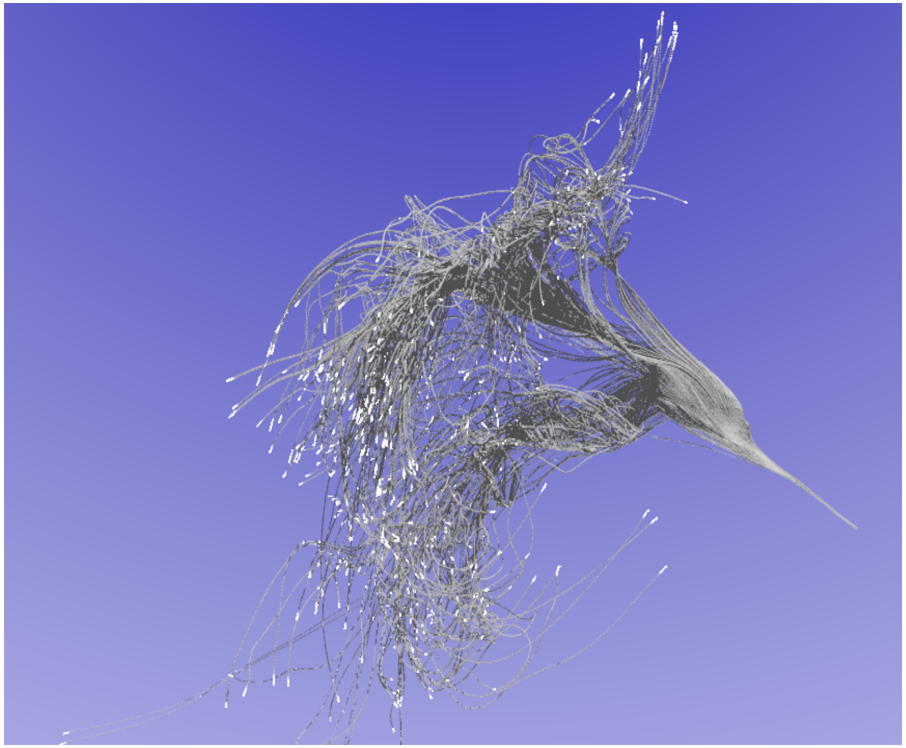
Dispersion of particles from localized sources in turbulence



Riccardo Scatamacchia

University of Rome "Tor Vergata"

&

Eindhoven University of Technology

With:

Luca Biferale, Dept. Physics, Univ. Tor Vergata (Italy)

Alessandra S. Lanotte, Institute for Atmospheric Science and Climate CNR-ISAC (Italy)

Federico Toschi, Eindhoven University of Technology (The Netherlands)

Where we can find point-source-like emissions

Volcano eruptions

Burning and pollutants dispersion

Plan of the talk

- *How pairs of tracer particles separate in homogeneous and isotropic turbulence*

- *DNS results and comparison with Richardson's PDF*

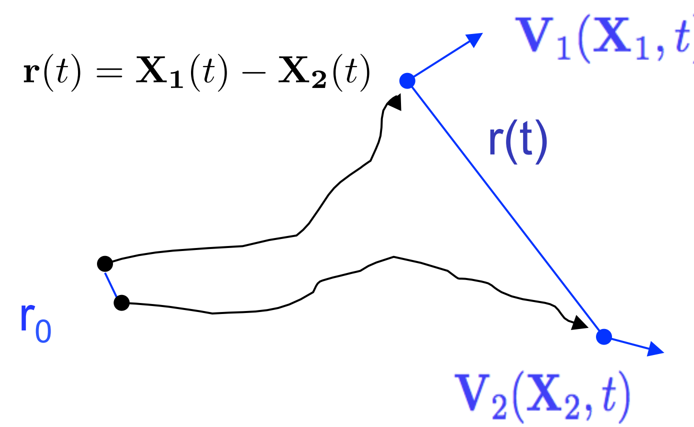
- *Simple model of the eddy-diffusivity to characterize the importance of finite Reynolds number effects* on tracer particles dispersion

