European Research Council
Established by the European Commission

Slide of the Seminar

DROPLETS & EMULSIONS: dynamics & rheology with LBM numerical simulations

Prof. Mauro Sbragaglia

ERC Advanced Grant (N. 339032) "NewTURB"
(P.I. Prof. Luca Biferale)

Università degli Studi di Roma Tor Vergata
C.F. n. 80213750583 – Partita IVA n. 02133971008 - Via della Ricerca Scientifica, 1 – 00133 ROMA

DROPLETS & EMULSIONS:

dynamics & rheology with LBM numerical simulations

Mauro Sbragaglia

Department of Physics and INFN, University of Rome "Tor Vergata"

sbragaglia@roma2.infn.it

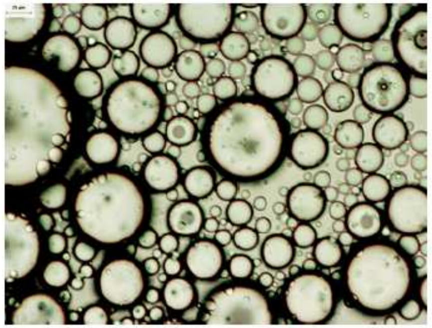
http://people.roma2.infn.it/sbragaglia

Credits:

R. Benzi, M. Lulli (Dept. Physics, University of Rome "Tor Vergata", Italy)
A. Scagliarini (Helmholtz Institute, Nunberg, Germany)
S. Succi, M. Bernaschi (IAC-CNR, Rome, Italy)

Complex Fluids & Soft Glassy Materials

Emulsions, Foams, Gels, Pastes, etc



Composition

- Multiple Phases
- Multiple Components
- Often Both

Structural Complexity

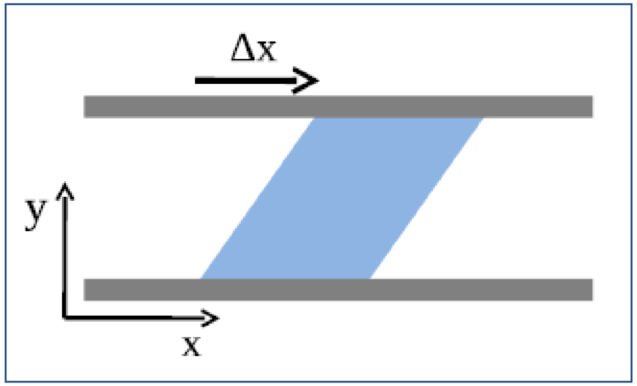
- Dynamical Arrest
- Clustering Networks
- Heterogeneities

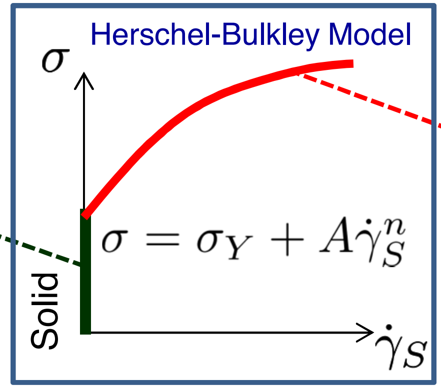
Rheological Complexity

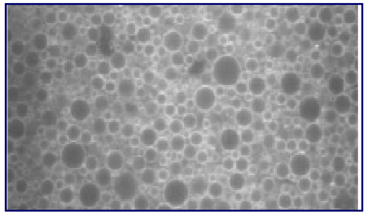
- Yield Stress Fluids
- Viscoelasticity
- Cooperativity

Flow-Curves of Soft Glassy Materials

Solid Behaviour

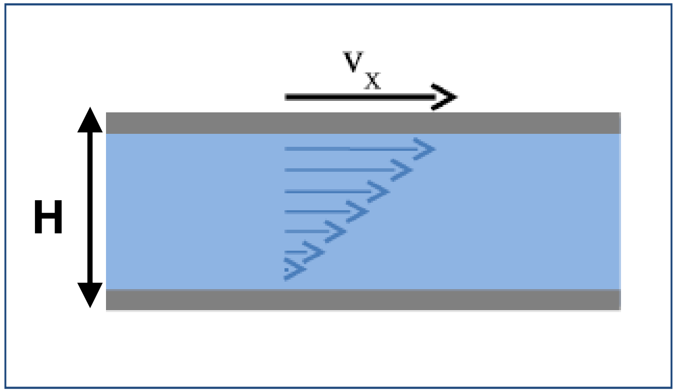






(e.g. concentrated emulsion)

Fluid Behaviour



Hookean (Elastic) solid displaced by a distance Δx :

The Stress needed to do this is:

$$\sigma = G\frac{\Delta x}{H} = G\gamma_S$$

γ_S : Strain G : Storage Modulus

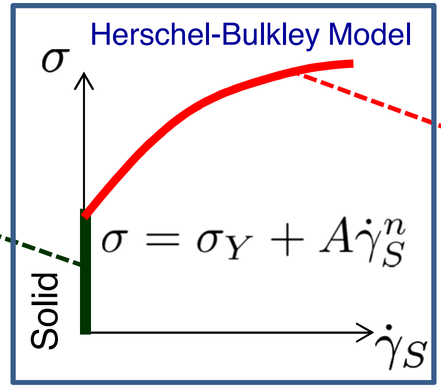
Viscous Fluid driven by a constant velocity :

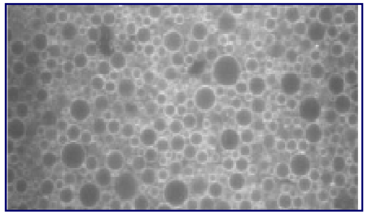
The Stress needed to do this is:

$$\sigma = \eta\frac{\partial v_x}{\partial y} = \eta\frac{v_x}{H} = \eta\dot{\gamma}_S$$

$\dot{\gamma}_S$: Shear η : Viscosity

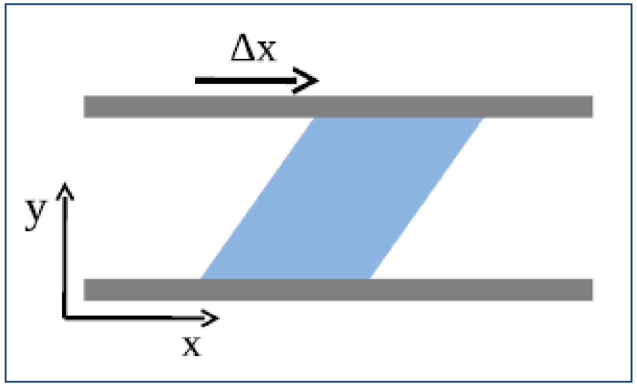
Flow-Curves of Soft Glassy Materials



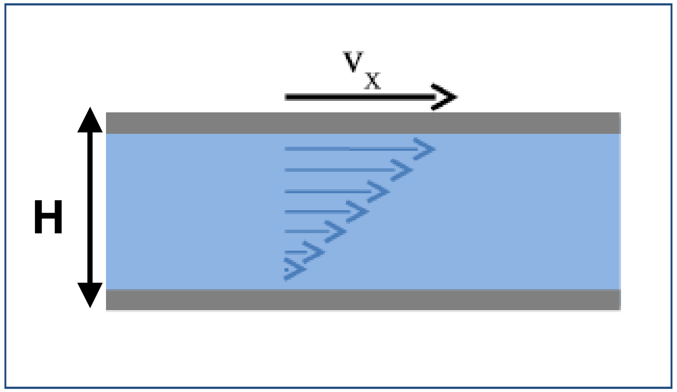


(e.g. concentrated emulsion)

Solid Behaviour



Fluid Behaviour



Hookean (Elastic) solid displaced by a distance Δx :

The Stress needed to do this is:

$$\sigma = G\frac{\Delta x}{H} = G\gamma_S$$

γ_S : Strain G : Storage Modulus

Viscous Fluid driven by a constant velocity :

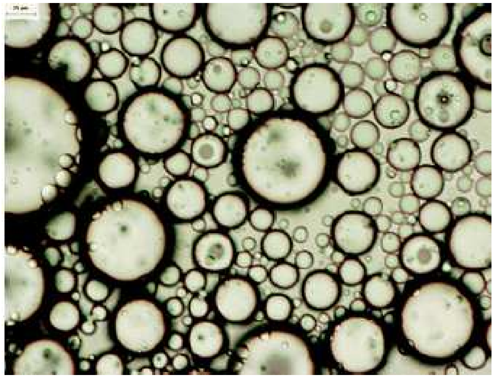
The Stress needed to do this is:

$$\sigma = \eta\frac{\partial v_x}{\partial y} = \eta\frac{v_x}{H} = \eta\dot{\gamma}_S$$

$\dot{\gamma}_S$: Shear η : Viscosity

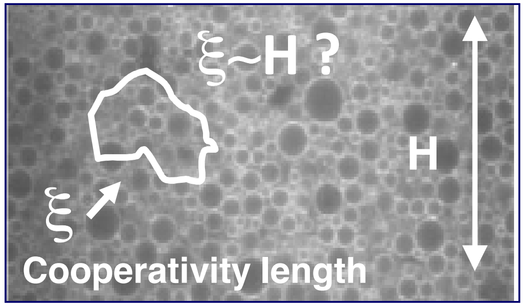
How does motion take place at the level of **Micro-Structural Constituents** (i.e. Droplets) **?**

Soft Glassy Materials & Fluidity Models



Droplets of a dispersed phase in a continuous phase

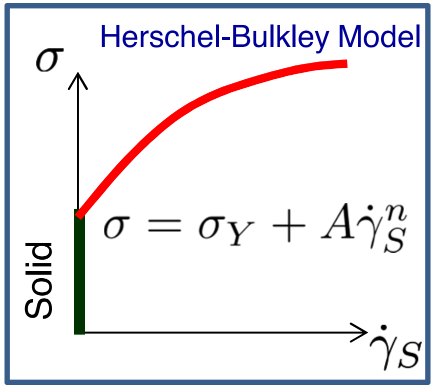
concentrated emulsion



✓ **Yield Stress** (Solid below)

✓ Non-Newonian Fluid (Above Yield)

✓ Effects of **Confinement** ?



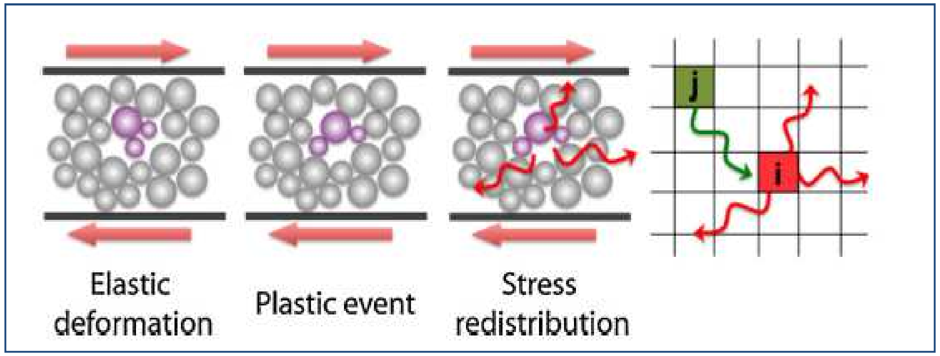
$\dot{\gamma}_S$: Shear σ : Stress

σ_Y : Yield Stress

ξ : **Cooperativity** length

$f = \frac{\dot{\gamma}_S}{\sigma}$ **Fluidity**

Bocquet, Ajdari, Colin, Phys Rev. Lett **103**, 0360011 (2009)



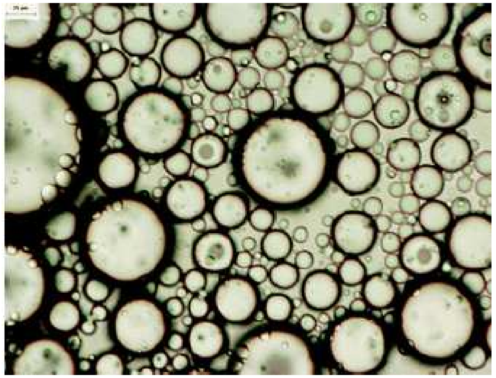
$$\xi^2 \Delta f + (f_b - f) = 0$$

Relaxation-Diffusion equation for the Fluidity

$$f_b = f_b(\sigma(\vec{r}))$$

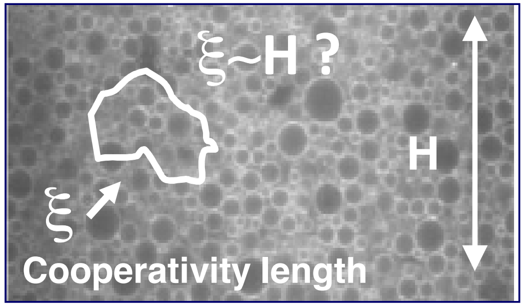
"Bulk" Fluidity

Soft Glassy Materials & Fluidity Models

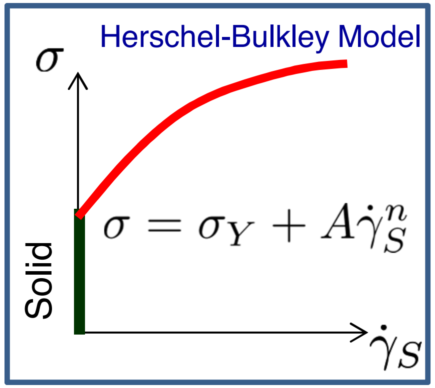


Droplets of a dispersed phase
in a continuous phase

concentrated emulsion



- ✓ **Yield Stress** (Solid below)
- ✓ Non-Newonian Fluid (Above Yield)
- ✓ Effects of **Confinement** ?



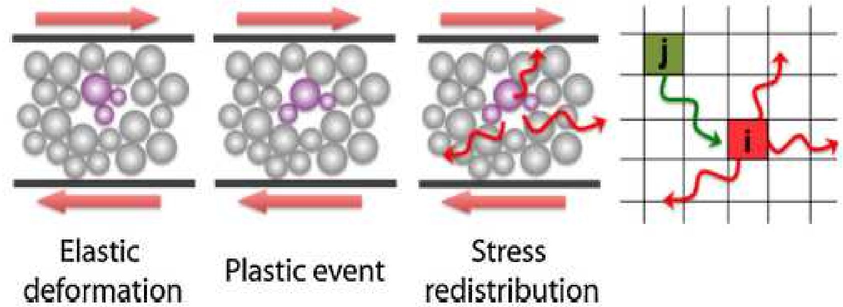
$\dot{\gamma}_S$: Shear σ : Stress

σ_Y : Yield Stress

ξ : **Cooperativity** length

$f = \frac{\dot{\gamma}_S}{\sigma}$ **Fluidity**

Bocquet, Ajdari, Colin, Phys Rev. Lett **103**, 0360011 (2009)



- ✓ **Ab Initio** Proof of "Plasic Flows" ?!?
- ✓ **Test** Fluidity Models ?!?
- ✓ **Extend** the picture of plastic flows ?!?

$$\xi^2 \Delta f + (f_b - f) = 0$$

Relaxation-Diffusion equation for the Fluidity

$$f \propto \Gamma$$

Proportionality Between Fluidity and
RATE of Plastic Events (Γ)

Where to Sit Across the scales?


MACROSCOPIC

(NAVIER-STOKES)

$$\vec{\nabla} \cdot \vec{u} = 0$$

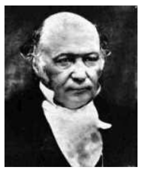
$$\rho D_t \vec{u} = -\vec{\nabla} p + \eta \Delta \vec{u}$$

....

