Spatiotemporal dynamics of turbulent flows under the presence of waves

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Wave Turbulence and Extreme Events
Udine, Italia
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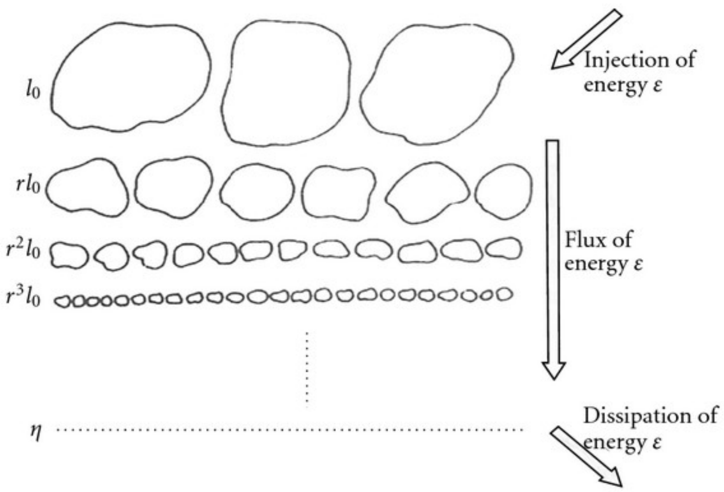
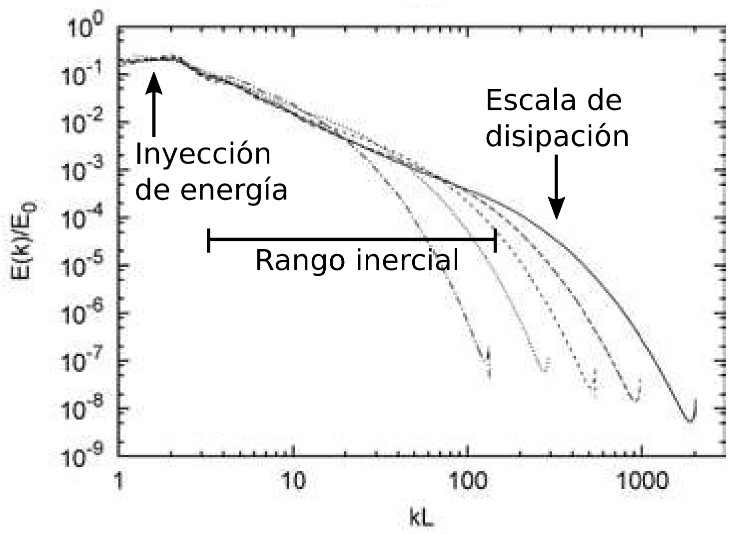
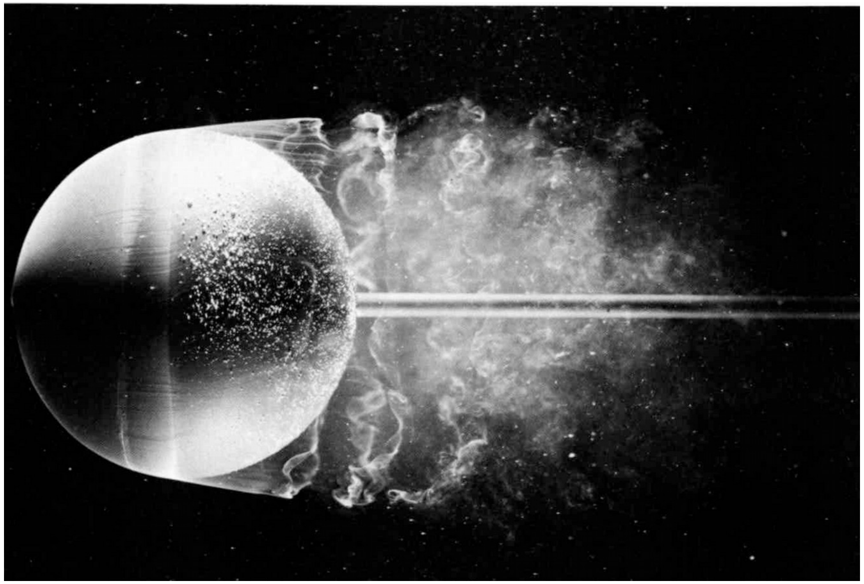
The classical picture of turbulence

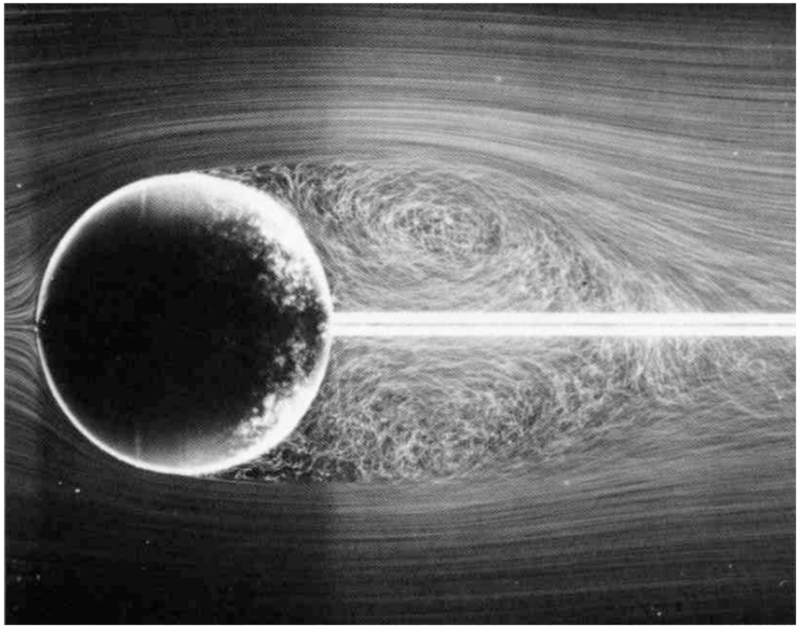




The analysis of turbulent flows often focuses only on spatial properties (in simulations) or only on temporal properties (in experiments).

The spatiotemporal nature of turbulence



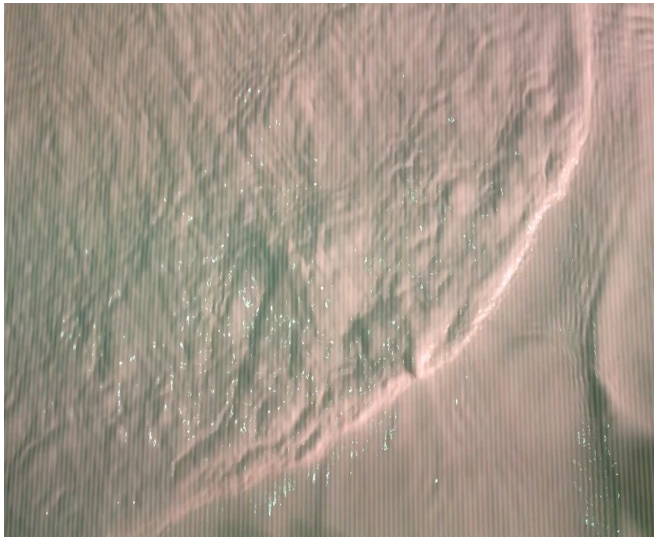


- Turbulence is a spatiotemporal phenomenon.
- Its study should take this into account.
- Of crucial importance when studying flows where there are waves present.

Turbulent flows where waves are present

Internal waves in the atmosphere

Surface gravity waves



Kelvin waves in superfluids
[Video from Lathrop's lab]

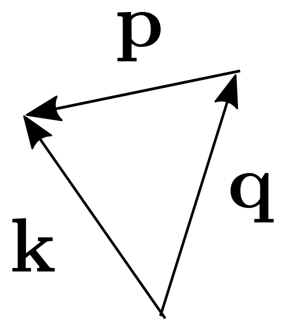
How can we differentiate the impact of waves from that of other structures?

Turbulence fundamentals
Nonlinear triads

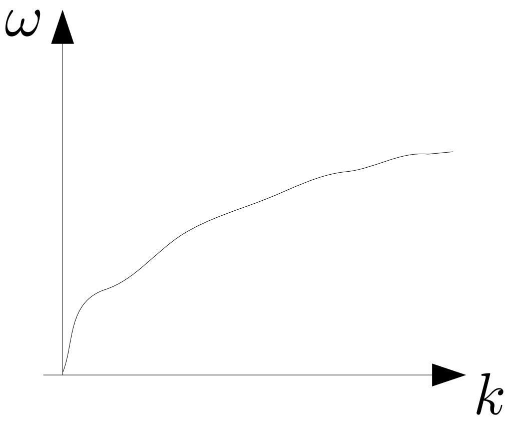
$$\frac{\partial \mathbf{u}}{\partial t} + \nu \nabla^2 \mathbf{u} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla p + \mathbf{f}$$

Fourier transform

$$\left(\frac{\partial}{\partial t} + \nu k^2\right)\mathbf{u}_{\mathbf{k}} = -i\mathbb{P}_{\mathbf{k}} \underbrace{\int (\mathbf{u}_{\mathbf{k}-\mathbf{q}}\cdot\mathbf{q})\mathbf{u}_{\mathbf{q}} d\mathbf{q}}_{\text{triad } \mathbf{k}=\mathbf{p}+\mathbf{q}} + \mathbf{f}_{\mathbf{k}}$$



Turbulence fundamentals
Flows with waves and resonant triads



Given the dispersion relation $\omega(\mathbf{k})$ the spatiotemporal structure of waves is defined

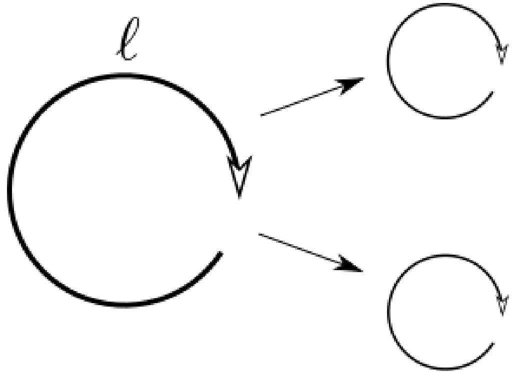
We can propose a multiscale expansion of the form $\mathbf{u_k}(t) = \mathbf{U_k}(t)e^{i\omega(\mathbf{k})t}$

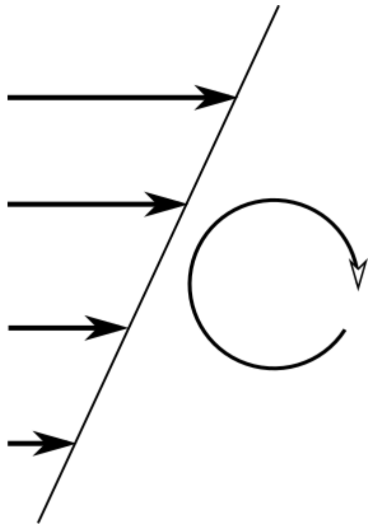
$$\left(\frac{\partial}{\partial t} + \nu k^2\right)\mathbf{U_k} = -i\mathbb{P}_\mathbf{k}\int(\mathbf{U_p}\cdot\mathbf{q})\mathbf{U_q}e^{i[\omega(\mathbf{p})+\omega(\mathbf{q})-\omega(\mathbf{k})]t}d\mathbf{q} + \mathbf{f_k}$$

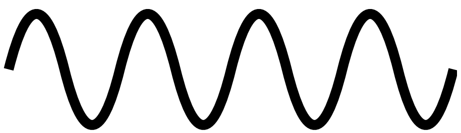
This way only resonant triads survive

$$\mathbf{k} = \mathbf{p} + \mathbf{q}$$
$$\omega(\mathbf{k}) = \omega(\mathbf{p}) + \omega(\mathbf{q})$$

Turbulence fundamentals
Time scales







Nonlinear time

$$\tau_{NL} \sim \frac{\ell}{u}$$

Sweeping time

$$\tau_{sw} \sim \frac{\ell}{U}$$

Wave period

$$\tau_{\omega} \sim \frac{1}{\omega(\mathbf{k})}$$

We will analyse the relation between the different spatial and temporal scales via spatiotemporal spectra

$$E(\mathbf{k},\omega) = \frac{1}{2}\left|\int \mathbf{u}_{\mathbf{k}}(t)e^{-i\omega t}t\right|^2$$

