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Assessing Entropic LBM as an implicit Large Eddy Simulation

Lesson from the 2D case

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This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No' 642069 and was conducted within the activity of ERC Grant No' 339032

Motivations

Lattice Boltzmann Method:

- Adapted to a wide range of physical simulations
- Intrinsic scalability, well suited for HPC
- Can handle very complex (moving) geometry

Large Eddy Simulation:

- Enable cost-effective highly turbulent flow simulations
- Popular in commercial CFD softwares

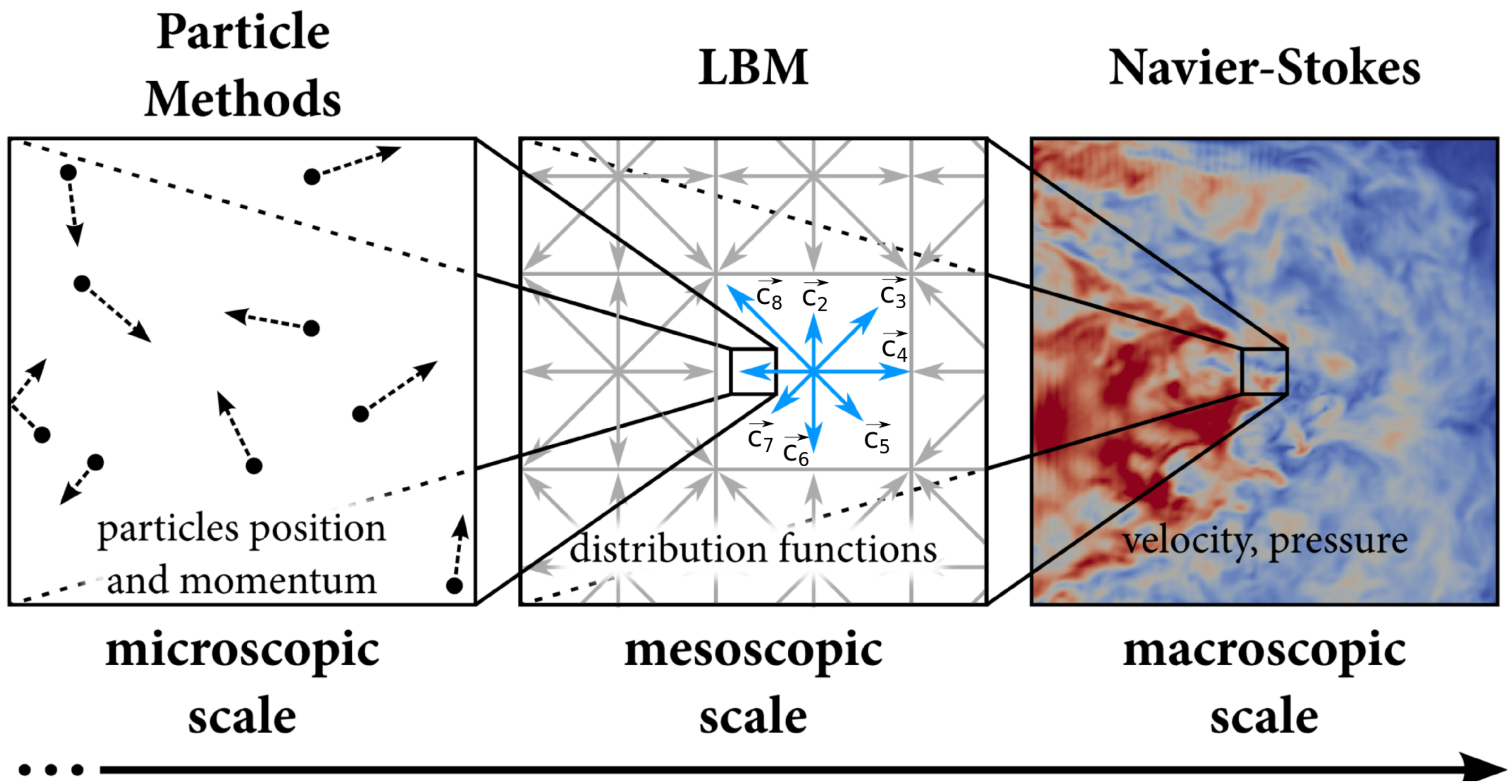
Study of a Large Eddy Simulation within the Lattice Boltzmann framework

Introduction to LBM

LBM Equation with a relaxation time $\tau \equiv \tau_0$ fixed (LBGK)

$$f_i(\vec{x} + \vec{c}_i \Delta t, t + \Delta t) - f_i(\vec{x}, t) = -\frac{1}{\tau_0}\left[f_i(\vec{x}, t) - f_i^{eq}(\vec{x}, t)\right]$$

Macroscopic quantities: Density $\rho = \sum_i f_i$ Momentum $\rho\vec{u} = \sum_i f_i \vec{c}_i$



Introduction to LBM

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↓

Chapman-Enskog expansion $Ma = \frac{u_{RMS}}{c_s^2}$

$\nu = c_s^2(\tau - 0.5)\Delta t$ $Kn = \frac{\lambda}{L}$

Weakly compressible Navier-Stokes with viscosity $\nu \equiv \nu_0$ fixed

$$\partial_t(\rho u_i) + \partial_j(\rho u_i u_j) = -\partial_i p + \partial_j \rho\nu\left(\partial_j u_i + \partial_i u_j\right) + \mathcal{O}(M_a^3) + \mathcal{O}(K_n^2)$$