Plastic Flows of Emulsions Live – **by definition** - at **Mesoscales**

Can we handle the Problem with a **Mesoscopic Method**?

YES !!! A possibility is the **Lattice Boltzmann Method**



MICROSCOPIC

(MOLECULAR DYNAMICS)

$$\mathcal{H} = \sum_k \frac{p_k^2}{2m} + \sum_{i,k} V_{i,k}$$

$$\dot{q}_k = \frac{\partial \mathcal{H}}{\partial p_k} \qquad \dot{p}_k = -\frac{\partial \mathcal{H}}{\partial q_k}$$

Where to Sit Across the scales?

MACROSCOPIC

(NAVIER-STOKES)

$$\vec{\nabla} \cdot \vec{u} = 0$$

$$\rho D_t \vec{u} = -\vec{\nabla} p + \eta \Delta \vec{u}$$

....

Continuum Description

Chapman-Enskog
(Multi-Scale Lattice Theory)

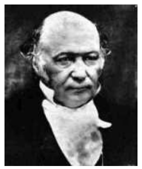
MESOSCOPIC

(LATTICE BOLTZMANN)

$$n_i(\vec{r} + \vec{c}_i, t+1) = n_i(\vec{r}, t) + \Delta_i$$

Kinetic '*Inspired*' Equations

Supra-molecular
Framework for lattice
interactions



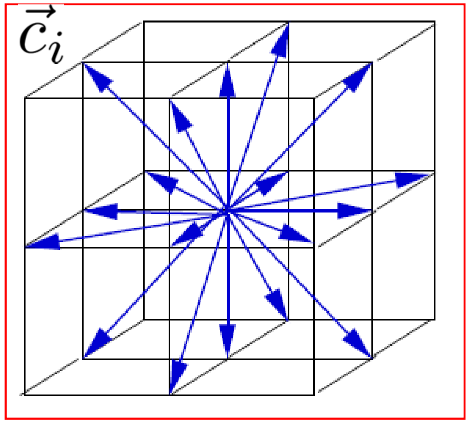
MICROSCOPIC

(MOLECULAR DYNAMICS)

$$\mathcal{H} = \sum_k \frac{p_k^2}{2m} + \sum_{i,k} V_{i,k}$$

$$\dot{q}_k = \frac{\partial \mathcal{H}}{\partial p_k} \qquad \dot{p}_k = -\frac{\partial \mathcal{H}}{\partial q_k}$$

Lattice Boltzmann: Basics & Conservation Laws



- Linearized Boltzmann Equation
- Fully Discretized (time and space)
- Sites $\vec{r}$, and Lattice Velocities $\vec{c}_i$
- Time t, time step $\Delta t = 1$

$$n_i(\vec{r} + \vec{c}_i, t+1) = n_i(\vec{r}, t) + \Delta_i\{n_i(\vec{r}, t)\}$$

"Streaming"

"Local **Relaxation** towards equilibrium"
Example: single time (τ) relaxation

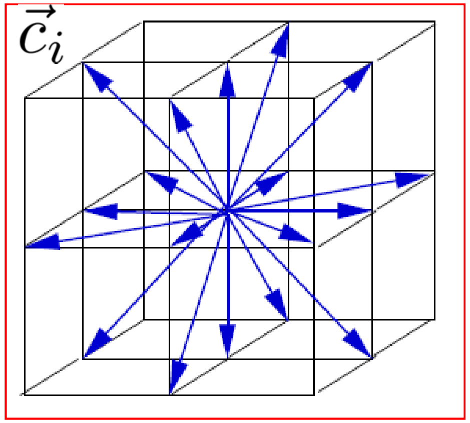
$$\Delta_i\{n_i(\vec{r}, t)\} \propto -\frac{1}{\tau}(n_i - n_i^{(eq)})$$

- $\vec{c}_i$ **Small Set** of Velocities
- $\vec{c}_i$ Connects 2 sites…**exact streaming** preserved
- $n_i(\vec{r}, t)$ Real Number: particle pdf on space-time location $(\vec{r}, t)$

Succi, *"The Lattice Boltzmann Equation: for fluid dynamics and beyond"* (**2001**)
Wolf Gladrow, "Lattice Gas Cellular Automata and LBM: an introduction" (**2000**)

How to access coarse grained variables?

Lattice Boltzmann: Basics & Conservation Laws



- Linearized Boltzmann Equation
- Fully Discretized (time and space)
- Sites $\vec{r}$, and Lattice Velocities $\vec{c}_i$
- Time t, time step $\Delta t = 1$

$$n_i(\vec{r} + \vec{c}_i, t+1) = n_i(\vec{r}, t) + \Delta_i\{n_i(\vec{r}, t)\}$$

$$\sum_i \Delta_i = \sum_i \vec{c}_i \Delta_i = 0$$

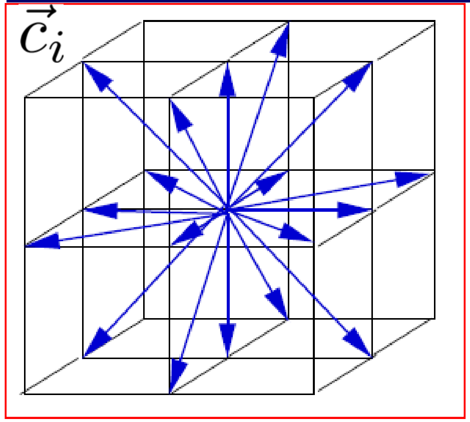
Mass Conservation!
Momentum Conservation!!
Locality !!!

Coarse Grained Fields → Hydrodynamic Equations

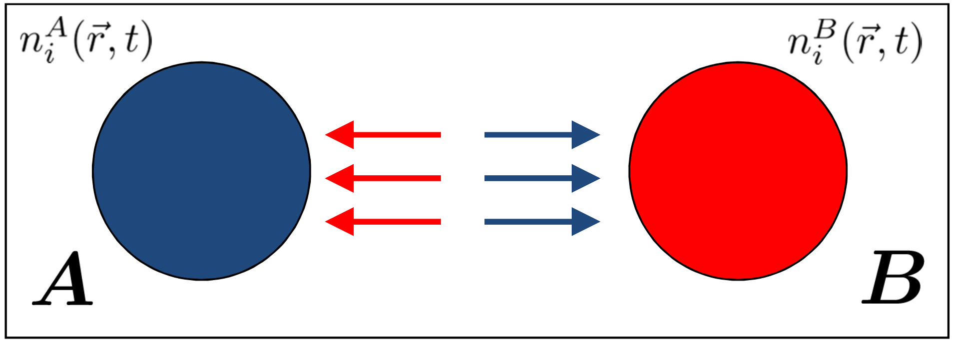
$$\rho(\vec{r}, t) = \sum_i n_i(\vec{r}, t)$$ (Density)

$$\vec{u}(\vec{r}, t) = \frac{1}{\rho(\vec{r}, t)} \sum_i \vec{c}_i n_i(\vec{r}, t)$$ (Velocity)

Non Ideal Lattice Fluids: Phase Segregation



How ?



$$\vec{F}(\vec{r}) = -g_{AB}\rho_A(\vec{r})\sum_i w_i \rho_B(\vec{r}+\vec{c}_i)\vec{c}_i + (A \leftrightarrow B)$$

LBM Phase Segregating Interactions

(g_{AB} is a coupling strength)

Interaction Range **Crucial** for OUR Application

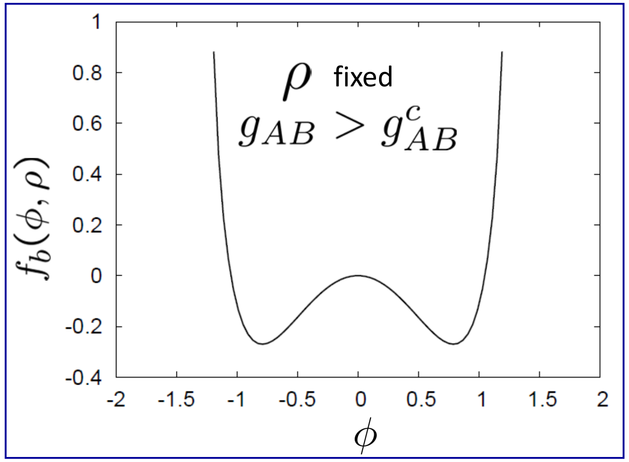
$$\rho = \rho_A + \rho_B$$

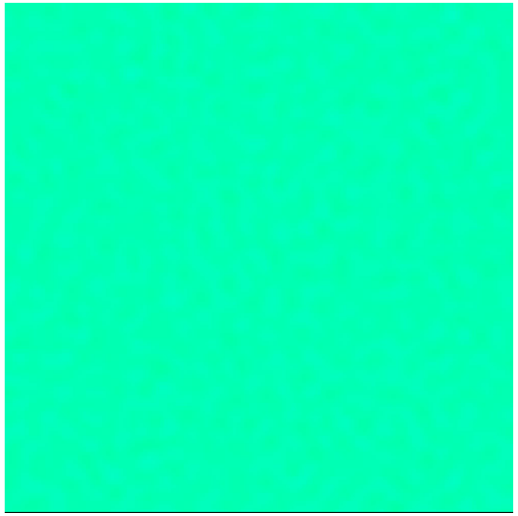
Total Density

$$\phi = \rho_A - \rho_B$$

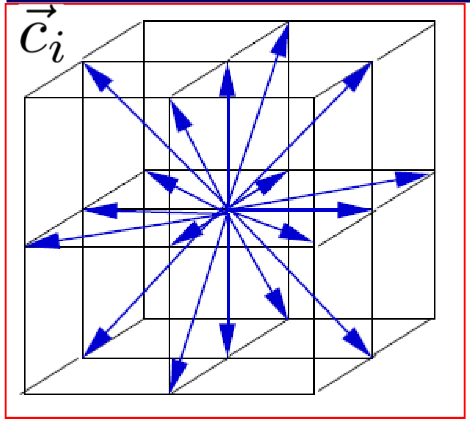
Order Parameter

Model (bulk) Free Energy

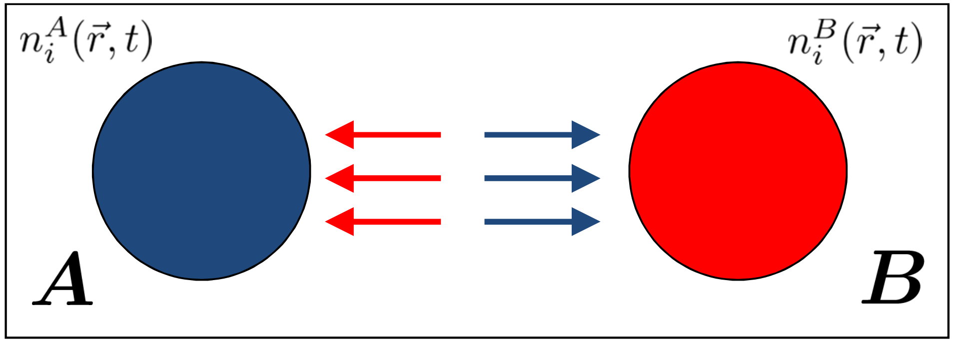




Non Ideal Lattice Fluids: Phase Segregation



How ?



$$\vec{F}(\vec{r}) = -g_{AB}\rho_A(\vec{r})\sum_i w_i \rho_B(\vec{r}+\vec{c}_i)\vec{c}_i + (A \leftrightarrow B)$$

LBM Phase Segregating Interactions
(g_{AB} is a coupling strength)

Interaction Range **Crucial** for OUR Application

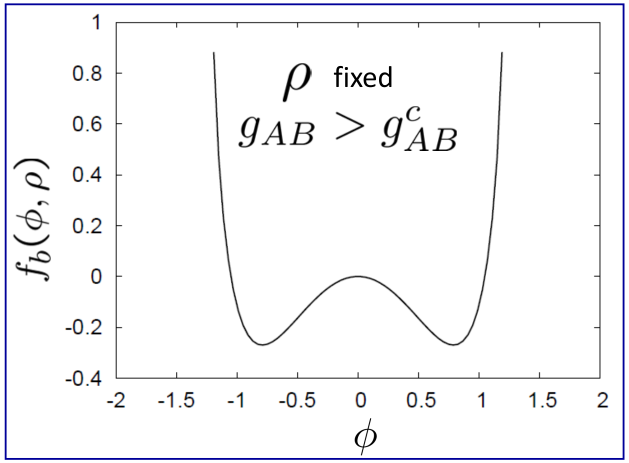
$$\rho = \rho_A + \rho_B$$

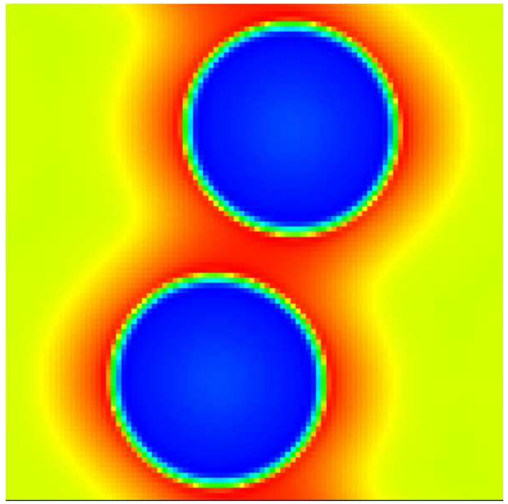
Total Density

$$\phi = \rho_A - \rho_B$$

Order Parameter

Model (bulk) Free Energy

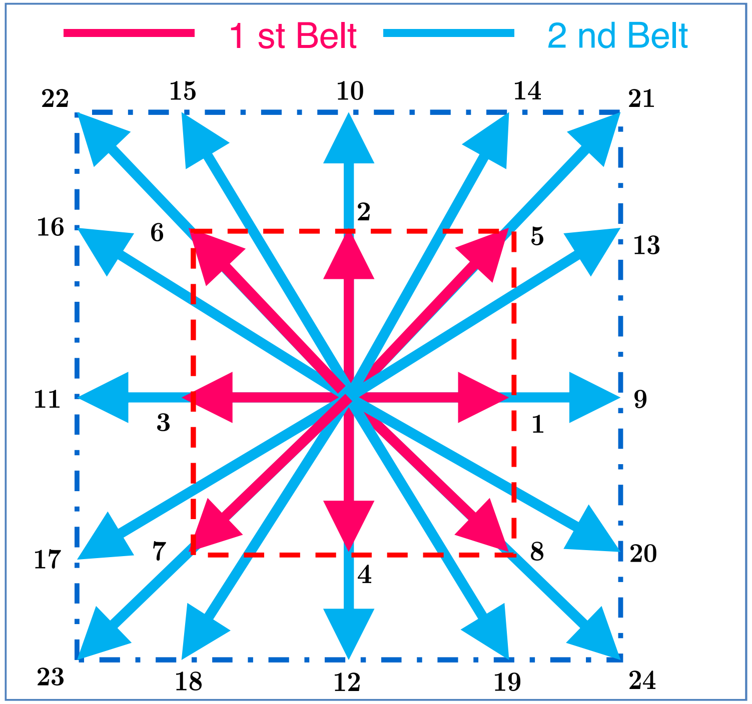




(Lattice) Disjoining Pressure (Π): How ?