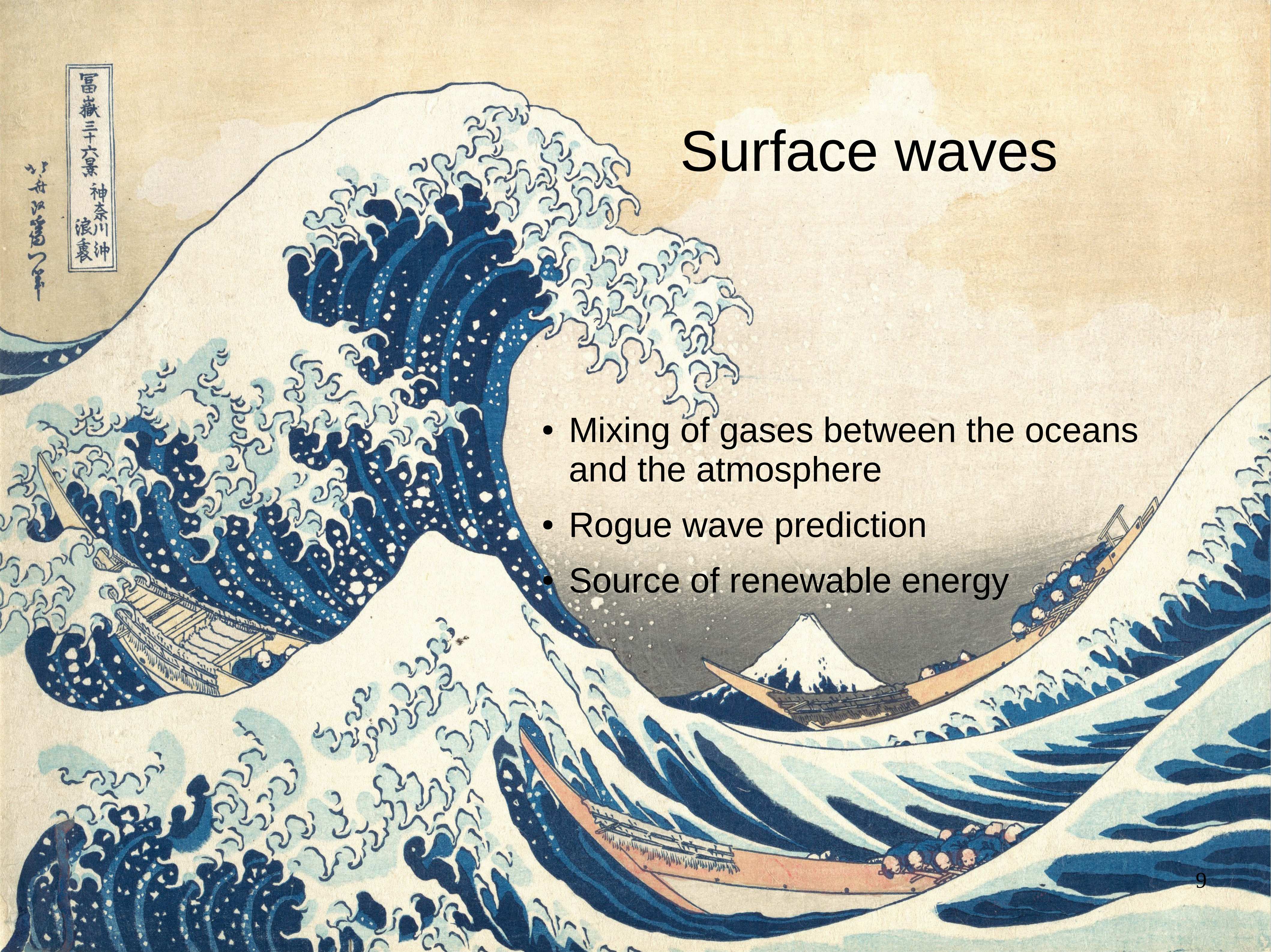
Problems tackled

- Surface waves
  - Experiments on gravito-capillary waves
  - Simulations of the Boussinesq model for surface waves
- Atmospheric flows
  - Rotating turbulence and stratified turbulence
  - Simulations of the Boussinesq model
- Quantum turbulence
  - Simulations of the Gross-Pitaevskii equation

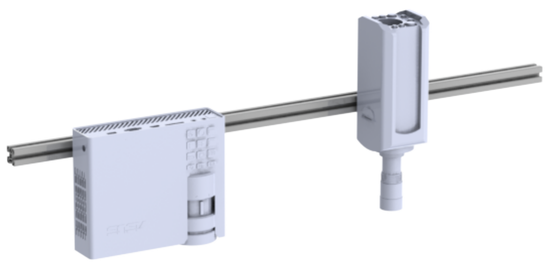
- All simulations were performed using a pseudospectral method which does not introduce neither dispersion nor dissipation.

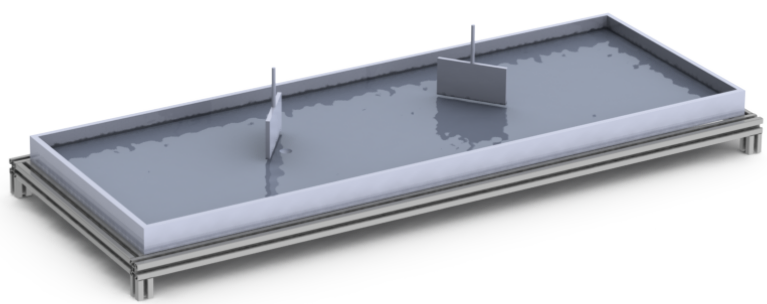
Surface waves

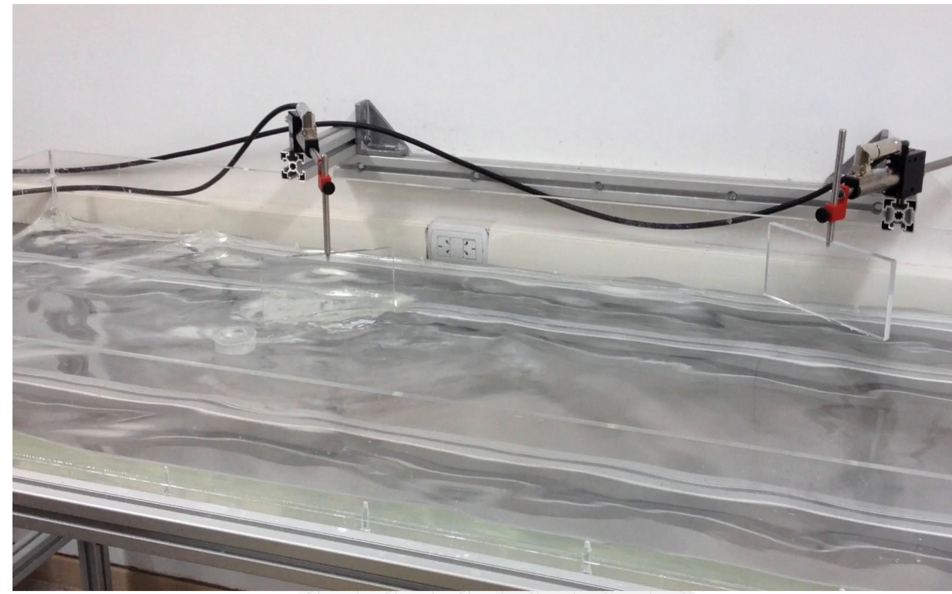
- Mixing of gases between the oceans and the atmosphere
- Rogue wave prediction
- Source of renewable energy

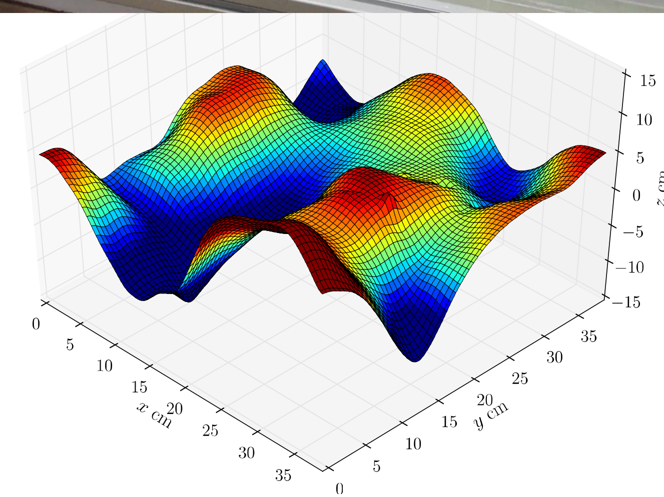


Surface waves
Experimental set-up





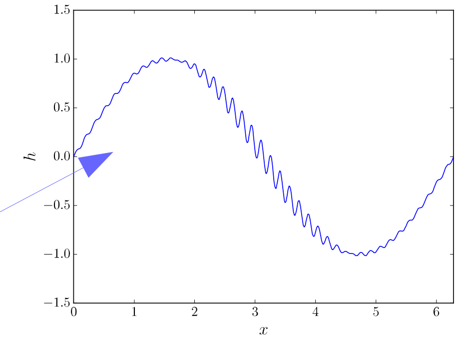




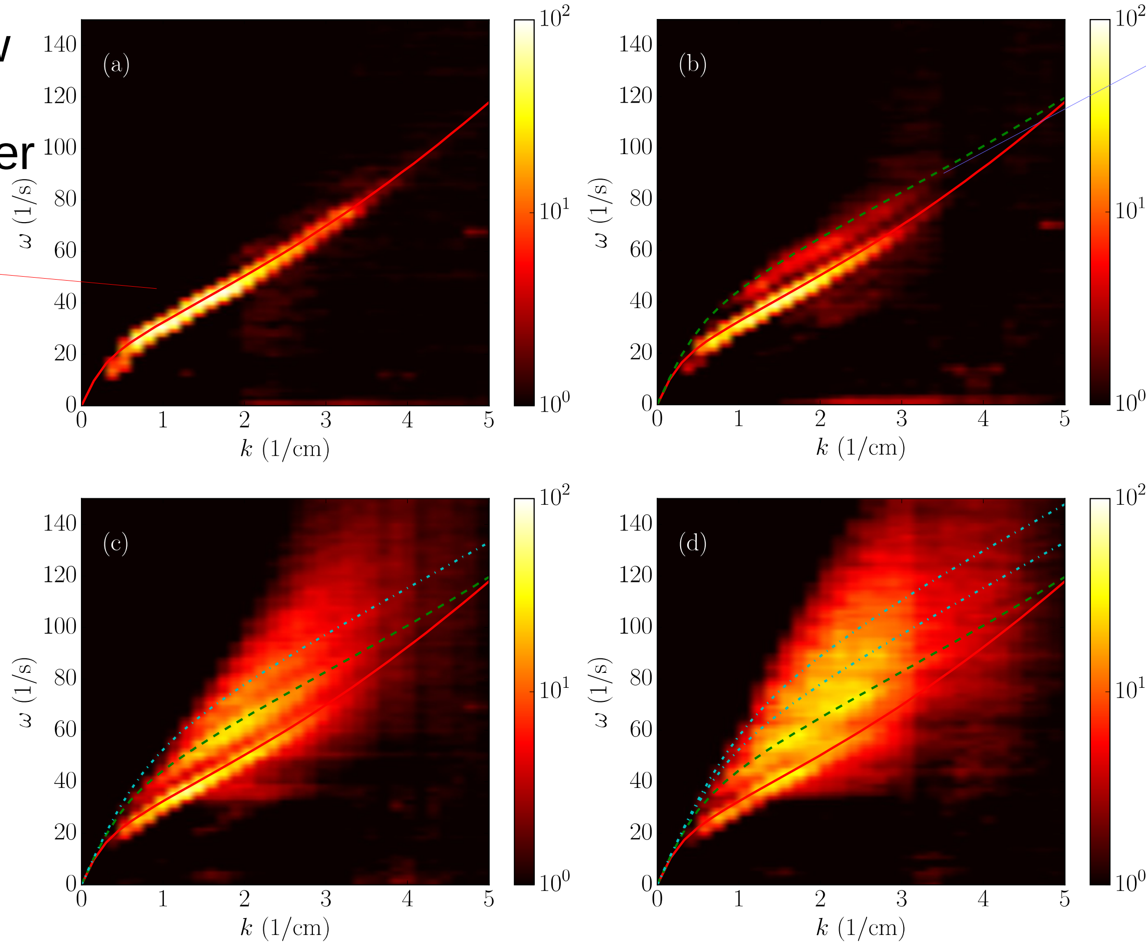
Pattern projection profilometry [Maurel et al. (2009); Cobelli et al. (2009)]

Surface waves

Experiments of gravito-capillary waves



We can see how energy is accumulated over the system's dispersion relation!