We want to use LBM to simulate highly turbulent flows

Simulation of 2D Forced Turbulence

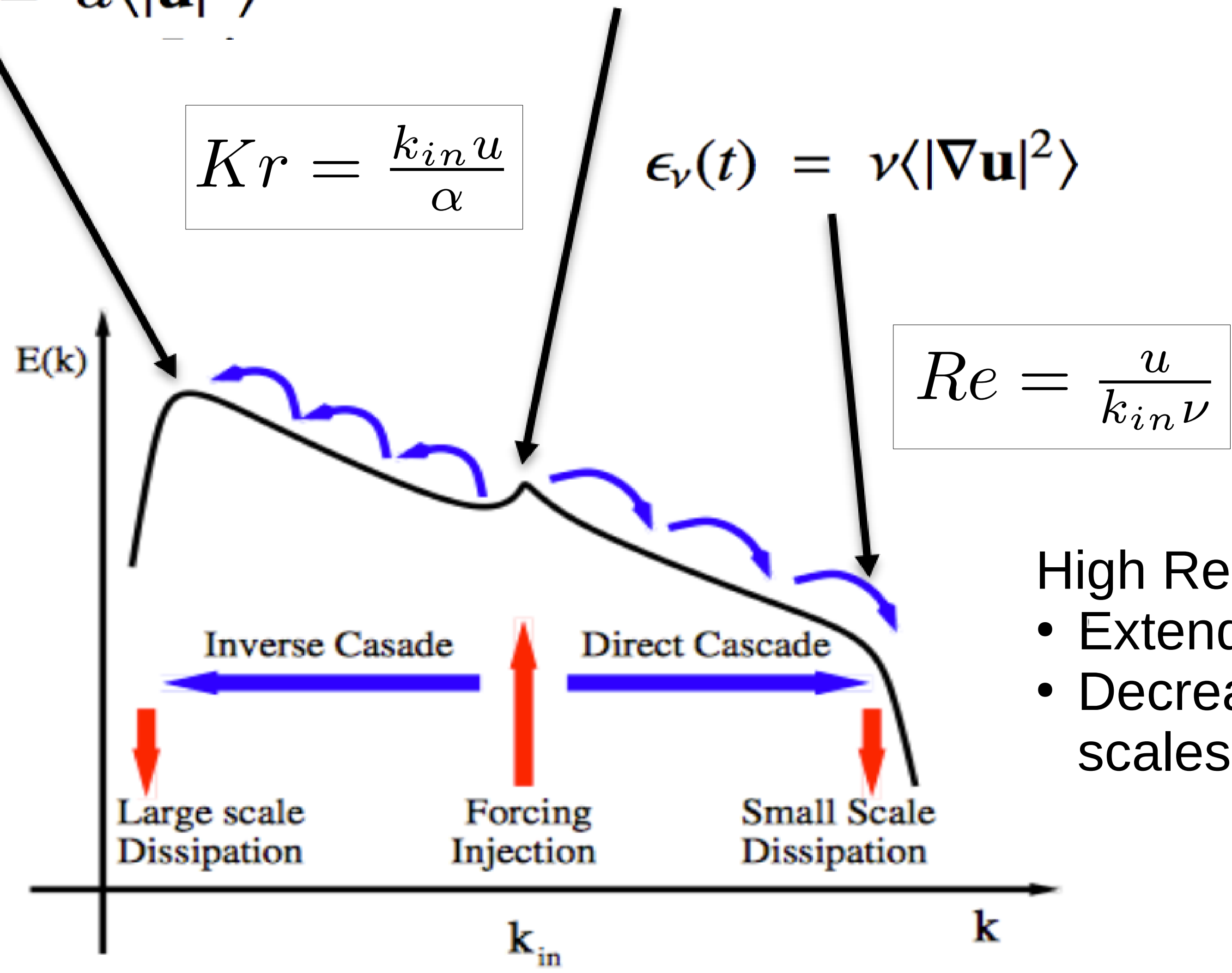
$$\epsilon_\alpha(t) = \alpha\langle|\mathbf{u}|^2\rangle$$