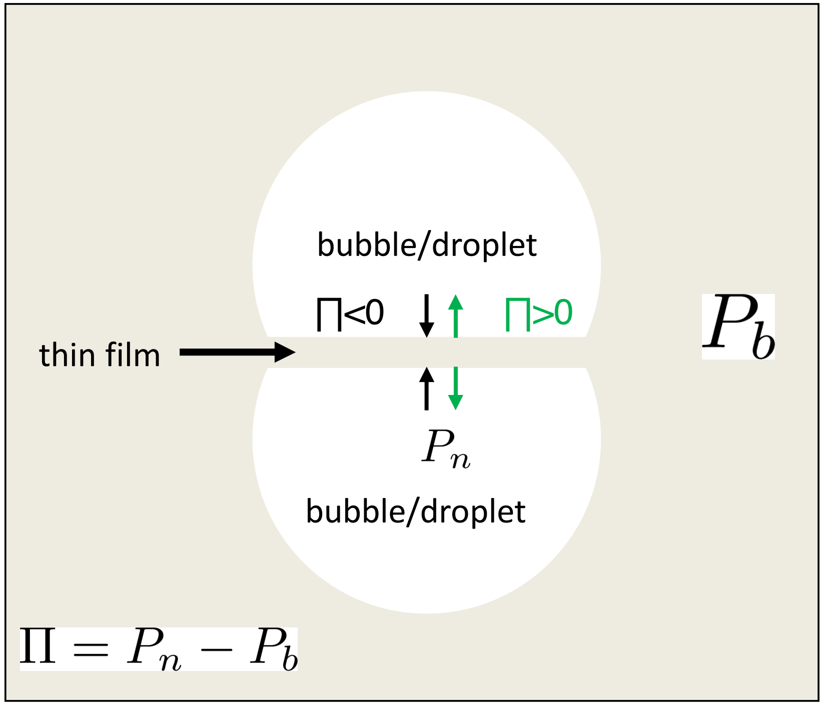
"Multirange" Potentials in LBM:

Competing Nearest Neighbours (NN) +
Next to Nearest Neighbours (NNN) interactions



"Frustration" Mechanism :

"Frustrate" the tendency of the interface to minimize the area...**does it work ?**



$$\vec{F}(\vec{r}) = -\mathcal{G}_1 \psi(\vec{r}) \sum_{i \in NN} \psi(\vec{r}+\vec{c}_i)\vec{c}_i - \mathcal{G}_2 \psi(\vec{r}) \sum_{i \in NNN} \psi(\vec{r}+\vec{c}_i)\vec{c}_i$$

$$\psi(\vec{r}) = \psi[\rho(\vec{r})]$$

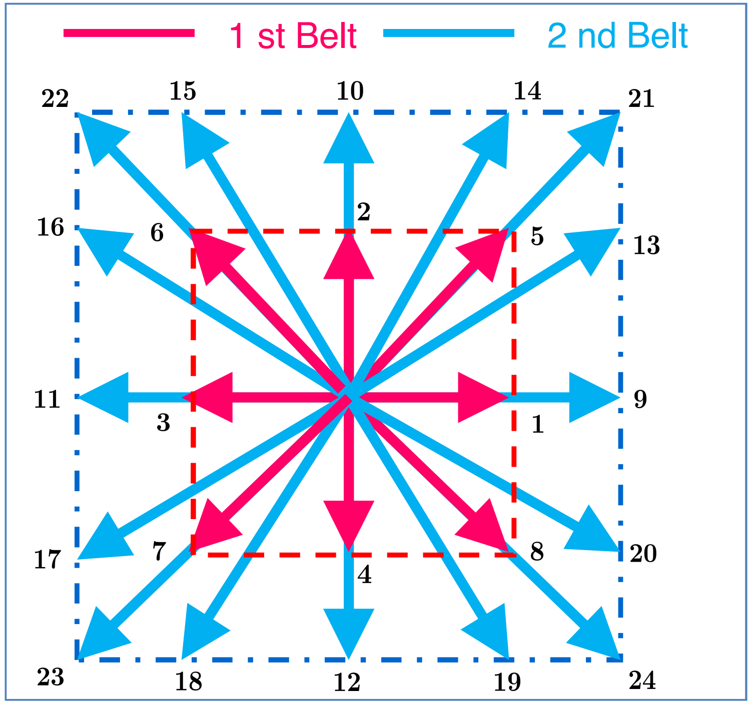
Pseudo-Potential

J.D. Shore, M. Holzer, J.P. Sethna, *Phys. Rev. B* **46**, 11376 (1992)
M. Sbragaglia, R. Benzi, M. Bernaschi & S. Succi, *Soft Matter* ***8***, 10773-10782 (2012)

(Lattice) Disjoining Pressure (Π): How ?

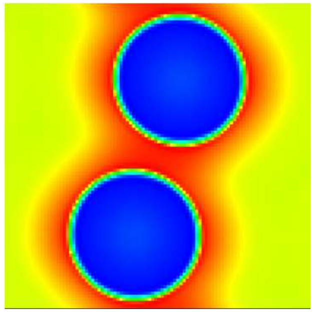
"Multirange" Potentials in LBM:

Competing Nearest Neighbours (NN) +
Next to Nearest Neighbours (NNN) interactions

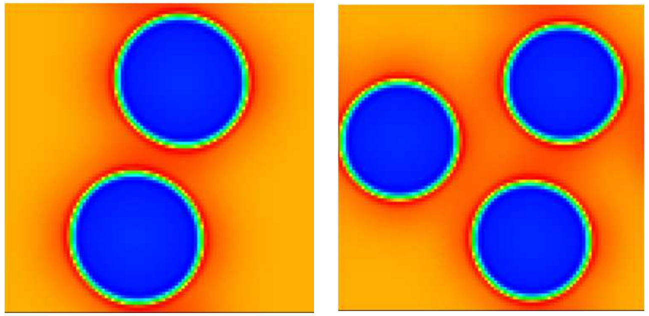


"Frustration" Mechanism :

"Frustrate" the tendency of the interface to minimize the area...**YES!!!!**



Droplets Coalescence (**Traditional LBM** implementation)



Inhibition of Coalescence (**Multirange LBM**)

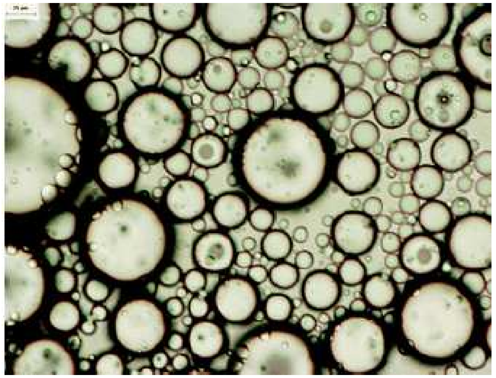
$$\vec{F}(\vec{r}) = -\mathcal{G}_1 \psi(\vec{r}) \sum_{i \in NN} \psi(\vec{r}+\vec{c}_i)\vec{c}_i - \mathcal{G}_2 \psi(\vec{r}) \sum_{i \in NNN} \psi(\vec{r}+\vec{c}_i)\vec{c}_i \qquad \psi(\vec{r}) = \psi[\rho(\vec{r})]$$

Pseudo-Potential

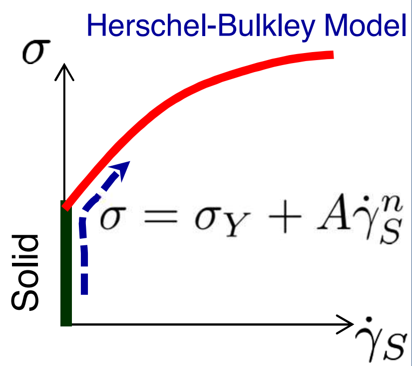
J.D. Shore, M. Holzer, J.P. Sethna, *Phys. Rev. B* **46**, 11376 (1992)
M. Sbragaglia, R. Benzi, M. Bernaschi & S. Succi, *Soft Matter* ***8***, 10773-10782 (2012)

Yield Stress Fluids & Plastic Rearrangements

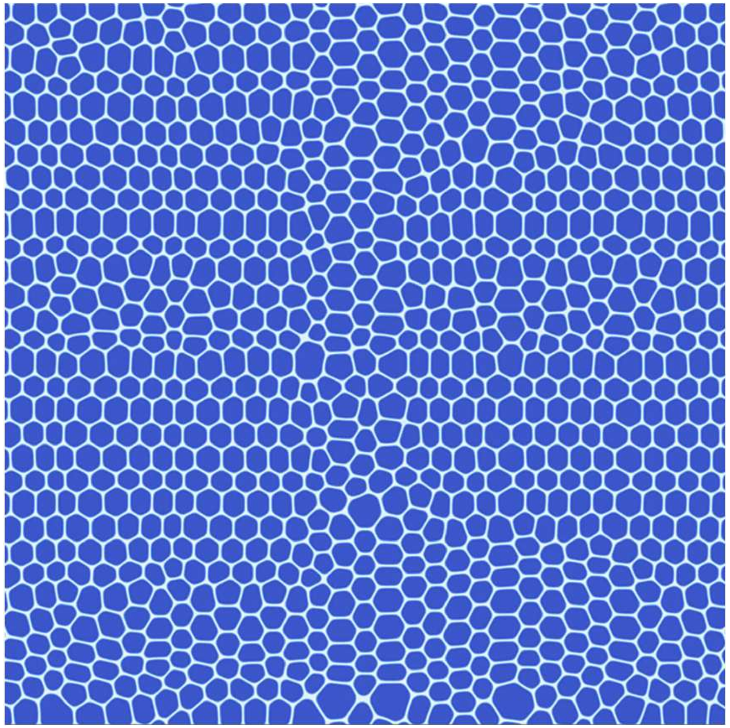
R. Benzi, M. Sbragaglia, P. Perlekar, M. Bernaschi, S. Succi & F. Toschi, *Soft Matter* **10**, 4615-4624 (2014)



Droplets of a dispersed phase in a continuous phase



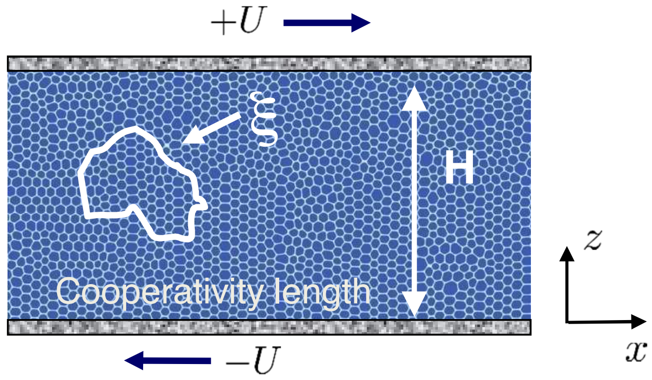
- ✓Yield Stress (Solid below)
- ✓Non-Newtonian (above yield)
- ✓Cooperativity Effects & Plastic Rearrangements

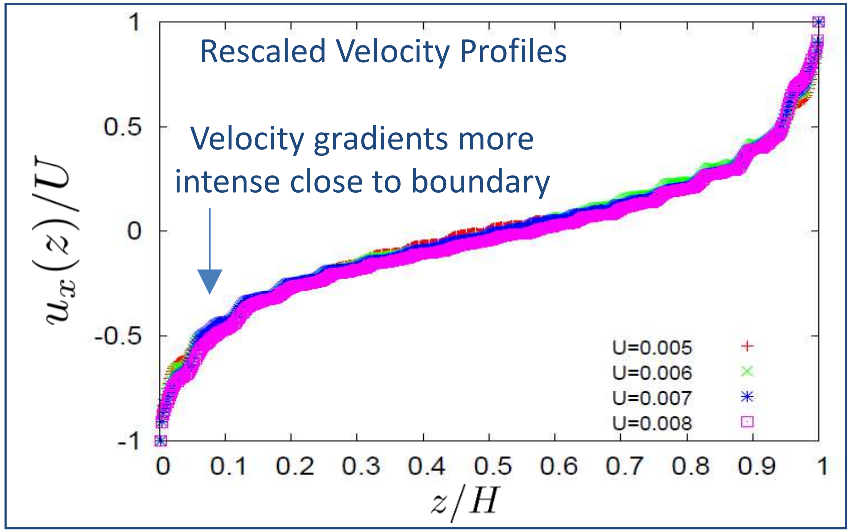


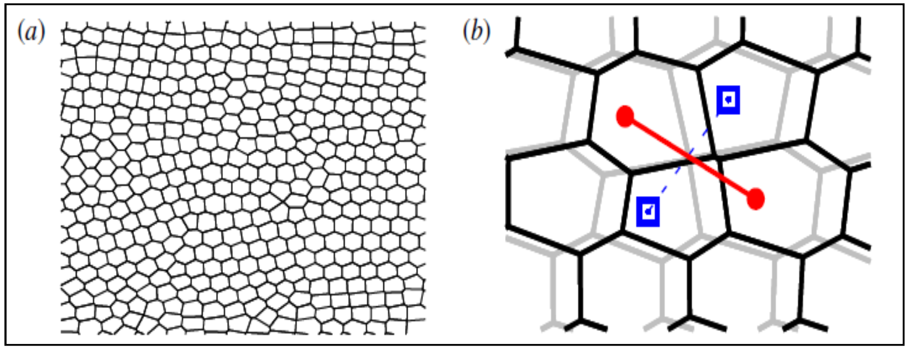
Kolmogorov Flow (full periodic system)
step-by-step increase of the driving force **(solid to liquid transition)**

Testing Fluidity Models with LBM

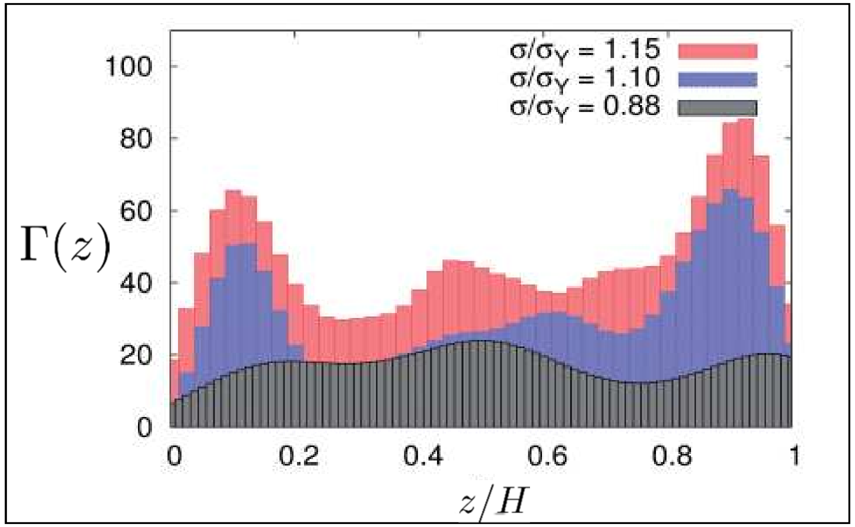
M. Sbragaglia, R. Benzi, P. Perlekar, M. Bernaschi, S. Succi & F. Toschi, *Soft Matter* ***10***, 4514-4524 (2014)