When the energy injected is increased, bound modes appear!

Increasing the amount of energy injected creates slave modes
[Clark di Leoni, Cobelli & Mininni, EPJE (2015)]
First reported by [Mordant et al, PRL (2010)]

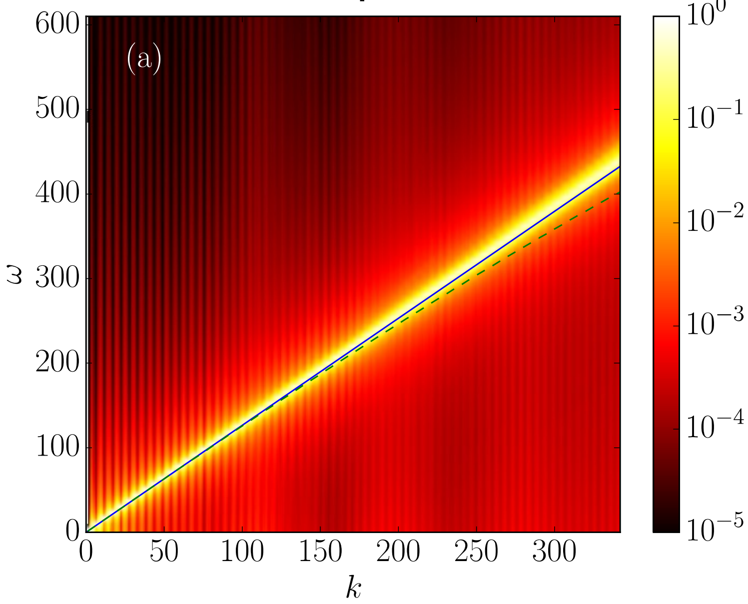
Surface waves

Simulations of the Boussinesq model

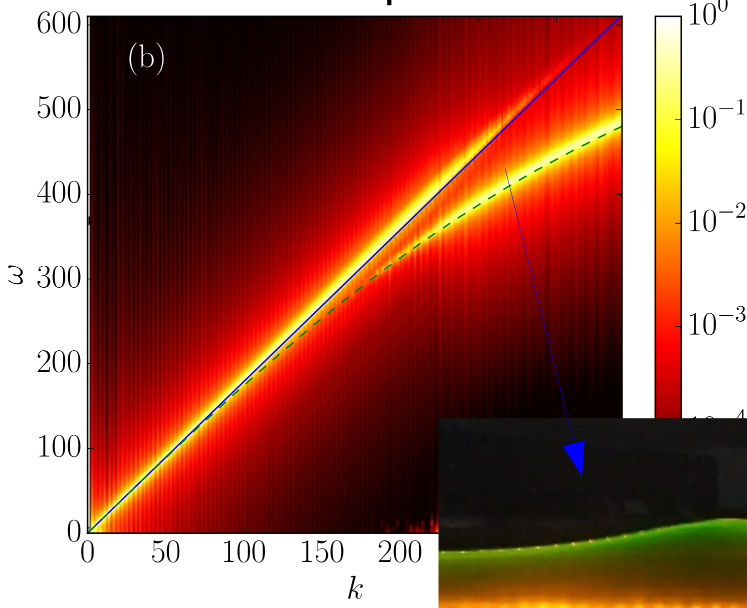
$$\omega = \frac{c_0 k}{\sqrt{1+\frac{h_0^2 k^2}{3}}}$$

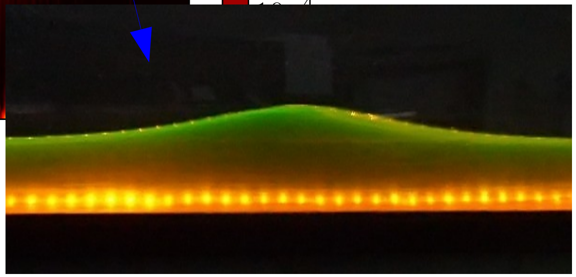
Model for not-so-shallow waves

No dispersion



With dispersion



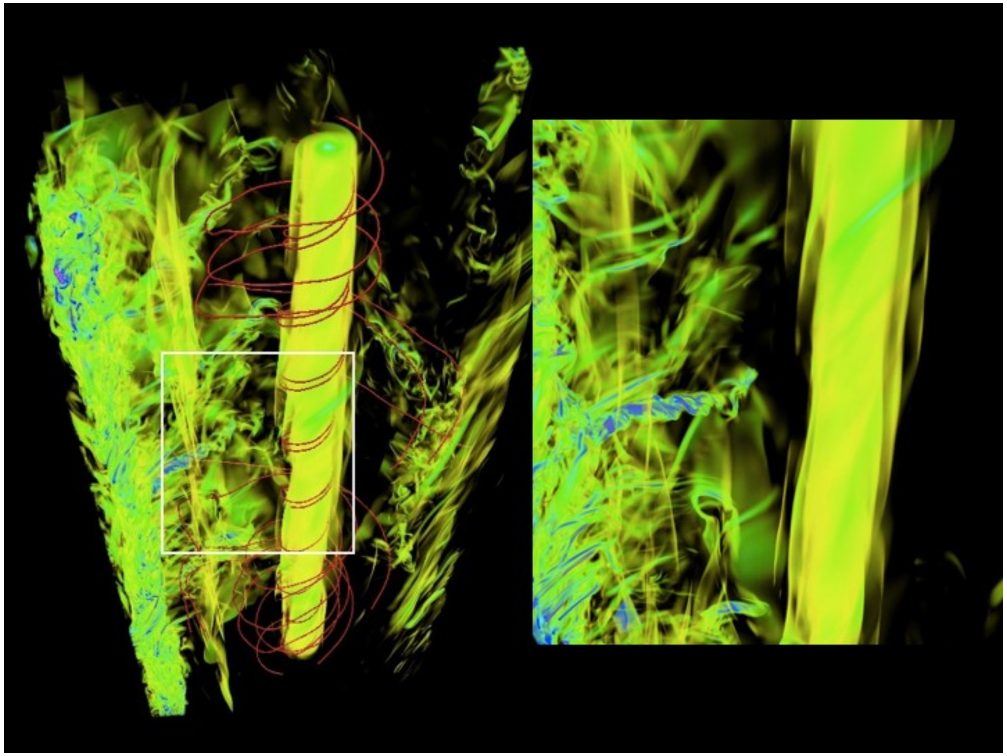


Changing the intensity of dispersion makes the system go from a wave turbulence regime to a mixed soliton and wave regime
[Clark di Leoni, Cobelli & Mininni, PRE (2014)]
Seen in experiments too [Hassaini & Mordant, PRF (2017)]

Atmospheric turbulence

- Understanding turbulence is important in order to develop both forecast and climate models
- We studied rotating flows and stratified flows
- In the scale of the ~1000 km rotation dominates the dynamics
- In the scale of the ~100 km stratification dominates the dynamics
- They are involed in problems such as mixing and blockage
- Three-dimensional!
- Strong interaction between vortices and waves

Rotating turbulence
Introduction



Columns in rotating turbulence
[Mininni et al. (2009)]

Inertial waves

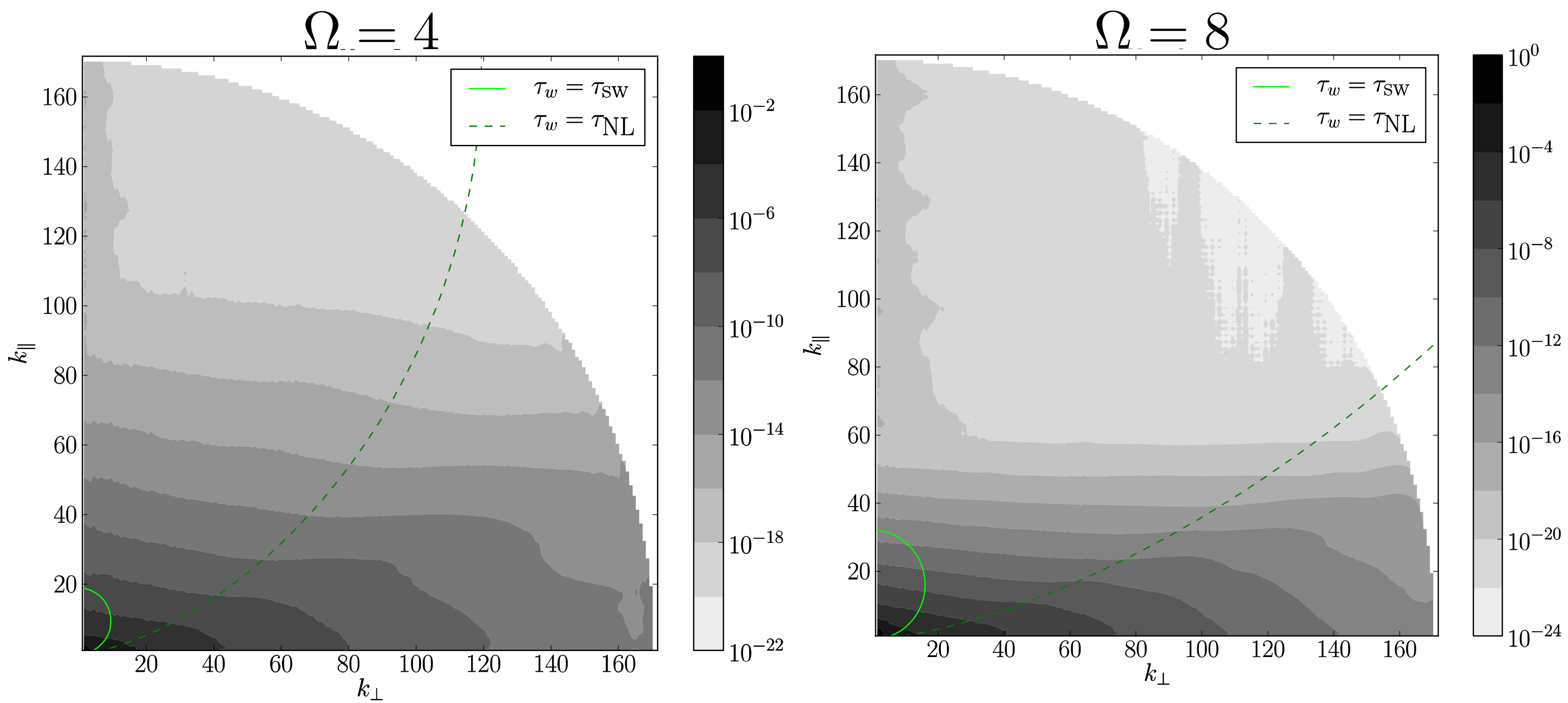
$$\omega_R(\mathbf{k}) = \frac{2\Omega k_{\|}}{k}$$

Flow bidimensionalization due to the effect of resonant triads
[Waleffe (1993)]

We performed simulations at different rotation rates Ω and quantified the presence of waves