$$\epsilon_{in}(t) = \langle\mathbf{u}\cdot\mathbf{f}\rangle$$

$$Kr = \frac{k_{in}u}{\alpha}$$

$$\epsilon_\nu(t) = \nu\langle|\nabla\mathbf{u}|^2\rangle$$

$$Re = \frac{u}{k_{in}\nu}$$

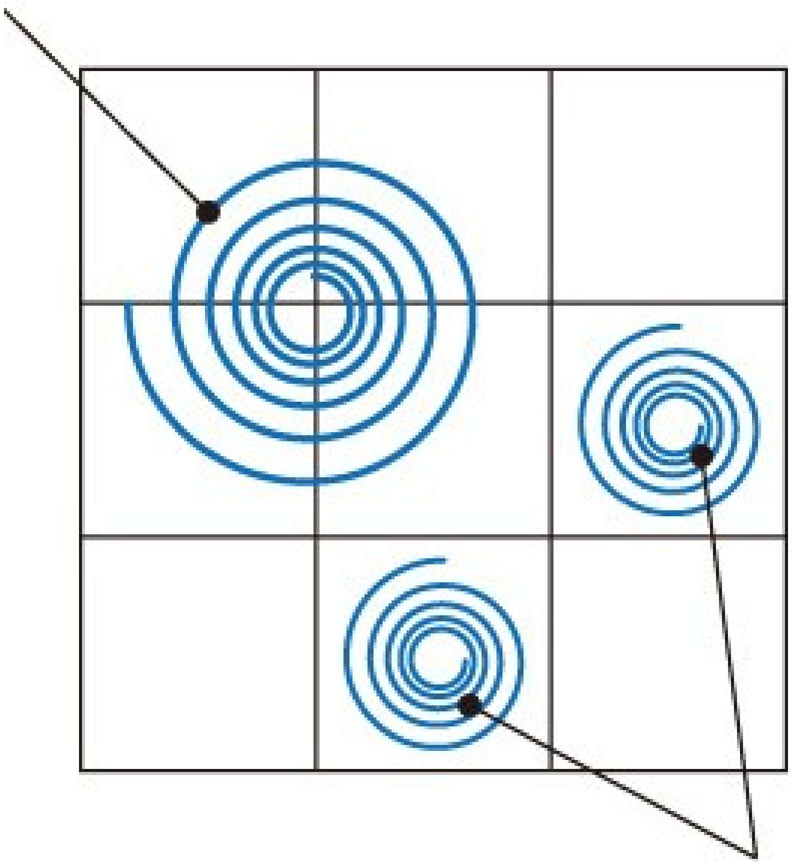


High Re

- Extended direct cascade
- Decreased smallest scales

(Implicit) Large Eddy Simulation (LES)

Grid scale:
Resolved



Sub-Grid Scale (SGS):
Not captured by the grid
Needs to be modeled

- Direct Num. Sim. (DNS)

  All scales of the flow are solved (expensive)
- Large Eddy Sim. (LES)

  All scales up to a cut-off are resolved, a SGS is used to model small scales effect

Good SGS?

- Small scales dissipation
- Allows intermittent transfer of energy to grid scales (backscatter)

No SGS =>small scale instabilities

Entropic Lattice Boltzmann Method (ELBM)

With LBM Instabilities arise as $\tau_0 \to 0.5 \;\; (\nu_0 \to 0, \mathrm{Re} \to \infty)$:

Can we get rid of those unstabilities?

- Non-linear stabilization of LBM has been linked to the existence of a H-functional acting as a Lyapunov functional
- Entropic LBM equips a H-theorem by locally adapting

$$\tau = \tau_{eff}(\vec{x}, t) = K(f_i)\tau_0$$

[Karlin *et. al.*, EPL, 1999]

- ELBM is unconditionally stable and recover N-S with

$$\nu = \nu_{eff}(\vec{x}, t) = c_s^2(\tau_{eff}(\vec{x}, t) - 0.5)\Delta t$$
$$= \nu_0 + c_s^2 \tau_0 (K - 1)\Delta t = \nu_0 + \nu_t(\vec{x}, t)$$

$$\nu_t(\vec{x}, t) = c_s^2 \tau_0 (K - 1)\Delta t \qquad K(\vec{x}, t) = K(\{f_i(\vec{x}, t)\})$$

Non-linear dependency

ELBM: macroscopic formulation of the SGS

- Assuming $\mathrm{K} \approx 1$, one can derive an approximation of $\nu_t(\vec{x}, t)$ using Chapman-Enskog expansion: [Malaspinas & Sagaut, PRE, 2008]

where $$\nu_t^M(\vec{x}, t) = -\frac{4c_s^2}{3}\tau_0^2 \Delta t^2 \frac{S_{\theta\kappa} S_{\kappa\gamma} S_{\gamma\theta}}{S_{\lambda\mu} S_{\lambda\mu}} \propto -\frac{Tr(S^3)}{Tr(S^2)}$$

$$\mathrm{S}_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$$ **NOT DEFINITE-POSITIVE**

Change sign (allows backscatter events)