- Intermittency in tracer pairs separation

Richardson's law (1926)

Diffusive process in inertial subrange characterized by an effective turbulent diffusivity

$$D_{Ric}(r) = \frac{1}{2}\frac{d\langle r^2\rangle}{dt} \sim \tau(r)\langle(\delta_r v)^2\rangle \sim r^{4/3}$$



Richardson's approach can be reinterpreted as the evolution of a particle pair in a stochastic Gaussian and delta-correlated in time velocity field

$$\partial_t P(r,t) = \frac{1}{r^2}\partial_r r^2 D_{||}(r)\partial_r P(r,t)$$

If the eddy-diffusivity has a power law behavior $\mathbf{D_{||}(r) = D_1 r^{\xi}}$ **with** $\mathbf{0 \le \xi \le 2}$ **we obtain an asymptotic form of *P(r,t)***

$$\begin{cases} P(r,t) \propto \frac{r^2}{\langle r^2(t)\rangle^{\frac{3}{2}}} \exp\left\{-b\left(\frac{r}{\langle r^2(t)\rangle^{\frac{1}{2}}}\right)^{2-\xi}\right\} \\ \langle r^2(t)\rangle \propto t^{2/(2-\xi)} \end{cases}$$

ξ = 4/3 → ***Richardson's expression***

ξ = 2 → ***Log-normal expression***

Numerical simulation details

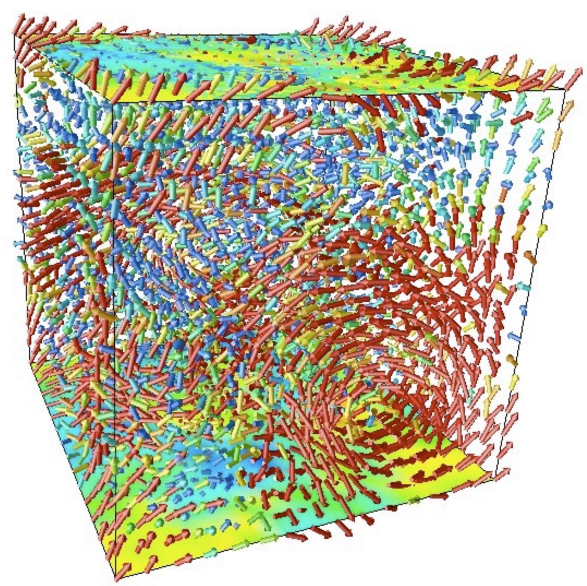
Fluid

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\, \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{F}$$