Rotating turbulence results

Axisymmetric spatial spectra

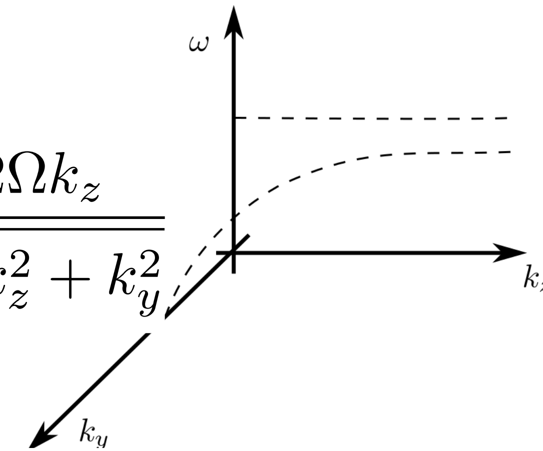


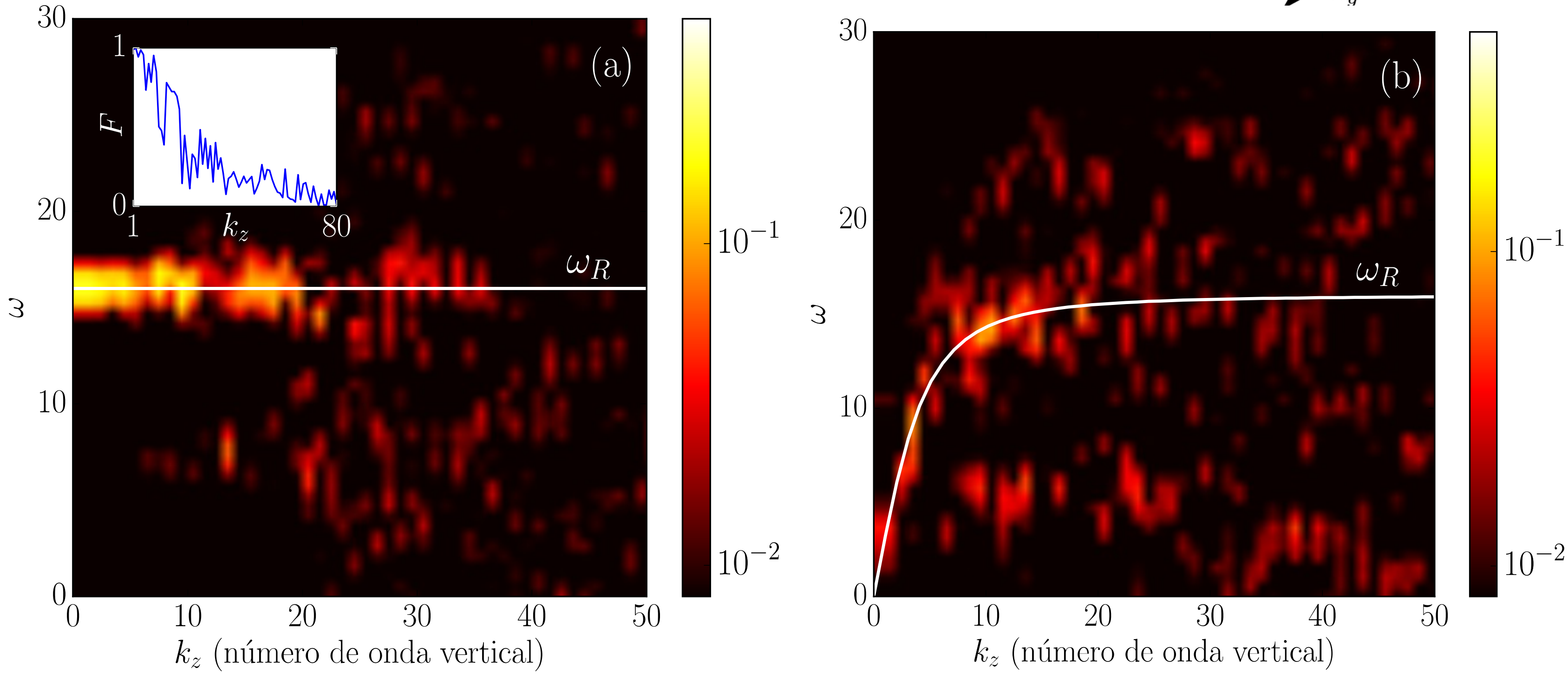
We can see the anisotropic effects on the flow,
but can be observe waves directly?

Rotating turbulence results

Spatiotemporal spectra

$$\omega_R(\mathbf{k}) = \frac{2\Omega k_z}{\sqrt{k_z^2 + k_y^2}}$$



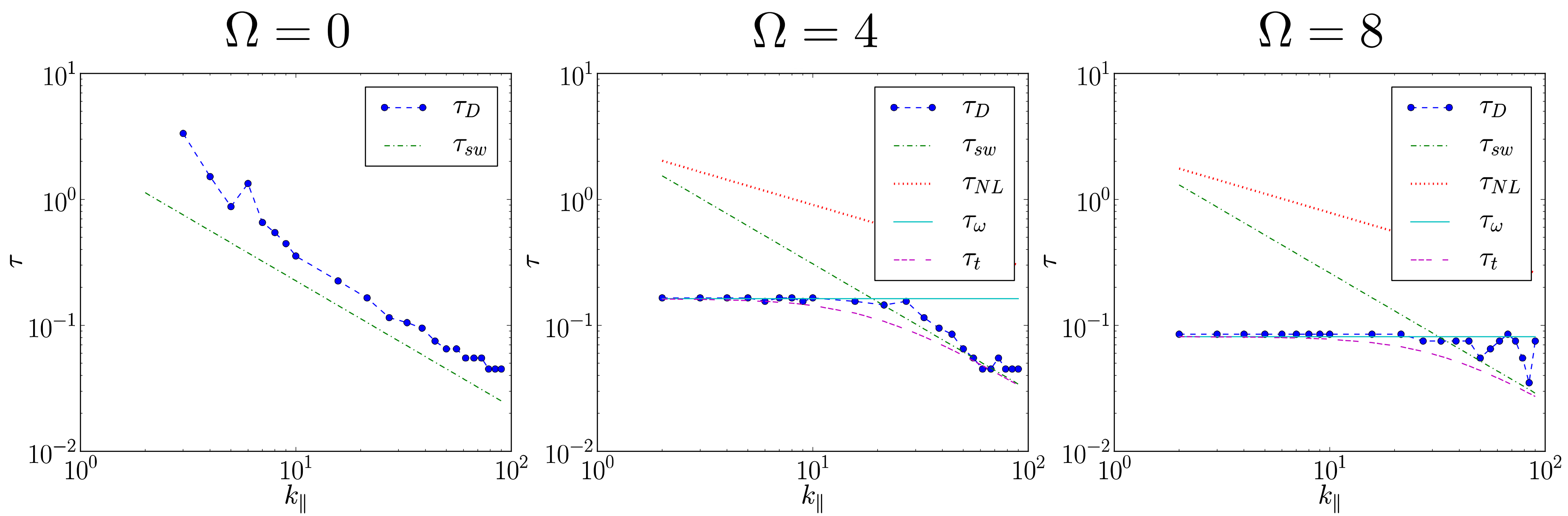


Energy is concentrated along the dispersion relation
[Clark di Leoni et al, PoF (2014)]
Seen experimentally too [Yarom & Sharon (2014)]

Rotating turbulence

Decorrelation times

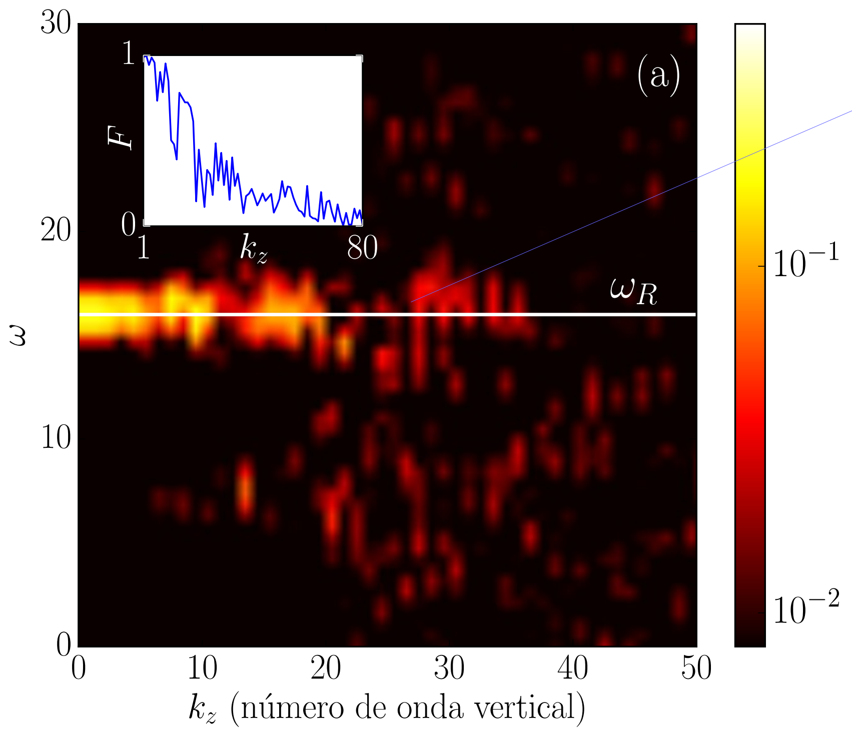
$$\Gamma_{xx}(\mathbf{k},\tau) = \frac{\langle \hat{u}_x^*(\mathbf{k},t)\hat{u}_x(\mathbf{k},t+\tau)\rangle_t}{\langle \hat{u}_x^*(\mathbf{k},t)\hat{u}_x(\mathbf{k},t)\rangle_t} \quad \Rightarrow \quad \tau_d$$



When the sweeping mechanisms become faster than the waves,
they start to dominate the dynamics.

Interactions between triads

Motivations



How does energy get here?

Are there “special” triads?

Can we quantify resonance?

We derived and used equations to measure the contribution and syntonization of each triad.

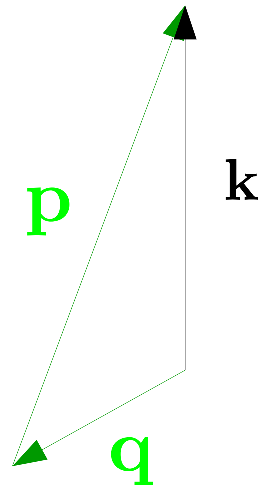
[Clark di Leoni & Mininni, JFM (2016)]

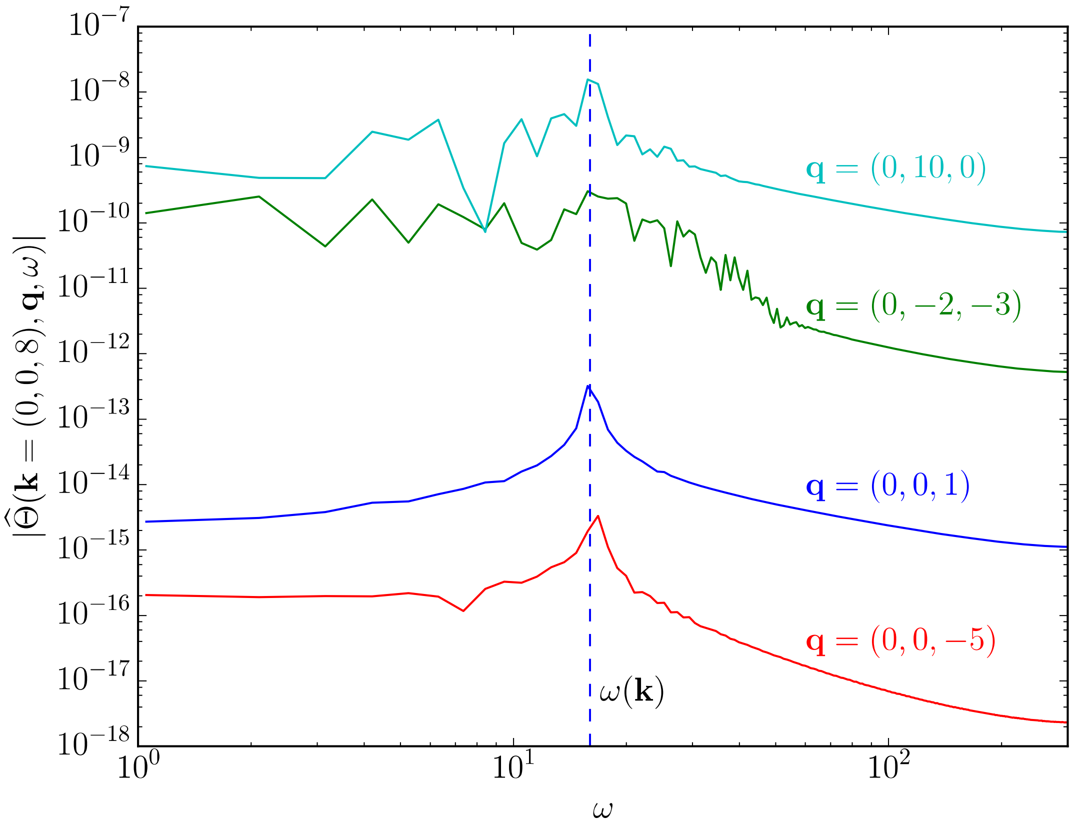
Interaction between triads

Contribution functions

$$\Theta(\mathbf{k},\mathbf{p},\mathbf{q},\tau) = \langle \mathbf{u}^*_{\mathbf{k}}(t') \cdot [\mathbf{u}_{\mathbf{p}}(t'+\tau)\cdot\mathbf{q}]\,\mathbf{u}_{\mathbf{q}}(t'+\tau)\rangle_{t'}$$