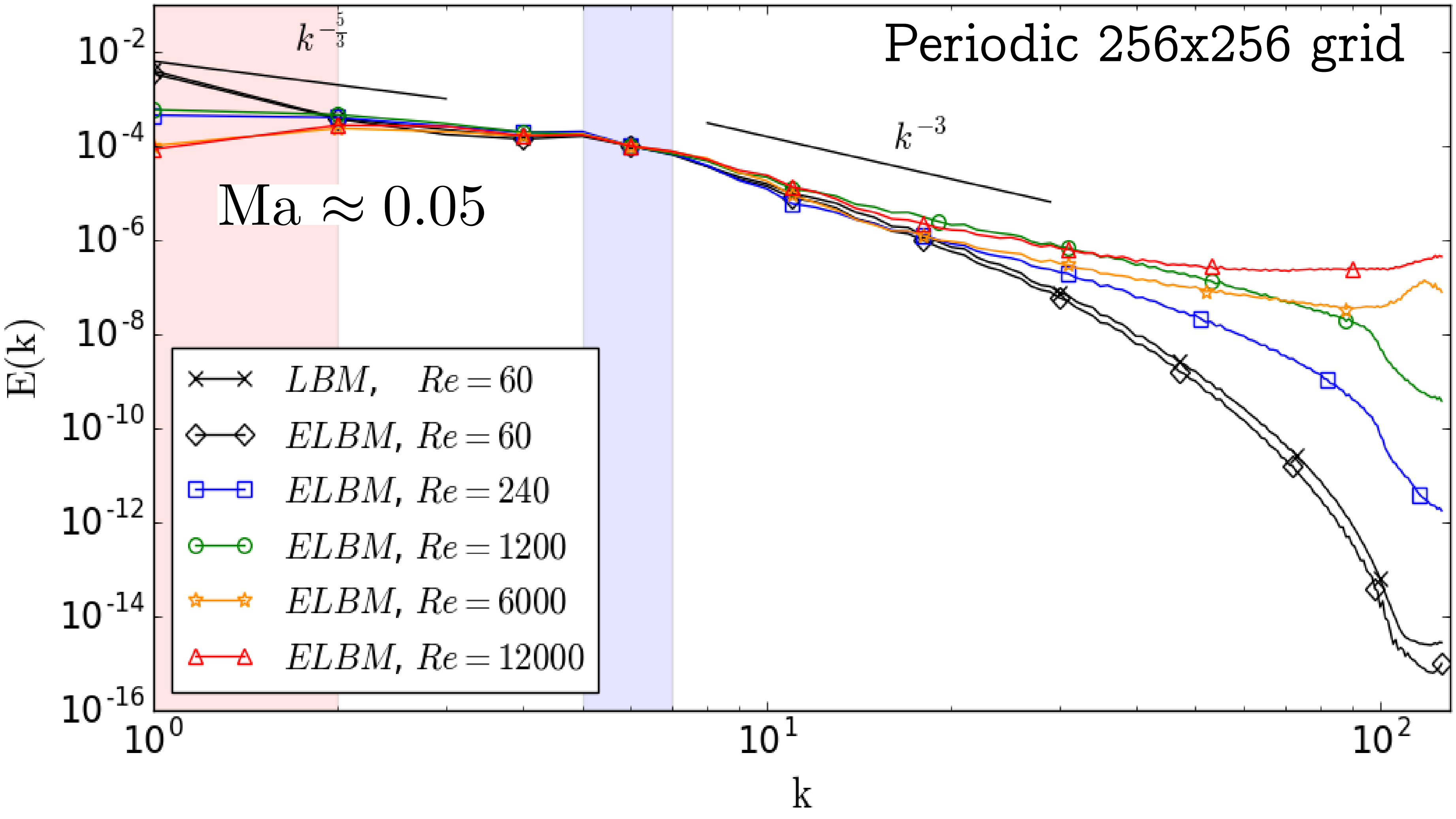
Scale as $|\mathbf{S}|$ like the Smagorinsky SGS [Smagorinsky, 1963]

$$\nu_t^S(\vec{x},\, t) = C^S \sqrt{S_{\theta\kappa} S_{\theta\kappa}}$$ **DEFINITE-POSITIVE**

Objectives of this work:

- Is the Malaspinas approximation valid? (check numerically)
- Artifact of the stabilization or SGS stemming from Kinetic theory?