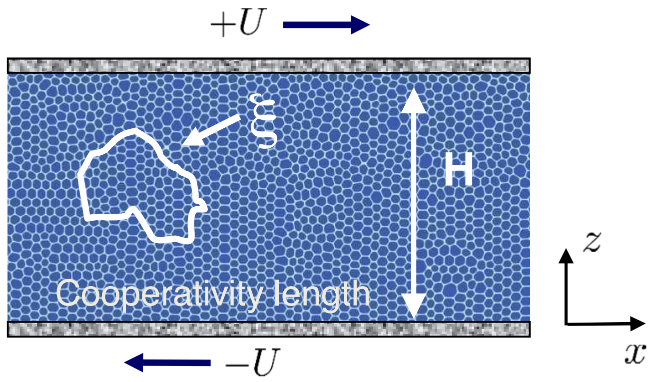


Rate (Γ) of plastic rearrangements estimated from Voronoi Tesselation



Testing Fluidity Models with LBM

M. Sbragaglia, R. Benzi, P. Perlekar, M. Bernaschi, S. Succi & F. Toschi, *Soft Matter* ***10***, 4514-4524 (2014)

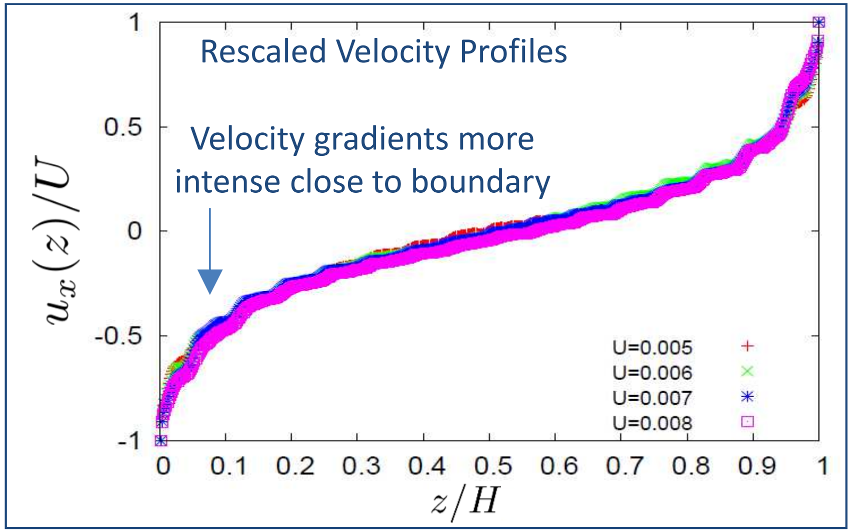


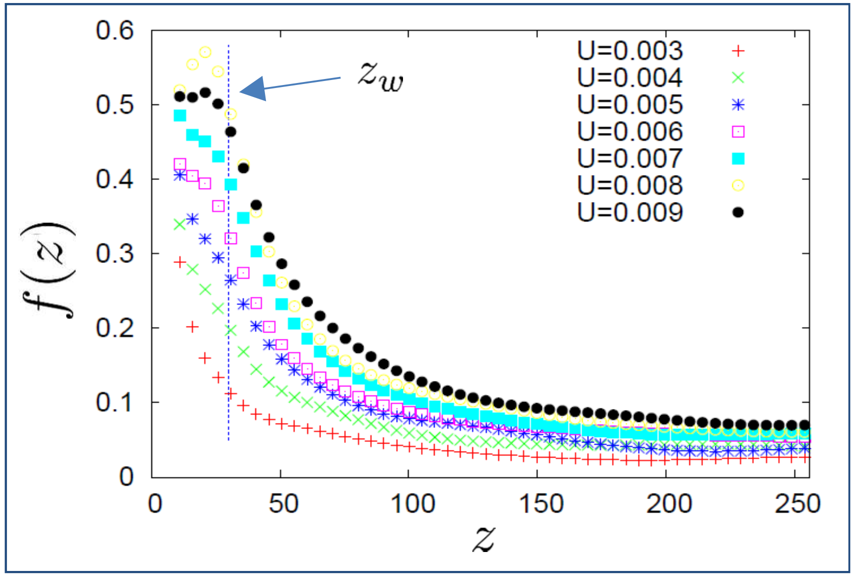
$$f = \frac{\dot{\gamma}_S}{\sigma}$$

Fluidity

$$\xi^2 \frac{d^2 f(z)}{dz^2} + (f_b - f(z)) = 0$$

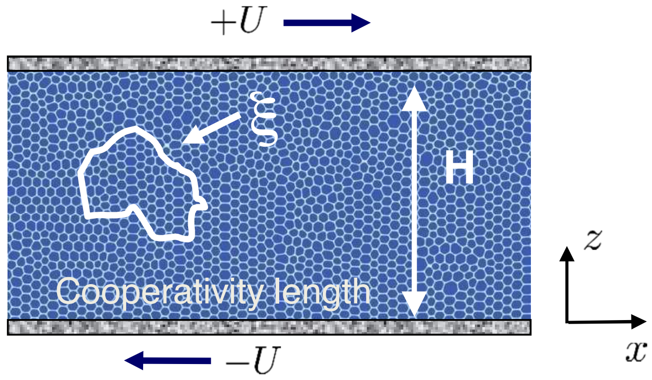
Relaxation-Diffusion equation for the Fluidity





Testing Fluidity Models with LBM

M. Sbragaglia, R. Benzi, P. Perlekar, M. Bernaschi, S. Succi & F. Toschi, *Soft Matter* ***10***, 4514-4524 (2014)

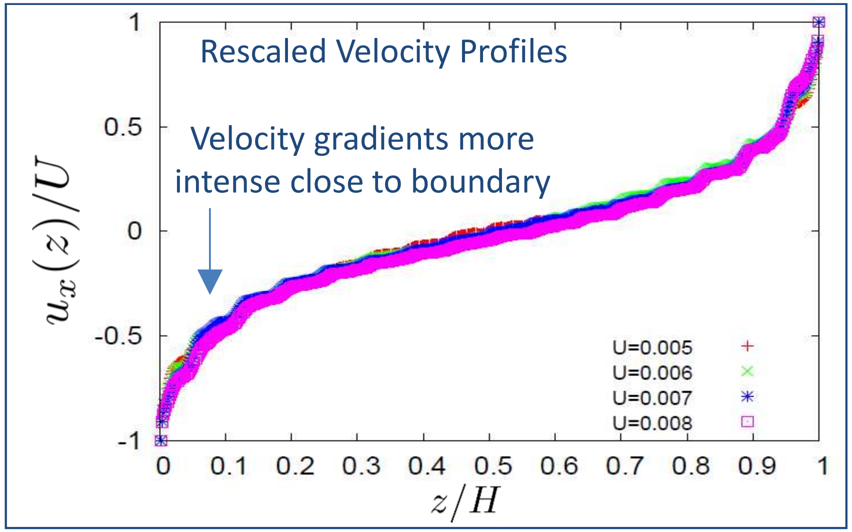


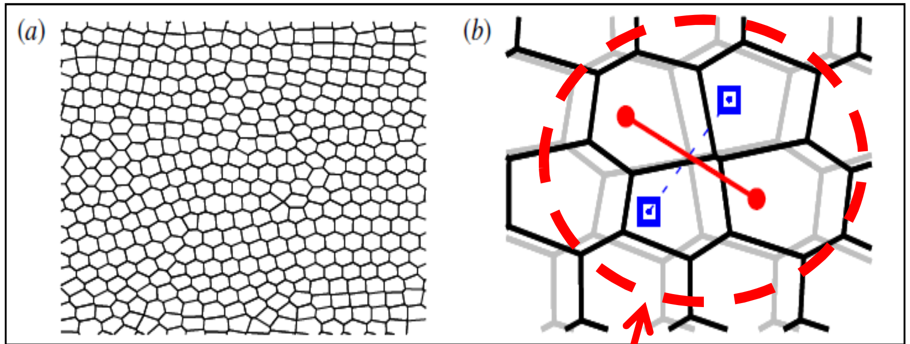
$$f = \frac{\dot{\gamma}_S}{\sigma}$$

Fluidity

$$\frac{(f(z) - f_b)}{(f_w - f_b)} = e^{-(z - z_w)/\xi}$$

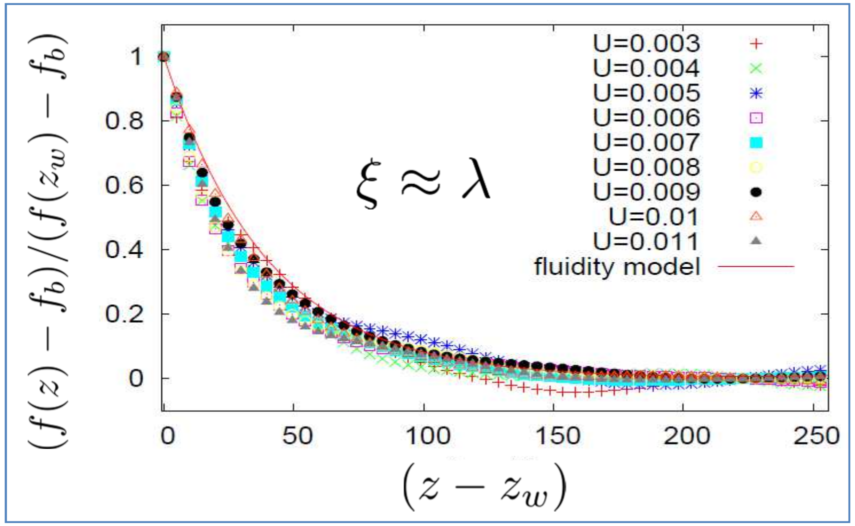
Exact Solution for Couette Flow
(f_b is constant)





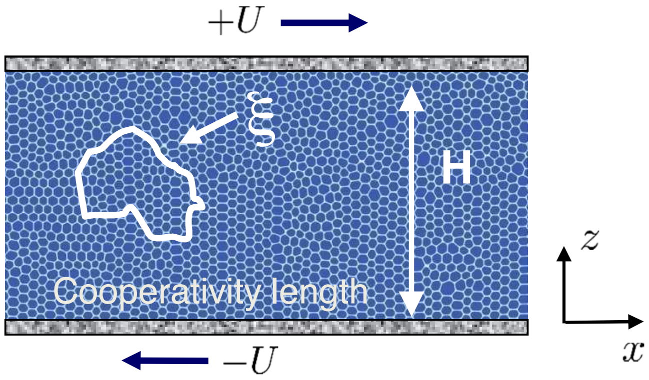
Characteristic size of plastic rearrangements estimated from Voronoi Tesselation

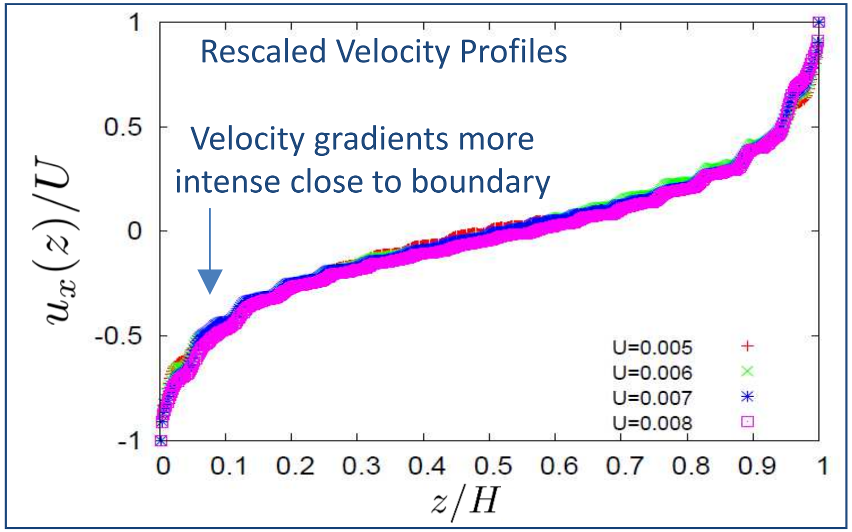
$$A \sim \pi\lambda^2$$



Testing Fluidity Models with LBM

M. Sbragaglia, R. Benzi, P. Perlekar, M. Bernaschi, S. Succi & F. Toschi, *Soft Matter* ***10***, 4514-4524 (2014)





$$f = \frac{\dot{\gamma}_S}{\sigma}$$

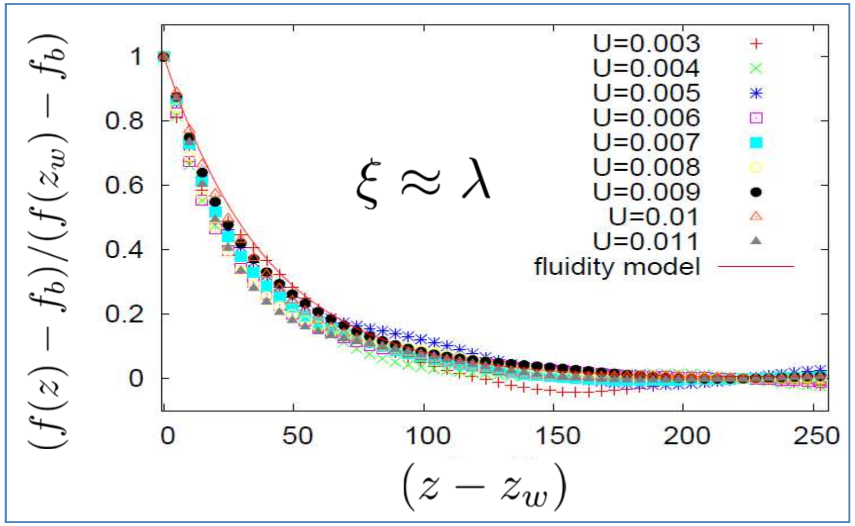
Fluidity

$$\frac{(f(z) - f_b)}{(f_w - f_b)} = e^{-(z - z_w)/\xi}$$

Exact Solution for Couette Flow

(f_b is constant)

f_w ← Wall Fluidity



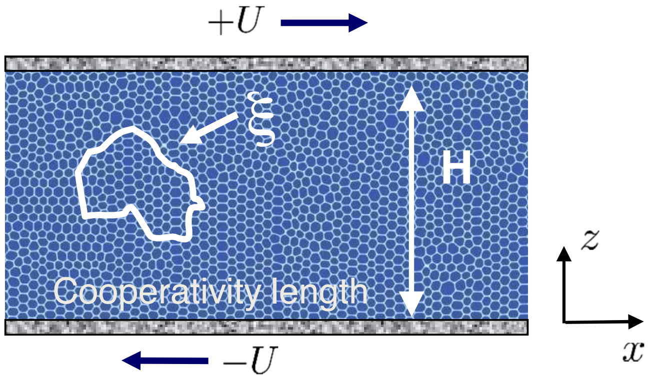
- ✓ **Wall Fluidity**: "Phenomenological" Parameter
- ✓ **Importance:** Engineering appropriate surface
- ✓ **Quantify** the Impact of:

  1) Wettability J. Paredes, *Phys. Rev. E* ***95***, 042313 (2015)

  2) Roughness M. Sbragaglia et al., *Europhys. Lett.* ***114***, 64003 (2016)

