67th Annual Meeting of the **APS Division of Fluid Dynamics**, **November 23th-25th 2014, San Francisco**

Burgers Turbulence on a Fractal Fourier set

M. Buzzicotti[1], L. Biferale[1], U. Frisch[2], S. S. Ray[3]

[1]**University of Rome Tor Vergata, Italy**

[2]**CNRS Observatory of Nice, France**

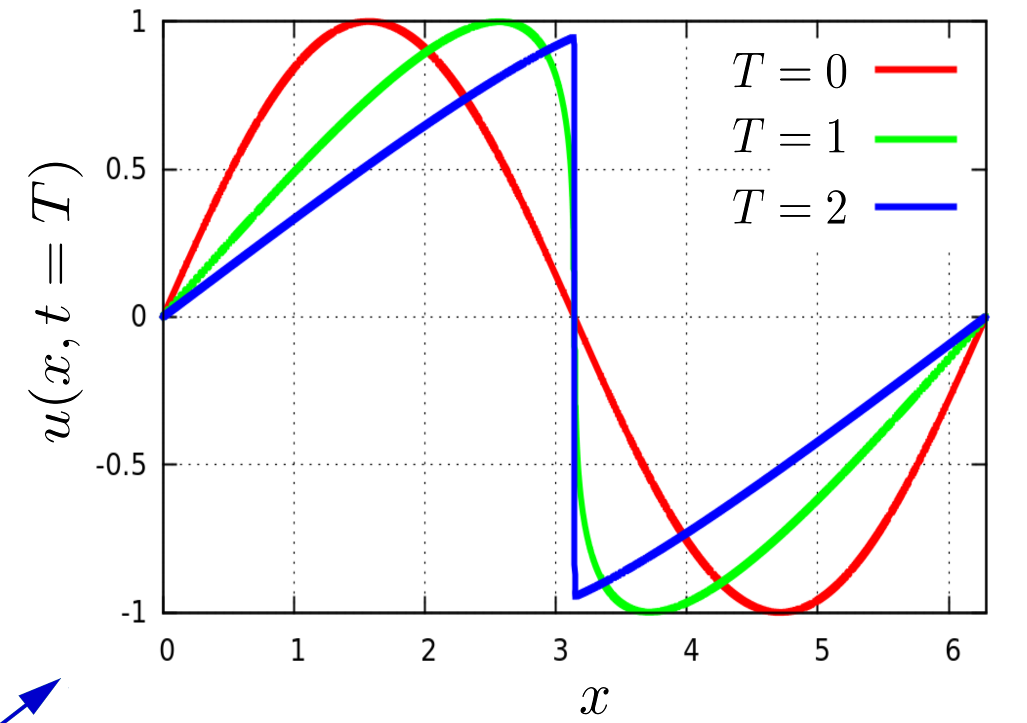
[3]**Tata Institute of Fundamental Research of Bangalore, India**

We present a systematic investigation of the effects introduced by a fractal decimation in Fourier space on stochastically forced Burgers equations in 1d. The aim is to understand the statistical robustness of the shock singularity under different reduction of the number of the degrees of freedom. We performed a series of direct numerical simulations using a pseudo-spectral code with resolution up to 16384 points and at varying the dimension of the fractal set of Fourier modes, D_F <1. We present results concerning the scaling properties on real space and the probability distribution functions of local and non-local triads in Fourier space.

ERC Advanced Grant (N. 339032) “NewTURB”

Burgers' equation:

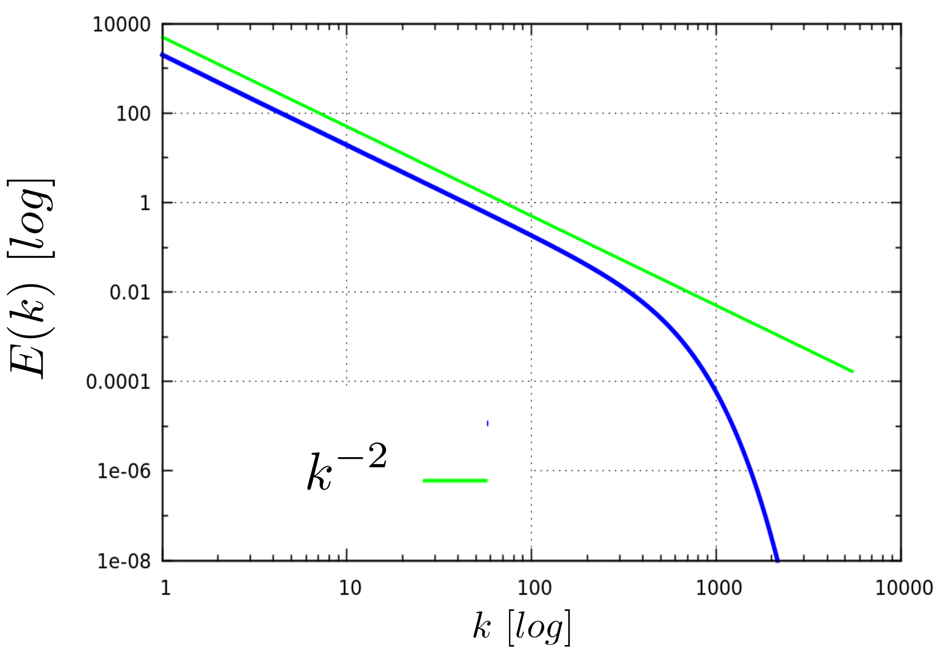
$$\boxed{\frac{\partial u(x,t)}{\partial t} + u\frac{\partial u(x,t)}{\partial x} = \nu\frac{\partial^2 u(x,t)}{\partial x^2}}$$

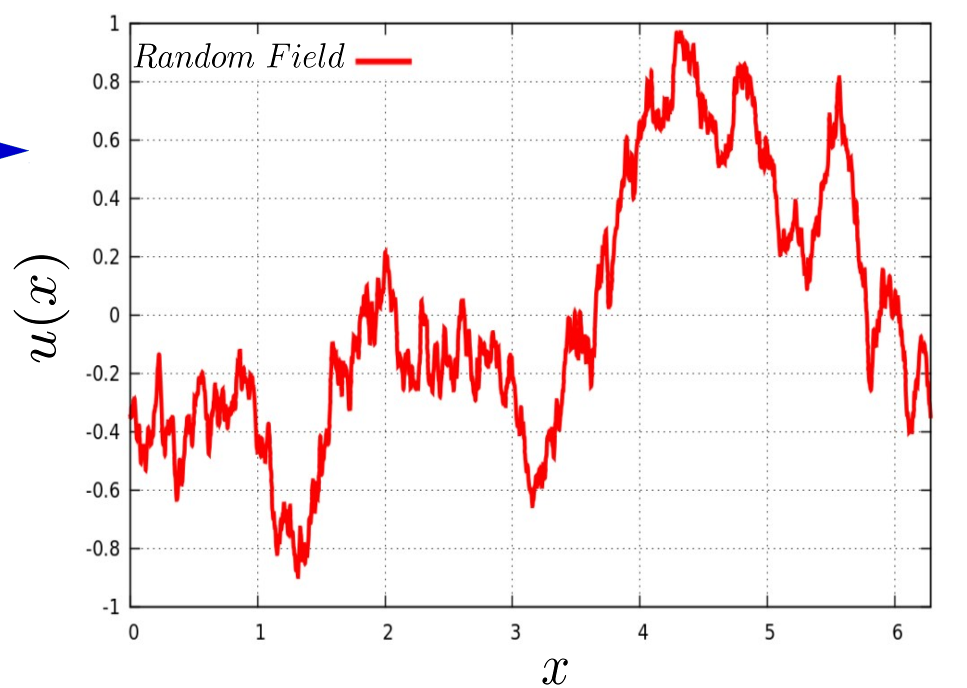


Burgers' evolution **produces a singularity**, (**shock**)