Superposed energy spectra of the simulations

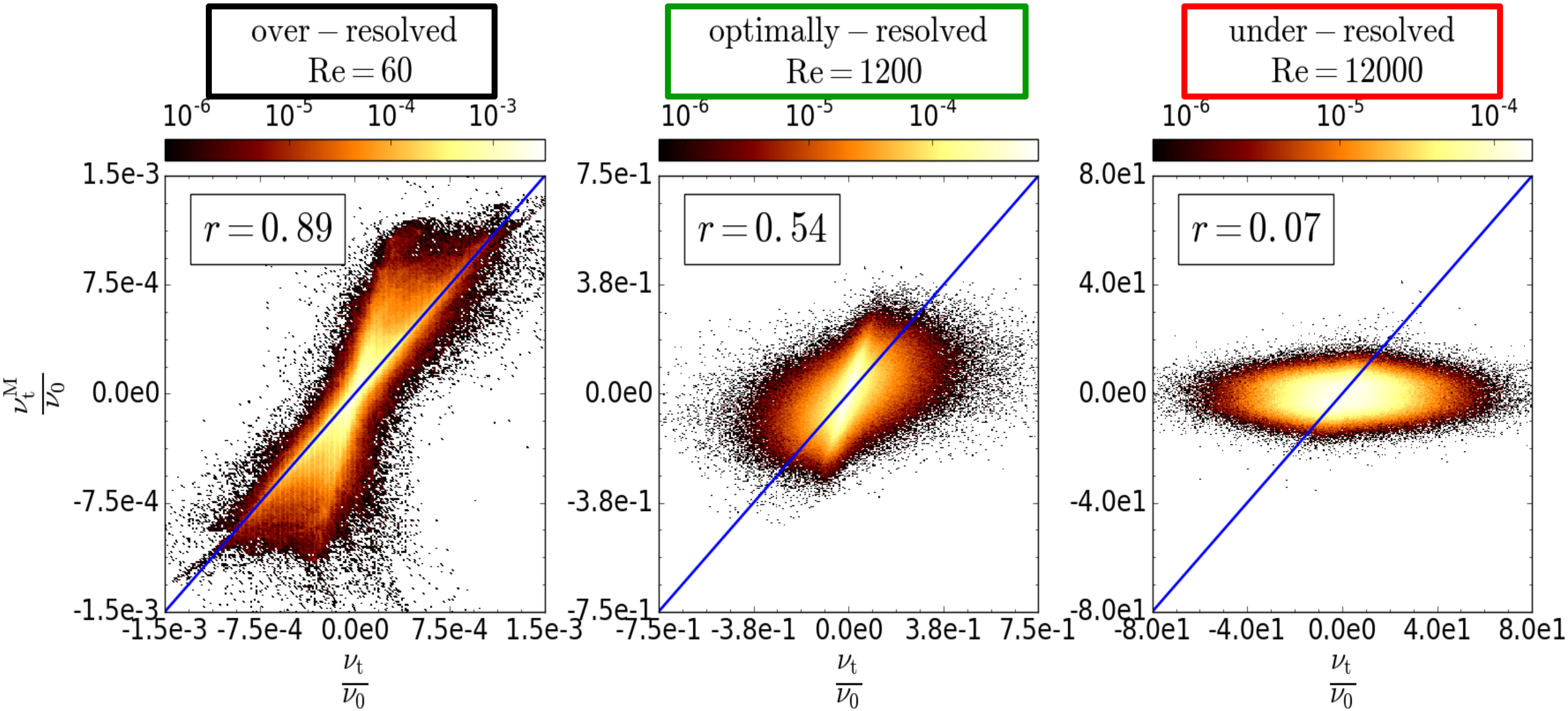


over-resolved
$Re = 60$

optimally-resolved
$Re = 1200$

under-resolved
$Re = 12000$

Numerical check of Malaspinas formulation ν_t^M



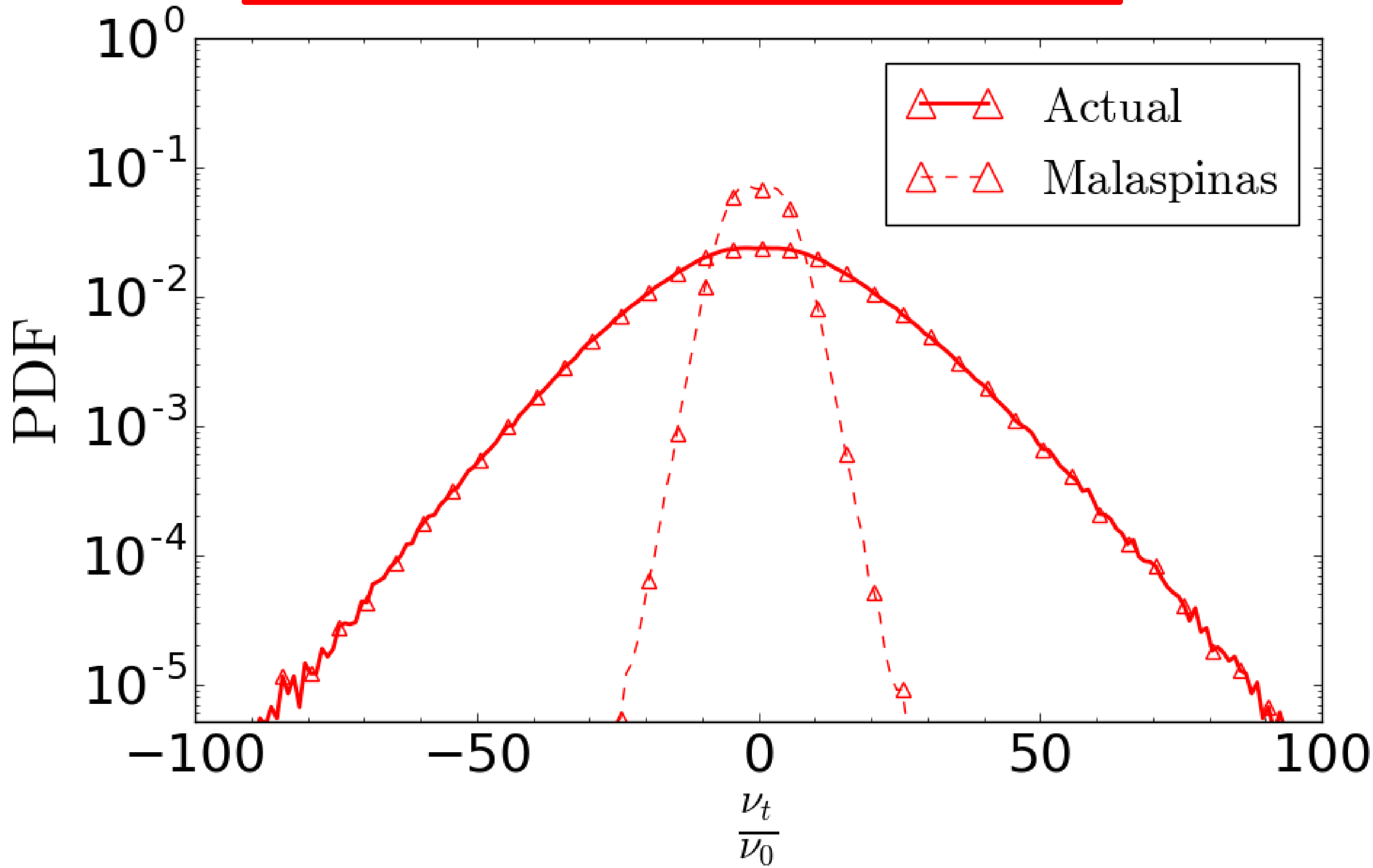
Actual turbulent viscosity

$$\nu_t(\vec{x}, t) = c_s^2 \tau_0 (K - 1) \Delta t$$

Malaspinas approximation

$$\nu_t^M(\vec{x}, t) = -\frac{4c_s^2}{3} \tau_0^2 \Delta t^2 \frac{Tr(S^3)}{Tr(S^2)}$$