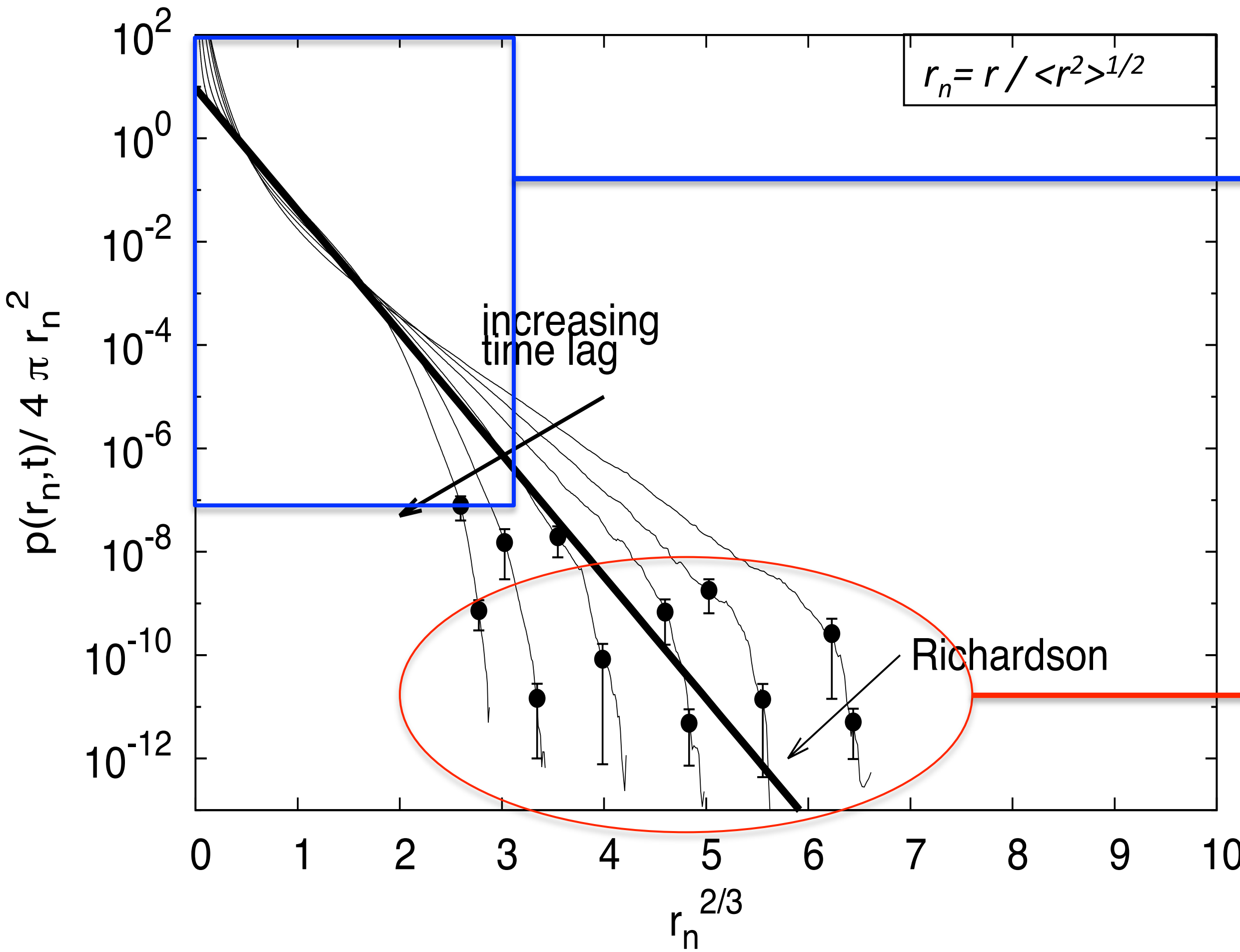
$$\nabla \cdot \mathbf{u} = 0$$

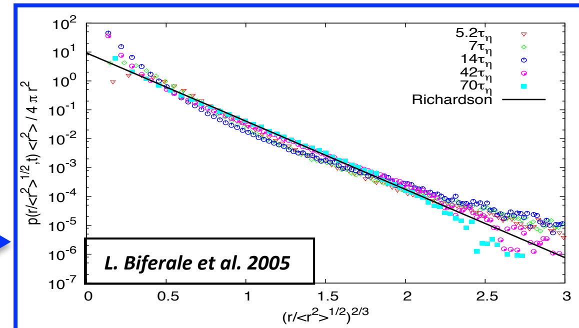
$$\dot{\mathbf{x}}(t) = \mathbf{u}(\mathbf{x}(\mathbf{t}), \mathbf{t})$$ → ***Tracer particles***