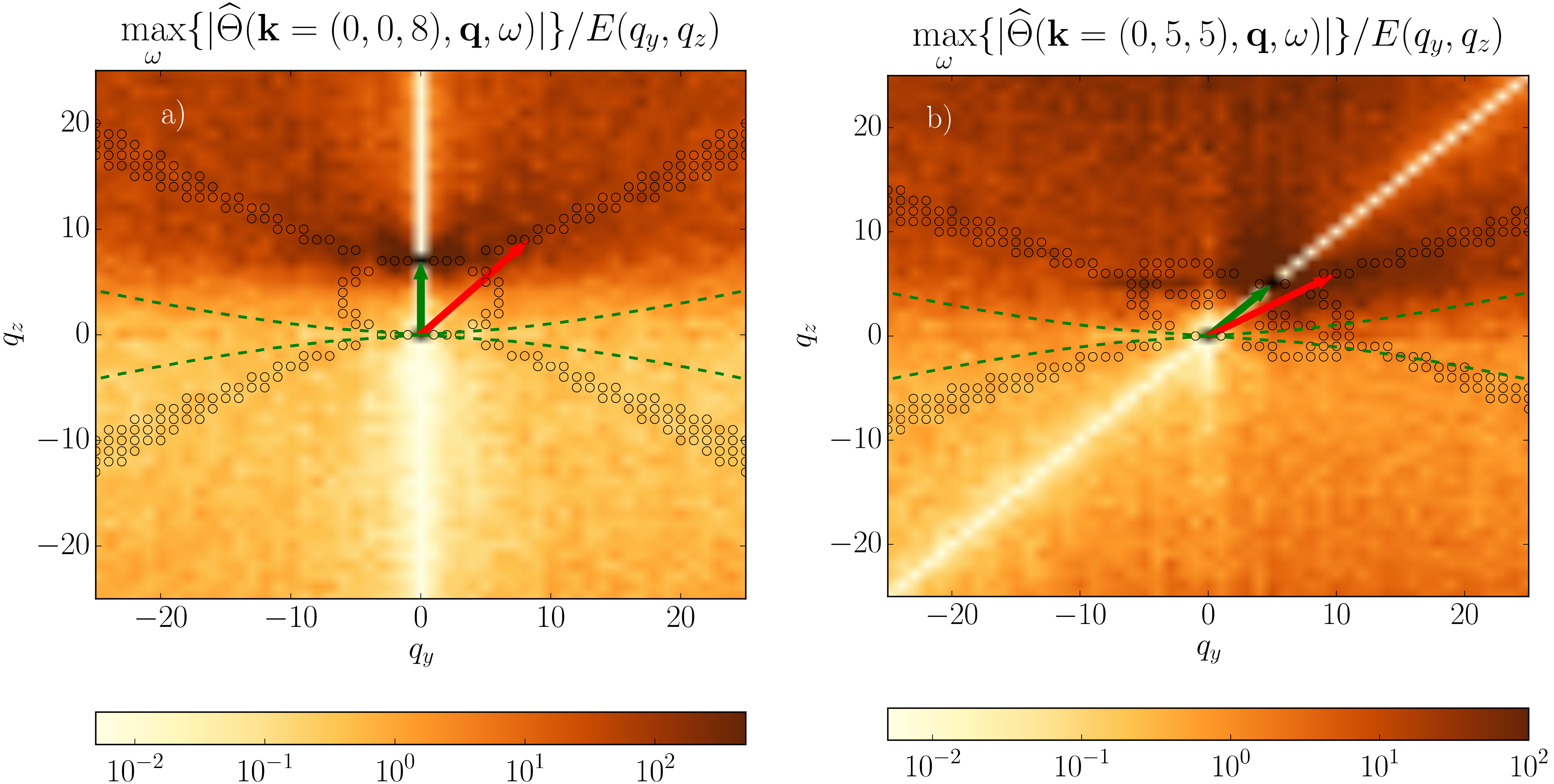
$$2\omega E(\mathbf{k},\omega) = \sum_{\mathbf{p}+\mathbf{q}=\mathbf{k}} \widehat{\Theta}(\mathbf{k},\mathbf{q},\mathbf{p},\omega).$$





Interaction between triads

Contribution functions

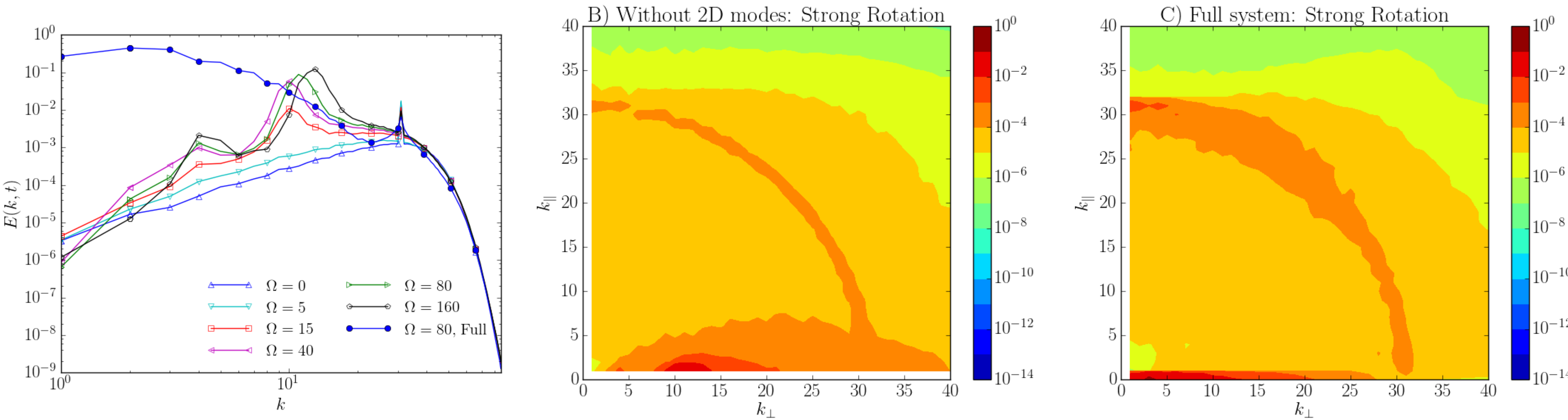


Both resonant and quasi-resonant triads are important!

3D (fast) vs 2D (slow) modes

$$\omega_R(\mathbf{k}) = \frac{2\Omega k_{\|}}{k}$$

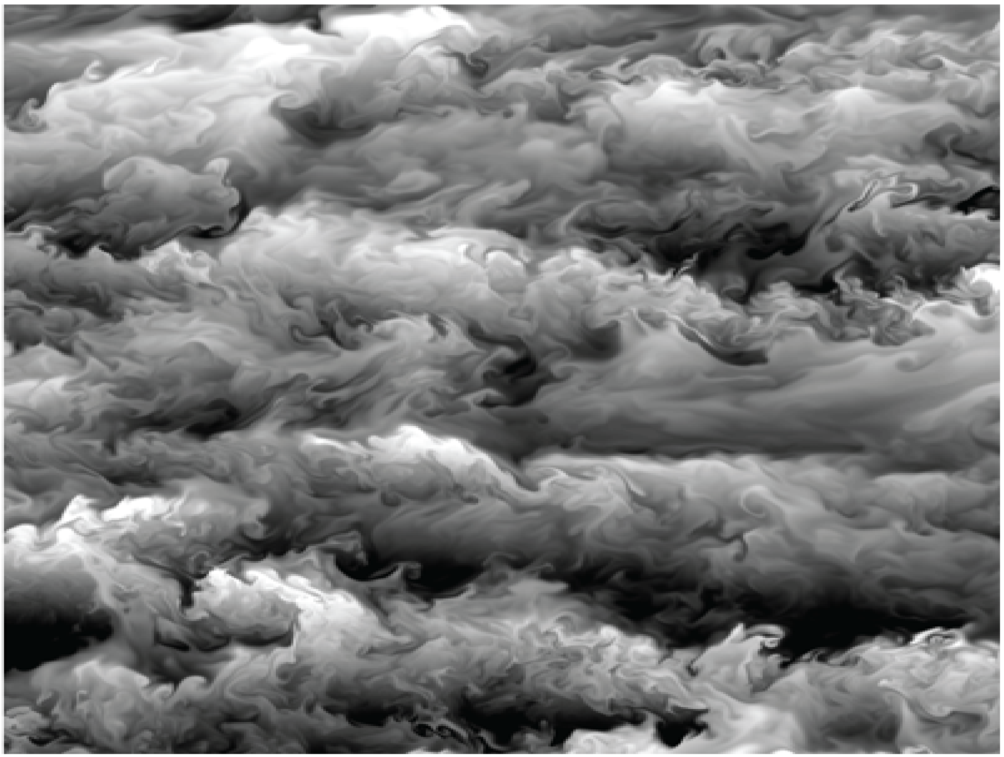
- Under strong rotation 3D and 2D get decouple [Waleffe (1993)]
- Under strong rotation an inverse cascade can form too
- This is typically thought as 2D problem
- Do 3D modes have any say in the inverse cascade?
- We removed the 2D modes in order to find out!
- [Buzzicotti, Clark Di Leoni & Biferale, arXiv (2018)]



While there is some transfer of energy backwards, a zero flux regime is formed and no inverse cascade is seen.
Quasi-resonant modes are important to couple the 2D and 3D dynamics

Stratified turbulence

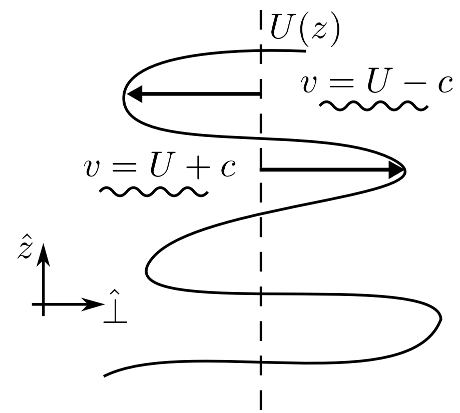
Introduction



Vertically sheared horizontal winds
[Smith & Waleffe (2002); Rorai et al. (2014)]

Internal waves

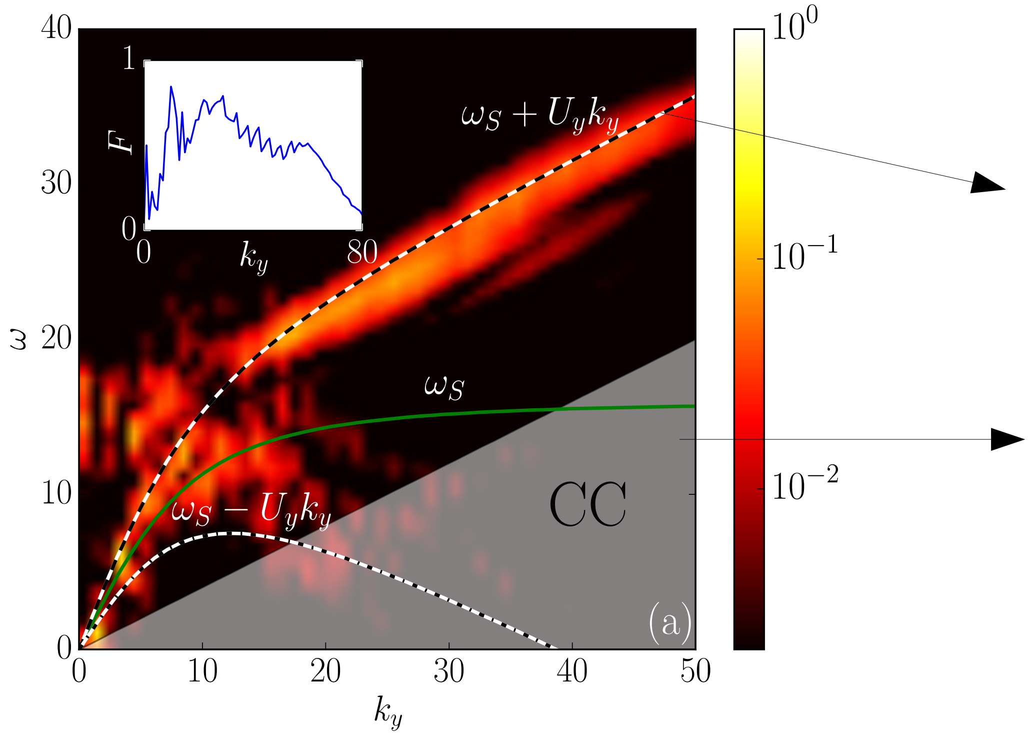
$$\omega_S(\mathbf{k}) = \frac{Nk_\perp}{k}$$