Under-resolved case



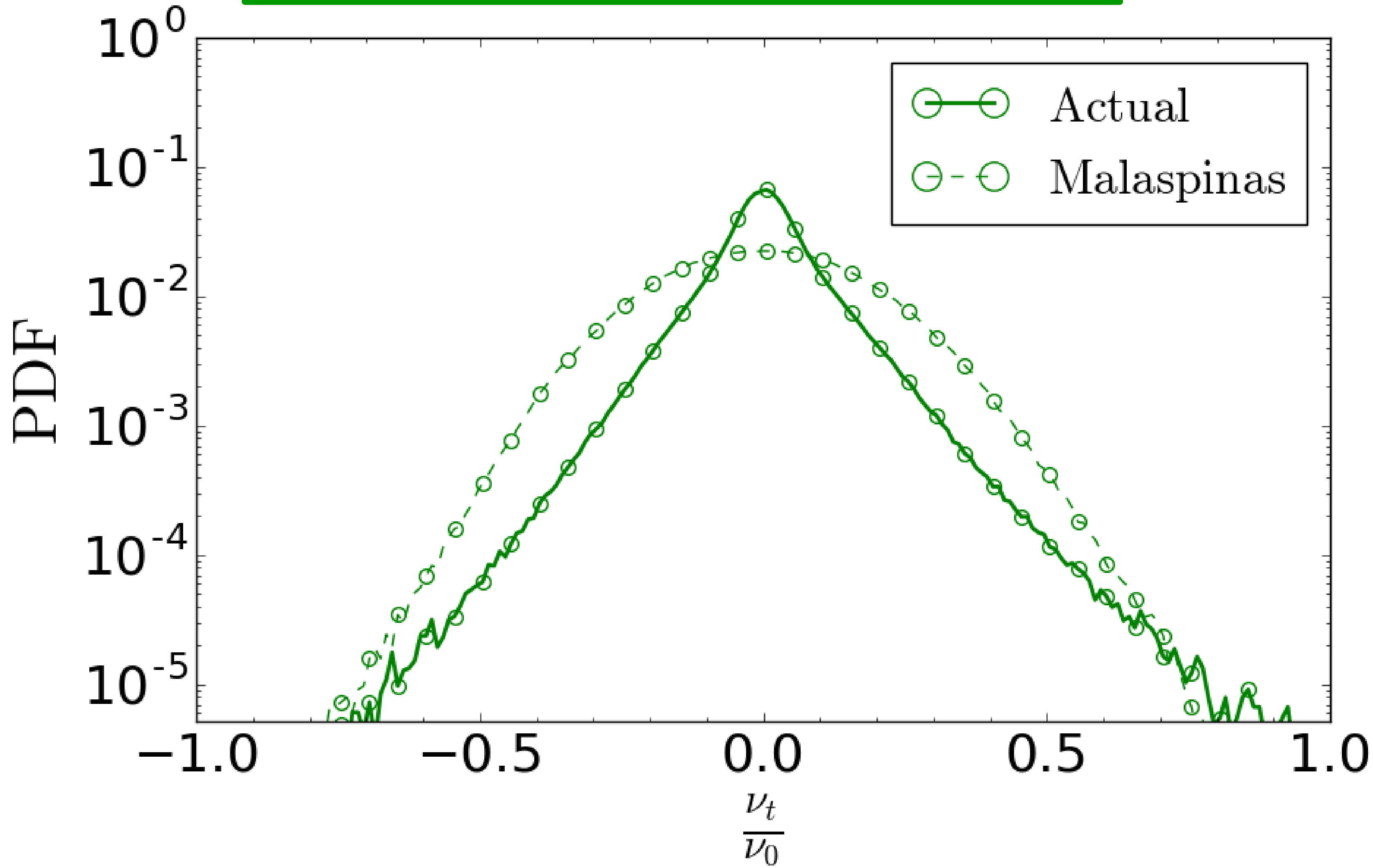
Actual turbulent viscosity

$$\nu_t(\vec{x}, t) = c_s^2 \tau_0 (K - 1)\Delta t$$

Malaspinas approximation

$$\nu_t^M(\vec{x}, t) = -\frac{4c_s^2}{3}\tau_0^2 \Delta t^2 \frac{Tr(S^3)}{Tr(S^2)}$$