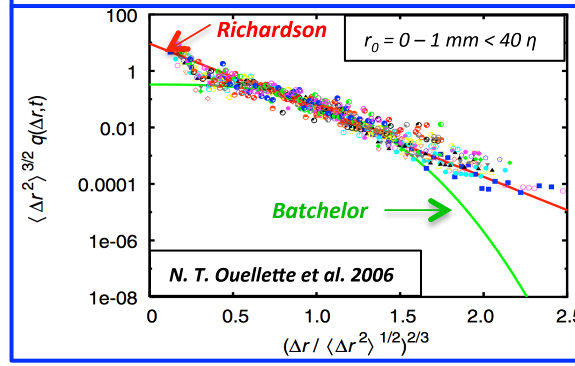
- *3-D homogeneous isotropic flow at $Re_\lambda \sim 300$*
- *Regular cubic box (1024^3 grid points) with periodic BC*
- *256 sources where anyone emits 2000 tracers every τ_η*
- *4×10^{11} particle pairs*
- *Parallel pseudo-spectral code*



Comparison with Richardson's PDF







Strong departures from the ideal self-similar Richardson distribution

These unideal effects can be either due to finite Reynolds effects or by neglected temporal correlations, or both

Eddy-diffusivity model (finite Reynolds effects)

R.Scatamacchia, L.Biferale and F. Toschi. PRL 109, 144501 (2012)

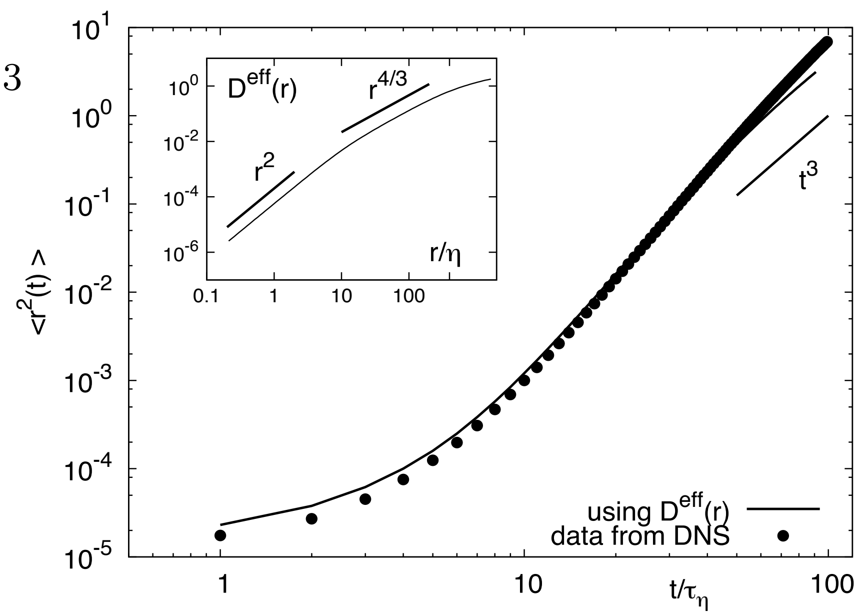
Numerical integration of Richardson diffusive equation using an effective turbulent eddy-diffusivity that keeps in account the viscous and large scale cut-offs

$$D_{||}^{eff}(r) \sim \tau(r)\langle(\delta_r v)^2\rangle \quad \Longrightarrow \quad \begin{cases} D_{||}^{eff}(r) \sim r^2 & r \ll \eta \\ D_{||}^{eff}(r) \sim r^{4/3} & \eta \ll r \ll L_0 \\ D_{||}^{eff}(r) \sim const. & r \gg L_0 \end{cases}$$