$$\omega = \omega_S + \mathbf{U} \cdot \mathbf{k}$$

If U=c a critical layer is formed
where waves are absorbed

Stratified turbulence results
Spatio-temporal spectra



Energy is not accumuled directly on the dispersion relation

Doppler shift!
U_y is the horizontal rms velocity

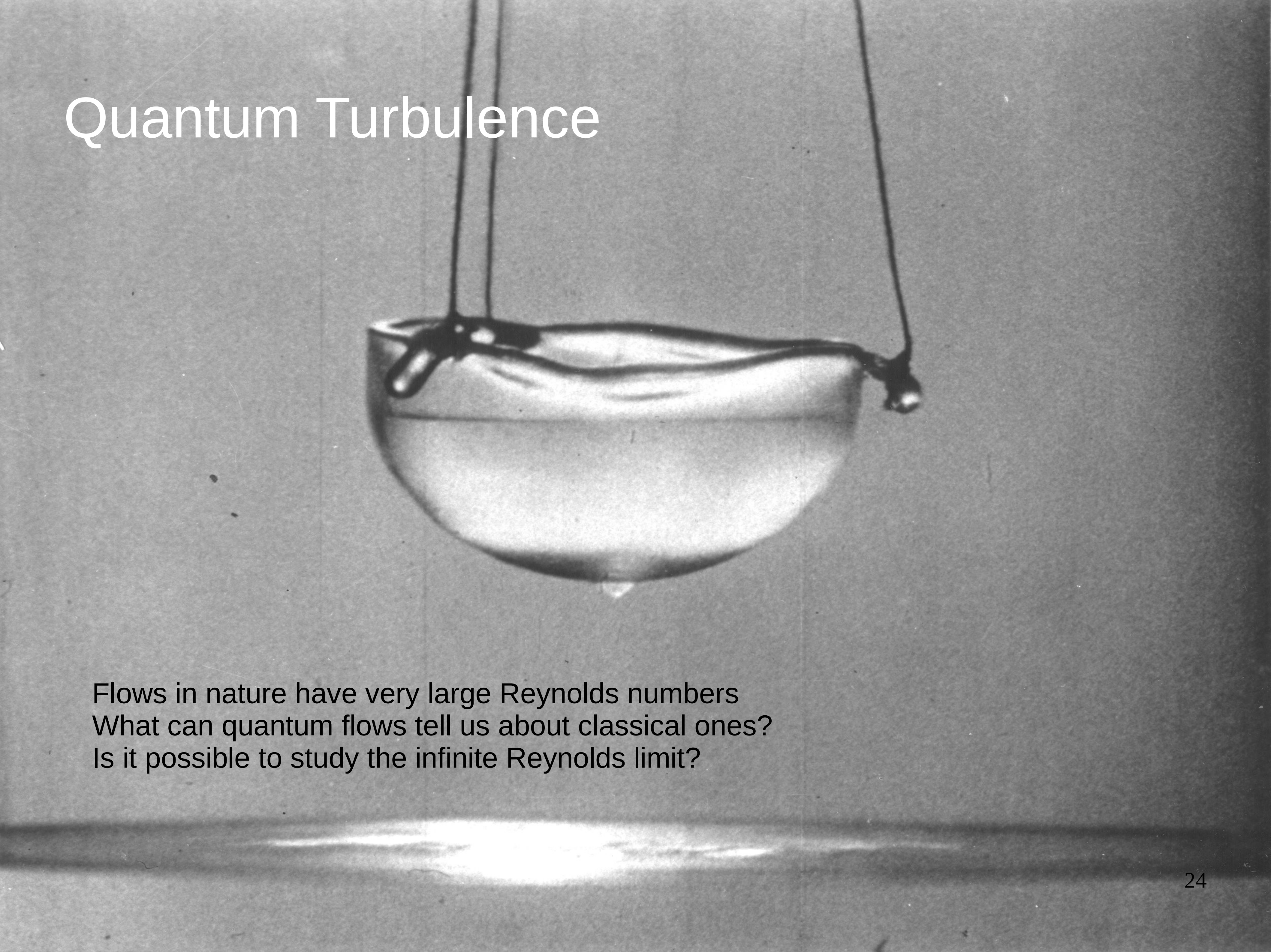
$$\omega \leq U_y k$$

Critical Layer absorption

These effects were studied previously in linear models
[Hines (1971); Booker & Bretherton (1967)]

But until now they have not been studied in turbulent flows
[Clark di Leoni & Mininni, PRE (2015)]

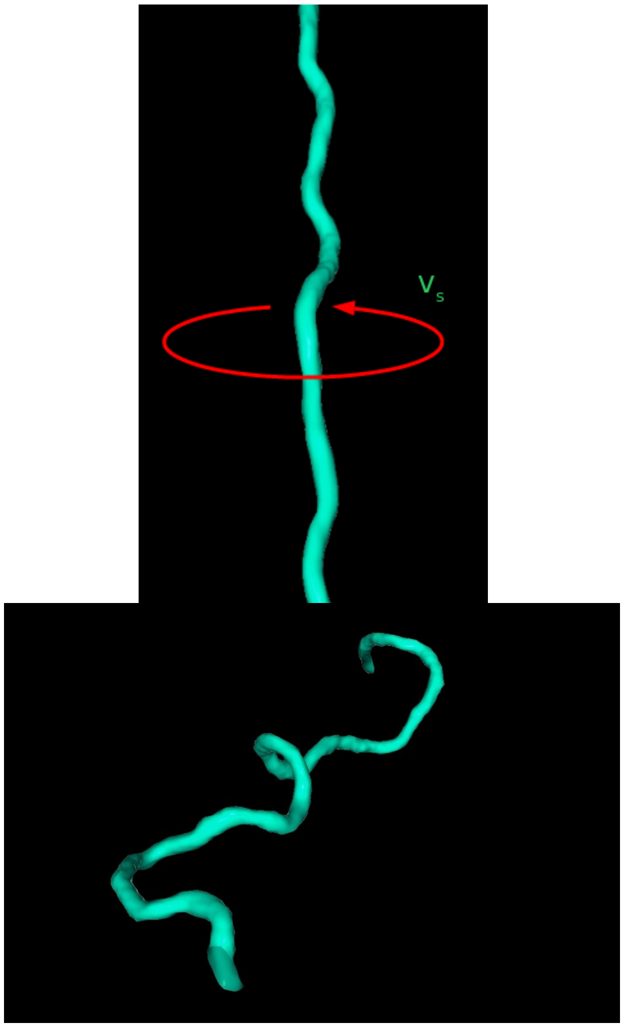
Quantum Turbulence



Flows in nature have very large Reynolds numbers
What can quantum flows tell us about classical ones?
Is it possible to study the infinite Reynolds limit?

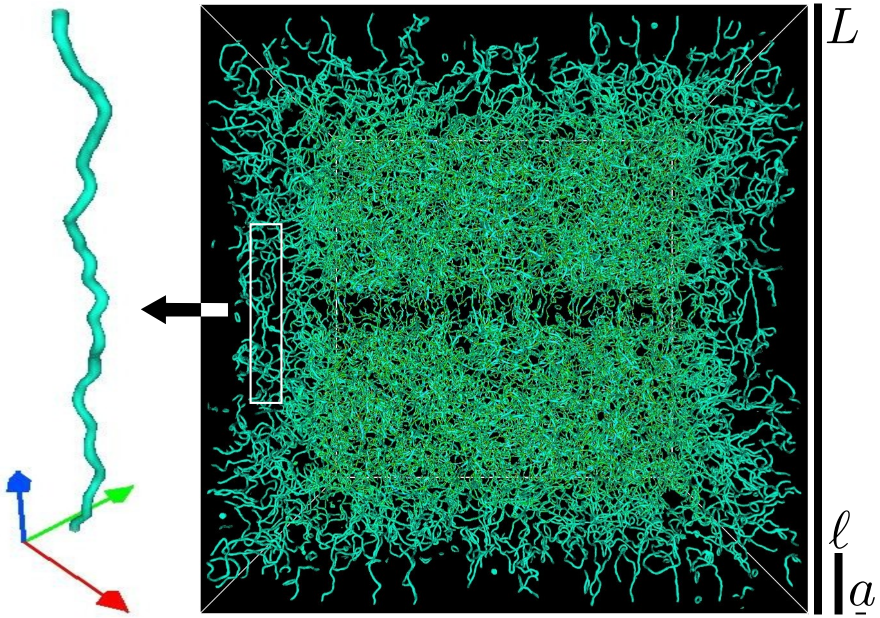
Quantum turbulence

Gross-Pitaevskii equation

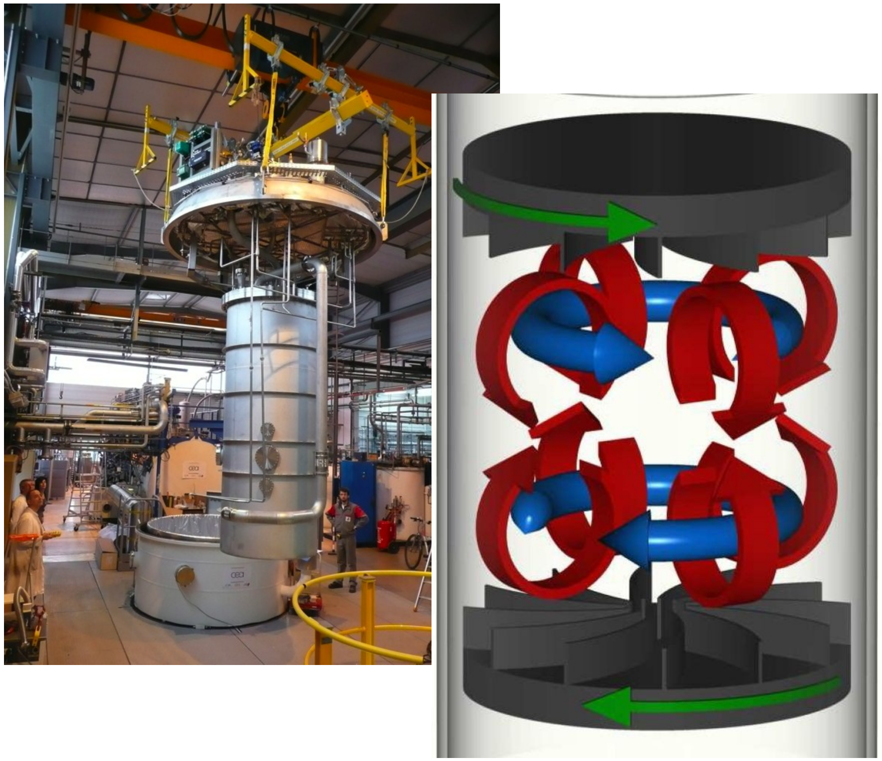


- All vorticity is concentrated along topological line defects
- Except along these lines, the flow can be understood as that of an ideal fluid
- There are known similarities between classical and quantum turbulence
- Vortices can sustain Kelvin waves, which are thought to generate a turbulent cascade [Lvov & Nazarenko (2010)]
- Previous studies have focused on a small number of vortices [Krstulovic (20012), Baggaely & Laurie (2014)]
- Phonons are left with the task of “dissipating” energy [Vinen & Niemela (2002), Proment et al (2009)]

Quantum turbulence
Simulations of the Gross-Pitaevskii equation



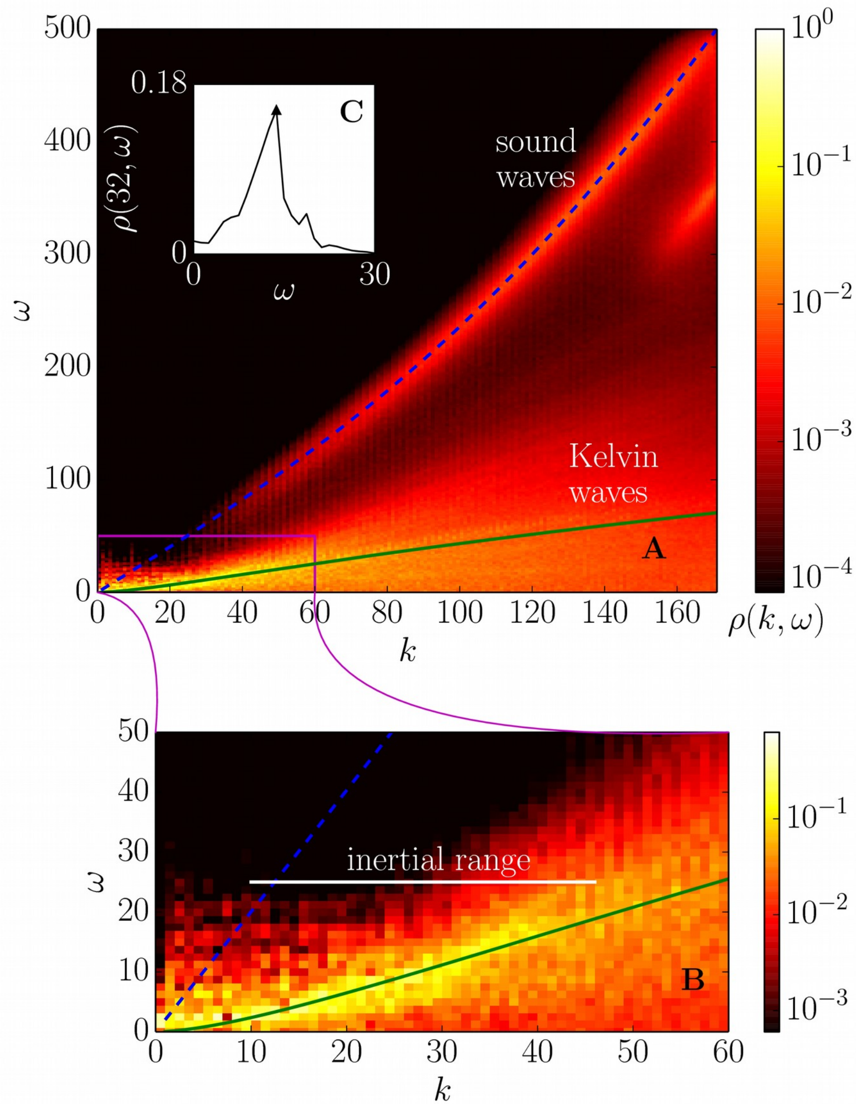
We have to extract Kelvin waves
out of this mess!