Testing Fluidity Models with LBM

M. Sbragaglia, R. Benzi, P. Perlekar, M. Bernaschi, S. Succi & F. Toschi, *Soft Matter* ***10***, 4514-4524 (2014)



$$f = \frac{\dot{\gamma}_S}{\sigma}$$

Fluidity

$$\frac{(f(z) - f_b)}{(f_w - f_b)} = e^{-(z - z_w)/\xi}$$

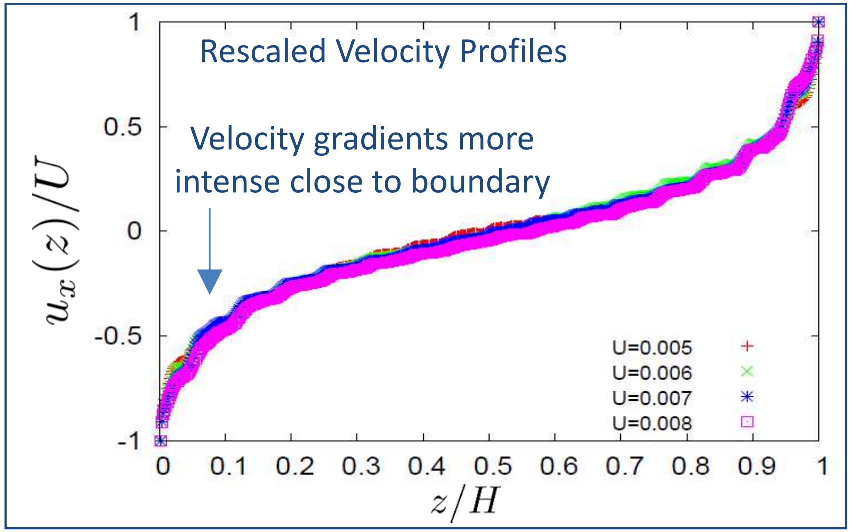
Exact Solution for Couette Flow (f_b is constant)

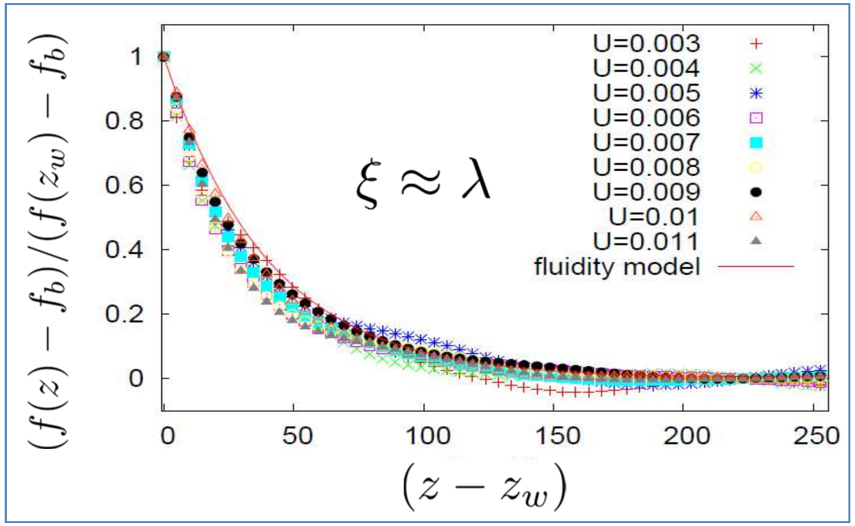
f_w ← Wall Fluidity

- ✓ **Wall Fluidity**: "Phenomenological" Parameter
- ✓ **Importance:** Engineering appropriate surface
- ✓ **Quantify** the Impact of:

1) Wettability J. Paredes, *Phys. Rev. E* ***95***, 042313 (2015)

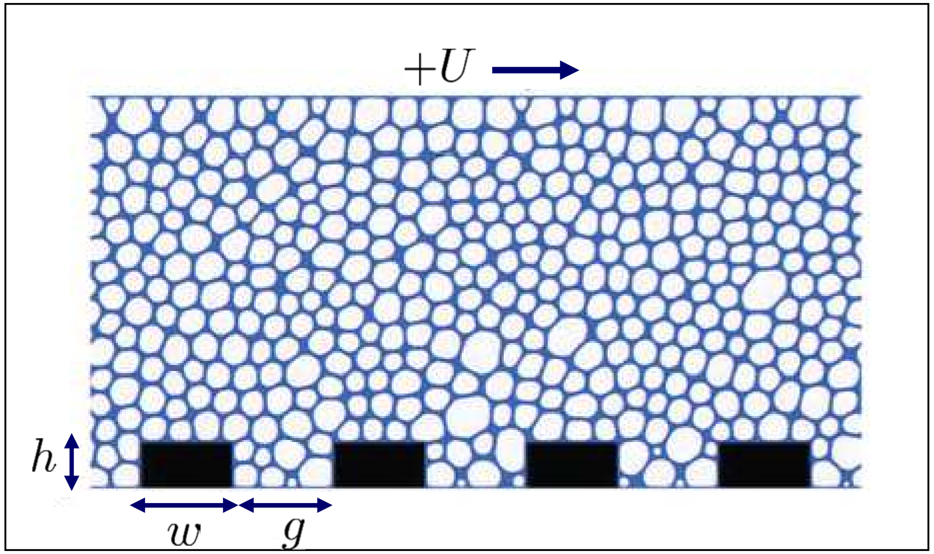
→ 2) Roughness M. Sbragaglia et al., *Europhys. Lett.* ***114***, 64003 (2016)





Rough Channels: Study of Boundary Fluidity

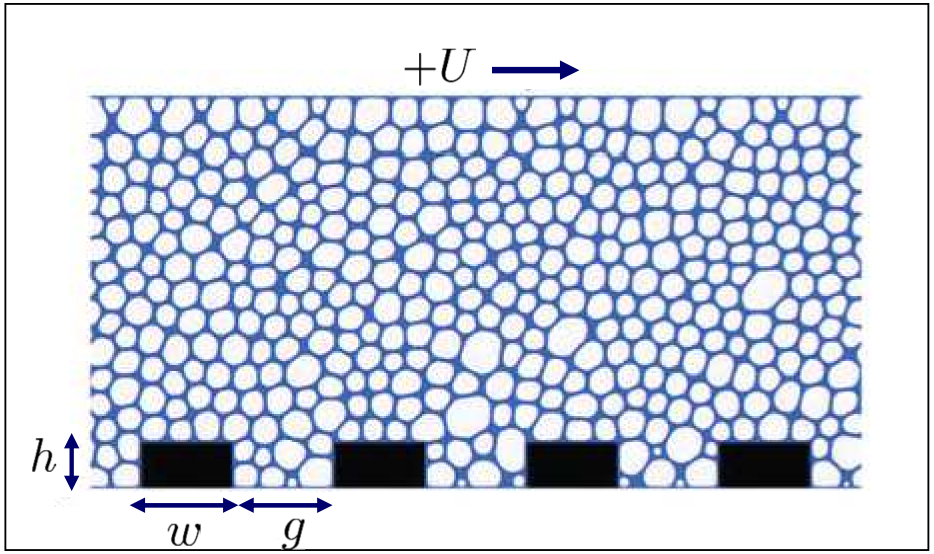
A. Scagliarini, M. Lulli, M. Sbragaglia and M. Bernaschi, *Europhys. Lett.* ***114***, 64003 (2016)



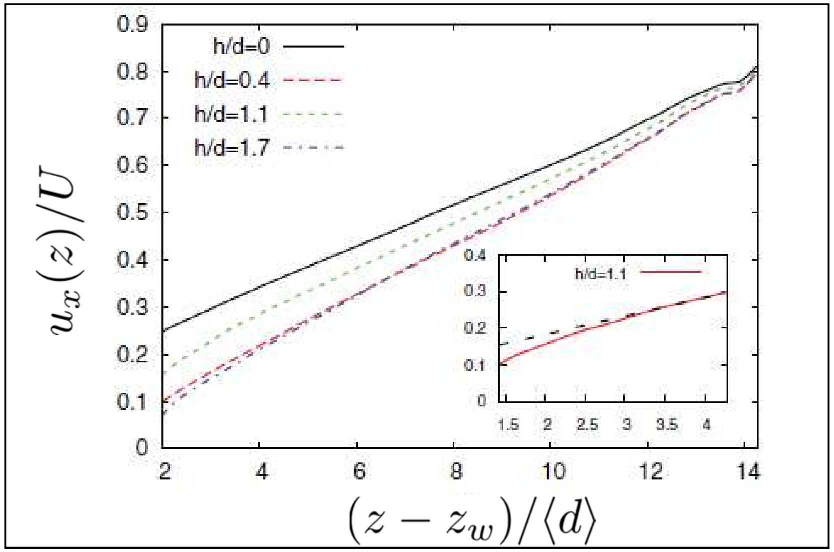
- ✓ Couette Cell with **controlled** Roughness
- ✓ **Fixed** Gap (g) and Width (w)
- ✓ **Variable** height (h)
- ✓ Interplay between **h** and droplet diameter **<d>**

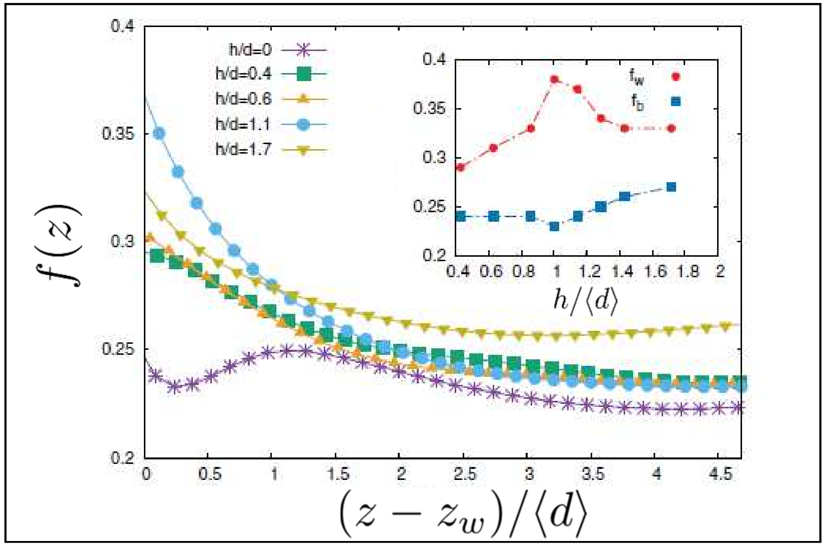
Rough Channels: Study of Boundary Fluidity

A. Scagliarini, M. Lulli, M. Sbragaglia and M. Bernaschi, *Europhys. Lett.* ***114***, 64003 (2016)



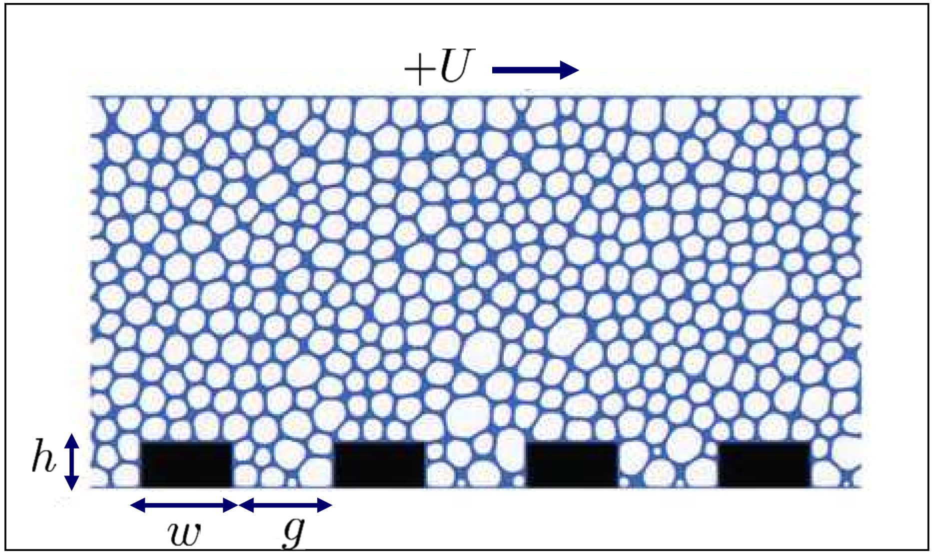
- ✓ Couette Cell with **controlled** Roughness
- ✓ **Fixed** Gap (g) and Width (w)
- ✓ **Variable** height (h)
- ✓ Interplay between **h** and droplet diameter **<d>**





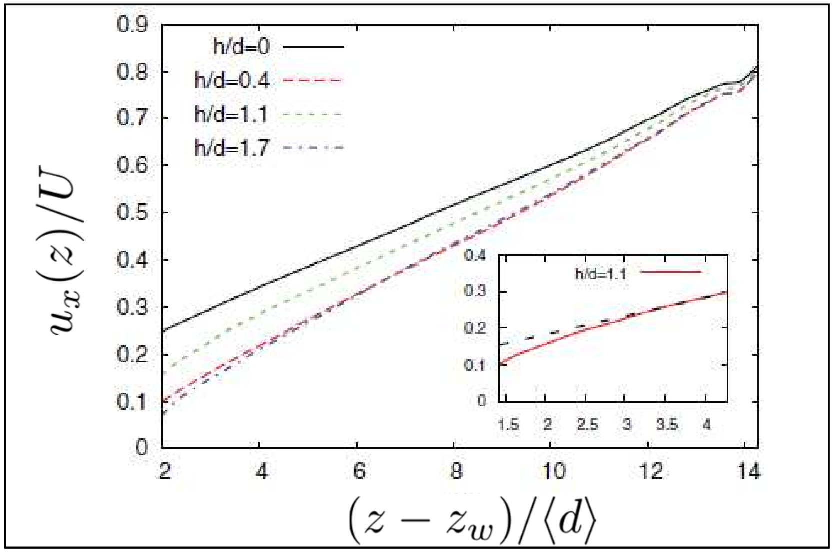
Rough Channels: Study of Boundary Fluidity

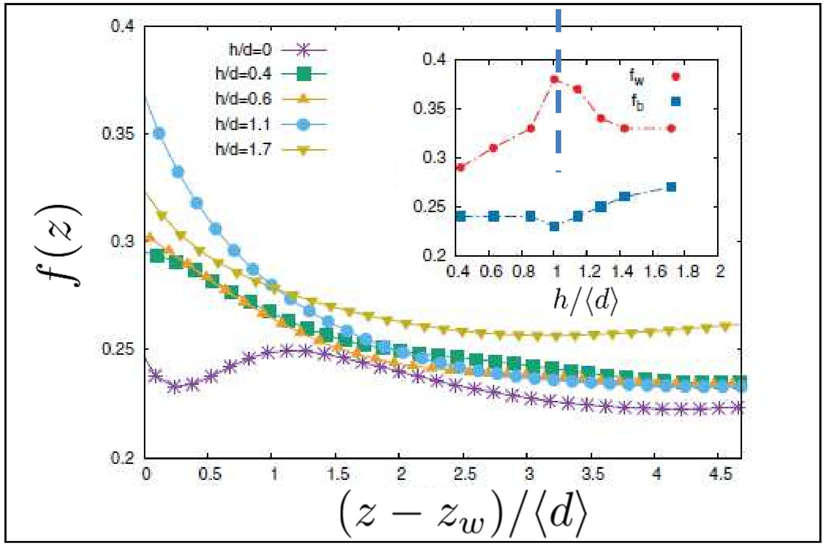
A. Scagliarini, M. Lulli, M. Sbragaglia and M. Bernaschi, *Europhys. Lett.* ***114***, 64003 (2016)