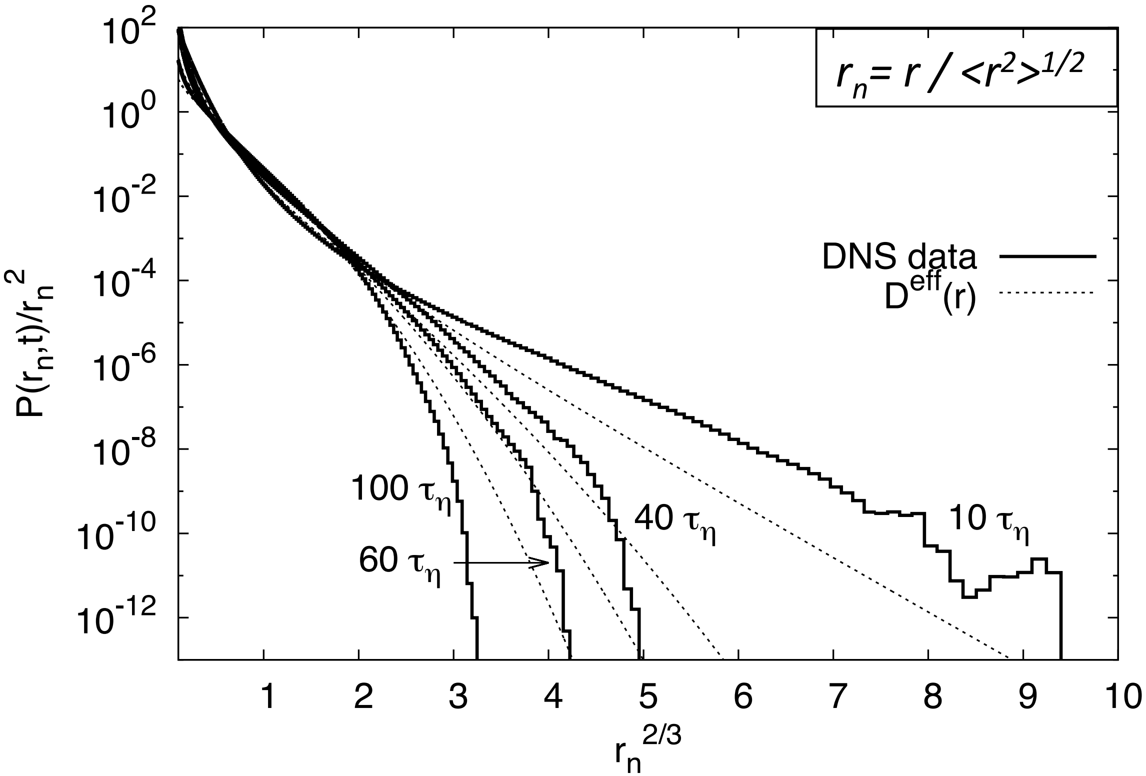
Fitting formula that matches the expected UV and IR scaling for both $\tau(r)$ and $<(\delta_r v)^2>$

$$\begin{cases} \langle(\delta_r v)^2\rangle = c_0 \frac{r^2}{((r/\eta)^2 + c_1)^{(2/3)}} \left[1 + c_2 (\frac{r}{L})^2\right]^{-1/3} \\ \tau(r) = \frac{\tau_\eta}{((r/\eta)^2 + c_1)^{-1/3}} \left[1 + d_2 (\frac{r}{L})^2\right]^{-2/3} \end{cases}$$

c_0, c_1, c_2 are fitted from Eulerian statistics while d_2 is adjusted to reproduce a good agreement with $<r^2(t)>$ data



Model-DNS compared



The model breaks the ideal self-similar PDF behavior

The model is not able to describe the sharp change

It is not enough to impose a saturation in the effective eddy-diffusivity to reproduce the fastest-cases: something strongly different from a delta-correlated in time must be used

Multifractal prediction for pairs separation

L.Biferale , A.S. Lanotte, R.Scatamacchia, and F. Toschi. Accepted in JFM (2014)

Let's start from the following exact relation

$$\frac{d}{dt}\langle r^p \rangle = p \langle r^{p-1}(\delta_r u) \rangle$$

Let's suppose that the correlation can be estimated with Eulerian quantities

$$\langle r^{p-1}(\delta_r u) \rangle \propto \int dh r^{3-D(h)} r^{p-1} r^h$$

Using the bridge relation $t \sim r/\delta_r u \sim r^{1-h}$ we can relate the Eulerian and Lagrangian statistics

$$\langle r^{p-1}(\delta_r u) \rangle \propto \int dh t^{\frac{2-D(h)+p+h}{1-h}}$$