Similar flow configuration
as that of the SHREK experiment

Quantum turbulence results

Spatiotemporal spectrum of the density



Detection of Kelvin waves in a complex turbulent flow

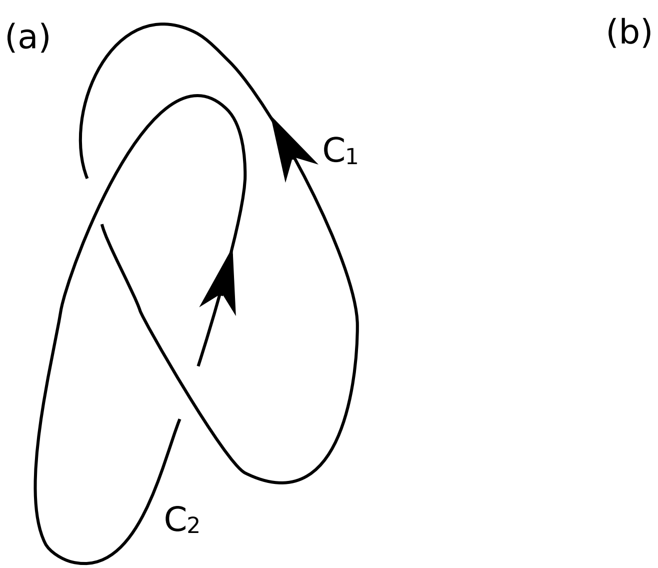
Phonon emission was also detected

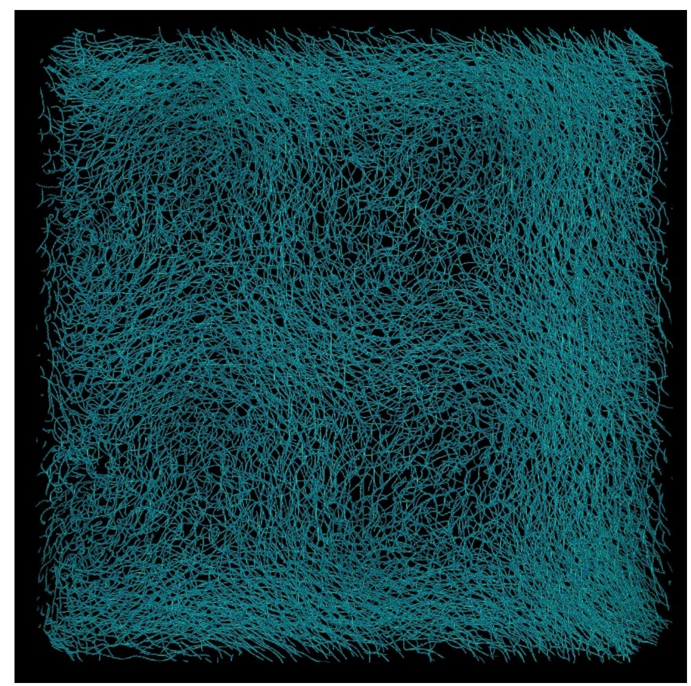
This a characteristic trait of quantum turbulence

[Clark di Leoni, Mininni & Brachet, PRA (2015)]

Quantum turbulence
Helicity in quantum flows

- $\mathrm{H} = \int u \times \omega dV$
- It’s a measure of how knotted the field lines are [Moffat (1968)]
- Difficult to calculate in quantum flows because along vortex lines, as both velocity and vorticity are singular
- Some authors have calculated helicity with topological tools, others have filtered the fields, we have regularized them!

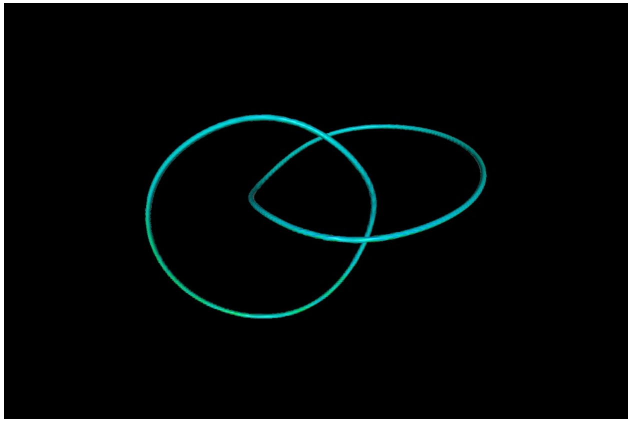


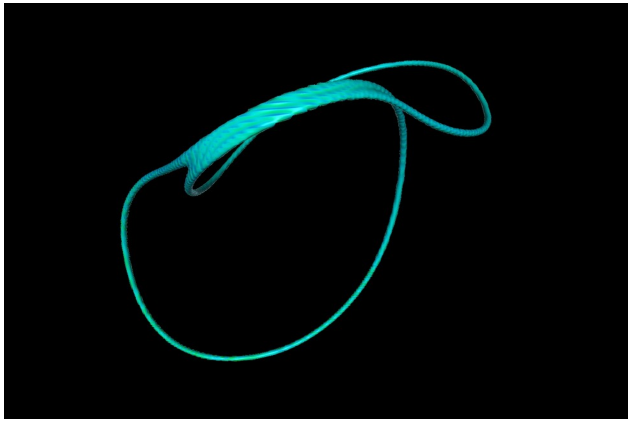
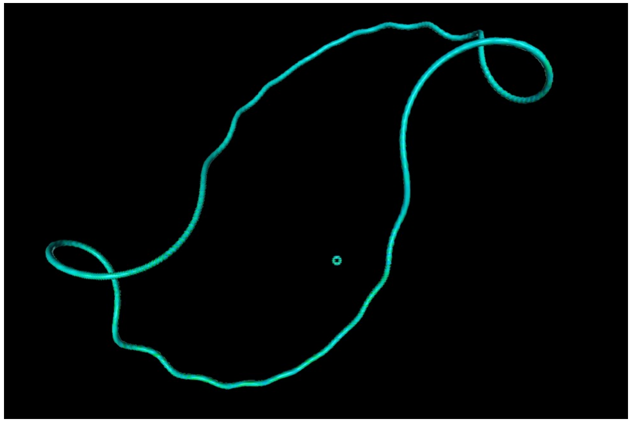


2048^3 grid points

Quantum turbulence
Helicity in quantum flows

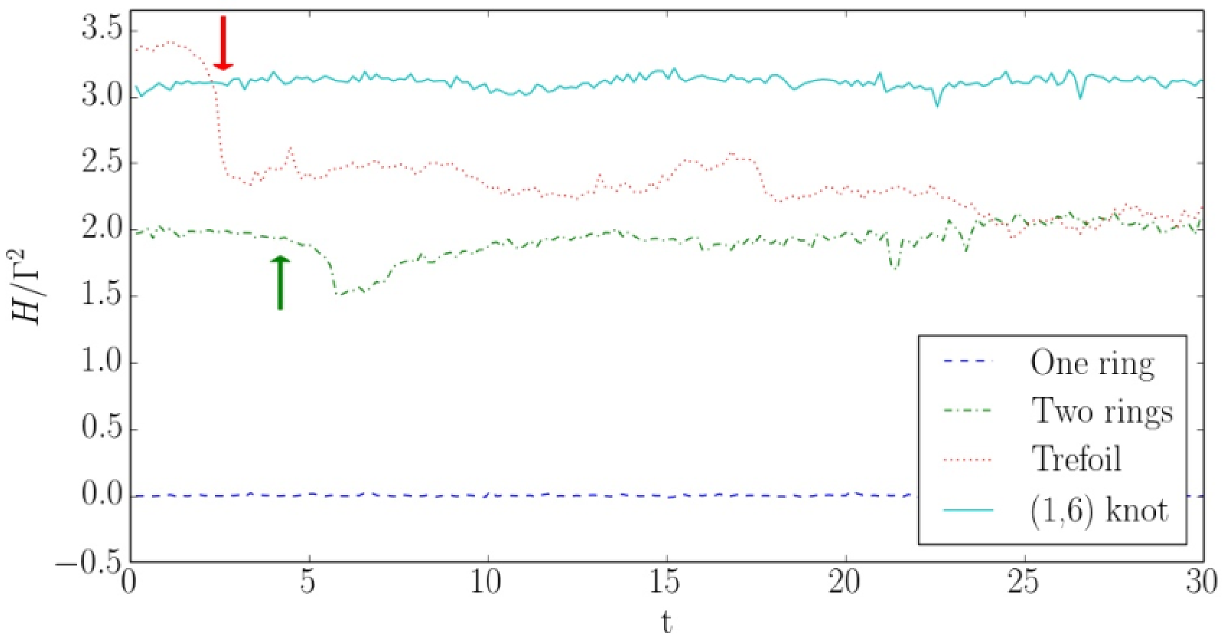
We start by studying simple flow configurations