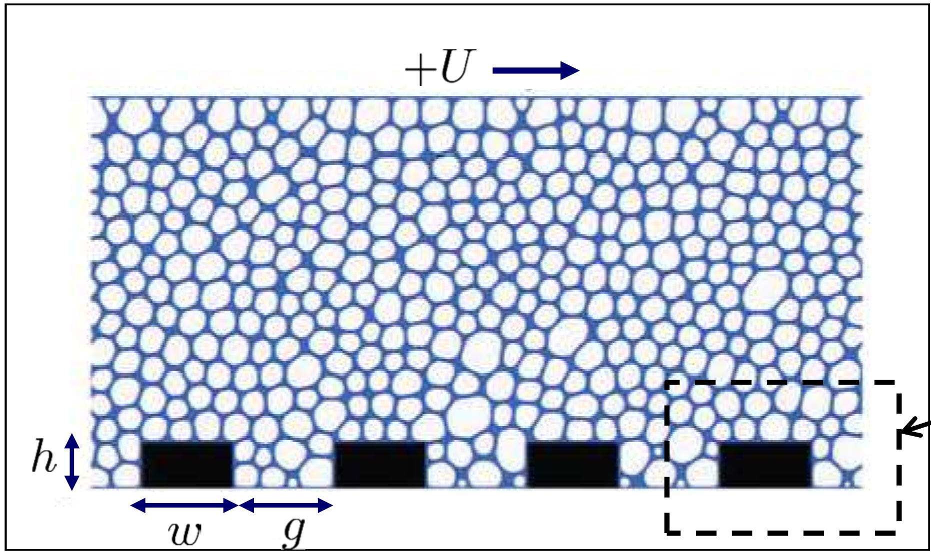
- ✓ Couette Cell with **controlled** Roughness
- ✓ **Fixed** Gap (g) and Width (w)
- ✓ **Variable** height (h)
- ✓ Interplay between **h** and droplet diameter **<d>**

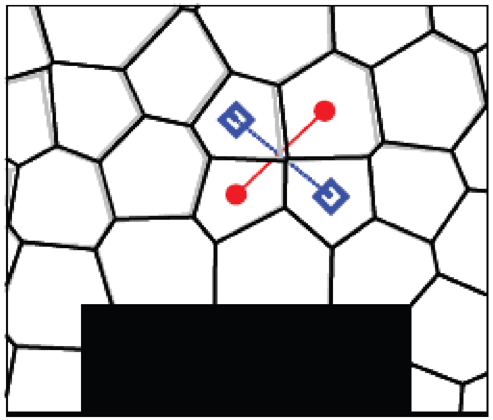
Optimal Height $h \approx \langle d \rangle$



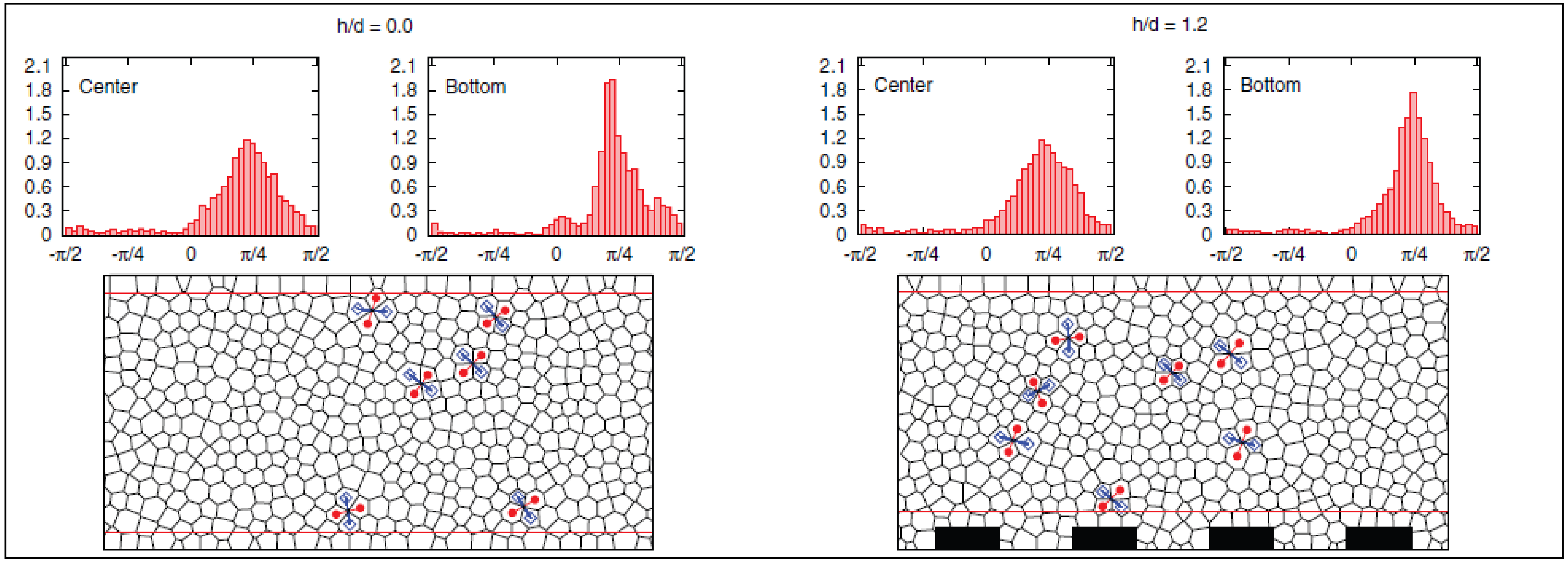


Orientational Statistics





M. Lulli, M. Sbragaglia *et al.*, *arXiv1607.00908* (2016)



Conclusions & Perspectives

LBM For Complex Hydrodynamics at Mesoscales

- ✓ Direct Link with Hydrodynamics (Chapman-Enskog Analysis)
- ✓ Flexibility in modelling Non-Ideal interactions
- ✓ Easy Handling of Boundary Conditions

Applications: Plastic Flows of Foams and Emulsions

- ✓ **Plastic Activity** of Jammed Systems and (corr)relation to **hydrodynamics**
- ✓ Impact of **Boundary Conditions**
- ✓ Impact of **Orientation** of Plastic Events (Quadrupoles not just "points")
- ✓ Importance of **Heterogeneities** within Fluidity Models (not shown…but see refs.)

References:

M. Sbragaglia, R. Benzi, M. Bernaschi & S. Succi, *Soft Matter* ***8**,* 10773-10782 (2012)

M. Sbragaglia, R. Benzi, M. Bernaschi & S. Succi, *Europhys. Lett.* ***104**,* 48006 (2013)

M. Sbragaglia, R. Benzi, P. Perlekar, M. Bernaschi, S. Succi & F. Toschi, *Soft Matter* ***10**,* 4514-4524 (2014)

B. Dollet, A. Scagliarini & M. Sbragaglia, *Jour. Fluid Mech.* ***766**, 556-589* (2015)

R. Benzi, M. Sbragaglia, M.Bernaschi, S. Succi & F. Toschi, *Soft Matter* **12**, 514-530 (2016)

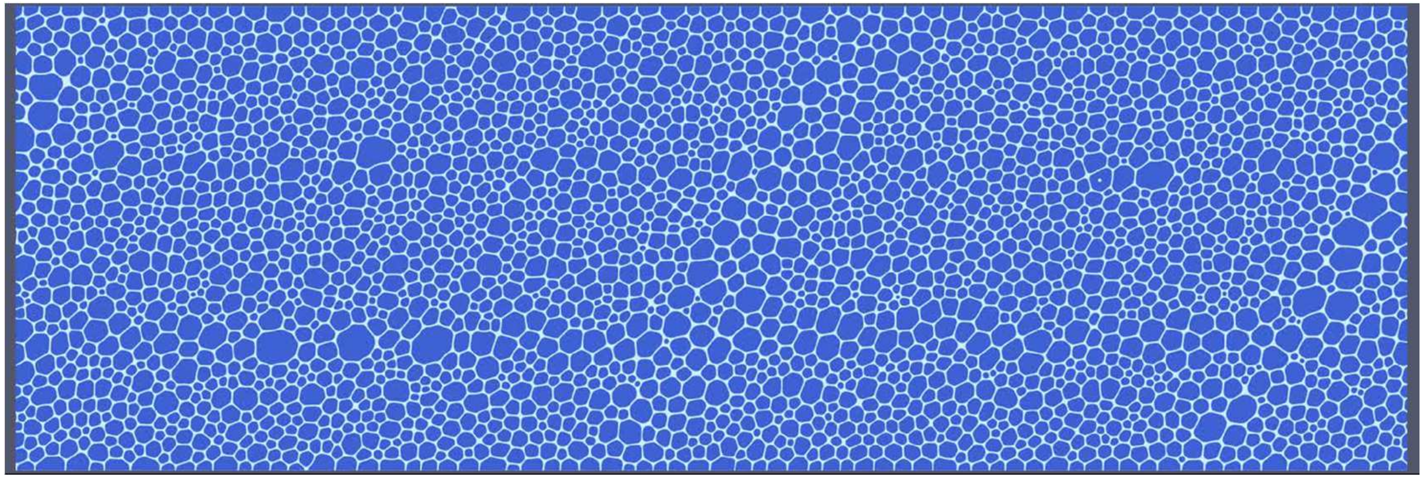
A. Scagliarini, M. Lulli, M. Sbragaglia and M. Bernaschi, *Europhys. Lett.* ***114**,* 64003 (2016)

M. Lulli, M. Sbragaglia and M. Bernaschi, *arXiv1607.00908* (2016)

Plastic Events: LBM with Different Load Conditions

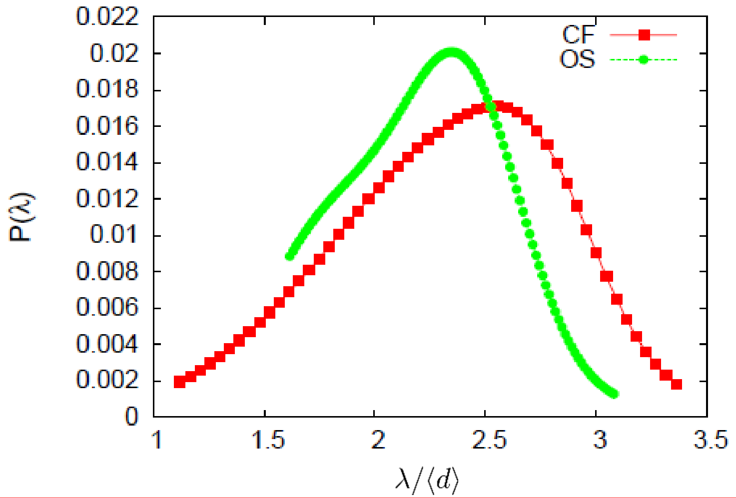
M. Sbragaglia, R. Benzi, P. Perlekar, M. Bernaschi, S. Succi & F. Toschi, *Soft Matter* ***10***, 4514-4524 (2014)

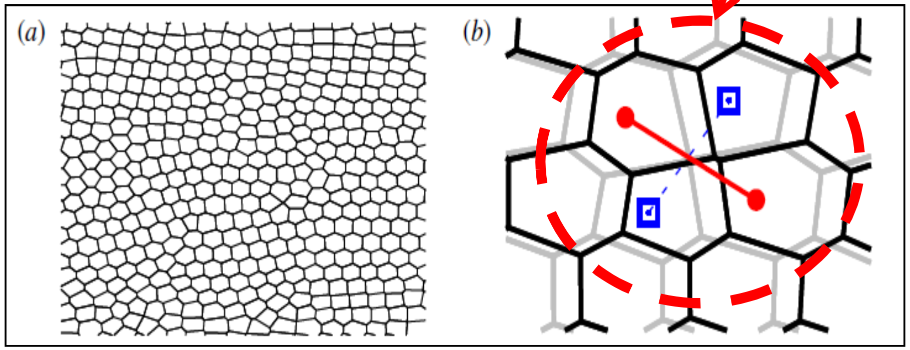
Oscillating Strain (OS) Boundary Conditions



Characteristic size of plastic rearrangements estimated from Voronoi Tesselation

$\sim \pi\lambda^2$





Benzi R., Sbragaglia M., Perlekar P., Bernaschi M., Succi S. Toschi F. *Soft Matter* 10, 4615-4624 (2014)

Probability Distribution of size of Plastic Rearrangement

(Different Flow Conditions: **CF (Couette Flow); OS (Oscillatory Strain)**)

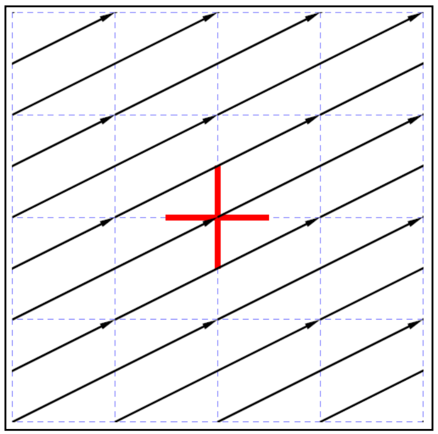
Mechanical Pressure Tensor: How?

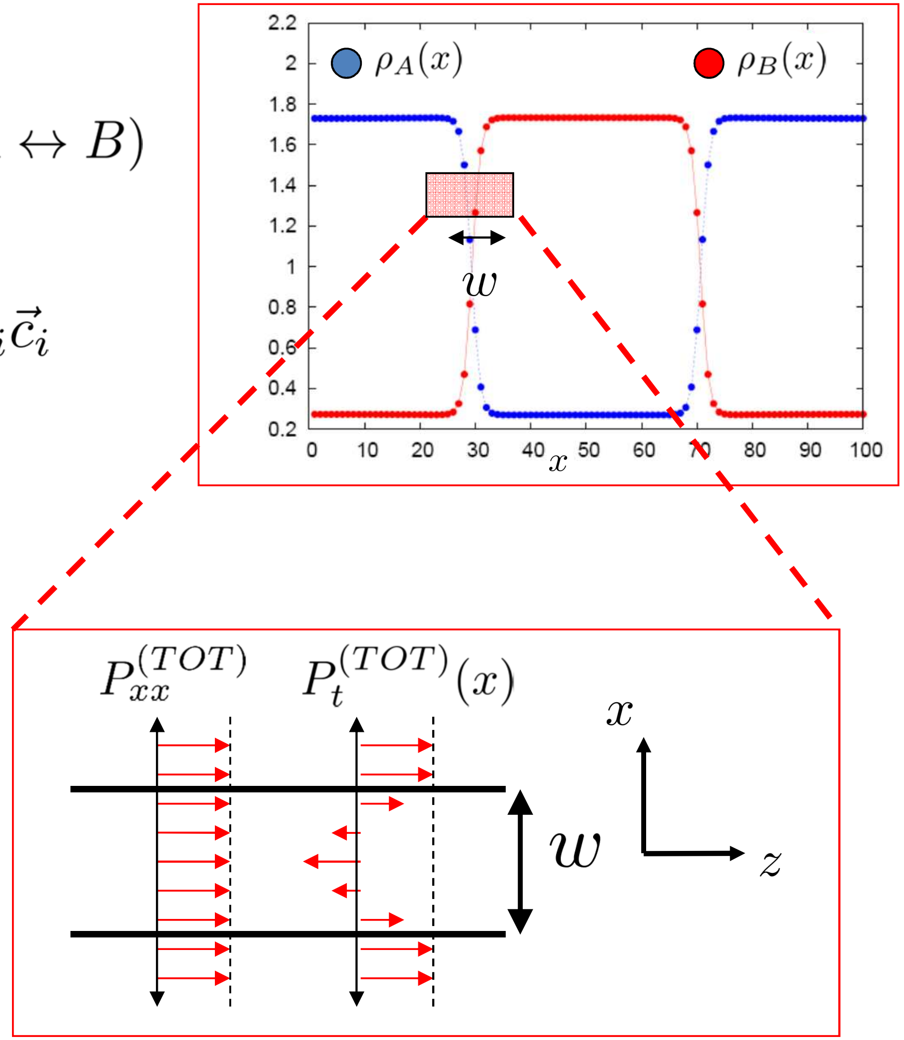
Forces

$$\vec{F} = -g_{AB}\rho_A(\vec{r})\sum_i w_i \rho_B(\vec{r}+\vec{c}_i)\vec{c}_i + (A \leftrightarrow B)$$