After a time integration and using a saddle point approximation, we get

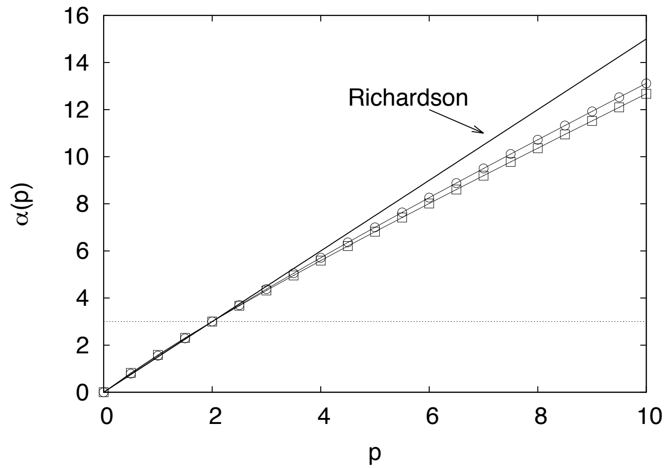
$$\langle r^p(t) \rangle \propto t^{\alpha(p)}, \qquad \alpha(p) = \min_h \frac{(3 - D(h) + p)}{(1-h)}$$

Multifractal prediction for pairs separation

To measure the scaling behaviours we use the Extended Self Similarity (ESS)

$$F_p(t) = \frac{\langle r^p(t) \rangle}{\langle r^2(t) \rangle^{\frac{\alpha(p)}{3}}}$$

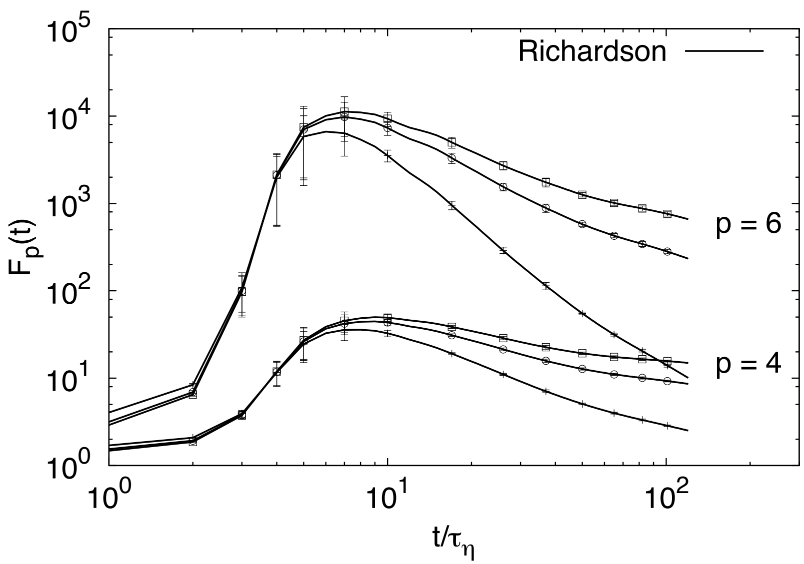
$$\alpha_{Rich}(p) = 3p/2$$



$D_L(h)$ (○)

$D_T(h)$ (□)

Using the multifractal prediction for α(p)



The multifractal prediction works better than the dimensional one.

Because the plateau is very narrow it is necessary waiting for data at high Re before making any firm conclusion.

Conclusion

- *We showed for the first time that both extremal "fast" and "slow" separations events DO NOT FOLLOW Richardson-like inertial and self-similar behavior.*

- *By using a model that keeps into account viscous and integral scale physics, we got a qualitative agreement with DNS data.*

- *The multifractal approach for the scaling behaviours of* $\langle r^p(t) \rangle$ *goes in the right direction but, due to viscous contaminations of the inertial range we don't observe a clear proof.*