$$\partial_t u(k,t) = -ik\int u(p,t)u(p-k,t)dp - \nu k^2 u(k,t)$$

$$E(k) = u(k)u^*(k)$$

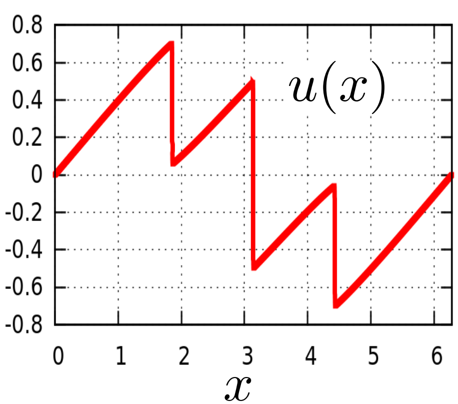


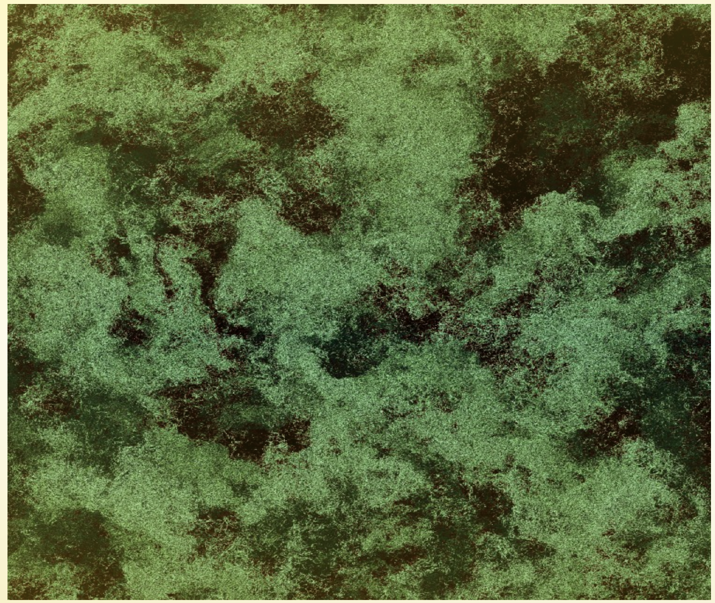


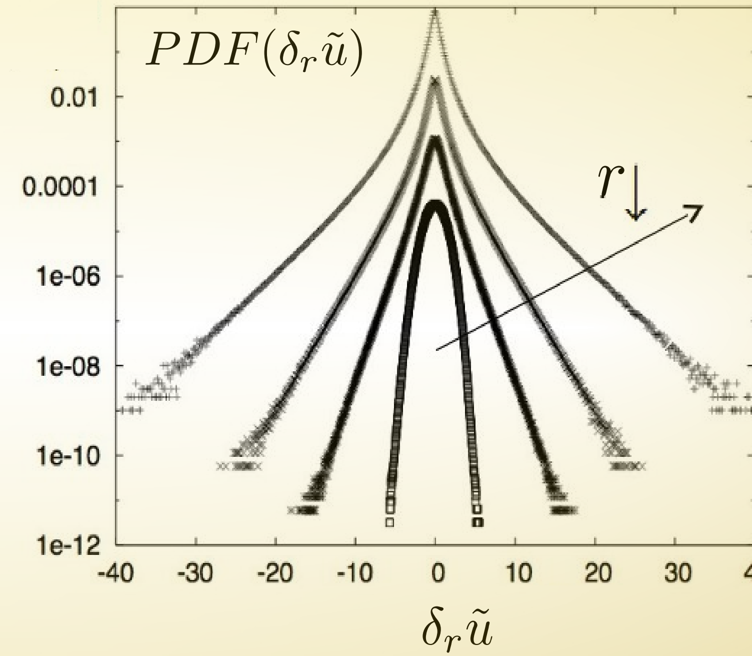
..Burgers equation + Random Forcing:

$$\delta_r u = u(x+r) - u(x)$$

$$\delta_r \tilde{u} = \frac{\delta_r u}{< (\delta_r u)^2 >^{1/2}}$$

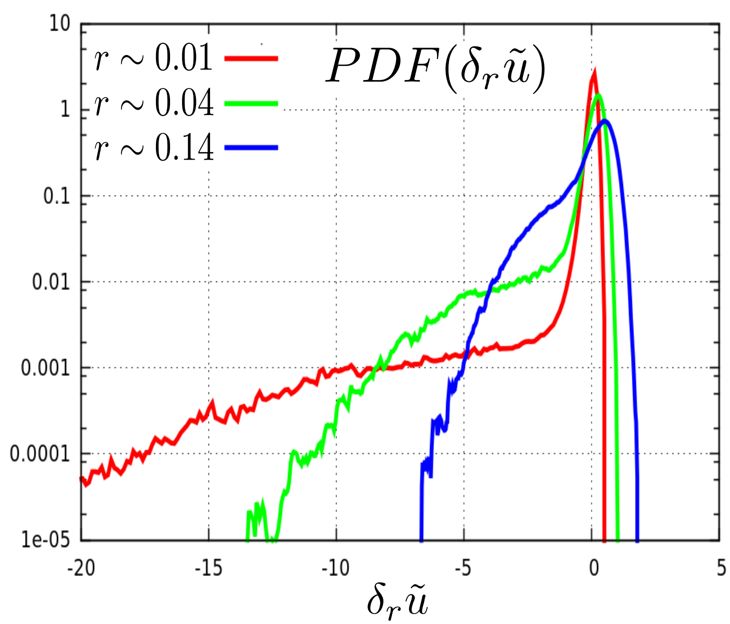


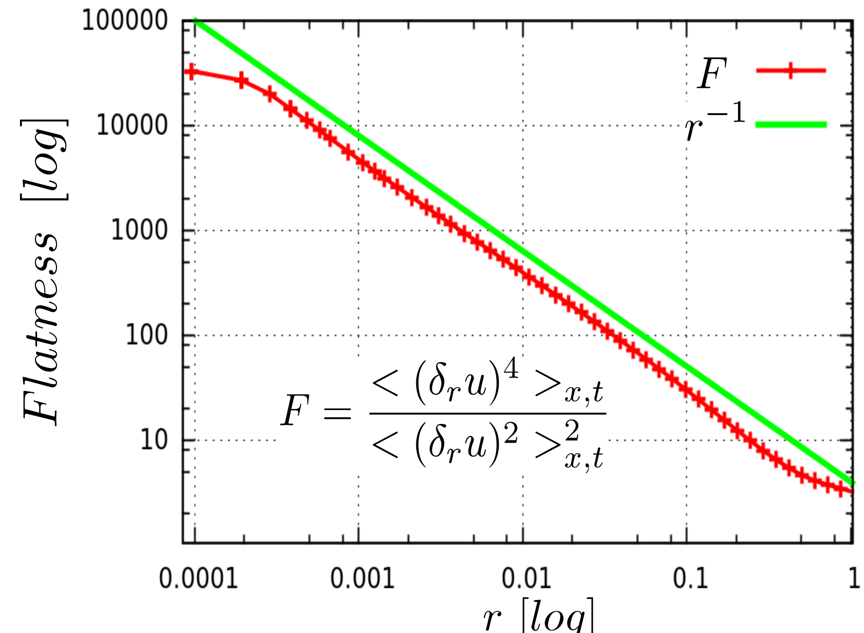




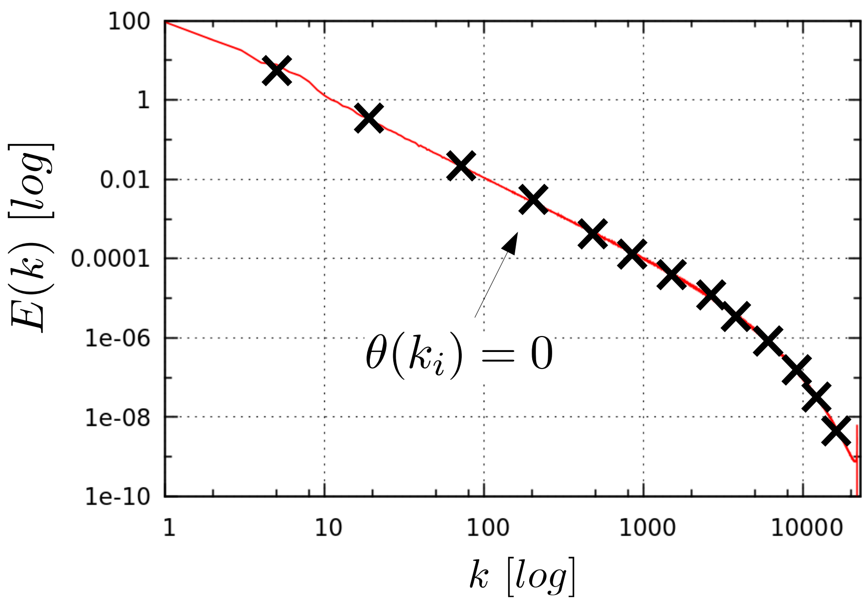
- Kaneda et al., ARFM (2011).

As vortices in real 3D turbulence, **shock produces a non-trivial statistics in the Burgers' velocity field.**





Fractal Fourier decimation:



Motivation:

- Understanding of how many degrees of freedom are related to the coherent structure and shock formation.
- Quantification of the statistical properties at different fractal dimensions.
- Comparison with data from the real Navier-Stokes system.