Non-ideal **Lattice** Pressure Tensor

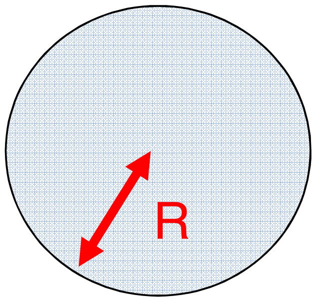
$$\boldsymbol{P}^{(NI)} = \frac{g_{AB}}{2}\rho_A(\vec{r})\sum_i w_i \rho_B(\vec{r}+\vec{c}_i)\vec{c}_i\vec{c}_i + (A \leftrightarrow B)$$





X. Shan, Physical Review E 77, 066702 (2008)
M. Sbragaglia & X. Shan, Physical Review E 84, 036703 (2011)
M. Sbragaglia & D. Belardinelli, Physical Review E 91, 023313 (2013)

Surface Tension at the non ideal interfaces



Laplace Law

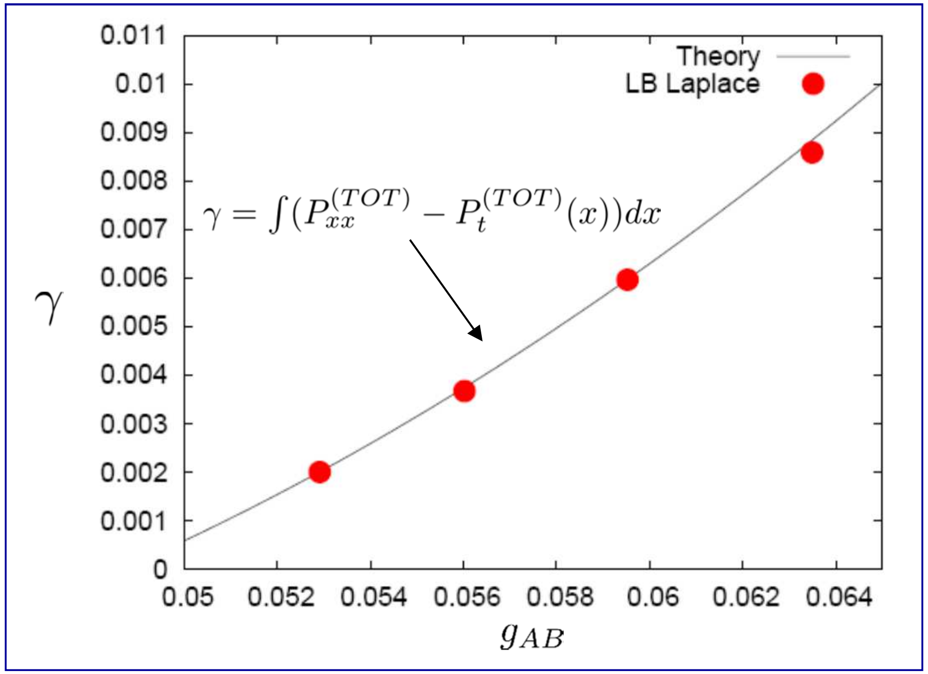
$$\Delta P = P^{(TOT)}_{bulk,in} - P^{(TOT)}_{bulk,out} = \frac{\gamma}{R}$$

Inner-Outer pressures for equilibrium droplets

YES!

$$\gamma = \int (P^{(TOT)}_{xx} - P^{(TOT)}_{t}(x))dx$$

Surface Tension
(Mechanical Definition)



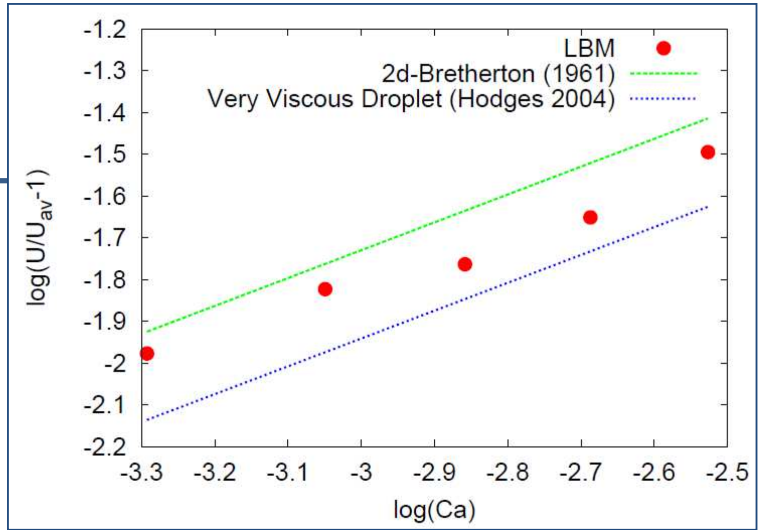
Benchmark Computation: Motion of "Long" Droplets in a Tube

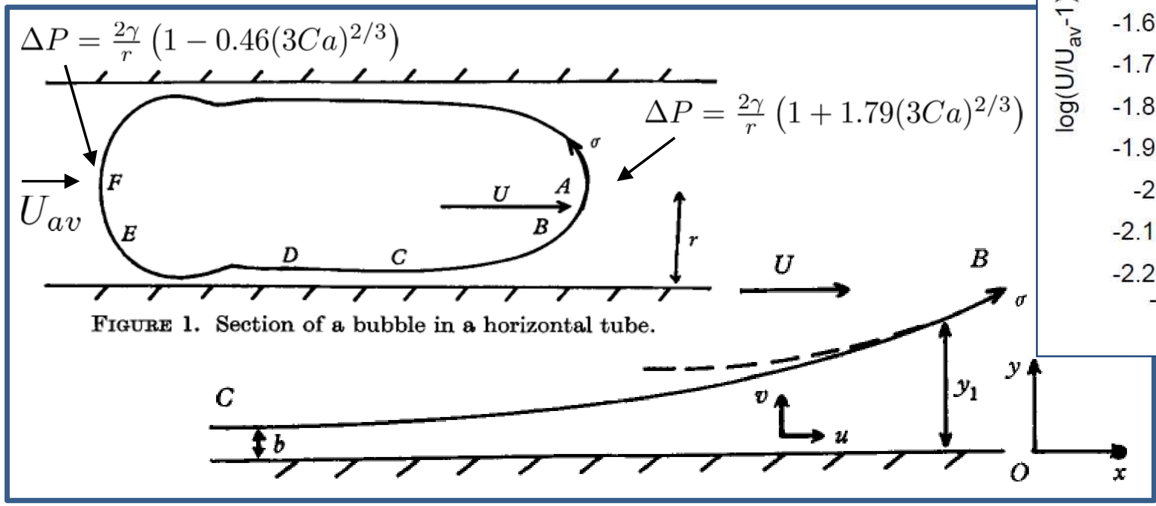
$$Ca = \eta U / \gamma$$

Capillary number (viscous/surface)

$$U/U_{\rm av} = 1 + 0.643\, Ca^{2/3}$$

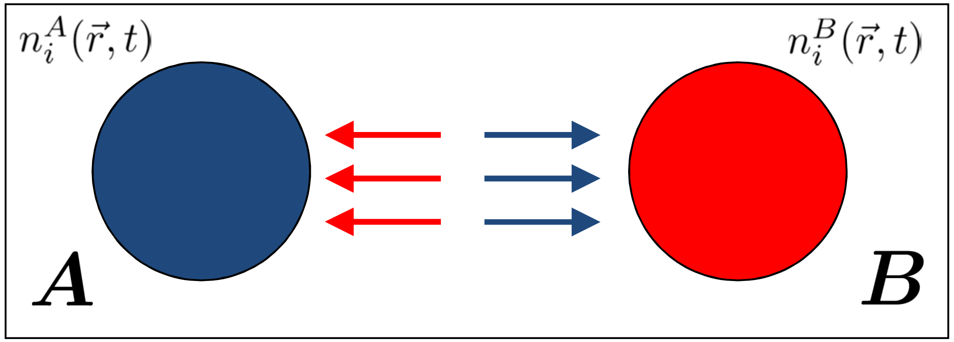
Bretherton Scaling

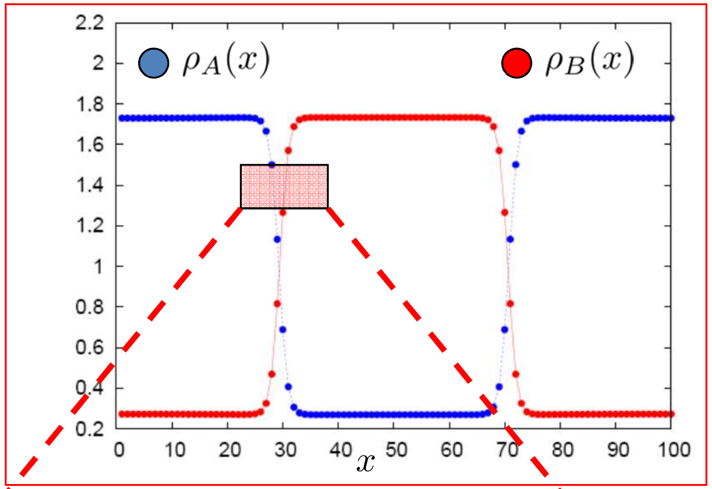




Bretherton, *Jour. Fluid Mech.* (**1961**)

Non Ideal Forces & Coalescence

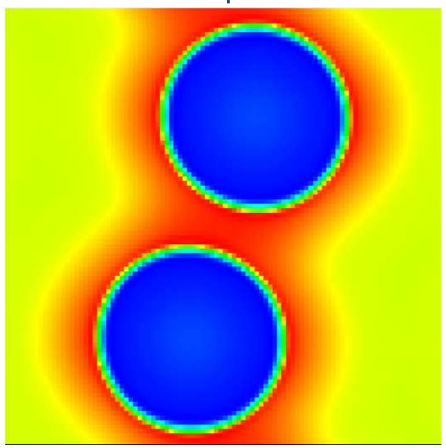




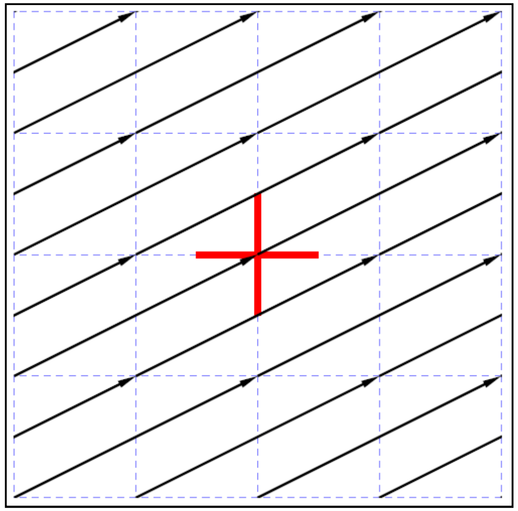
Non-ideal (NI) Lattice Pressure Tensor

$$P_{jk}^{(NI)} = \frac{g_{AB}}{2}\rho_A(\vec{r})\sum_i w_i \rho_B(\vec{r}+\vec{c}_i)c_i^j c_i^k + (A \leftrightarrow B)$$

Minimization of interface Free-Energy leads to film rapture



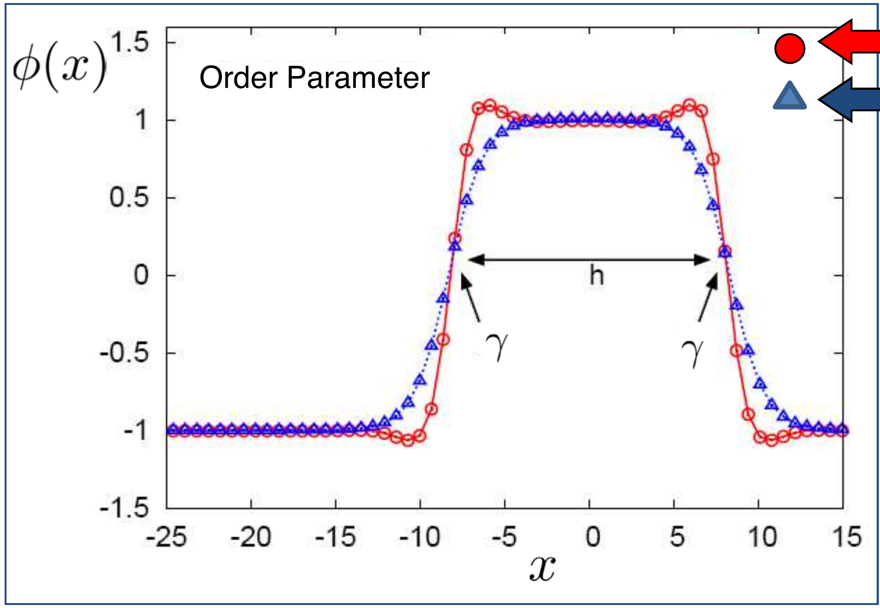
Droplets Coalescence

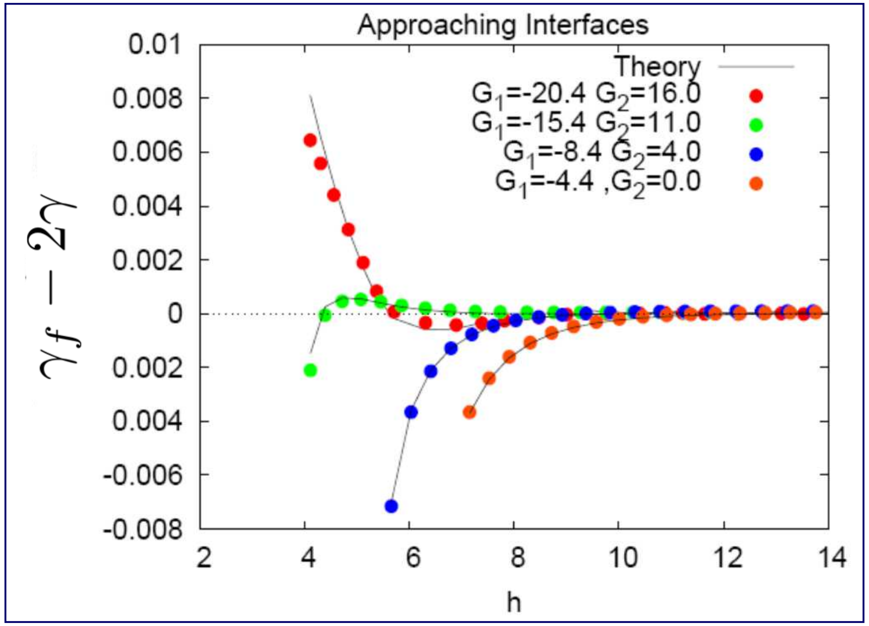


Disjoining Pressure From Lattice Kinetic models

(Competing NN & NNN) $\mathcal{G}_1 < 0; \mathcal{G}_2 > 0$

(bare NN) $\mathcal{G}_1 < 0; \mathcal{G}_2 = 0$





M. Sbragaglia et al., *Soft Matter* **8**, 10773-10782 (2012)

$$\gamma_f = \int_{-\infty}^{+\infty} (P_n - P_t(x))\, dx$$

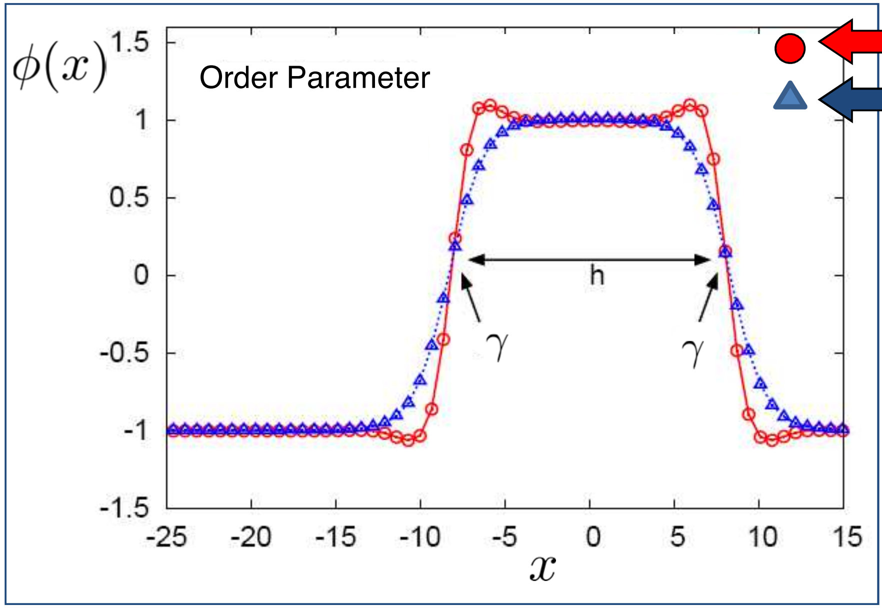
'film tension'