Optimally-resolved case



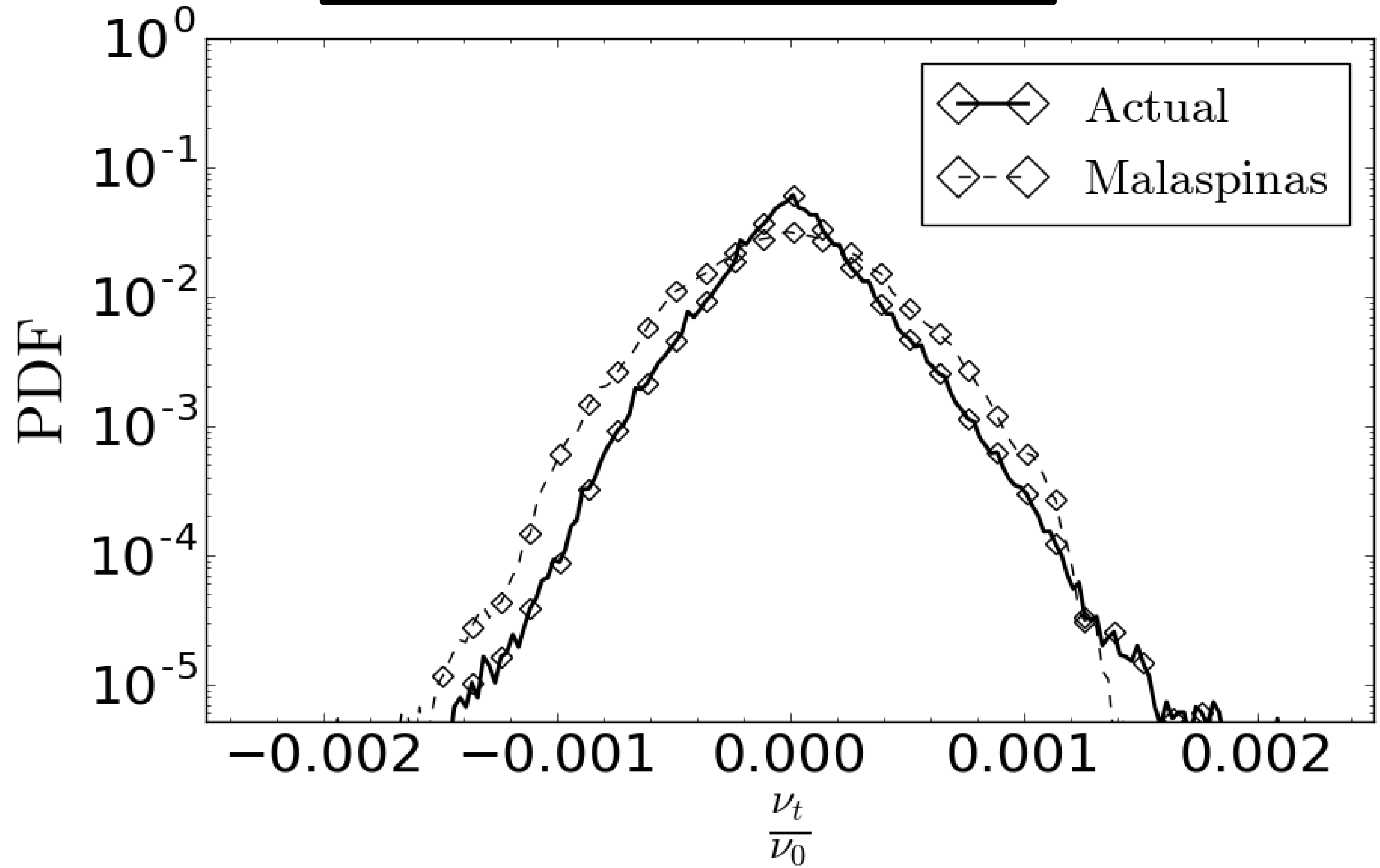
Actual turbulent viscosity

$$\nu_t(\vec{x}, t) = c_s^2 \tau_0 (K - 1)\Delta t$$

Malaspinas approximation

$$\nu_t^M(\vec{x}, t) = -\frac{4c_s^2}{3}\tau_0^2 \Delta t^2 \frac{Tr(S^3)}{Tr(S^2)}$$

Over-resolved case



Actual turbulent viscosity

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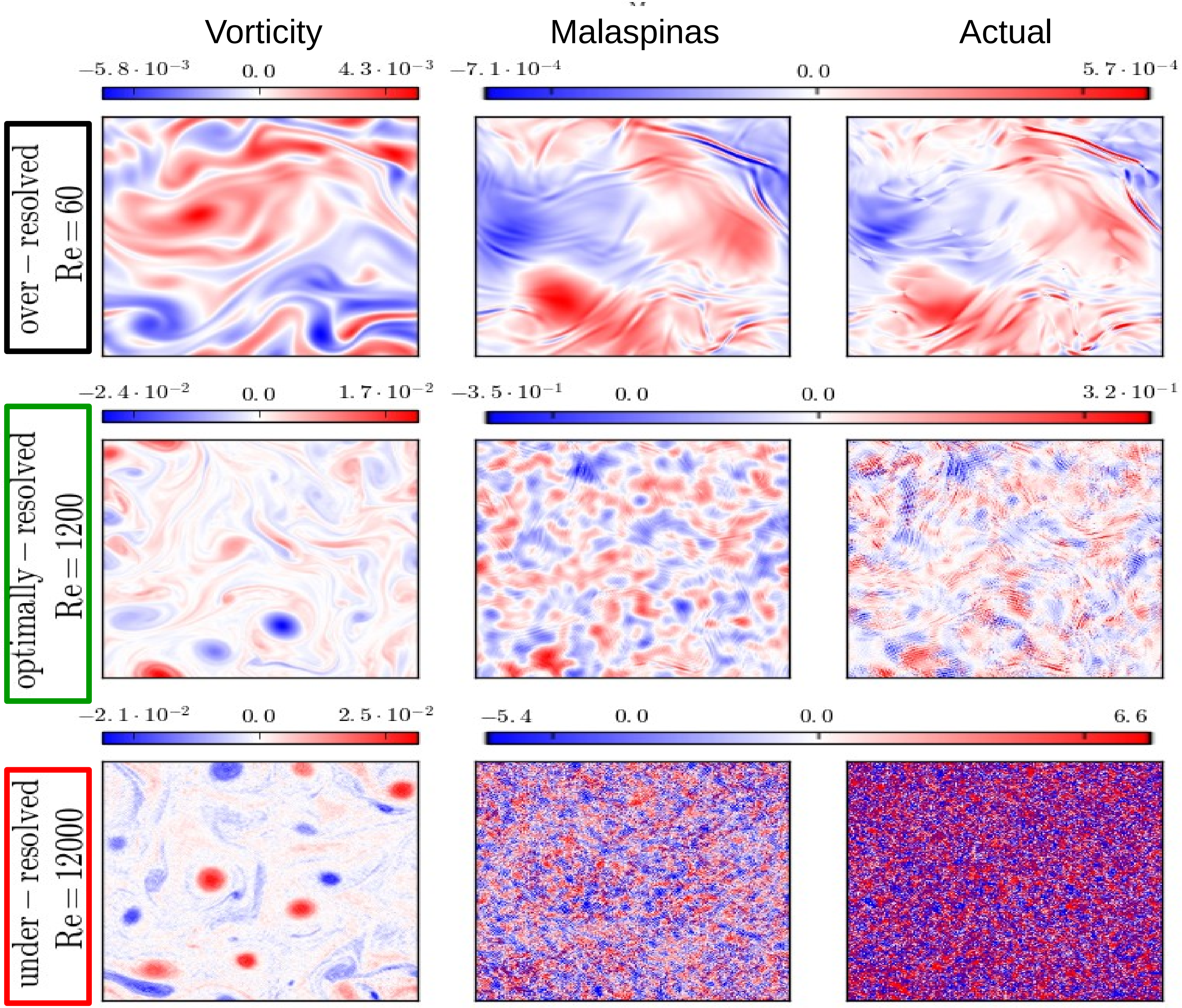

