^2)$

(3) $\rho_p \simeq \rho_0 \rightarrow \beta \simeq 1 + \frac{2N^2}{3g}(z - z_0)$

(4) $\frac{du}{dt} \ll g$

- We get:

$$\mathbf{v} - \mathbf{u} = \tau_s \frac{d\mathbf{u}}{dt} - \tau_s \beta \frac{d\mathbf{u}}{dt} + O(\tau_s^2) - \tau_s (1 - \beta) g \hat{\mathbf{z}}$$

$$\mathbf{v} - \mathbf{u} = \tau_s (1 - \beta)\left(\frac{d\mathbf{u}}{dt} - g\hat{\mathbf{z}}\right)$$

$$\mathbf{v} - \mathbf{u} = \tau_s \frac{2N^2}{3}(z - z_0)\hat{\mathbf{z}}$$