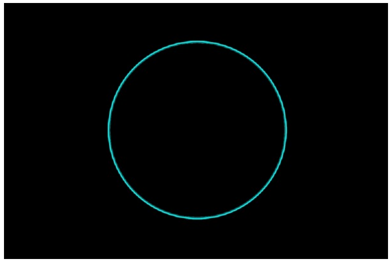


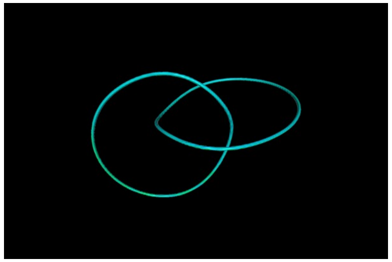



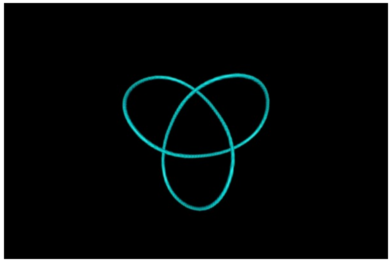
Quantum turbulence results

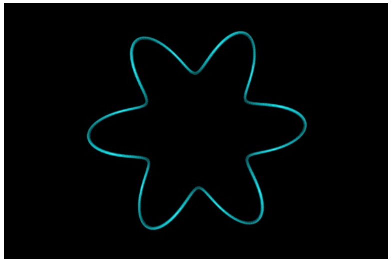
Helicity in quantum flows







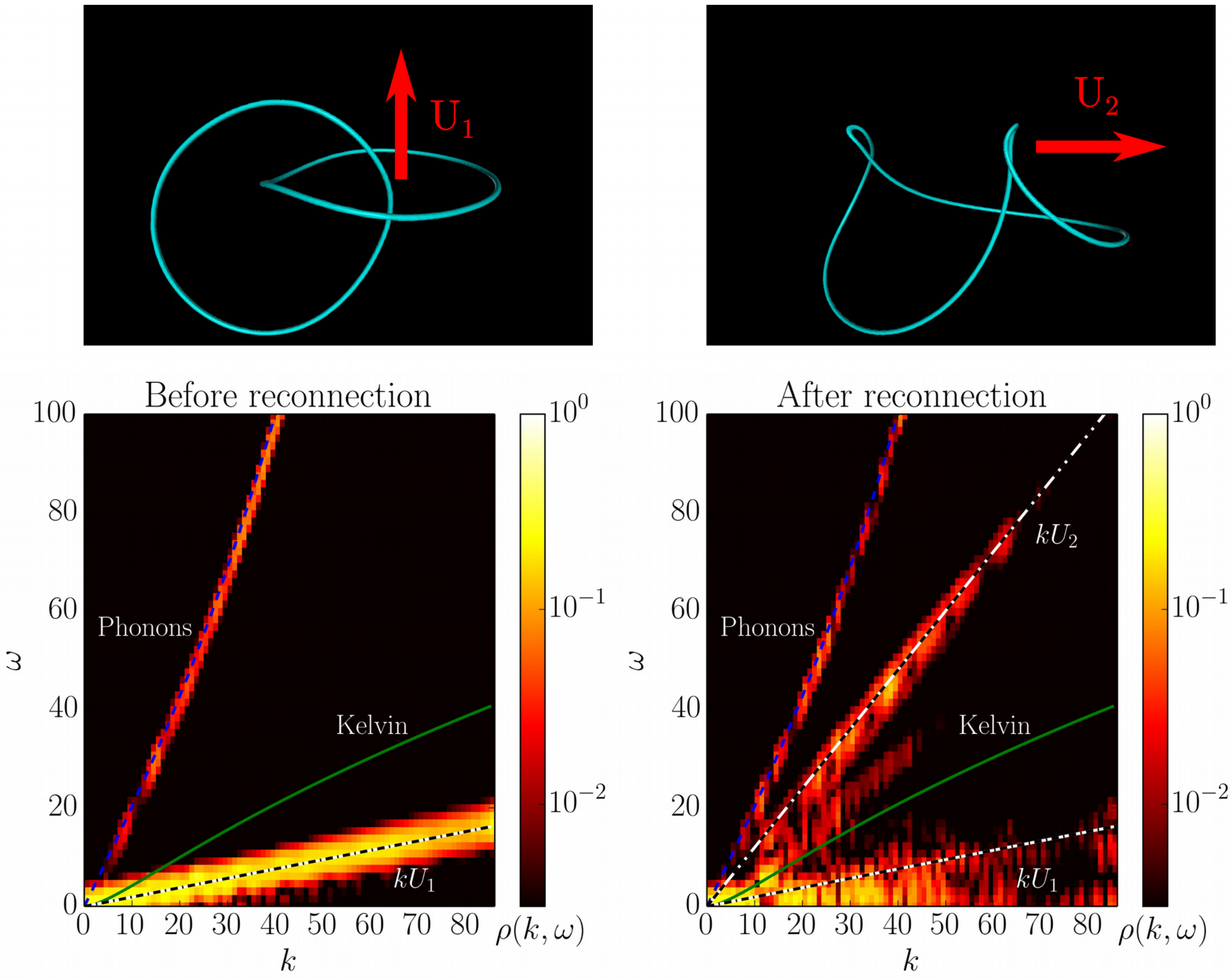




We can study the evolution of helicity.
The values coincide with those calculated via topological methods.
Not all reconnection events conserve helicity.
[Clark di Leoni, Mininni & Brachet, PRA (2016)]

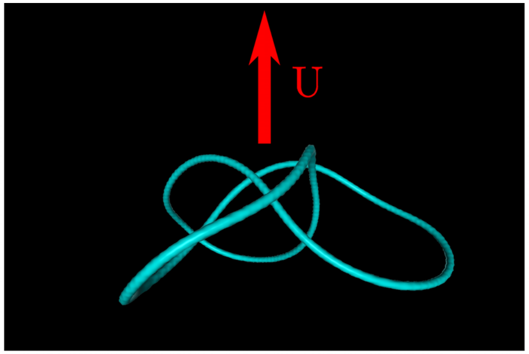
Quantum turbulence

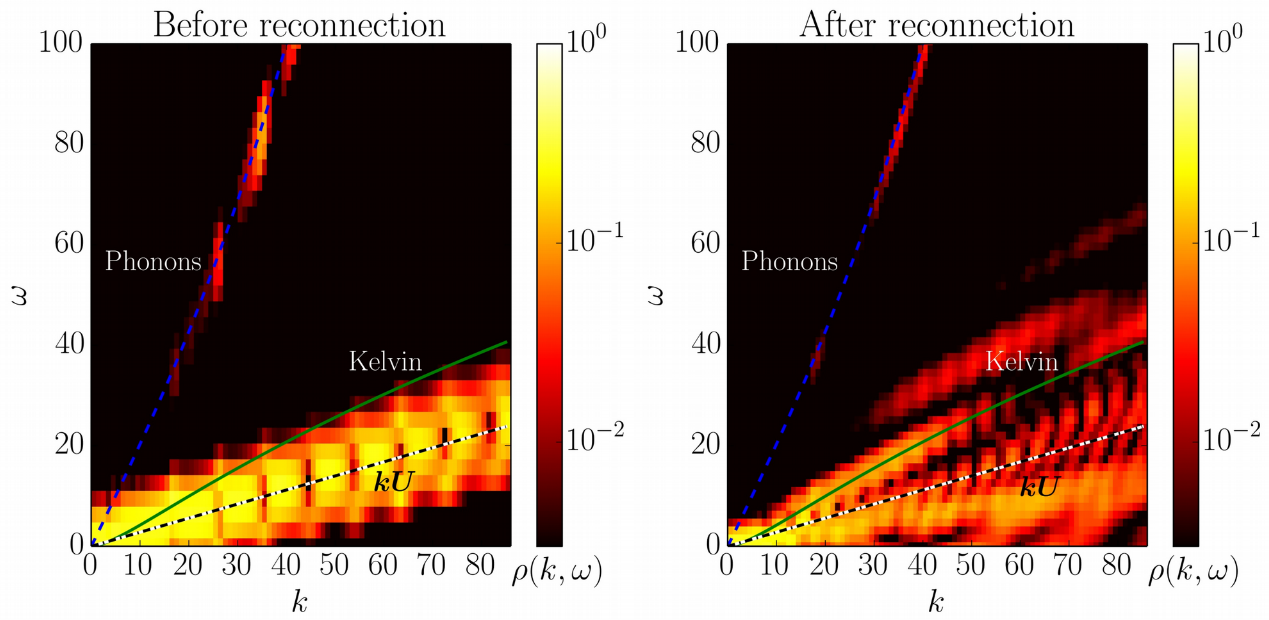
What happens when two rings reconnect?



Quantum turbulence

What about the trefoil knot?

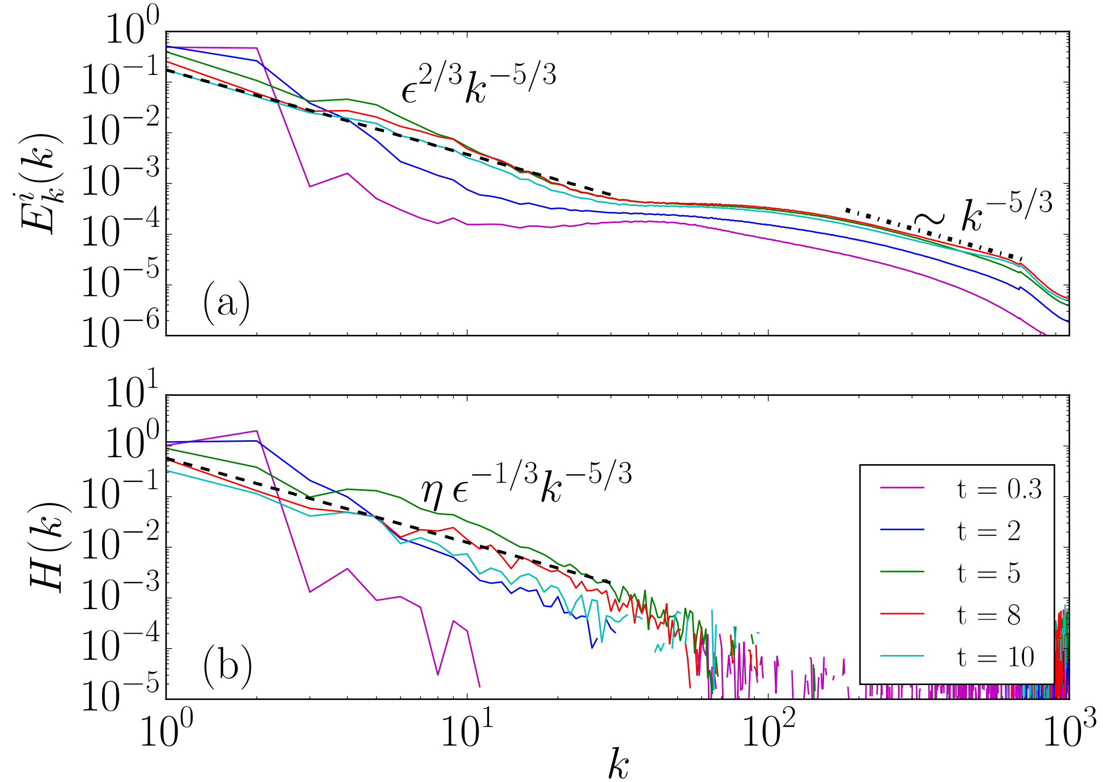


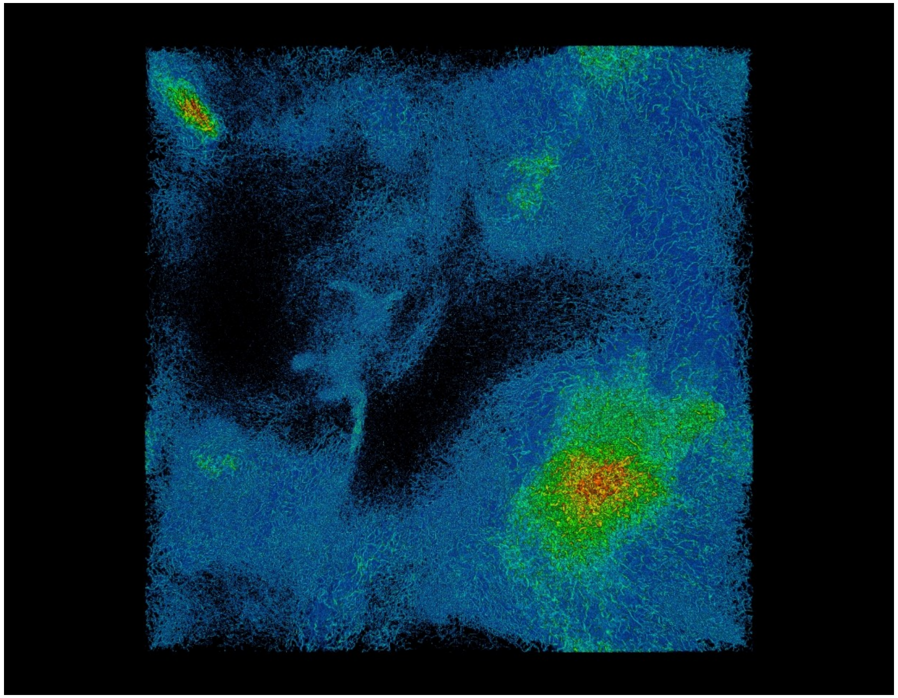


When helicity is not conserved, Kelvin waves are excited and phonons are emitted
[Clark di Leoni, Mininni & Brachet, PRA (2016)]

Quantum turbulence
Helicity in quantum flows

We can use the regularized helicity to study turbulent flows
In the classical case there’s a dual cascade of energy and helicity [Brissaud et al (1973)].
What about the quantum case?





Quantum tornadoes!

A dual cascade is also present in the quantum case.
Kelvin wave presence confirmed via spatio-temporal spectra.
Look at that bottleneck!
[Clark di Leoni, Mininni & Brachet, PRA (2017)]

Closing remarks

Thanks to spatiotemporal spectra we were able to:

- Detect mixed wave and soliton turbulence states in gravity surface waves
- Spot bound modes in gravito-capillary turbulence
- Determine which scales are dominated by waves in rotating and stratified flows
- Detect Doppler shift and critical layer absoption in stratified flows
- Extract Kelvin waves out of complex turbulent flows