Vorticity
Malaspinas
Actual
−5.8 · 10^−3
0.0
4.3 · 10^−3
−7.1 · 10^−4
0.0
5.7 · 10^−4
over − resolved
Re = 60
−2.4 · 10^−2
0.0
1.7 · 10^−2
−3.5 · 10^−1
0.0
0.0
3.2 · 10^−1
optimally − resolved
Re = 1200
−2.1 · 10^−2
0.0
2.5 · 10^−2
−5.4
0.0
0.0
6.6
under − resolved
Re = 12000

Conclusions

- Conducted 2D Homogeneous Isotropic Turbulence simulations at increasing Reynolds number
- ELBM enables an extension of the inertial range
- The implicit turbulence models gets increasingly active with Re
- Malaspinas model is in fair agreement but fails to capture the skewness of the actual turbulent viscosity

Future work

Is ELBM a mere stabilization or an implicit physical model of the sub-grid scales stemming from kinetic theory?

- Development of a tool to check numerically the balance of kinetic energy and enstrophy across scales
- Systematic statistical analysis of hydrodynamics recovery for Entropic LBM with the implicit SGS included

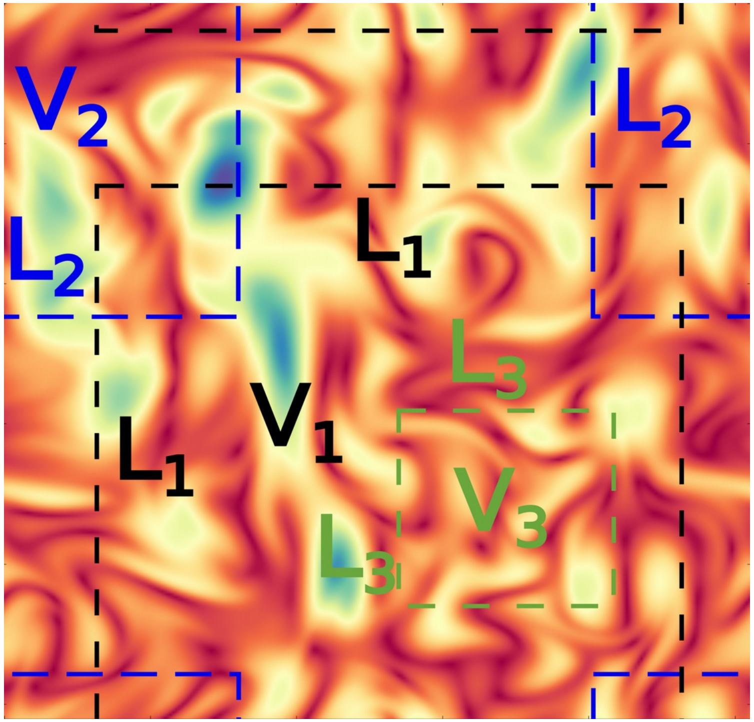
Statistics of hydrodynamic recovery at scale L

Averaged kinetic energy balance equation for $\nu = \nu^{eff}(\vec{x}, t) = \nu_0 + \nu_t(\vec{x}, t)$

$$\partial_t \langle \frac{\rho u_i u_i}{2} \rangle_V$$
$$= -\langle u_i \partial_i p \rangle_V - \nu_0 \langle \rho (\partial_j u_i + \partial_i u_j) \partial_j u_i \rangle_V + \nu_0 \langle \partial_j \rho u_i (\partial_j u_i + \partial_i u_j) \rangle_V$$
$$- \langle \partial_j \frac{\rho u_i u_i}{2} u_j \rangle_V + \langle u_i F_i \rangle_V - \langle \nu_t \rho (\partial_j u_i + \partial_i u_j) \partial_j u_i \rangle_V + \langle \partial_j \nu_t \rho u_i (\partial_j u_i + \partial_i u_j) \rangle_V$$

For a scale L, we calculate each term of the balance eq. For random sub-volumes of size

$V = L \times L$ and evaluate the hydrodynamic recovery



Acknowledgement

Thank you for your attention
Any question?

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