Exactly computed on the lattice

$$P_n = P_b(x) + C_1 \psi \frac{d^2\psi}{dx^2} + \ldots = \text{const}$$

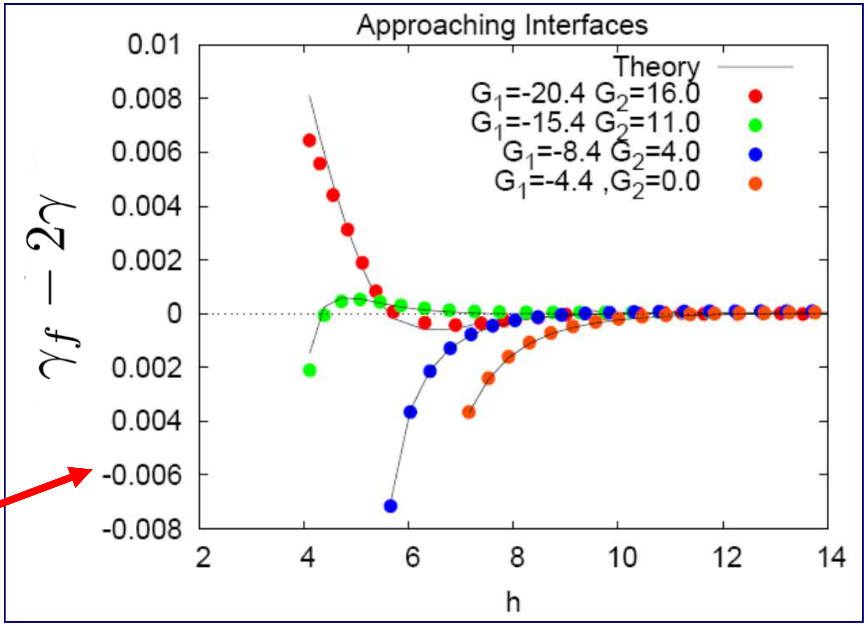
Analytical with the exact pressure tensor
(differential equation-> Profile)

Disjoining Pressure From Lattice Kinetic models



(Competing NN & NNN) $\mathcal{G}_1 < 0; \mathcal{G}_2 > 0$

(bare NN) $\mathcal{G}_1 < 0; \mathcal{G}_2 = 0$



$$\gamma_f = 2\gamma + \int_{\Pi(h=\infty)}^{\Pi(h)} h\, d\Pi$$

Disjoining Pressure definition based on 'film tension'

M. Sbragaglia et al., *Soft Matter* **8**, 10773-10782 (2012)

Exactly computed on the lattice

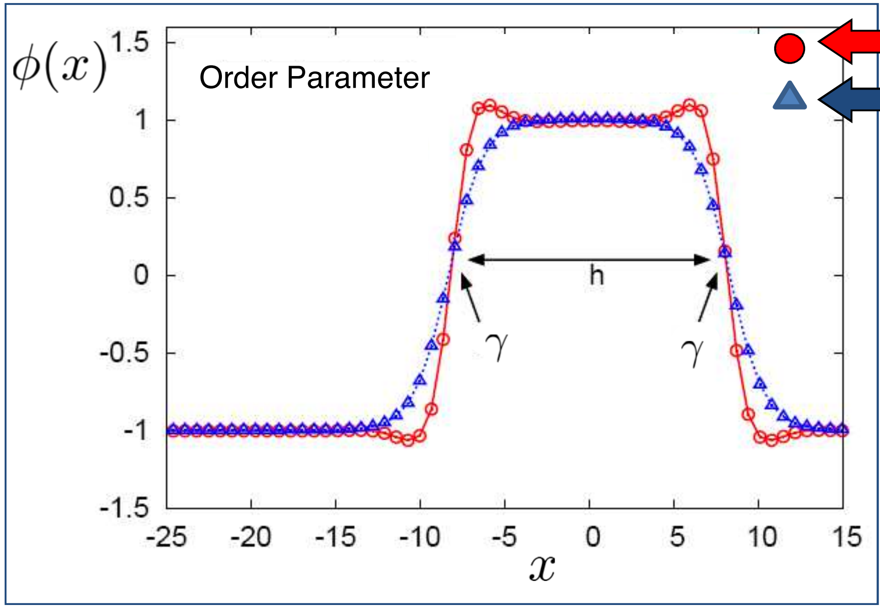
$$P_n = P_b(x) + C_1 \psi \frac{d^2\psi}{dx^2} + \ldots = \text{const}$$

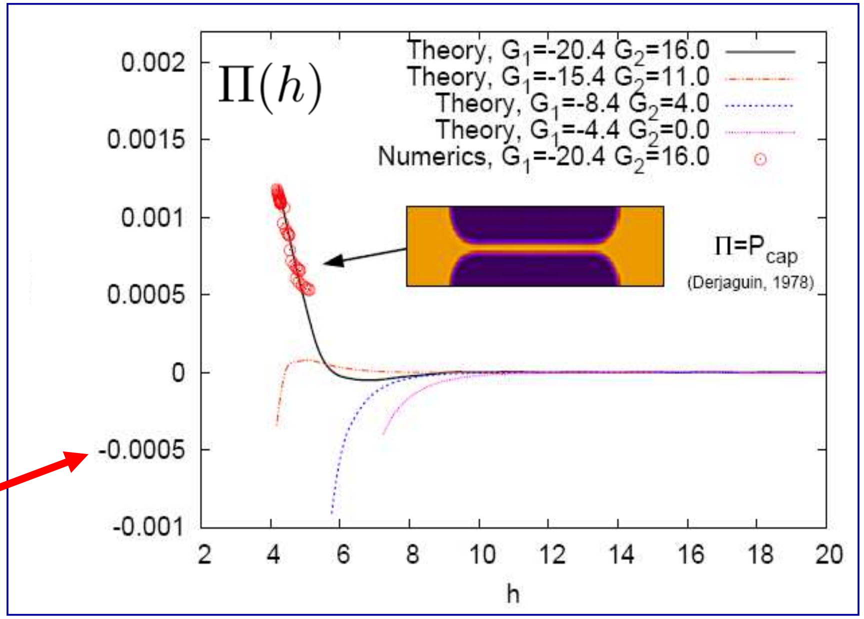
Analytical with the exact pressure tensor
(differential equation-> Profile)

Disjoining Pressure From Lattice Kinetic models

(Competing NN & NNN) $\mathcal{G}_1 < 0; \mathcal{G}_2 > 0$

(bare NN) $\mathcal{G}_1 < 0; \mathcal{G}_2 = 0$





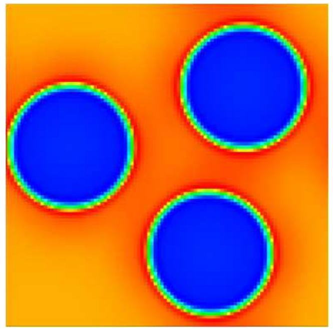
$$\gamma_f = 2\gamma + \int_{\Pi(h=\infty)}^{\Pi(h)} h \, d\Pi$$

Disjoining Pressure definition based on ‘film tension’

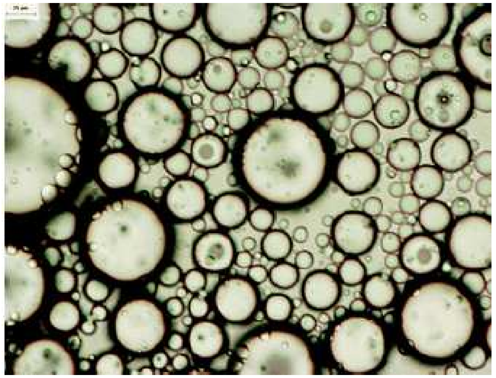
Exactly computed on the lattice

$$P_n = P_b(x) + C_1 \psi \frac{d^2\psi}{dx^2} + \ldots = \text{const}$$

Analytical with the exact pressure tensor
(differential equation-> Profile)

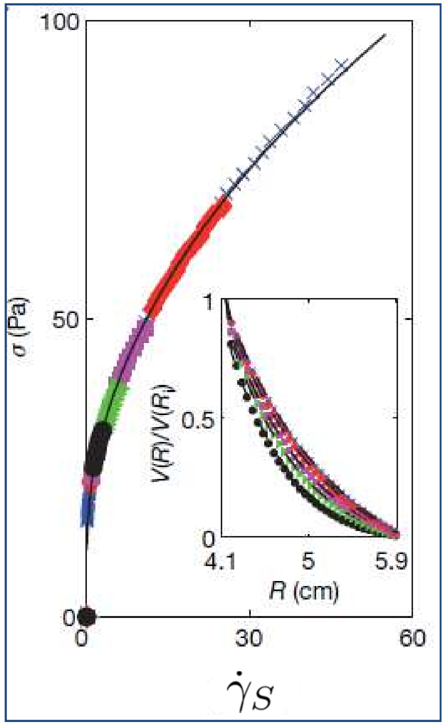


Universality Under Confinement ?

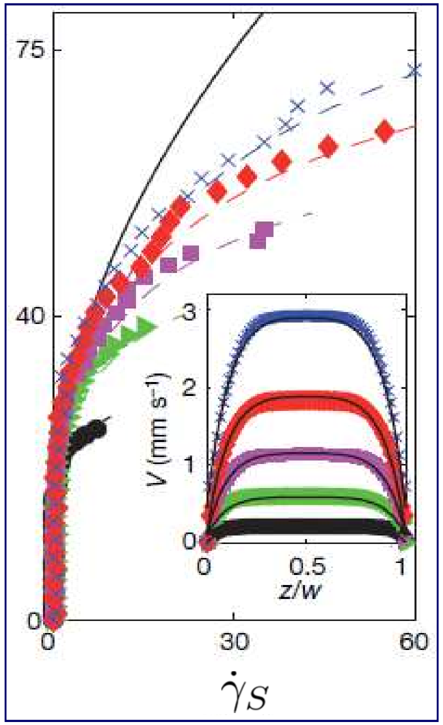


Droplets of a dispersed phase
in a continuous phase

- ✓Yield Stress (Solid below)
- ✓Non-Newtonian (above yield Stress)
- ✓Effect of Confinement (Cooperativity Effects)



Rheology in a **wide** Rheometer

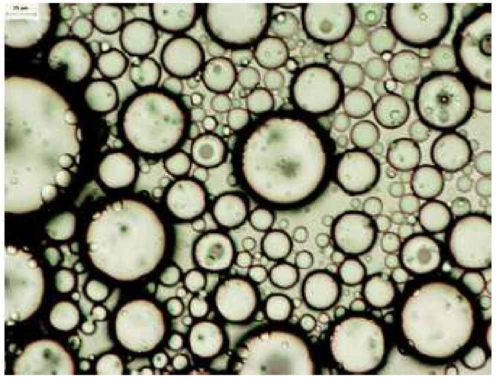


Rheology in **thin** Rheometer

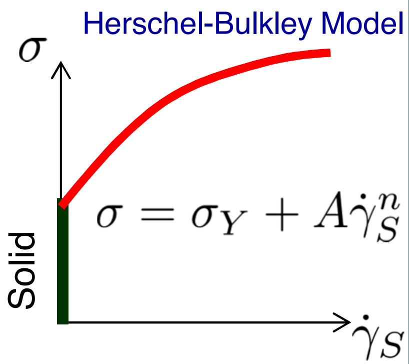
Goyon et al, Nature (**2008**)

Yield Stress Fluids & Plastic Rearrangements

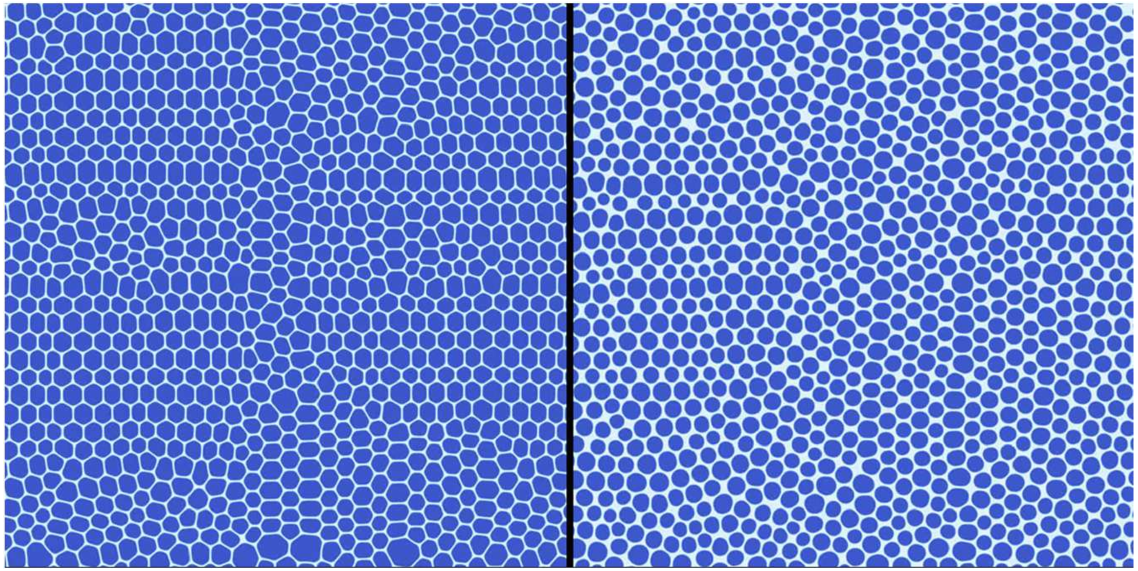
R. Benzi, M. Sbragaglia, P. Perlekar, M. Bernaschi, S. Succi & F. Toschi, *Soft Matter* **10**, 4615-4624 (2014)



Droplets of a dispersed phase in a continuous phase



- ✓Yield Stress (Solid below)
- ✓Non-Newtonian (above yield)
- ✓Cooperativity Effects & Plastic Rearrangements



High Packing Fraction   Lower Packing Fraction