Numerical approach:

$$u_{D_f}(x,t) = \sum_{k \in Z} e^{ikx}\theta(k)u(k,t)$$

$$\theta(k) = \begin{cases} 1 & with \quad probability \quad h_k \sim (k/k_0)^{D_f - 1} \\ 0 & with \quad probability \quad 1 - h_k \end{cases}$$

$$0 < D_f \leq 1$$

Decimation Main Properties:

1) The masks are generated randomly

2) Quenched on time

3) Galerkin Truncation

4) The number of active modes depends on the fractal dimension:

$$N(k) \sim k^{D_f}$$

Decimated Burgers equation:

$$\partial_t u_{D_f}(x,t) + P_{D_f}\left[u_{D_f}\partial_x u_{D_f}(x,t)\right] =$$
$$= \nu \partial^2_{xx} u_{D_f}(x,t) + F_{D_f}$$

- Frisch, Pomyalov, Procaccia, and Ray. *Turbulence in non-integer dimensions by fractal Fourier decimation.* Phys. Rev. Lett. 108, (2012)
- S. S. Ray. *Thermalised solutions, statistical mechanics and turbulence: An overview of some recent results.* Persp. in Nonlin. Dyn., in press (2014)

Real space and temporal evolution:

$D_f = 1$

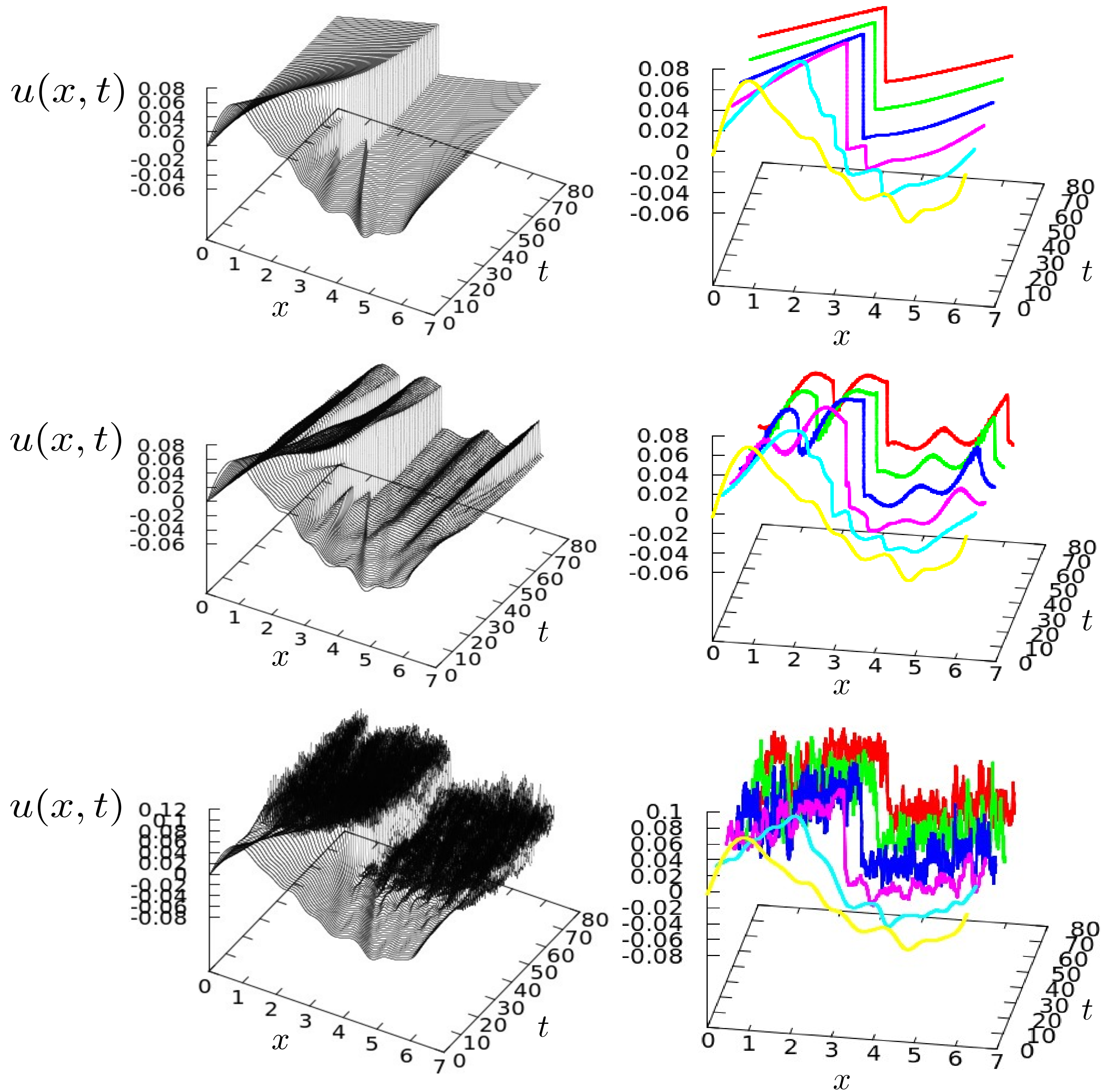
Shock evolution in energy decaying

$D_f = 0.99$

Formation of different shocks

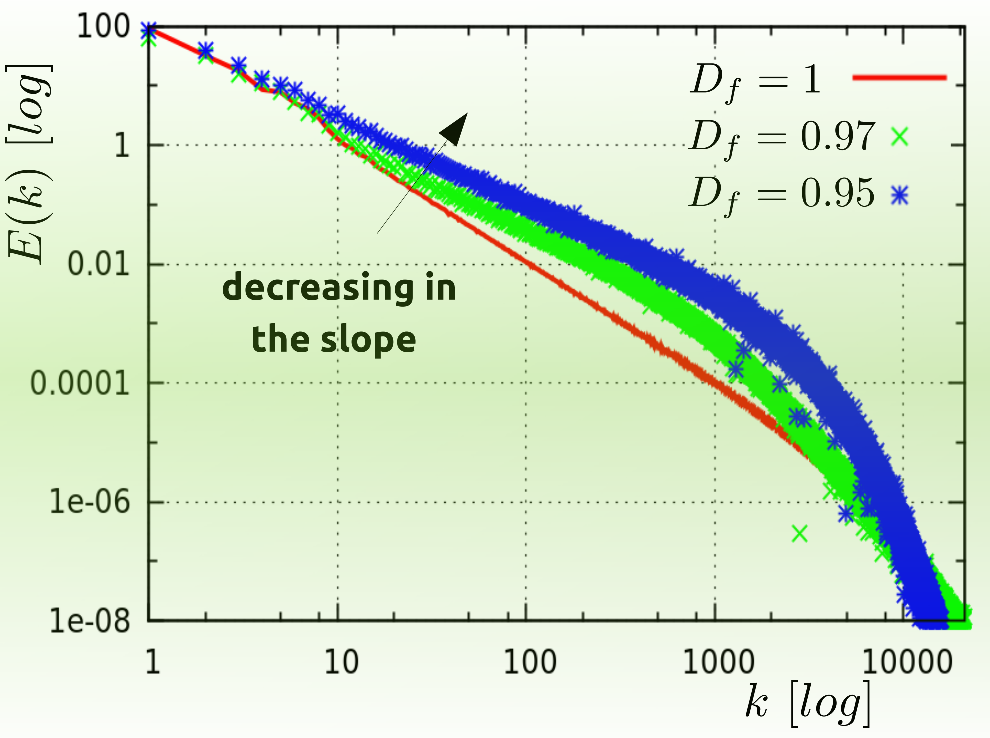
$D_f = 0.85$

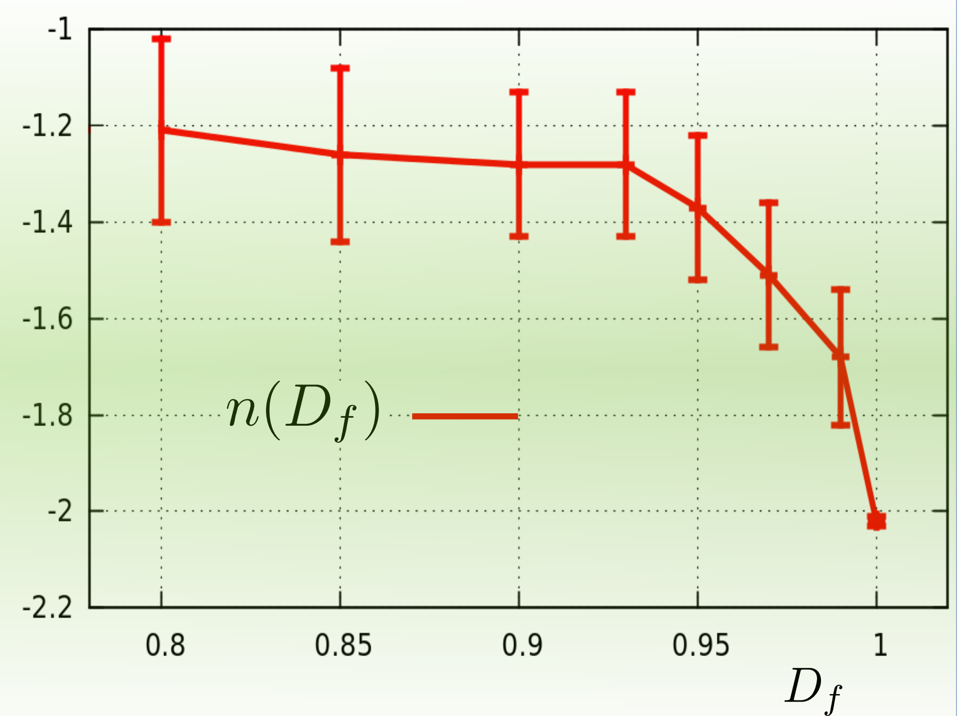
Random small scales structures



Energy Spectra:

$$E_{D_f}(k) = \theta(k)|u_{D_f}(k)|^2 \sim k^{n(D_f)}$$

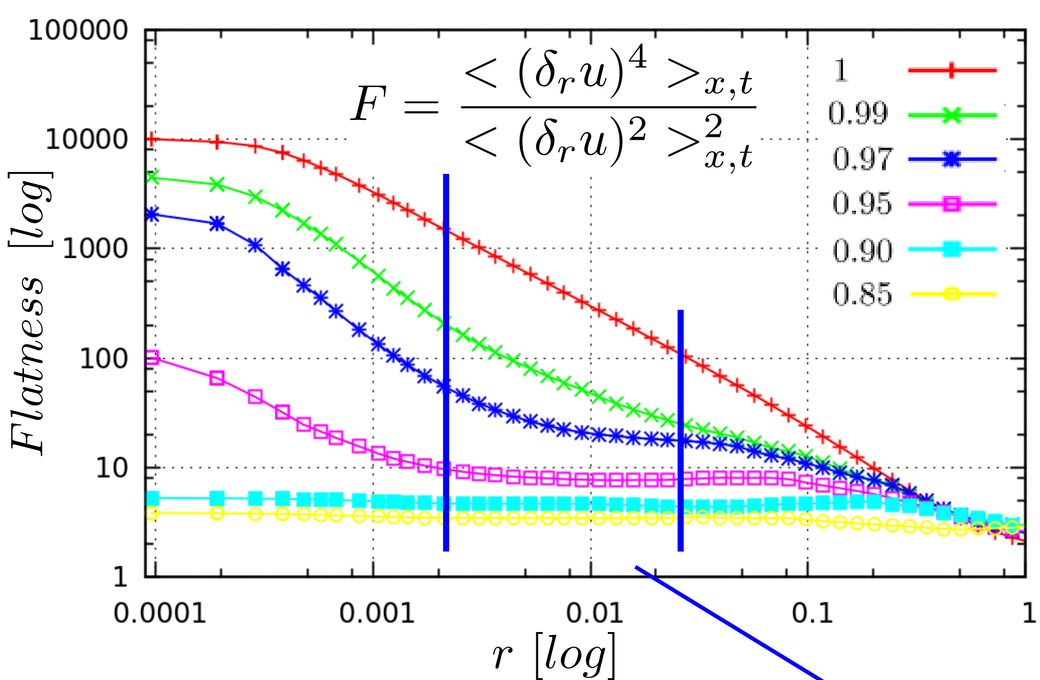


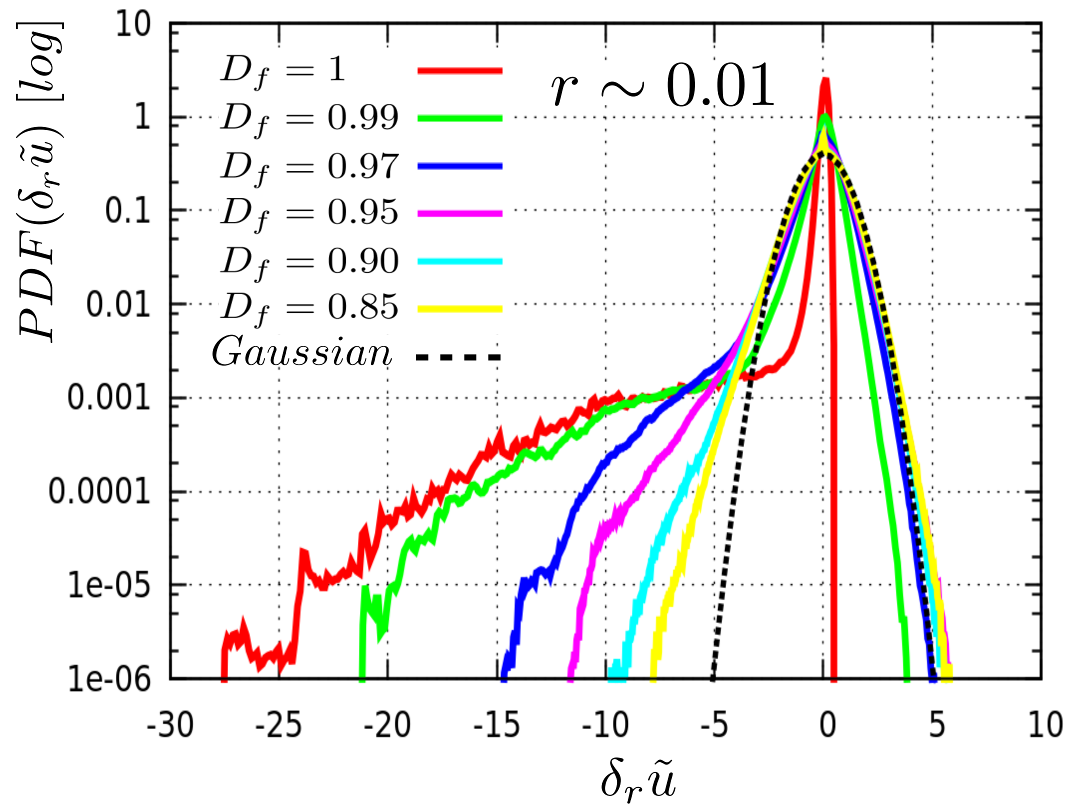


Results show a **dependence only on the dimension** and not on the random masks.

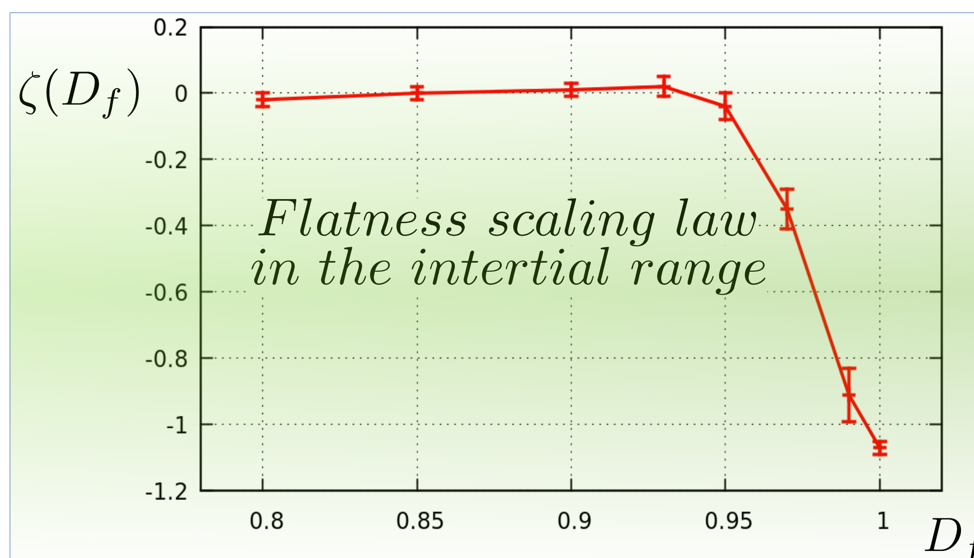
The energy spectrum slope decreases with the dimension up to $D_f \sim 0.95$

Statistical properties at different Fractal dimensions:





$F \sim r^{\zeta(D_f)}$



$$\delta_r u = u(x+r) - u(x)$$

$$\delta_r \tilde{u} = \frac{\delta_r u}{< (\delta_r u)^2 >^{1/2}}$$

Around dimension 0.95 intermittency is strongly reduced.

Conclusions

1) We performed a random decimation of **Burgers' velocity field in Fourier space**.

2) We are interested in the understanding of the **shock robustness at changing the dimension.**

3) We find that for dimension less than 0.95 the **intermittency is strongly reduced**.

4) We find a non-trivial **changing in the slope of the energy spectra** in function of the dimension, up to $D_f = 0.95$.

5) We see that **results** are not **dependent** neither on Reynolds nor on the random masks, but seem to depend **only on the system fractal dimension**.

6) Connection with phase correlation in Fourier space?